2026 MC Tennis Parity Puzzle

Charles Tan 2028'

Setup and Observations

- Let $p \in (0,1)$ be the probability that Player A wins a *service* game (when A serves)
- Let $q \in (0,1)$ be the probability that Player A wins a *return* game (when B serves)
- The same player serves first in *every* set; denote this first server by $S \in \{\mathrm{A}, \mathrm{B}\}$
- A set ends at score (a,b) with $\max(a,b) = 6$ and $|a-b| \geq 2$ (no 7+ games occur)
- Empirical constraints (the "parity observations"):
  (a) If A wins the set, then $a+b$ is *odd*
  (b) If B wins the set, then $a+b$ is *even*
- A won the match (best of $2n+1$ sets)

Single-Set DP Likelihood

I will model each set as a Markov chain on states (a,b,t), where $a,b \in \{0,\dots,6\}$ are games won by A and B, and $t \in \{0,1\}$ is the next server ($t{=}0 \Rightarrow$ A serves, $t{=}1 \Rightarrow$ B serves) The initial server is determined by S: start at $(0,0,t{=}0)$ if $S{=}\mathrm{A}$, or $(0,0,t{=}1)$ if $S{=}\mathrm{B}$

Transitions:

$$\begin{aligned} t{=}0 &: (a,b,0) \to (a{+}1,b,1) \text{ w.p. } p, \quad (a,b{+}1,1) \text{ w.p. } 1{-}p, \\ t{=}1 &: (a,b,1) \to (a{+}1,b,0) \text{ w.p. } q, \quad (a,b{+}1,0) \text{ w.p. } 1{-}q \end{aligned}$$

A terminal set occurs at any (a,b) with $\max(a,b) = 6$ and $|a-b| \geq 2$. Let $\mathbb{P}((a,b) \mid S)$ be the total absorbed mass at terminal (a,b) given S

Filter by parity observations, we define the following indicators:

$$I^{\mathrm{A}}_{(a,b)} = \mathbf{1}[\max(a,b) = 6,\ |a-b| \geq 2,\ a > b,\ (a{+}b) \text{ odd}]\,,$$

$$I^{\mathrm{B}}_{(a,b)} = \mathbf{1}[\max(a,b) = 6,\ |a-b| \geq 2,\ b > a,\ (a{+}b) \text{ even}]$$

Then the likelihood that a single set is consistent with the observations and is won by A (resp. B) under first server S is

$$\pi^{(S)}_{\mathrm{set,A}} \ = \ \sum_{a,b} \mathbb{P}((a,b) \mid S)\, I^{\mathrm{A}}_{(a,b)}, \qquad \pi^{(S)}_{\mathrm{set,B}} \ = \ \sum_{a,b} \mathbb{P}((a,b) \mid S)\, I^{\mathrm{B}}_{(a,b)}$$

Match Likelihood and Posterior

For a best-of $m = 2n+1$ sets with independent sets conditional on S, the probability that A wins the match *and* every set satisfies the observations is

$$L_S \;=\; \sum_{k=(m+1)/2}^{m} \binom{m}{k} \left(\pi_{\text{set,A}}^{(S)}\right)^k \left(\pi_{\text{set,B}}^{(S)}\right)^{m-k}$$

With prior $\mathbb{P}(S{=}\mathrm{A}) = \mathbb{P}(S{=}\mathrm{B}) = \frac{1}{2}$, the posterior is

$$\mathbb{P}(S{=}\mathrm{A} \mid \mathcal{O}) \;=\; \frac{L_{\mathrm{A}}}{L_{\mathrm{A}} + L_{\mathrm{B}}}$$

Proposition 1 (Symmetry)**.** *For any $p, q \in (0,1)$ and any integer $n \geq 0$, $L_{\mathrm{A}} = L_{\mathrm{B}}$. Consequently,*

$$\mathbb{P}(S{=}\mathrm{A} \mid \mathcal{O}) = \tfrac{1}{2}$$

Sketch. If $S = \mathrm{A}$, admissible A-won sets (odd length) end on A's serve; B-won sets (even length) end on B's serve. If $S = \mathrm{B}$, the same terminal scorelines occur but the serving labels are swapped at each game. The parity filter depends only on (winner, total-games parity), and there is a measure-preserving bijection between paths under $S = \mathrm{A}$ and $S = \mathrm{B}$. Thus the filtered mass is identical, giving $L_{\mathrm{A}} = L_{\mathrm{B}}$. □

Algorithms

Please find full code at my github: https://github.com/CharlesTanShi/Tennis-MC.git

Exact DP for One Set

Algorithm 1 SetLikelihoods(p, q, S)

Initialize DP tensor $P[a,b,t] \leftarrow 0$ for $a, b \in \{0, \ldots, 6\}$, $t \in \{0, 1\}$.
$t_0 \leftarrow 0$ if $S{=}\mathrm{A}$ else 1; $P[0,0,t_0] \leftarrow 1$.
function Terminal(a,b) **return** $(\max(a,b) = 6) \wedge (|a-b| \geq 2)$
for $a = 0..6$ **do**
  for $b = 0..6$ **do**
    for $t \in \{0,1\}$ **do**
      $w \leftarrow P[a,b,t]$; **if** $w = 0$ or Terminal(a,b) **then continue**
      if $t = 0$ **then** ▷ A serves
        $P[a{+}1,b,1] \mathrel{+}= w \cdot p$; $P[a,b{+}1,1] \mathrel{+}= w \cdot (1-p)$
      else ▷ B serves
        $P[a{+}1,b,0] \mathrel{+}= w \cdot q$; $P[a,b{+}1,0] \mathrel{+}= w \cdot (1-q)$
$\pi_{\mathrm{A}}, \pi_{\mathrm{B}} \leftarrow 0, 0$
for $a = 0..6$ **do**
  for $b = 0..6$ **do**
    if Terminal(a,b) and $\max(a,b) = 6$ **then**
      $w \leftarrow P[a,b,0] + P[a,b,1]$
      if $a > b$ and $(a{+}b)$ odd **then** $\pi_{\mathrm{A}} \mathrel{+}= w$
      if $b > a$ and $(a{+}b)$ even **then** $\pi_{\mathrm{B}} \mathrel{+}= w$
return $(\pi_{\mathrm{A}}, \pi_{\mathrm{B}})$