

Table of the successful solvings:

ω	Successful solvings out of 1000
1	84
2	968
3	1000
4	999

Sample solution tracing:

The puzzle is represented by a 9 element array. So for example, [0,1,2,3,4,5,6,7,8,-1] refers to the following puzzle setup (which is also the winning setup):

1	2	3
4	5	6
7	8	

Now the following listing, shows the listing of the program as it is solving the puzzle [4, 3, 0, -1, 2, 5, 7, 6, 1]. Each line shows the nodes that the program kept after keeping only the best $\omega=3$ nodes. The highlighted puzzle setups are form the path from the winning position all the way back to the initial setup.

[4, 3, 0, -1, 2, 5, 7, 6, 1]

[[-1, 3, 0, 4, 2, 5, 7, 6, 1], [4, 3, 0, 7, 2, 5, -1, 6, 1], [4, 3, 0, 2, -1, 5, 7, 6, 1]]
 [[4, 3, 0, 7, 2, 5, 6, -1, 1], [3, -1, 0, 4, 2, 5, 7, 6, 1], [4, -1, 0, 2, 3, 5, 7, 6, 1]]
 [[4, 3, 0, 7, -1, 5, 6, 2, 1], [4, 3, 0, 7, 2, 5, 6, 1, -1], [3, 2, 0, 4, -1, 5, 7, 6, 1]]
 [[4, -1, 0, 7, 3, 5, 6, 2, 1], [4, 3, 0, -1, 7, 5, 6, 2, 1], [3, 2, 0, -1, 4, 5, 7, 6, 1]]
 [[-1, 2, 0, 3, 4, 5, 7, 6, 1], [-1, 4, 0, 7, 3, 5, 6, 2, 1], [4, 0, -1, 7, 3, 5, 6, 2, 1]]
 [[2, -1, 0, 3, 4, 5, 7, 6, 1], [7, 4, 0, -1, 3, 5, 6, 2, 1], [4, 0, 5, 7, 3, -1, 6, 2, 1]]
 [[2, 0, -1, 3, 4, 5, 7, 6, 1], [7, 4, 0, 3, -1, 5, 6, 2, 1], [2, 4, 0, 3, -1, 5, 7, 6, 1]]
 [[7, -1, 0, 3, 4, 5, 6, 2, 1], [7, 4, 0, 3, 2, 5, 6, -1, 1], [2, 0, 5, 3, 4, -1, 7, 6, 1]]
 [[-1, 7, 0, 3, 4, 5, 6, 2, 1], [7, 0, -1, 3, 4, 5, 6, 2, 1], [7, 4, 0, 3, 2, 5, 6, 1, -1]]
 [[3, 7, 0, -1, 4, 5, 6, 2, 1], [7, 0, 5, 3, 4, -1, 6, 2, 1], [7, 4, 0, 3, 2, -1, 6, 1, 5]]
 [[7, 0, 5, 3, 4, 1, 6, 2, -1], [3, 7, 0, 6, 4, 5, -1, 2, 1], [3, 7, 0, 4, -1, 5, 6, 2, 1]]
 [[7, 0, 5, 3, 4, 1, 6, -1, 2], [3, 7, 0, 6, 4, 5, 2, -1, 1], [3, -1, 0, 4, 7, 5, 6, 2, 1]]
 [[7, 0, 5, 3, -1, 1, 6, 4, 2], [7, 0, 5, 3, 4, 1, -1, 6, 2], [3, 7, 0, 6, 4, 5, 2, 1, -1]]
 [[7, -1, 5, 3, 0, 1, 6, 4, 2], [7, 0, 5, 3, 1, -1, 6, 4, 2], [7, 0, 5, -1, 3, 1, 6, 4, 2]]
 [[7, 0, -1, 3, 1, 5, 6, 4, 2], [-1, 7, 5, 3, 0, 1, 6, 4, 2], [7, 5, -1, 3, 0, 1, 6, 4, 2]]
 [[7, -1, 0, 3, 1, 5, 6, 4, 2], [7, 5, 1, 3, 0, -1, 6, 4, 2], [3, 7, 5, -1, 0, 1, 6, 4, 2]]
 [[7, 1, 0, 3, -1, 5, 6, 4, 2], [-1, 7, 0, 3, 1, 5, 6, 4, 2], [7, 5, 1, 3, 0, 2, 6, 4, -1]]
 [[7, 1, 0, 3, 4, 5, 6, -1, 2], [7, 1, 0, -1, 3, 5, 6, 4, 2], [7, 1, 0, 3, 5, -1, 6, 4, 2]]
 [[7, 1, 0, 3, 4, 5, 6, 2, -1], [7, 1, 0, 3, 4, 5, -1, 6, 2], [-1, 1, 0, 7, 3, 5, 6, 4, 2]]
 [[7, 1, 0, 3, 4, -1, 6, 2, 5], [7, 1, 0, -1, 4, 5, 3, 6, 2], [1, -1, 0, 7, 3, 5, 6, 4, 2]]
 [[7, 1, -1, 3, 4, 0, 6, 2, 5], [7, 1, 0, 3, -1, 4, 6, 2, 5], [-1, 1, 0, 7, 4, 5, 3, 6, 2]]
 [[7, -1, 1, 3, 4, 0, 6, 2, 5], [7, 1, 0, 3, 2, 4, 6, -1, 5], [7, -1, 0, 3, 1, 4, 6, 2, 5]]
 [[-1, 7, 1, 3, 4, 0, 6, 2, 5], [7, 1, 0, 3, 2, 4, 6, 5, -1], [7, 4, 1, 3, -1, 0, 6, 2, 5]]
 [[7, 1, 0, 3, 2, -1, 6, 5, 4], [3, 7, 1, -1, 4, 0, 6, 2, 5], [7, 4, 1, 3, 2, 0, 6, -1, 5]]
 [[7, 1, -1, 3, 2, 0, 6, 5, 4], [7, 1, 0, 3, -1, 2, 6, 5, 4], [7, 4, 1, 3, 2, 0, 6, 5, -1]]
 [[7, 1, 0, 3, 5, 2, 6, -1, 4], [7, -1, 1, 3, 2, 0, 6, 5, 4], [7, -1, 0, 3, 1, 2, 6, 5, 4]]

[[7, 1, 0, 3, 5, 2, 6, 4, -1], [7, 1, 0, 3, 5, 2, -1, 6, 4], [7, 2, 1, 3, -1, 0, 6, 5, 4]]
[[7, 2, 1, 3, 5, 0, 6, -1, 4], [7, 2, 1, 3, 0, -1, 6, 5, 4], [7, 1, 0, -1, 5, 2, 3, 6, 4]]
[[7, 2, 1, 3, 5, 0, 6, 4, -1], [7, 2, -1, 3, 0, 1, 6, 5, 4], [7, 2, 1, 3, 0, 4, 6, 5, -1]]
[[7, -1, 2, 3, 0, 1, 6, 5, 4], [7, 2, 1, 3, 5, -1, 6, 4, 0], [7, 2, 1, 3, 0, 4, 6, -1, 5]]
[[7, 0, 2, 3, -1, 1, 6, 5, 4], [-1, 7, 2, 3, 0, 1, 6, 5, 4], [7, 2, 1, 3, -1, 5, 6, 4, 0]]
[[7, 2, 1, 3, 4, 5, 6, -1, 0], [7, 0, 2, 3, 5, 1, 6, -1, 4], [7, 0, 2, 3, 1, -1, 6, 5, 4]]
[[7, 2, 1, 3, 4, 5, 6, 0, -1], [7, 2, 1, 3, 4, 5, -1, 6, 0], [7, 0, 2, 3, 5, 1, 6, 4, -1]]
[[7, 2, 1, 3, 4, -1, 6, 0, 5], [7, 0, 2, 3, 5, -1, 6, 4, 1], [7, 2, 1, -1, 4, 5, 3, 6, 0]]
[[7, 0, 2, 3, -1, 5, 6, 4, 1], [7, 2, -1, 3, 4, 1, 6, 0, 5], [7, 2, 1, 3, -1, 4, 6, 0, 5]]
[[7, 0, 2, 3, 4, 5, 6, -1, 1], [7, -1, 2, 3, 0, 5, 6, 4, 1], [7, -1, 2, 3, 4, 1, 6, 0, 5]]
[[7, 0, 2, 3, 4, 5, 6, 1, -1], [7, 0, 2, 3, 4, 5, -1, 6, 1], [-1, 7, 2, 3, 0, 5, 6, 4, 1]]
[[7, 0, 2, 3, 4, -1, 6, 1, 5], [7, 0, 2, -1, 4, 5, 3, 6, 1], [3, 7, 2, -1, 0, 5, 6, 4, 1]]
[[7, 0, -1, 3, 4, 2, 6, 1, 5], [7, 0, 2, 3, -1, 4, 6, 1, 5], [-1, 0, 2, 7, 4, 5, 3, 6, 1]]
[[0, -1, 2, 7, 4, 5, 3, 6, 1], [7, -1, 0, 3, 4, 2, 6, 1, 5], [7, -1, 2, 3, 0, 4, 6, 1, 5]]
[[0, 4, 2, 7, -1, 5, 3, 6, 1], [0, 2, -1, 7, 4, 5, 3, 6, 1], [-1, 7, 0, 3, 4, 2, 6, 1, 5]]
[[0, 4, 2, 7, 6, 5, 3, -1, 1], [0, 4, 2, -1, 7, 5, 3, 6, 1], [0, 4, 2, 7, 5, -1, 3, 6, 1]]
[[0, 4, 2, 3, 7, 5, -1, 6, 1], [0, 4, 2, 7, 6, 5, -1, 3, 1], [0, 4, 2, 7, 6, 5, 3, 1, -1]]
[[0, 4, 2, 3, 7, 5, 6, -1, 1], [0, 4, 2, -1, 6, 5, 7, 3, 1], [0, 4, 2, 7, 6, -1, 3, 1, 5]]
[[0, 4, 2, 3, -1, 5, 6, 7, 1], [0, 4, 2, 3, 7, 5, 6, 1, -1], [0, 4, 2, 6, -1, 5, 7, 3, 1]]
[[0, -1, 2, 3, 4, 5, 6, 7, 1], [0, 4, 2, -1, 3, 5, 6, 7, 1], [0, 4, 2, 3, 5, -1, 6, 7, 1]]
[[-1, 0, 2, 3, 4, 5, 6, 7, 1], [0, 2, -1, 3, 4, 5, 6, 7, 1], [0, 4, 2, 3, 5, 1, 6, 7, -1]]
[[3, 0, 2, -1, 4, 5, 6, 7, 1], [0, 2, 5, 3, 4, -1, 6, 7, 1], [0, 4, 2, 3, 5, 1, 6, -1, 7]]
[[0, 2, 5, 3, 4, 1, 6, 7, -1], [3, 0, 2, 6, 4, 5, -1, 7, 1], [3, 0, 2, 4, -1, 5, 6, 7, 1]]
[[0, 2, 5, 3, 4, 1, 6, -1, 7], [3, -1, 2, 4, 0, 5, 6, 7, 1], [3, 0, 2, 6, 4, 5, 7, -1, 1]]
[[-1, 3, 2, 4, 0, 5, 6, 7, 1], [0, 2, 5, 3, -1, 1, 6, 4, 7], [0, 2, 5, 3, 4, 1, -1, 6, 7]]
[[4, 3, 2, -1, 0, 5, 6, 7, 1], [0, -1, 5, 3, 2, 1, 6, 4, 7], [0, 2, 5, 3, 1, -1, 6, 4, 7]]
[[4, 3, 2, 0, -1, 5, 6, 7, 1], [0, 2, -1, 3, 1, 5, 6, 4, 7], [4, 3, 2, 6, 0, 5, -1, 7, 1]]
[[0, -1, 2, 3, 1, 5, 6, 4, 7], [4, -1, 2, 0, 3, 5, 6, 7, 1], [4, 3, 2, 0, 7, 5, 6, -1, 1]]
[[0, 1, 2, 3, -1, 5, 6, 4, 7], [-1, 0, 2, 3, 1, 5, 6, 4, 7], [-1, 4, 2, 0, 3, 5, 6, 7, 1]]
[[0, 1, 2, 3, 4, 5, 6, -1, 7], [0, 1, 2, -1, 3, 5, 6, 4, 7], [0, 1, 2, 3, 5, -1, 6, 4, 7]]
[[0, 1, 2, 3, 4, 5, 6, 7, -1], [0, 1, 2, 3, 4, 5, -1, 6, 7], [0, 1, 2, 3, 5, 7, 6, 4, -1]]

```

from random import shuffle

#Heuristic asked in HW2, Counts number of puzzle parts that are not in their place
def SetDif(puzzle):
    count = 0
    for i in xrange(len(puzzle)):
        if(puzzle[i] is not -1):
            if(puzzle[i] is not i):
                count = count + 1
    return count

#A better heuristic which sums up total manhattan distance of all parts to their actual place
def Manhattan(puzzle):
    distance = 0
    for i in range(len(puzzle)):
        if(puzzle[i] is not -1):
            #print puzzle[i] , "-----"
            distance = distance + abs(puzzle[i] % 3 - i % 3) + abs(puzzle[i] / 3 - i / 3)

    return distance

#Generates Puzzles first creates alist -1 to 7 then shuffles it if it is solveable return else try again
def PuzzleGenerator():
    lis = [-1,0,1,2,3,4,5,6,7]#Represents value+1
    shuffle(lis)
    #print lis

    okashina = 0
    for i in range(len(lis)):
        if lis[i] is not -1:
            for j in range(i, len(lis)):
                if lis[j] is not -1:
                    if(lis[j] < lis[i]):
                        okashina = okashina + 1

    if( okashina % 2 is 1):#Not Solvable
        return PuzzleGenerator()
    else:#Solvable so return
        return lis

#A function that converts comparator to Key object
def cmp_to_key(mycmp):
    'Convert a cmp= function into a key= function'
    class K(object):
        def __init__(self, obj, *args):
            self.obj = obj
        def __lt__(self, other):
            return mycmp(self.obj, other.obj) < 0
        def __gt__(self, other):
            return mycmp(self.obj, other.obj) > 0
        def __eq__(self, other):
            return mycmp(self.obj, other.obj) == 0
        def __le__(self, other):
            return mycmp(self.obj, other.obj) <= 0
        def __ge__(self, other):
            return mycmp(self.obj, other.obj) >= 0
        def __ne__(self, other):
            return mycmp(self.obj, other.obj) != 0
    return K

#A comparator that uses Manhattan Heuristic
def CmpPuzzle2(p1,p2):#My Hurustic works better
    return Manhattan(p1) - Manhattan(p2)

#A comparator that uses SetDif Heuristic
def CmpPuzzle(p1,p2):#Bad Hurustic of HW
    return SetDif(p1) - SetDif(p2)

#Generates A list of Possible Moves From a game state
def PossibleMoves(puzzle):
    al = []
    empty = puzzle.index(-1);

```

```

row = empty //3
col = empty %3
#print (str(row)+      "-" +str(col))
tmp = list(puzzle)
if row > 0:
    tmp[empty] = tmp[(row -1)*3+col]
    tmp[(row -1)*3+col] = -1
    al.append(list(tmp))
tmp = list(puzzle)
if row < 2:
    tmp[empty] = tmp[(row +1)*3+col]
    tmp[(row +1)*3+col] = -1
    al.append(list(tmp))
tmp = list(puzzle)
if col > 0:
    tmp[empty] = tmp[row*3+col-1]
    tmp[row*3+col-1] = -1
    al.append(list(tmp))
tmp = list(puzzle)
if col < 2:
    tmp[empty] = tmp[row*3+col+1]
    tmp[row*3+col+1] = -1
    al.append(list(tmp))
return al
#A toString method, which can be changed later on to represent Puzzles
def ToString(puzzle):
    return str(puzzle)

#Beam Search Method
def BeamItUp(puzzle, w):

    l = []#Temp List to keep new puzzles
    ll = []#A list to keep puzzles
    ll.append(puzzle)#begin
    count = 0#counter, Probably not necessary anymore
    dic = {}#A dictionary to keep track of visited states to prevent looping
    dic[ToString(puzzle)] = 1
    try:#A try catch mechanism to detect index errors, This happens when ll is empty, so search
failed
        while(Manhattan(ll[0]) is not 0 and count < 1000):#Loop until finish puzzle
            l = []#empty temp list
            for i in range(w):
                if i >= len(ll):#This looks stupid atm, I dont know what I thought, I
could change for loop :P
                    break
                moves = PossibleMoves(ll[i])#Get Possible moves
                dic[ToString(ll[i])] = 1#Add current state to dictionary
                for pz in moves:#Check if new moves are in the dictionary, else add to
l
                    if ToString(pz) not in dic:
                        l.append(pz)

            #l.extend()
            ll = []#Reset the ll, since we expanded all w
            ll.extend(l)# Append l to ll

            ll = sorted(ll, key= cmp_to_key(CmpPuzzle))#Sort ll according to chosen
Heuristic

            #print ll
            ll = ll[0:w]#Remove states except first w
            print ll
            count = count + 1#increment counter

    except IndexError:
        #print "Failed: ",puzzle
        return False
    return True

#Main Function
def main():

```

```
count = 0#Counter to keep track of number of puzzles to solve
puzzles = {}#Dictionary to be sure that all puzzles are distinct
while count < 500:#1000 puzzles asked in hw
    l = PuzzleGenerator()#Generate a puzzle
    if ToString(l) not in puzzles:#if not in dictionary
        puzzles[ToString(l)] = l#Added it to dictionary
        count = count + 1#increment
#puzzles = puzzles.items()
puzzles = [v for k,v in puzzles.items()]#convert dictionary to a list to iterate
l = [0,0,0,0]
for p in puzzles:#for each puzzle
    for w in xrange(1,5):#for each omega
        #print p
        if BeamItUp(p,w):#BeamSearch
            l[w-1] = l[w-1] + 1

    print l
def main2():
    l = PuzzleGenerator()
    print l
    BeamItUp(l,3)
main2()#Call Main
```