Introduction

Game theory is the analysis of interactive decision making (Dixit, Skeath and Reiley, 2015). It involves modelling the interaction between individuals (players) in a strategic interdependence scenario with laid down rules and outcomes. Game theory is fundamental to the analysis of personal relationships, industries, economies, and international relations.

Game theory is largely attributed to the work of mathematician John von Neumann and economist Oskar Morgenstern in the 1940s and was developed extensively by many other researchers and scholars in the 1950s (Hayes, 2022).

The many tools developed in this space to analyse game scenarios is often called; The Science of Game Theory. The Art of Game Theory, on the other hand, is the ability of a researcher to abstract a real-world situation into a scenario that can be analysed with the tools of game theory.

In this paper, we will employ both the science and art of game theory to abstract a simple model of the stock market. The objective is to understand equilibrium outcomes for individuals participating in the stock market. We will employ tools such as the normal form, mixed strategies and the extensive form to understand the Nash equilibrium in the market.

General Assumptions of Game Theory

According to Romp (1997), there are three main assumptions of game theory, namely;

- Individuality
- Rationality
- Mutual Interdependence

The assumption of individuality in game theory implies that players in the game cannot enter into binding and enforceable agreements with each other. This means no permanence in cooperation between players, leaving them to make independent decisions still.

The second characteristic of rationality implies that individuals, firms or governments act in their self-interest. This would mean that individuals have complete information and preference about their courses of action and act to maximise their utility.

Perhaps the most crucial assumption of game theory is that of mutual interdependence. This assumption implies that any one individual's welfare (outcome) is partially dependent on the actions of the other players in the game.

Identification of Real-World Environment

A stock market is an aggregation of buyers and sellers of equity instruments (stocks) representing ownership shares in a company in a country. Individuals participate in the stock market to receive dividends at the end of the financial year, but mostly to sell their shares if the prices increase.

The stock market is a great real-world scenario to analyse using game theory because the price of a share is dependent on the strength of demand for the share. If the number of people buying

the shares is higher than the number of people selling the shares, then the price of the shares will increase. The reverse is also the case if the selling pressure dominates in the market. Other valuation determinants affect the perceived worth of a share; government policy, economic condition, company's past performance, customer base, and industry.

We will employ game theory to model the outcome for profit-seeking individuals in the market to understand the equilibrium outcome for a stock. The basic assumption is that demand and supply are the only determinants of the stock's price.

Model Abstraction

Using the art of game theory, we will develop our game scenario by abstracting from the real-world scenario. Our aim here is to use the principles of Occam's Razor to ensure that our model is parsimonious. Below are our assumptions;

- We are in the New York Stock Exchange Market (NYSE) for Apple Inc. shares.
- There are only two players in our stock market model; Charles vs Other Investors. This is a simple abstraction, given that there are numerous players in a regular market.
- Each player can buy or sell (short[1]) the shares of Apple Inc. in the market.
- The strategies for our players are to BUY or SELL (SHORT).
- Both players in this game adopt choose their actions simultaneously.
- The payoff for the game is the profit (increased valuation) from the trade decision taken by our players in the market.
- A decision to buy/ sell increases/ decreases share prices.

Application of Game Theory for Analysis

We will describe the stock market profit outcome for Charles as follows. First, we present a table of the joint impact on the market price of our stock by the actions of both players;

Table 1: Charles' Payoff Based on Market Prices.

		Other Investors	
		BUY	**SELL**
Charles	**BUY**		
	SELL		

If Charles coordinates his actions with other investors in the market, his payoffs are positive as indicated by the green arrow in Table 1 above. If he goes against the actions of the general market, then his payoffs are negative, indicated by the red arrow in Table 1. However, Charles does not know whether other players will buy or sell in the market, given that our game is simultaneously played.

[1] A "short" position is generally the sale of a stock you do not own. Investors who sell short believe the price of the stock will decrease in value, hence they borrow the stock and sell. If the price drops, investors can buy the stock at the lower price and make a profit.

Based on the assumptions presented above, we have the following game;

Table 2; Simultaneous Move Payoff Matrix for Two Players

		Other Investors	
		BUY	**SELL**
Charles	**BUY**	5, 5	-10,15
	SELL	-10, 15	5, 5

The actions which both players take jointly determine the price of Apple Inc. (the value of their portfolio). The important issue to note here is that the actions of Other Investors (all other participants in the stock market) have a more substantial effect on the price (valuation) of Apple Inc. shares than those of Charles alone. The combination of actions and the associated payoffs shown in table 2 are as follows;

- If Charles follows market sentiments and BUYS when Other Investors are BUYING, then all participants in the market get a payoff of 5 each.
- If Charles SELLS when Other Investors are SELLING, all market participants make a profit and get a payoff of 5 each.
- However, if Charles goes against the market sentiment and BUYS when other investors are SELLING Apple Inc shares, he makes a loss of 10. This is because the other investors jointly move the market, depressing the share price.
- Similarly, if Charles SELLS when other investors are BUYING, he makes a loss of 10. This is because he sold (shorted) at a low price; he now has to purchase the shares at a higher price to cover his short. The actions of the other market participants will increase the market price.
- For all the other market investors, if they succeed in playing a different strategy to Charles, his loss becomes their gain, resulting in a payoff of 15.

In this stock market game, Charles (who can represent any single player in the market) is playing against all other players. If he goes with the market sentiment, he makes a profit. If he does not, he makes a loss. On the other hand, the other investors are looking to not coordinate with Charles to make more profit.

Nash Equilibrium Under a Simultaneous Move-game

We begin to apply the science of game theory in our game by assuming that both players decide to buy and sell simultaneously. Using the best-response analysis, we obtain the following Nash Equilibrium;

Table 3: Nash Equilibrium for Simultaneous Move Stock Market Game

		Other Investors	
		BUY	**SELL**
Charles	**BUY**	5, 5	-10, 15
	SELL	-10, 15	5, 5

From the above table, using the method of best-response (also called method of underlining), there is no unique Pure Strategy Nash Equilibrium nor a Dominant Strategy for both players.

Given that Other Investors in the market can exploit Charles by mismatching with him to make more profits, we can do not have a clear equilibrium. To solve this problem, we employ a Mixed Strategy probabilistic distribution. A mixed strategy is an assignment of a probability to each pure strategy. A mixed strategy Nash equilibrium involves at least one player playing a randomised strategy, and no player can increase their expected payoff by playing an alternate strategy.

Nash Equilibrium in a Mixed Strategy Simultaneous Game

Based on the equilibrium obtained above, we can extend our analysis by introducing the probability of actions. Given our assumption that players intend to maximise payoffs in the stock market, they can randomise over an available set of actions so that their opponent is indifferent between their actions.

Table 4: Mixed Strategy Payoff Matrix Setup for Players in the Stock Market

		Other Investors	
		BUY (x)	**SELL (1 – x)**
Charles	**BUY (a)**	5, 5	-10,15
	SELL (1 – a)	-10, 15	5, 5

To employ the method of mixed strategies, for Charles, we assign a probability (a) for BUY and (1-a) for SELL. Similarly, we assign (x) for BUY and (1-x) for SELL for Other Investors. We solve for the probabilities that will leave both players indifferent to each other's strategy.

Our resulting calculation is as follows;

For Player 1;

- ⇨ 5*x + (-10)(1-x) = (-10)(x) + 5*(1-x)
- ⇨ 15x - 10 = 5 – 15x
- ⇨ X = 0.5

For Player 2;

- ⇨ 5*a + (1-a)*15 = 15*a + (1-a)(5)
- ⇨ 15 – 10a = 10a + 5
- ⇨ a = 0.5

The resulting probability distribution matrix is shown in table 5 below.

Table 5: Mixed Strategy Probability Distribution for Players in the Stock market

		Other Investors	
		BUY (0.5)	**SELL (0.5)**
Charles	**BUY (0.5)**	0.25	0.25
	SELL (0.5)	0.25	0.25

Thus the payoff for each players' strategies are as follows;

For Player 1: **BUY;** (1/4)*(5) + (1/4)*(-10)= -1.25

SELL; (1/4)*(-10) + (1/4)*(5)= -1.25

For Player 2: **BUY;** (1/4)*(5) + (1/4)*(15) = 5

SELL; (1/4)*(15) + (1/4)*(5) = 5

The result of the mixed strategies shows that irrespective of the action that Charles (individual stock market participants) take, their MSNE will always be negative. Charles is playing against a market that does not want to coordinate with him. The Other Investors in our model, however, always have positive payoffs.

Reflection

According to Professor Bige Kahraman at the Saïd Business School, academic experts consistently advise retail investors not to invest in individual shares. She believes this phenomenon is because individual investors lack the education, knowledge, breadth of information and algorithmic skill that well informed finance professionals have (University of Oxford, 2021). Research has shown that 90% of retail investors lose money in the stock market (Taylor, 2020). This is because retail investors invest based on tips, lack patience or even access to the more complicated tools and hedging techniques employed by professionals in Hedge Funds[2] and Asset Management Firms[3].

Using a simple abstraction of the stock market, we have shown how this occurrence is valid for retail investors. While individuals can profit in the stock market, they only do so if they manage to coordinate with the rest of the market. We employed the Mixed Strategy probabilistic distribution to find a Nash Equilibrium for their trading strategies. We found that the payoff for individual or retail investors is negative, while for other investors, payoffs are positive irrespective of the strategy they employ. Thus, even if retail investors were to mixed up their actions in the market, their expected payoffs after numerous iterations is negative.

This analysis is useful because it presents in simple terms the potential outcomes for retail investors in stock markets. Apple Inc. shares can be replaced with any other share listed on any stock exchange globally. The player Charles can be assumed to be any retail investor trading in the market. This analysis explains why retail investors do not always participate in the stock market because they enter the game handicapped.

By utilizing MSNE we have used the science of game theory to describe an outcome in the stock market which retail investors do not realize at the beginning. The importance of Game Theory to anlysing real world scenarios cannot be underestimated. However, this process does

[2] Hedge funds are financial partnerships that use pooled funds and employ different strategies to earn active returns for their investors.

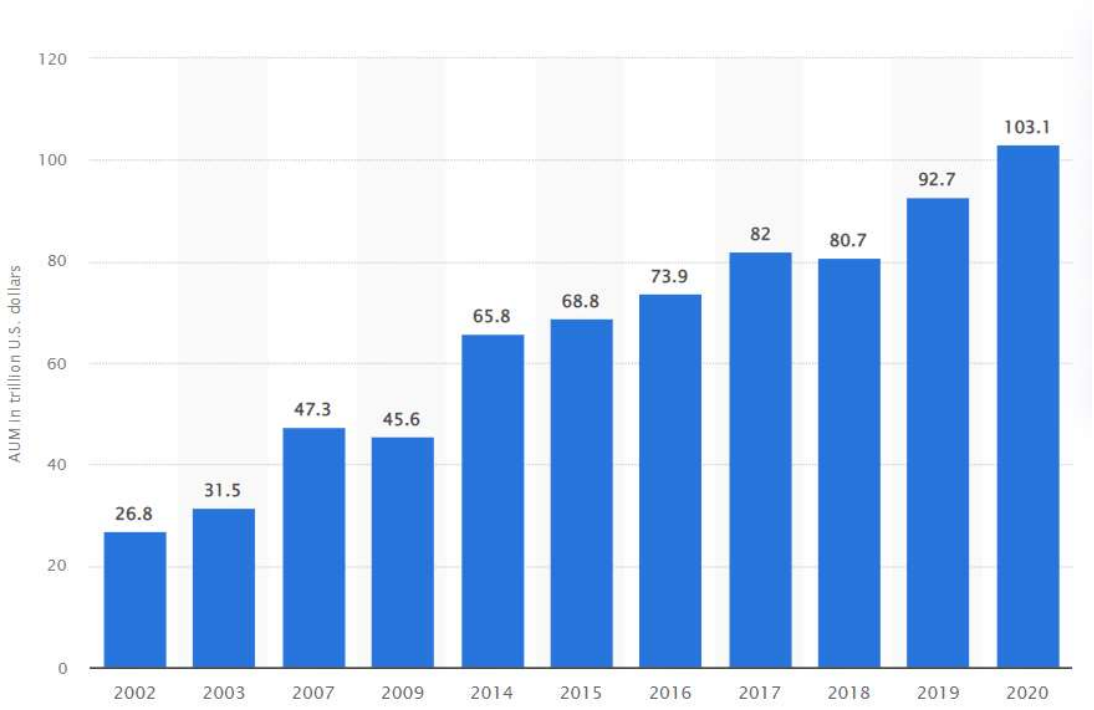
[3] Asset management firms manage funds for individuals and companies. They make well-timed investment decisions on behalf of their clients to grow their finances and portfolio.

have its shotcoming. The most significant being differences in estimation of payoffs for modelling a real world scenario.

Figure 1 below is a logical outcome of our game analysis above. Individuals and retail investors have continually realized that they cannot beat the market. Thus, they have moved their funds to financial trading firms to manage. The industry has seen an almost 5x growth in the past two decades.

Figure 1: Value of assets under management worldwide in selected years from 2002 to 2020

(in trillion U.S. dollars)



Source: Statistia.com

WORD COUNT: 1990

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