

Ecs 132 Homework 2

Problem 1:

1. Show, citing the mailing tubes, that if X and Y are independent random variables, then $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$.
2. Consider the bus ridership example. Intuitively, L_1 and L_2 are not independent, so we suspect (a) does not hold with $X = L_2$ and $Y = L_1$. Determine the difference. (Find the three variances analytically, **and confirm via simulation.**)

Problem 2:

The game is to toss a coin until we get r consecutive heads or reach a total of s tosses, whichever comes first.

Let X denote the number of tosses we make. We win $\$X$. Find the minimum fee that should be charged for this game if $r = 3$ and $s = 6$.

Confirm via simulation.

Problem 3:

Here we will be interested in *permutations*, which are like combinations but with all possible orderings. For instance, suppose we are looking at subsets of $1, 2, \dots, 8$, taken two at a time. Corresponding to the combination $(2, 7)$, the permutations are $(2, 7)$ and $(7, 2)$. For the combination $(1, 2, 7)$, the permutations are $(1, 2, 7)$, $(2, 1, 7)$, $(1, 7, 2)$, $(7, 2, 1)$, $(7, 1, 2)$ and $(2, 7, 1)$.

a. Write a function with call form

```
permn(x, n, FUN)
```

analogous to `combn()` but for permutations. `FUN` (which has no default, i.e. cannot be `NULL`) is applied to each permutation, so that `permn()` returns a vector consisting of the values of `FUN` at the various permutations of `x` taken `n` at a time.

You will NOT be required to save memory, i.e. if you wish, you can have your `permn()` first generate all permutations, then call `FUN()` on each one. However, `FUN` must be an argument of `permn()`, as above.

There are permutation generators in the `partitions` and `gtools` packages, available on CRAN.

Example:

```
> first <- function(z) z[1]
> permn(7:10, 2, first)
[1] 7 8 7 9 7 10 8 9 8 10 9 10
```

b. Apply your `permn()` to solve the following problem. We choose 8 numbers, X_1, \dots, X_8 from $1, 2, \dots, 12$. We are interested in the quantity $W = \sum_{i=1}^7 |X_{i+1} - X_i|$. Find EW .

Problem 4:

Let X be a random variable that denotes the sum of the values on a roll of 2 dice (4 sided dice with equal prob of getting any face).

1. What values does the random variable take?
2. Find the pmf.
3. What is the expected value of X ?

Problem 5:

5 The Binomial Distribution

Suppose that the microchip a company produces passes inspection at a 97% rate. If the company sends 10 chips out for inspection.

1. Use R to plot the probability of 0-10 chips passing inspection.
2. What is the probability that at least 2 of the chips fails inspection?