May 31, 2021

Movie	Alice	Bob	Carol	Dave
$x^{(1)}$	5	5	0	0
$x^{(2)}$	5	?	?	0
$x^{(3)}$	?	4	0	?
$x^{(4)}$	0	0	5	4
$x^{(5)}$	0	0	5	?

$n_m = 5, n_u = 4$

$$\Rightarrow Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

$n_m \times n_u$

Predicted Ratings:

$$Y = \begin{bmatrix} (\theta^{(1)})^T x^{(1)} & (\theta^{(2)})^T x^{(1)} & \cdots & (\theta^{(n_u)})^T x^{(1)} \\ (\theta^{(1)})^T x^{(2)} & & & \vdots \\ \vdots & & & \\ (\theta^{(1)})^T x^{(n_m)} & \cdots & \cdots & (\theta^{(n_u)})^T x^{(n_m)} \end{bmatrix}$$

$$\hookrightarrow X = \begin{bmatrix} -(x^{(1)})^T- \\ -(x^{(2)})^T- \\ \vdots \\ -(x^{(n_m)})^T- \end{bmatrix}, \quad \Theta = \begin{bmatrix} -(\theta^{(1)})^T- \\ -(\theta^{(2)})^T- \\ \vdots \\ -(\theta^{(n_u)})^T- \end{bmatrix}$$

Low Rank Matrices

$\$$Low Rank Matrix Factorization; $Y = X\Theta^T$

Finding Related Movies:

For each movie i, we learn a feature vector $x^{(i)} \in \mathbb{R}^n$.

→ x_1 = romance, x_2 = action, etc

To find movies j related to movie i:

small $||x^{(i)} - x^{(j)}||$ → movie i and j are "similar"

Find 5 most similar movies: find 5 movies j w/ smallest $||x^{(i)} - x^{(j)}||$.

Mean Normalization for New User Recommendations:

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 & ? \\ 5 & ? & ? & 0 & ? \\ ? & ? & 0 & ? & ? \\ 0 & 0 & 5 & ? & ? \\ 0 & 0 & 5 & 0 & ? \end{bmatrix} \rightarrow \mu = \begin{bmatrix} 2.5 \\ 2.5 \\ 2 \\ 2.25 \\ 1.25 \end{bmatrix}$$ → avg of row 1

new user

$$\rightarrow Y := Y - \mu = \begin{bmatrix} 2.5 & 2.5 & -2.5 & -2.5 & ? \\ 2.5 & ? & ? & -2.5 & ? \\ ? & 2 & -2 & ? & ? \\ -2.25 & -2.25 & 2.75 & 1.75 & ? \\ -1.25 & -1.25 & 3.75 & -1.25 & ? \end{bmatrix}$$

Every movie now has avg rating of 0

For user j on movie i, predict:

$(\theta^{(j)})^T x^{(i)} + \mu_i$

add back mean

$\therefore$ For new user $\theta^{(s)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $(\theta^{(s)})^T x^{(i)} + \mu_i = \mu_i$.

Assume average rating if no rating.