Course project: Reconstruction of flow past a cylinder

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Project statement

Navier–Stokes equations describe the physics of flows in diverse applications of scientific and engineering interest. They may be used to model the weather, ocean currents, water flow in a pipe and air flow around a wing. The Navier–Stokes equations in their full and simplified forms help with the design of aircrafts and cars, the study of blood flow, the design of power stations, the analysis of the dispersion of pollutants, and many other applications. Let us consider the Navier–Stokes equations in two dimensions (2D) given explicitly by

$$\boldsymbol{u}_t + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}, \qquad \text{in } \Omega \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \text{in } \Omega, \tag{2}$$

$$\tag{3}$$

where $\boldsymbol{u}(t, x, y)$ denotes the x and y components of the velocity field, and $p(t, x, y)$ is the pressure field. Solutions to the Navier-Stokes equations are searched in the set of divergence-free functions; i.e.,

$$u_x + v_y = 0 \tag{4}$$

Equation (4) is the continuity equation for incompressible fluids that describes the conservation of mass of the fluid. Here we consider the prototype problem of incompressible flow past a circular cylinder; a problem which exhibit rich dynamic behavior and transitions for different regimes of the Reynolds number $Re = u_\infty D/\nu$. Assuming a non-dimensional free stream velocity $u_\infty = 1$, cylinder diameter $D = 1$, and kinematic viscosity $\nu = 0.01$, the system exhibits a periodic steady state behavior characterized by a asymmetrical vortex shedding pattern in the cylinder wake, known as the Kármán vortex street.

Dataset

We provide the data in *.mat* format which were generated using spectral element method generated by applying uniform free stream velocity profile imposed at the left boundary, a zero pressure outflow condition imposed at the right boundary located 30 diameters downstream of the cylinder, and periodicity for the top and bottom boundaries of the $[-15, 25] \times [-8, 8]$ domain. An example of flow simulation representing u, v and p is shown in Figure 1.

Task

1. In this task, randomly sample 10,20,30, and 40 data points in a patch inside domain (e.g., as shown in Figure 2) for $\boldsymbol{u}$ and p and reconstruct the flow field $u = [u, v]$ for entire domain [1]. Report the L_2-norm of relative errors between reconstructed and actual fields. Write a brief summary on your observations.

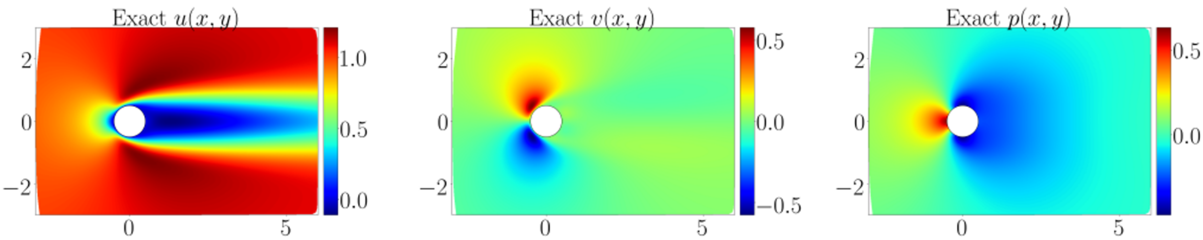

Figure 1: u, v, and p for flow past over cylinder

2. Use different sample strategies such as random *np.random* or Latin Hyper-cube sampling (LHS) [2] method from PyDOE package.

3. For the given flow conditions in the data file, use PINNs for solving the inverse problem of estimating the Reynolds's number (Re). Investigate the effect of sparsity and noise in data on the ability of PINN to retrieve Re. Provide your observations.

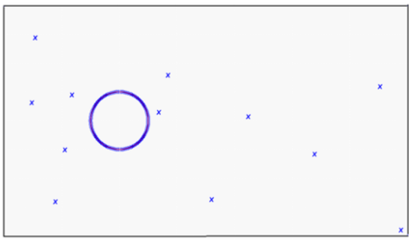

Figure 2: An example of patch and sample strategy.

Programming Options

You may use TensorFlow, PyTorch, or Modulus for completing the tasks.

References

[1] M. Raissi, P. Perdikaris, and G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational physics*, 378:686–707, 2019.

[2] M. Stein. Large sample properties of simulations using latin hypercube sampling. *Technometrics*, 29(2):143–151, 1987.