Warm up exercises: Numerical Method: 101

Numerical Differentiation

1. First Derivative using Forward Difference: Given the function $f(x) = x^2 + 2x + 1$ at the point $x = 2$, use the forward difference method with a step size $h = 0.01$ to estimate $f'(2)$.

2. Second Derivative using Central Difference: For the function $g(x) = \sin(x)$, compute the second derivative at $x = \pi/4$ using the central difference method. Use a step size $h = 0.1$.

3. Error Analysis in Differentiation: Compare the numerical and analytical derivatives of $h(x) = e^x$ at $x = 1$. Use the forward difference method with different step sizes ($h = 0.1, 0.01, 0.001$) and discuss how the error changes with the step size.

Numerical Integration

1. Trapezoidal Rule: Estimate the integral of $f(x) = x^3$ from $x = 1$ to $x = 2$ using the trapezoidal rule with 4 equal subdivisions.

2. Simpson's 1/3 Rule: Apply Simpson's 1/3 Rule to approximate the integral of $g(x) = \sqrt{x}$ from $x = 1$ to $x = 4$. Use 6 subdivisions and compare the result with the exact integral.

3. Calculate the integral of $h(x) = \ln(x)$ from $x = 1$ to $x = 2$ using both the trapezoidal rule and Simpson's 1/3 Rule with the same number of subdivisions (e.g., 4). Discuss the differences in the results.