· 瞬时功率与平均功率

瞬时功率： $p(t) = v(t)\,i(t)$ 　　元件吸收能量的功率

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

电路吸收的瞬时功率： $p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t+\theta_v)\cos(\omega t+\theta_i)$ 　由 $\cos A \cos B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v-\theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t+\theta_v+\theta_i)$ ，

$\begin{cases} p(t) \text{ 为正：电路吸收功率} \\ p(t) \text{ 为负：电源吸收功率} \end{cases}$

平均功率：一个周期内瞬时功率的平均值： $P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m I_m \cos(\theta_v-\theta_i)$

$\tilde{V} = V_m \angle\theta_v$ ， $\tilde{I} = I_m \angle\theta_i$

$\frac{1}{2}VI^* = \frac{1}{2}V_m I_m \angle\theta_v-\theta_i = \frac{1}{2}V_m I_m[\cos(\theta_v-\theta_i)+j\sin(\theta_v-\theta_i)]$

$P = \frac{1}{2}\mathrm{Re}[\tilde{V}\tilde{I}^*] = \frac{1}{2}V_m I_m \cos(\theta_v-\theta_i)$

$(\theta_v=\theta_i)$ 纯电阻电路或电阻性负载R： $P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\tilde{I}|^2 R$ ． $|\tilde{I}|^2 = I \times I^*$ 　任何时刻吸收功率

$(\theta_v-\theta_i=\pm 90^\circ)$ 纯电抗电路吸收的平均功率为零．（L 或 C）

· 最大平均功率传输

$Z_{Th} = R_{Th} + jX_{Th}$ 　　　$Z_L = R_L + jX_L$

过负载的电流： $I = \frac{V_{Th}}{Z_{Th}+Z_L} = \frac{V_{Th}}{(R_{Th}+jX_{Th})+(R_L+jX_L)}$

$P = \frac{1}{2}|I|^2 R_L$ ． $\frac{\partial P}{\partial R_L}$ ， $\frac{\partial P}{\partial X_L} = 0$

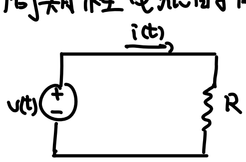
$\downarrow$ 　　$\downarrow$

$R_L = \sqrt{R_{Th}^2 + (X_{Th}+X_L)^2}$ 　　$X_L = -X_{Th}$

为得 P_{max} ， Z_L 必满足上式， $Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*$ 　此时 $P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}$

· 有效值

周期性电流的有效值是指与该周期性电流传递给电阻器的平均功率相等的直流电流值



$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt$

直流电路中，电阻器吸收的功率为 $P = I_{eff}^2 R$

$\Rightarrow I_{eff}=\sqrt{\frac{1}{T}\int_0^T i^2\,dt}$, $V_{eff}=\sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

有效值 也称 方均根值，(rms) $I_{eff}=I_{rms}$，$V_{eff}=V_{rms}$

对任意周期函数 $x(t)$，有效值 $X_{rms}=\sqrt{\frac{1}{T}\int_0^T x^2\,dt}$

仅对正弦信号， $i(t)=I_m\cos\omega t$ $I_{rms}=\sqrt{\frac{1}{T}\int_0^T I_m^2\cos^2\omega t\,dt}=\sqrt{\frac{I_m^2}{T}\int_0^T\frac{1}{2}(1+\cos2\omega t)\,dt}=\frac{I_m}{\sqrt{2}}$

同理 $v(t)=V_m\cos\omega t$，$V_{rms}=\frac{V_m}{\sqrt{2}}$ 平均功率：$P=\frac{1}{2}V_mI_m\cos(\theta_v-\theta_i)=V_{rms}I_{rms}\cos(\theta_v-\theta_i)$

电阻R吸收的平均功率：$P=I_{rms}^2R=\frac{V_{rms}^2}{R}$

• 视在功率与功率因数

$v(t)=V_m\cos(\omega t+\theta_v)$ $i(t)=I_m\cos(\omega t+\theta_i)$; $\tilde{V}=V_m\angle\theta_v$, $\tilde{I}=I_m\angle\theta_i$

$P=V_{rms}I_{rms}\cos(\theta_v-\theta_i)=S\cos(\theta_v-\theta_i)$

$S=V_{rms}I_{rms}$ 视在功率 apparent power (V·A)，$\cos(\theta_v-\theta_i)$ 称功率因数 pf power factor.

$pf=\frac{P}{S}=\cos\underbrace{(\theta_v-\theta_i)}_{\text{功率因数角}}$

功率因数是电压和电流的相位角之差的余弦值，也是负载阻抗辐角的余弦值

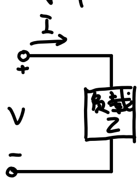
$$Z=\frac{\tilde{V}}{\tilde{I}}=\frac{V_m\angle\theta_v}{I_m\angle\theta_i}=\frac{V_m}{I_m}\angle\theta_v-\theta_i$$

$$\tilde{V}_{rms}=\frac{\tilde{V}}{\sqrt{2}}=V_{rms}\angle\theta_v$$

$$\tilde{I}_{rms}=\frac{\tilde{I}}{\sqrt{2}}=I_{rms}\angle\theta_i$$

$$Z=\frac{\tilde{V}}{\tilde{I}}=\frac{\tilde{V}_{rms}}{\tilde{I}_{rms}}=\frac{V_{rms}}{I_{rms}}\angle\theta_v-\theta_i$$

• 复功率



$\tilde{V}=V_m\angle\theta_v$, $\tilde{I}=I_m\angle\theta_i$

$\tilde{V}_{rms}=\frac{\tilde{V}}{\sqrt{2}}=V_{rms}\angle\theta_v$

$\tilde{I}_{rms}=\frac{\tilde{I}}{\sqrt{2}}=I_{rms}\angle\theta_i$

复功率 complex power. $\tilde{S}=\frac{1}{2}\tilde{V}\tilde{I}^*=\tilde{V}_{rms}\tilde{I}_{rms}^*$

$=V_{rms}I_{rms}\angle\theta_v-\theta_i=V_{rms}I_{rms}\cos(\theta_v-\theta_i)+jV_{rms}I_{rms}\sin(\theta_v-\theta_i)$

$Z=\frac{\tilde{V}}{\tilde{I}}=\frac{\tilde{V}_{rms}}{\tilde{I}_{rms}}=\frac{V_{rms}}{I_{rms}}\angle\theta_v-\theta_i$. $V_{rms}=ZI_{rms}$. $\therefore\tilde{S}=I_{rms}^2Z=\frac{V_{rms}^2}{Z^*}=\tilde{V}_{rms}\tilde{I}_{rms}^*$

$Z=R+jX$. $\tilde{S}=I_{rms}^2(R+jX)=P+jQ$

有功功率 average power $P=Re(\tilde{S})=I_{rms}^2R=V_{rms}I_{rms}\cos(\theta_v-\theta_i)$, W

无功功率 reactive power $Q=\mathrm{Im}(S)=I_{rms}^2X=V_{rms}I_{rms}\sin(\theta_v-\theta_i)$, var

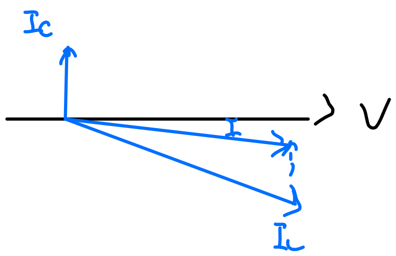
$pf=1 \quad Q=0$,

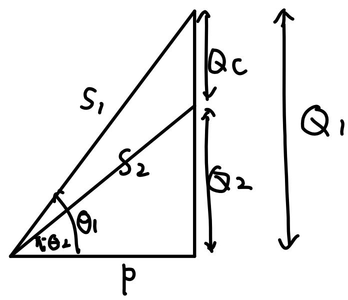
电容性负载（超前pf） $Q<0$.

电感性负载（滞后pf） $Q>0$

- power factor correction

在电感电路并一个电容，可以使它的pf变大





$$C=\frac{P(\tan\theta_1-\tan\theta_2)}{\omega V^2}$$