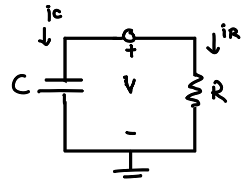
无源RC电路

$v(0) = V_0$、已被储存能量 $w(0) = \frac{1}{2}CV_0^2$



$i_C + i_R = 0$，$i_C = C\,dv/dt$

$\Rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0 \quad \Rightarrow v(t) = V_0 e^{-t/RC}$

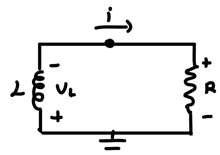
令 $\tau = RC$，$v(t) = V_0 e^{-t/\tau}$

$i_R(t) = \frac{V_0}{R} e^{-t/\tau}$

$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$，$w_R(t) = \frac{1}{2}CV_0^2(1 - e^{-2t/\tau})$

无源RL电路

$i(0) = I_0$，储存能量 $w(0) = \frac{1}{2}LI_0^2$



$v_L + V_R = 0$，$v_L = L\frac{di}{dt}$

$\Rightarrow L\frac{di}{dt} + Ri = 0. \quad \Rightarrow i(t) = I_0 e^{-Rt/L}$

令 $\tau = L/R$. $i(t) = I_0 e^{-t/\tau}$

$v_R(t) = iR = I_0 R e^{-t/\tau}$

$P = I_0^2 R e^{-2t/\tau}$，$w_R(t) = \frac{1}{2}LI_0^2(1 - e^{-2t/\tau})$

奇异函数

- 单位阶跃函数 $u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$

$$u(t-t_0) = \begin{cases} 0, & t<t_0 \\ 1, & t>t_0 \end{cases}$$

- 单位冲激函数 $\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t<0 \\ \text{未定义}, & t=0 \\ 0, & t>0 \end{cases}$

单位冲激可被视为电路中施加或得到的尖峰，是一个持续时间非常短的脉冲单位面积.

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1$$

冲激函数单位面积被称为冲激函数的强度. 当一个冲激函数强度大于单位冲激函数时，

它的面积等同于它的强度.

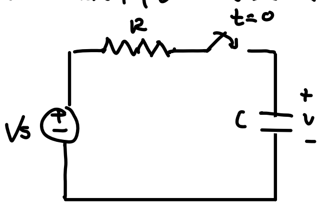
- 单位斜坡函数 $r(t)=\int_{-\infty}^{t} u(\lambda)\,d\lambda = t u(t) = \begin{cases} 0 & t\le 0 \\ t & t\geqslant 0 \end{cases}$

关系： $\delta(t) = \dfrac{d\,u(t)}{dt}$, $u(t) = \dfrac{d\,r(t)}{dt}$

$u(t) = \int_{-\infty}^{t} \delta(\lambda)\,d\lambda$, $r(t) = \int_{-\infty}^{t} u(\lambda)\,d\lambda$

RC 电路的阶跃响应

电路的阶跃响应是电路受到阶跃函数激励时的行为，激发它的可以是电压或电流源.



$v(0^-) = v(0^+) = V_0.$

$C\dfrac{dv}{dt} + \dfrac{v - V_s u(t)}{R} = 0$ 即: $\dfrac{dv}{dt} + \dfrac{v}{RC} = \dfrac{V_s}{RC}u(t)$

$t>0$ 时: $\dfrac{dv}{dt} + \dfrac{v}{RC} = \dfrac{V_s}{RC}$ $\Rightarrow$ $v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$

因此: $v(t) = V_s(1-e^{-t/\tau})u(t)$

$i(t) = \dfrac{V_s}{R}(1-e^{-t/\tau})u(t)$

全响应 = 自由响应（储存能量） + 强迫响应（独立源） $v = v_n + v_f = V_0 e^{-t/\tau} + V_s(1-e^{-t/\tau})$

全响应 = 暂态响应（暂时部分） + 稳态响应（永久部分） $v = v_t + v_{ss} = (V_0 - V_s)e^{-t/\tau} + V_s$

$\Rightarrow v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

RL 电路的阶跃响应

$i = i_t + i_{ss} = Ae^{-t/\tau} + \dfrac{V_s}{R}$

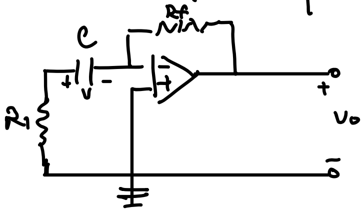
$i(0^+) = i(0^-) = I_0.$ $t=0$ $I_0 = A + \dfrac{V_s}{R}$

$i(t) = \dfrac{V_s}{R} + (I_0 - \dfrac{V_s}{R})e^{-t/\tau}$

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$

$v(t) = L\dfrac{di}{dt} = V_s e^{-t/\tau} u(t)$

First order Op Amp circuits



$v(0) = 3V$

$R_f = 80k\Omega$, $R_1 = 20k\Omega$

$C = 5\mu F$

$$\left(C\frac{dv}{dt}\right)R_1 + V = 0 \qquad t>0.$$

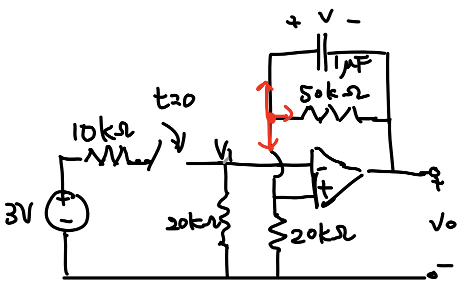
$$\tau = R_1 C = 20\times10^3 \times 5\times10^{-6} = 0.1s$$

$$v = V(0)e^{-t/\tau} = 3e^{-t/0.1} = 3e^{-10t}\ V$$

$$V_o = \left(-C\frac{dv}{dt}\right)R_f$$

$$= \left(-5\times10^{-6}\frac{d}{dt}(3e^{-10t})\right)\times 80\times10^3$$

$$= 12e^{-10t}\quad V$$



$t<0$

$V_1(t) = 0 \qquad V_o(t) = 0 \qquad V(t) = 0.$

$t>0.$

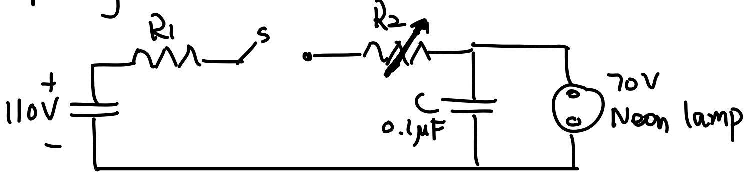
$$V_1(t) = 3\times\frac{20}{10+20} = 2V.$$

$$(1\times10^{-6})\frac{dv}{dt} + \frac{v}{50\times10^3} + \frac{2}{20\times10^3} = 0.$$

$$\Rightarrow V(t) = 5e^{-20t} - 5V$$

$$V_o(t) = -V(t) + 2 = 7 - 5e^{-20t}\ (V)$$

Delay Circuit



$R_1 = 1.5M\Omega \qquad R_2 = 2.5M\Omega.$

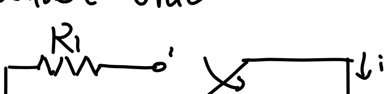
How long for the lamp to glow for the first time after the switch is closed

$$V_c(0) = 0 \qquad V_c(\infty) = 110 \qquad \tau = (1.5+2.5)\times10^6\times0.1\times10^{-6} = 0.4s$$

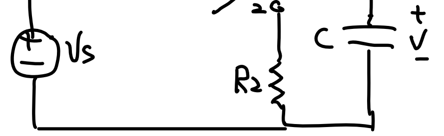
$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-t/\tau}$$

$$t = \tau\ln\left(\frac{V_c(0^+) - V_c(\infty)}{V_c(t) - V_c(\infty)}\right) \approx 0.4046s.$$

Photoflash Unit



position 1. capacitor V↑ from 0 to Vs



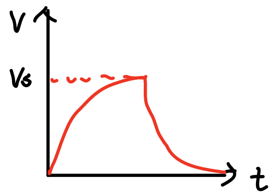
I ↓ from $\frac{V_s}{R_1}$ to 0.
peak

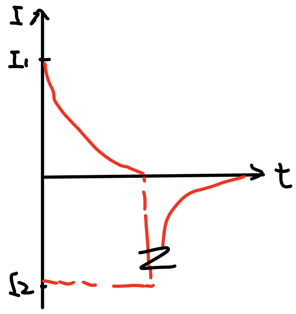
$t_{charge} = 5R_1C$

position 2. capacitor V ↓ from Vs to 0.

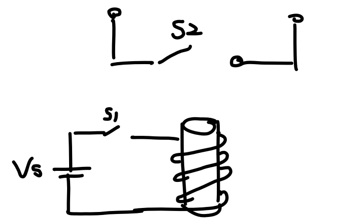
I ↓ from $\frac{V_s}{R_s}$ to 0.
peak

$t_{discharge} = 5R_2C$

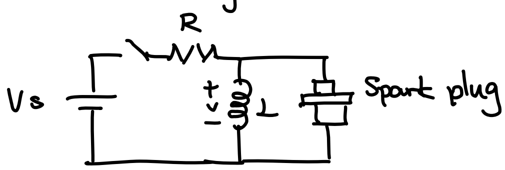




Relay Circuit 继电器



Automobile Ignition Circuit



$R = 4\Omega$ $L = 6mH$ $V_s = 12V$. Δt for switch open $1\mu s$

$i = V_s/R = 12/4 = 3A$

$W = \frac{1}{2}Li^2 = \frac{1}{2} \times 6\times10^{-3} \times 3^2 = 0.027 J$

$v = L\frac{di}{dt} = 6\times10^{-3} \cdot \frac{3}{1\times10^{-6}} = 18 kV$