- **传递函数**

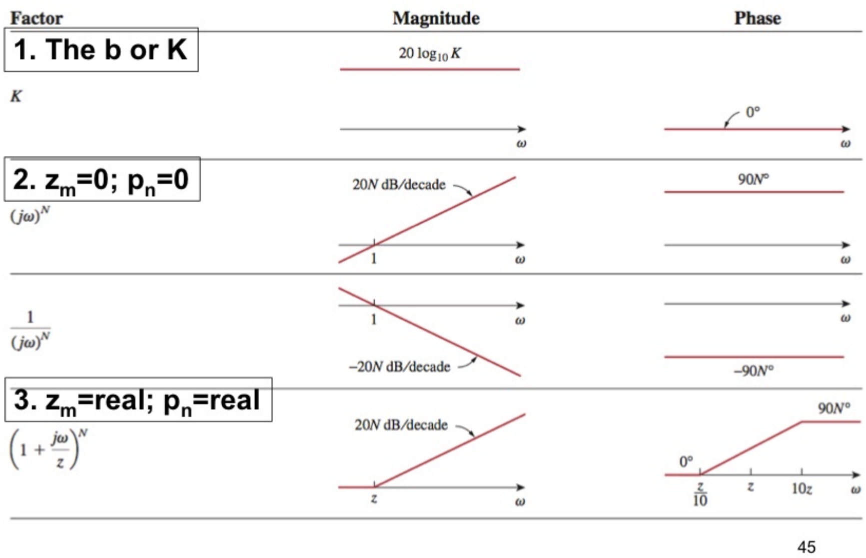
$$H(\omega)=\frac{Y(\omega)}{X(\omega)}=\frac{N(\omega)}{D(\omega)}$$

- **分贝表示法**

$$G_{dB}=10\log_{10}\frac{P_2}{P_1}=20\log_{10}\frac{V_2}{V_1}=20\log_{10}\frac{I_2}{I_1}$$

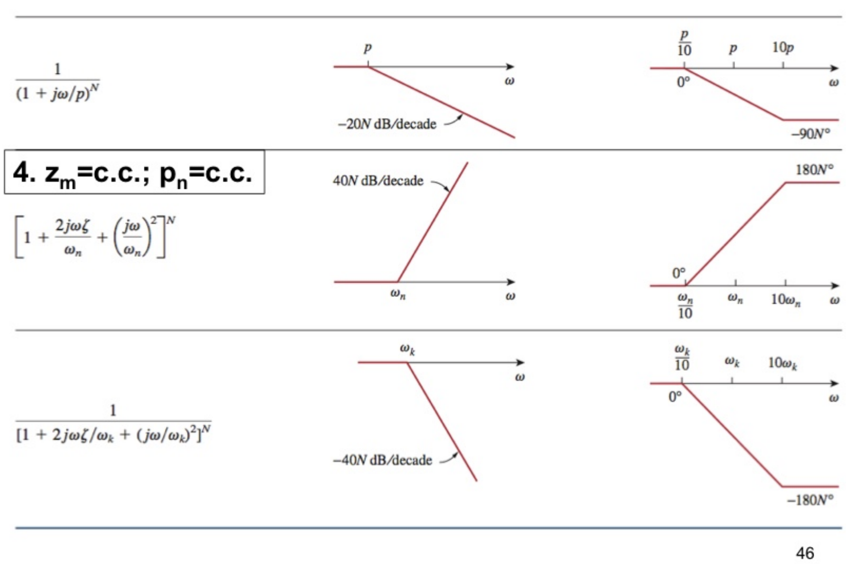
- **伯德图**

伯德图是传递函数的模(dB)与相位(°)关于频率的对数曲线图.




$k>0 \quad \phi=0°$

$k<0 \quad \phi=180°$




- **串联谐振电路.**

谐振是RLC电路中容性电抗与感性电抗大小相等时呈现的一种状态，此时该电路呈现出纯电阻的阻抗性质.

$$Z = H(\omega) = \frac{V_s}{I} = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

谐振时：$Im(Z) = \omega L - \frac{1}{\omega C} = 0$. 此 $\omega_0$ 称谐振频率. $\omega_0 L = \frac{1}{\omega_0 C}$. $\omega_0 = \frac{1}{\sqrt{LC}}$

又 $\omega_0 = 2\pi f_0$. $\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$ resonanct frequency

谐振性质：① $Z = R$

② 电压 $V_s$ 与电流 $I$ 同相. 功率因数为 1.

③ $H(\omega) = Z(\omega)$ 幅度最小

④ 电感器两端电压与电容器两端电压比电源电压高得多.

$$|V_L| = \frac{V_m}{R}\omega_0 L = QV_m, \quad |V_C| = \frac{V_m}{R}\frac{1}{\omega_0 C} = QV_m$$ Q：品质因数

电流幅度的频率响应 $I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

$\omega = \omega_1、\omega_2$ 电路消耗功率是最大功率的一半 $P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$

$I = U_m/R$ $\quad P(\omega_0) = \frac{V_m^2}{2R}$ max.

$$\begin{cases} \omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \\ \omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \end{cases} \qquad \omega_0 = \sqrt{\omega_1 \omega_2}$$

带宽 $B = \omega_2 - \omega_1$ half-power bandwidth

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2 R(1/f_0)} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$ quality factor

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega_0^2 CR$$

$Q \geq 10$ 称 高Q值电路 可近似为 $\omega_1 \approx \omega_0 - \frac{B}{2}$, $\omega_2 \approx \omega_0 + \frac{B}{2}$

- 并联谐振电路

$$Y = H(\omega) = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

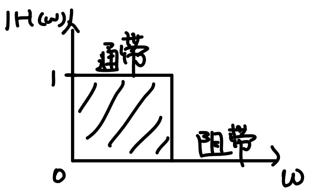
$B = \omega_2 - \omega_1 = \frac{1}{RC}$, $Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$ $\quad \omega = \pm\frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$

特性 串联电路 并联电路

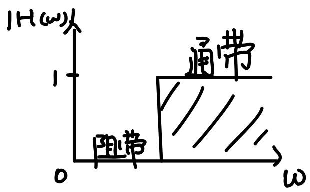
| | | |
|---|---|---|
| 谐振频率 $\omega_0$ | $\frac{1}{\sqrt{LC}}$ | $\frac{1}{\sqrt{LC}}$ |
| 频带宽度 $B$ | $\frac{\omega_0}{Q}$ | $\frac{\omega_0}{Q}$ |
| $Q\geq10$ 时 $\omega_1,\omega_2$ | $\omega_0\pm\frac{B}{2}$ | $\omega_0\pm\frac{B}{2}$ |
| 品质因数 $Q$ | $\frac{\omega_0 L}{R}$ 或 $\frac{1}{\omega_0 RC}$ | $\frac{R}{\omega_0 L}$ 或 $\omega_0 RC$ |
| 半功率频率 | $\omega_0\sqrt{1+(\frac{1}{2Q})^2}\pm\frac{\omega_0}{2Q}$ | $\omega_0\sqrt{1+(\frac{1}{2Q})^2}\pm\frac{\omega_0}{2Q}$ |

- 无源滤波器

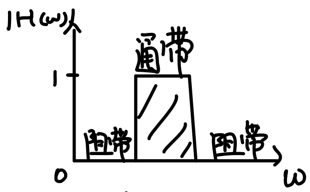
滤源器是一个使期望频率的信号通过，同时阻止或衰退其他频率信号的电路。



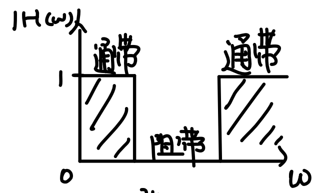
低通



高通



带通 bandpass

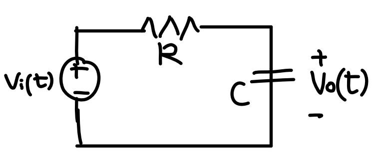


带阻 bandstop

| 滤波器类型 | $H(0)$ | $H(\infty)$ | $H(\omega)$ 或 $H(\omega_0)$ |
|---|---|---|---|
| 低通 | 1 | 0 | $1/\sqrt{2}$ |
| 高通 | 0 | 1 | $1/\sqrt{2}$ |
| 带通 | 0 | 0 | 1 |
| 带阻 | 1 | 1 | 0 |

1) 低通滤波器

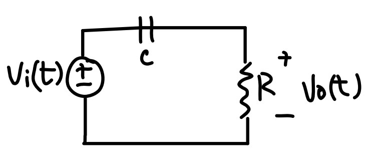
RC电路输出取自电容器两端电压




截止频率 $\omega_c=\frac{1}{RC}$

只允许从直流到 $\omega_c$.

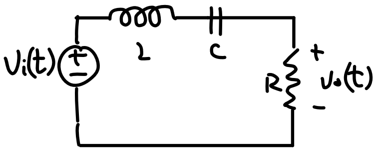
2) 高通滤波器




截止频率 $\omega_c=\frac{1}{RC}$

只允许从高于 $\omega_c$ 频率通过.

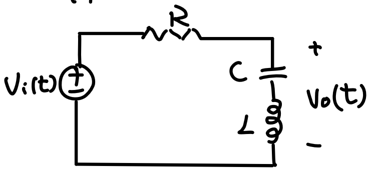
3) 带通滤波器




$\omega_0 = \frac{1}{\sqrt{LC}}$

允许 $\omega_1 < \omega < \omega_2$ 通过

4) 带阻滤波器




$\omega_0 = \frac{1}{\sqrt{LC}}$

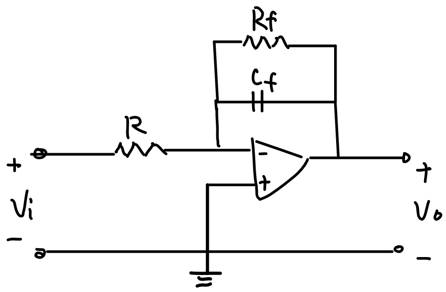
抑制或消除在 $\omega_1 < \omega < \omega_2$ 所有频率信号.

## • 有源滤波器.

1) 一阶低通滤波器.

$$H(\omega) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i} \qquad Z_i = R,\ Z_f = \frac{R_f}{1 + j\omega C_f R_f}$$

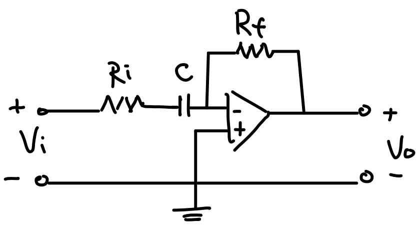
$$\therefore H(\omega) = -\frac{R_f}{R_i} \cdot \frac{1}{1 + j\omega C_f R_f} \qquad \text{转折频率} \quad \omega_c = \frac{1}{R_f C_f}$$




2) 一阶高通滤波器

$$H(\omega) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i} \qquad Z_i = R_i + 1/j\omega C_i. \quad Z_f = R_f$$

$$\therefore H(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}. \qquad \omega_c = \frac{1}{R_i C_i}$$




3) 带通滤波器.

$V_i \longrightarrow$ 低通（设 $\omega_2$） $\longrightarrow$ 高通（设 $\omega_1$） $\longrightarrow$ 反相器（增益） $\longrightarrow V_o$.

$$H(\omega) = \frac{V_o}{V_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

$$= -\frac{R_f}{R_i} \cdot \frac{1}{1 + j\omega C_1 R} \cdot \frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

$$\omega_2 = \frac{1}{RC_1}, \quad \omega_1 = \frac{1}{RC_2}$$

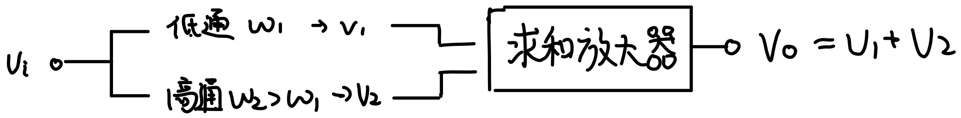
$$\omega_0 = \sqrt{\omega_1\omega_2}, \quad B = \omega_2 - \omega_1, \quad Q = \frac{\omega_0}{B}$$

代为标准形式 $H(\omega) = -\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)} = -\frac{R_f}{R_i}\frac{j\omega\omega_2}{(\omega_1+j\omega)(\omega_2+j\omega)}$

$$|H(\omega)| = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1+\omega_2}$$

$$K = \frac{R_f}{R_i}\cdot\frac{\omega_2}{\omega_1+\omega_2}$$

4) 带阻滤波器




$$H(\omega) = \frac{V_o}{V_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1+j\omega C_1 R} - \frac{j\omega C_2 R}{1+j\omega C_2 R}\right)$$

$$= \frac{R_f}{R_i}\left(\frac{1}{1+j\omega/\omega_2} + \frac{j\omega/\omega_2}{1+j\omega/\omega_1}\right)$$

$$= \frac{R_f}{R_i}\cdot\frac{(1+j2\omega/\omega_1+(j\omega)^2/\omega_1\omega_2)}{(1+j\omega/\omega_2)(1+j\omega/\omega_1)}$$

$$K = \frac{R_f}{R_i}$$

$$H(\omega_0) = \frac{R_f}{R_i}\cdot\frac{2\omega_1}{\omega_1+\omega_2}$$