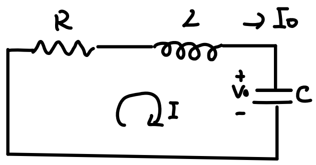
· 电感电流和电容电压不会突变

· 无源串联 RLC 电路




$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

解此方程需有两个初始条件 $\begin{cases} i\text{初值及一阶导} \\ i, V\text{初值} \end{cases}$

有 $V_0 = \frac{1}{C}\int_{-\infty}^{0} i\,dt$ , $i(0) = I_0 = C\frac{dv}{dt}$

又 $R\,i(0) + L\frac{di(0)}{dt} + V_0 = 0$. 即 $\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$

令 $i = Ae^{st}$. 代入并求一阶导 $\Rightarrow As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0$.

$\alpha$: neper frequency Np/s　　$\Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$　　两根 $s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$w_0$: resonant ·· rad/s　　$\alpha = \frac{R}{2L}$ , $w_0 = \frac{1}{\sqrt{LC}}$　　有 $s = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$

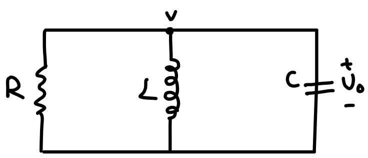
1. $\alpha > w_0$ 过阻尼　　2. $\alpha = w_0$ 临界阻尼　　3. $\alpha < w_0$ 欠阻尼

① s 为正实根　　$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$

② $i(t) = (A_1t + A_2)e^{-\alpha t}$

③ $i(t) = e^{-\alpha t}(B_1\cos w_d t + B_2\sin w_d t)$

· 无源并联 RLC 电路




$$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)\,dt$$

$$V(0) = V_0$$

顶点 KCL : $\frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau + C\frac{dv}{dt} = 0$

$$\Rightarrow \frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0 \Rightarrow s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$S = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

$\alpha = \frac{1}{2RC}$ , $w_0 = \frac{1}{\sqrt{LC}}$

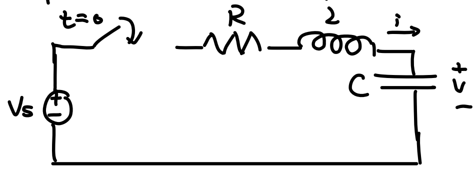
求初值 : $\frac{V_0}{R} + I_0 + C\frac{dV_0}{dt} = 0$. 即: $\frac{dV_0}{dt} = -\frac{(V_0 + RI_0)}{RC}$

① $v(t) = A_1e^{s_1t} + A_2e^{s_2t}$

② $v(t) = (A_1 + A_2t)e^{-\alpha t}$

③ $v(t) = e^{-\alpha t}(A_1\cos wt + A_2\sin wt)$

- 串联电路RLC阶跃响应




$$L\frac{di}{dt}+Ri+v=V_s. \quad i=C\frac{dv}{dt}$$

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{V_s}{LC}$$

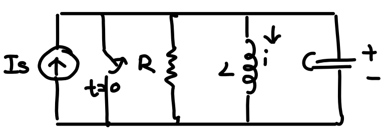
$$v(t)=v_t(t)+v_{ss}(t) \qquad v(\infty)=V_s$$

$$v(t)=V_s+A_1e^{s_1t}+A_2e^{s_2t}$$ (过阻尼)

$$v(t)=V_s+(A_1+A_2t)e^{-\alpha t}$$ (临界阻尼)

$$v(t)=V_s+(A_1\cos\omega t+A_2\sin\omega t)e^{-\alpha t}$$ (欠阻尼)

- 并联RLC电路的阶跃响应




$$\frac{v}{R}+i+C\frac{dv}{dt}=I_s. \quad 又\ v=L\frac{di}{dt}$$

$$\Rightarrow \frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{i}{LC}=\frac{I_s}{LC}$$

$$i(t)=I_s+A_1e^{s_1t}+A_2e^{s_2t}$$ (过阻尼)

$$i(t)=I_s+(A_1+A_2t)e^{-\alpha t}$$ (临界阻尼)

$$i(t)=I_s+(A_1\cos\omega_d t+A_2\sin\omega_d t)e^{-\alpha t}$$ (欠阻尼)