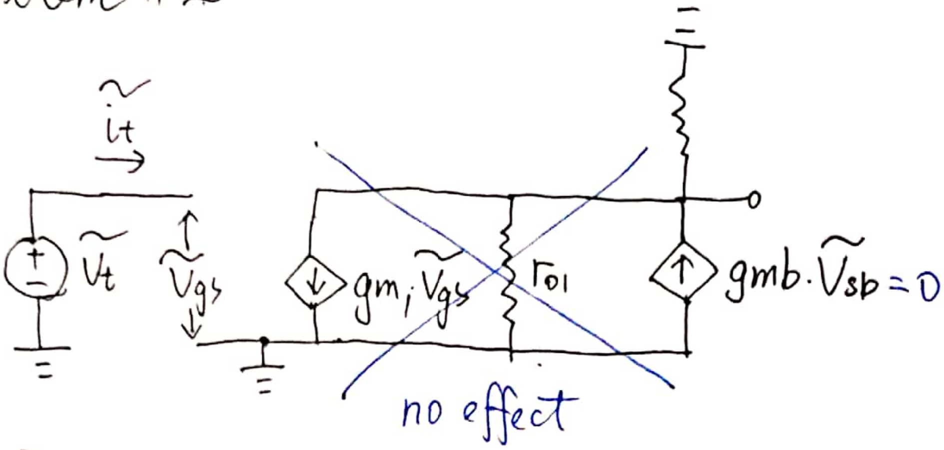
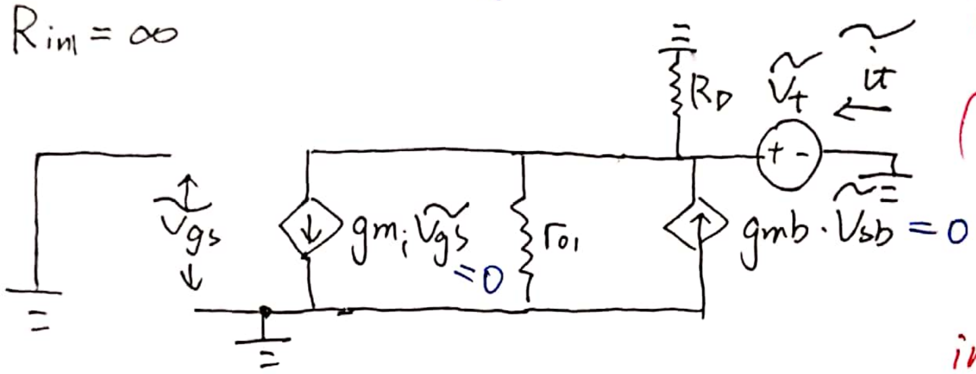
Problem #1

(1)



(Calculate R_{in}: connect a test voltage to the input node and output node do nothing)) $R_{in}=\frac{\tilde{V}_t}{\tilde{i}_t}$ (flow into the amplifier)

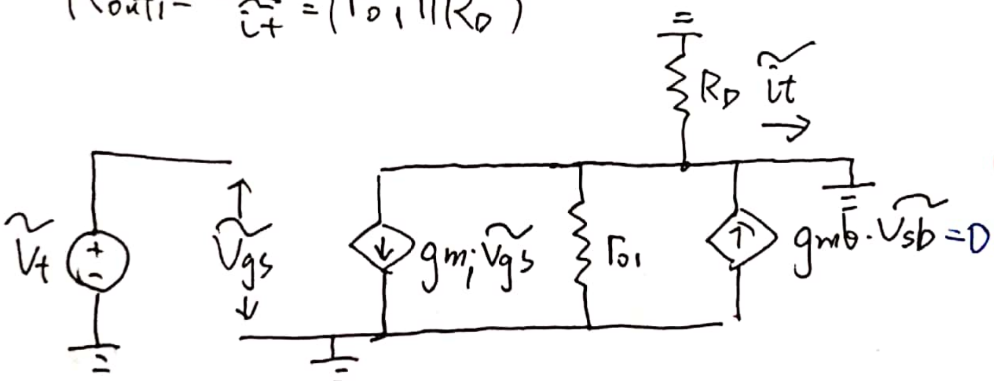
$R_{in}=\infty$



(Calculate R_{out}: connect a test voltage to the output node and connect input node to ground)

$R_{out1}=\frac{\tilde{V}_t}{\tilde{i}_t}=(r_{o1}\,||\,R_D)$

$R_{out}=\frac{V_t}{i_t}$ (flow into the amplifier)



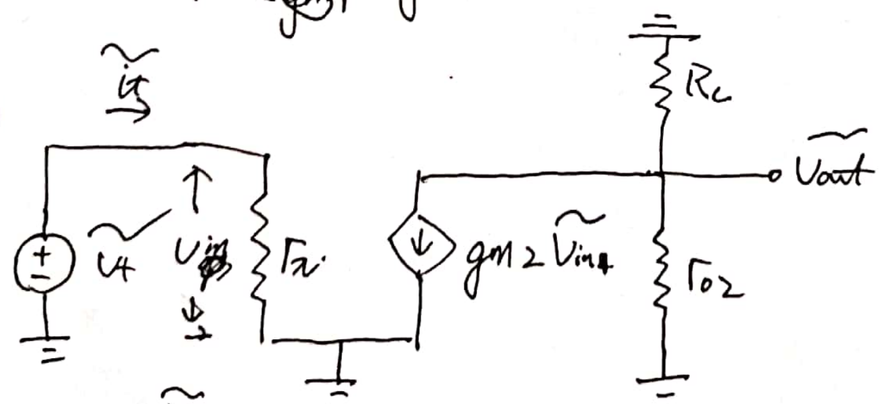
(Calculate G_m: connect a test voltage to the input node and connect output node to ground)

Since r_{o1} and R_D are shorted, then $\tilde{i}_t=-g_{m1}\tilde{V}_{gs}$

$G_m=\frac{i_t}{V_t}$ (flow out of the amplifier)

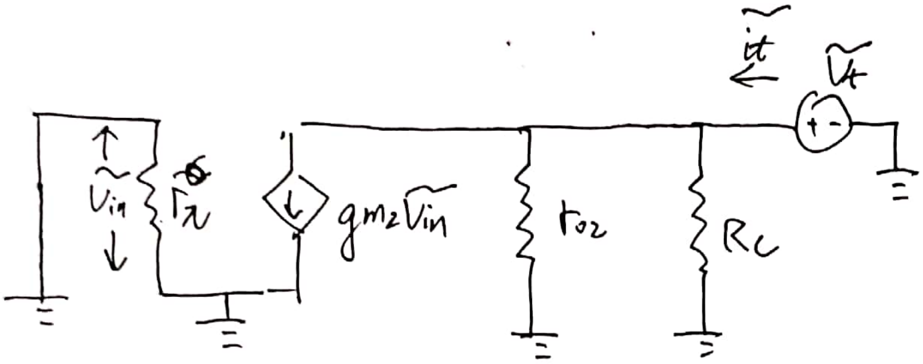
$G_{m1}=\frac{\tilde{i}_t}{\tilde{V}_t}=-g_{m1}$

(2)

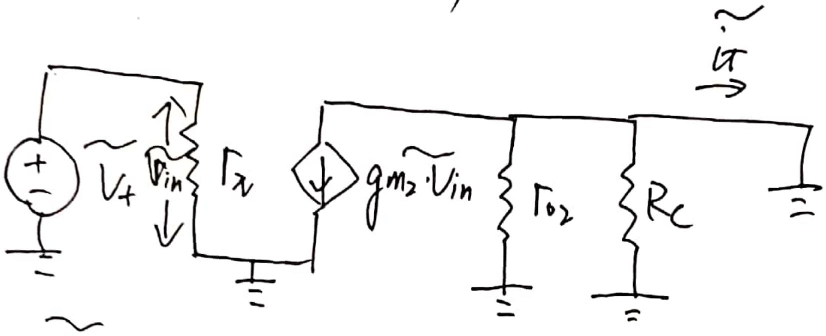


(Reasons are the same as (1))

$R_{in2}=\frac{\tilde{V}_t}{\tilde{i}_t}=r_\pi$

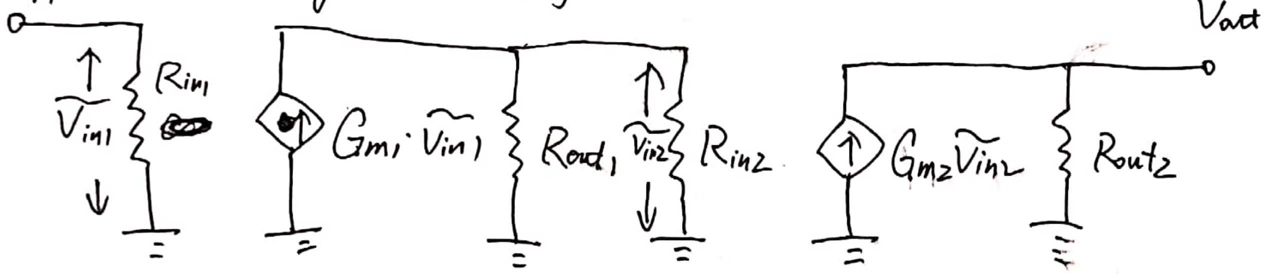


$$R_{out2} = \frac{\tilde{v}_t}{\tilde{i}_t} = (r_{o2} \| R_C)$$



$$\tilde{i}_t = -g_{m2}\cdot\tilde{v}_{in} \Rightarrow G_{m2} = \frac{\tilde{i}_t}{\tilde{v}_{in}} = -g_{m2}$$

(3) Draw the general amplifier model out as below:



$$\tilde{v}_{in1} = \tilde{v}_{in}$$

$$\tilde{v}_{in2} = G_{m1}\tilde{v}_{in1}\cdot(R_{out1}\|R_{in2}) = G_{m1}\tilde{v}_{in}(R_{out1}\|R_{in2})$$

$$\tilde{v}_{out} = G_{m2}\tilde{v}_{in2}R_{out2} = G_{m1}G_{m2}(R_{out1}\|R_{in2})R_{out2}\tilde{v}_{in}$$

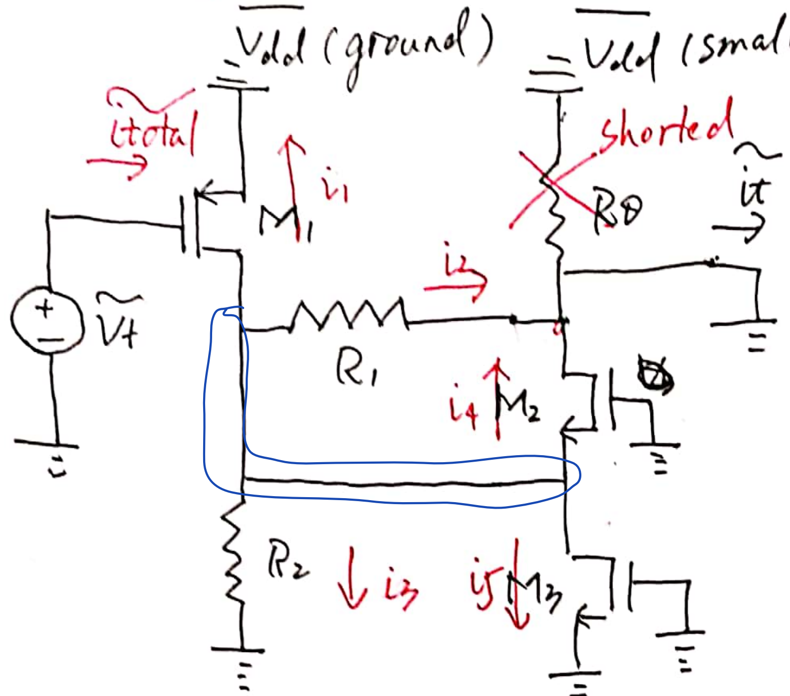
$$A_v = \frac{\tilde{v}_{out}}{\tilde{v}_{in}} = G_{m1}G_{m2}(R_{out1}\|R_{in2})R_{out2}$$

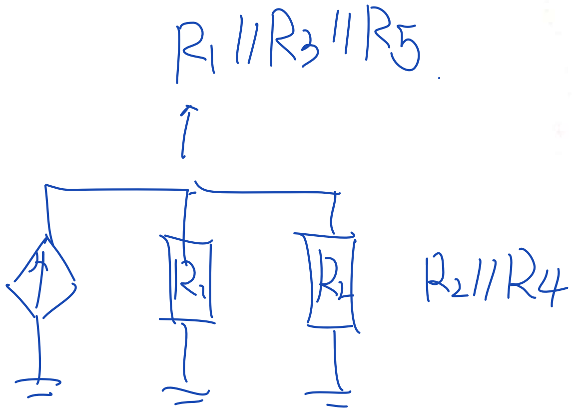
$$= (-g_{m1})(-g_{m2})\left[(r_{o1}\|R_D)\|r_\pi\right](r_{o2}\|R_C)$$

$$= g_{m1}g_{m2}(r_{o1}\|R_D\|r_\pi)(r_{o2}\|R_C)$$

Problem #2

(I) Identify the circuit connection, when calculating G_m, we have





R_D is shorted, M_1, R_2, R_1, M_2, M_3 are parallel connected.

Thus, $\tilde{i}_{total} = \tilde{i}_1 + \tilde{i}_2 + \tilde{i}_3 + \tilde{i}_4 + \tilde{i}_5$ 全是并的

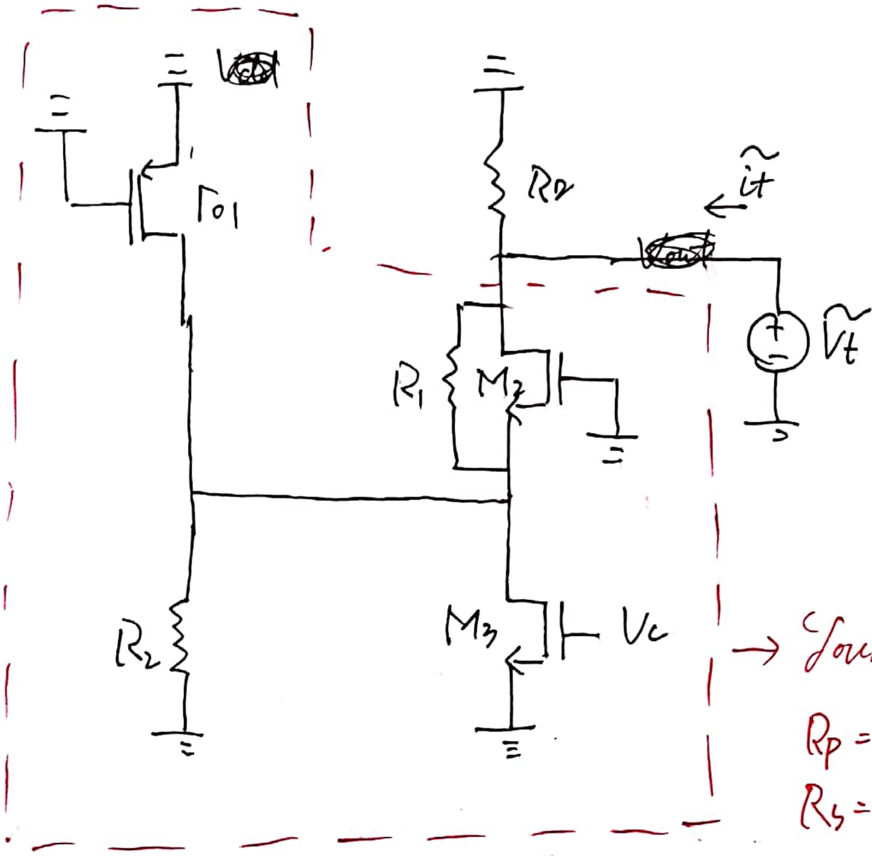
$\tilde{i}_t = \tilde{i}_2 + \tilde{i}_4$

$$\tilde{i}_t = -g_{m1}\tilde{v}_t \times \frac{\tilde{i}_2 + \tilde{i}_4}{\tilde{i}_1 + \tilde{i}_2 + \tilde{i}_3 + \tilde{i}_4 + \tilde{i}_5} = \frac{Z_{eq}(1\|3\|5) \cdot (-g_{m1} v_4)}{Z_{eq}(1\|3\|5) + Z_{eq}(2\|4)}$$ (Z stands for impedance)

$$= \frac{-g_{m1} v_t \cdot (r_{o1} \| r_{o3} \| R_2)}{(r_{o1} \| r_{o3} \| R_2) + (R_1 \| r_{o2} \| \frac{1}{g_{m2}} \| \frac{1}{g_{mb2}})}$$

Thus, $$G_m = \frac{i_t}{v_t} = -g_{m1} \frac{(r_{o1} \| r_{o3} \| R_2)}{(r_{o1} \| r_{o3} \| R_2) + (R_1 \| r_{o2} \| \frac{1}{g_{m2}} \| \frac{1}{g_{mb2}})}$$

(I) When calculating R_{out}, we have



→ Source Degeneration structure $R_{total\,sd}$

$R_p = R_1 \| r_{o2}$

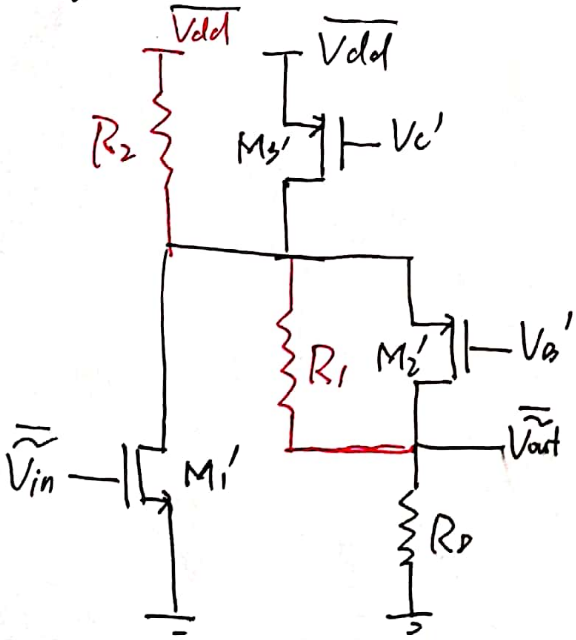
$R_s = r_{o3} \| r_{o1} \| R_2$

$$R_{out} = R_D \| R_{sd\,total}$$

$$= R_D \| \left[R_1 \| r_{o2} + r_{o3} \| r_{o1} \| R_2 + (g_{m2}+g_{mb2})(R_1 \| r_{o2})(r_{o3} \| r_{o1} \| R_2) \right]$$

$$A_V = G_m \cdot R_{out} = -g_{m1} \frac{(r_{o1} \| r_{o3} \| R_2)}{(r_{o1} \| r_{o3} \| R_2) + (R_1 \| r_{o2} \| \frac{1}{g_{m2}} \| \frac{1}{g_{mb2}})} \cdot \left\{ R_D \| \left[r_{o2} \| R_1 + r_{o1} \| r_{o3} \| R_2 + (g_{m2}+g_{mb2})(R_1 \| r_{o2})(r_{o3} \| r_{o1} \| R_2) \right] \right\}$$

(II)



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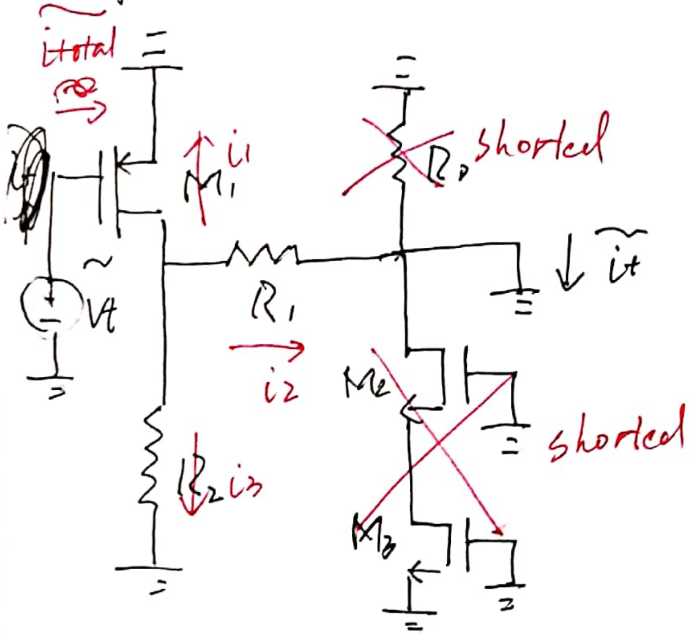
(III) The DC current source should be considered as open circuit.

For G_m:



$R_0 . M_2 . M_3$ are shorted, $R_1 . R_2 . M_1$ are connected in parallel.

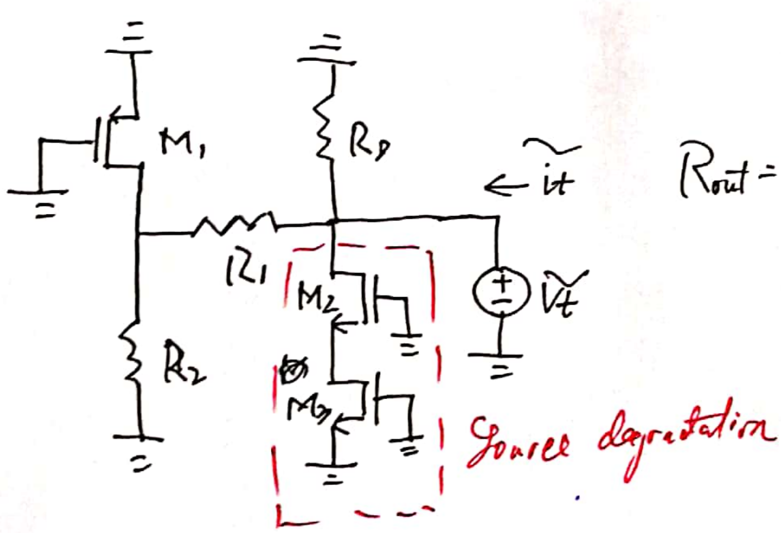
$$\tilde{i}_{total} = \tilde{i}_1 + \tilde{i}_2 + \tilde{i}_3 \qquad \tilde{i}_t = i_2$$

$$\tilde{i}_t = -g_{m2}\tilde{v}_t \cdot \frac{\tilde{i}_2}{\tilde{i}_1 + \tilde{i}_2 + \tilde{i}_3} = -g_{m2}\tilde{v}_t \cdot \frac{Z_{eq}(1\|3)}{Z_{eq}(1\|3) + Z_{eq}(2)}$$

$$= -g_{m2}\tilde{v}_t \cdot \frac{r_{o1} \| R_2}{r_{o1} \| R_2 + R_1}$$

$$G_m = \frac{\tilde{i}_t}{\tilde{v}_t} = -g_{m2}\frac{r_{o1} \| R_2}{r_{o1} \| R_2 + R_1}$$

For R_{out}:

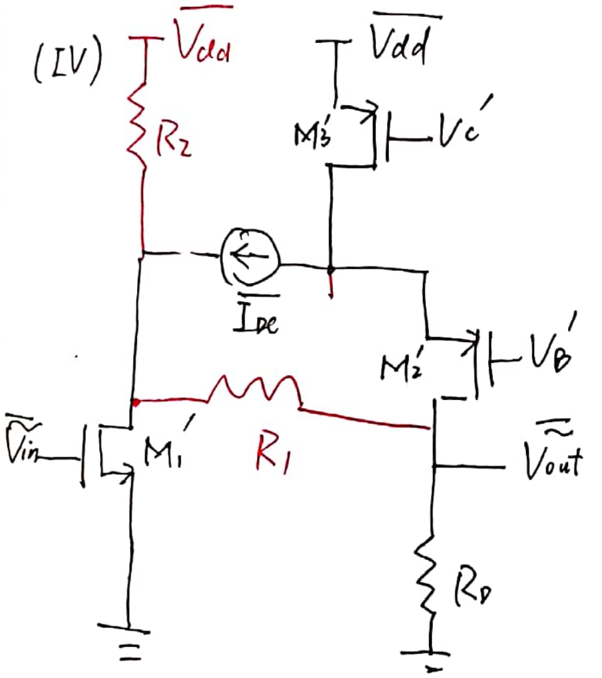


$$R_{out} = R_0 \| [R_1 + r_{o1} \| R_2] \| [r_{o3} + r_{o2} + (g_{m2} + g_{mb2}) r_{o2} r_{o3}]$$

$$A_v'' = G_m R_{out}$$

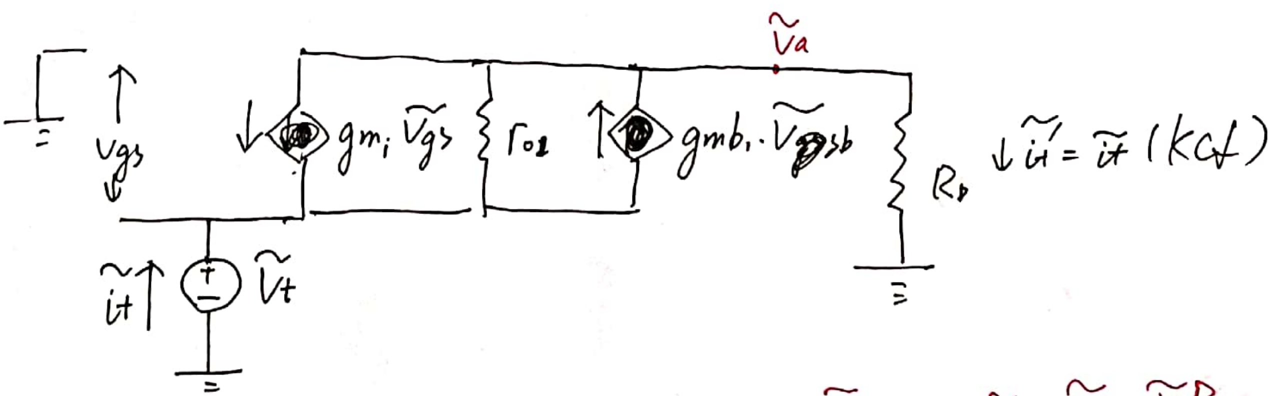
$$= -g_{m2}\frac{r_{o1} \| R_2}{r_{o1} \| R_2 + R_1}\left\{R_D \| (R_1 + r_{o1} \| R_2) \| [r_{o2} + r_{o3} + (g_{m2} + g_{mb2}) r_{o2} r_{o3}]\right\}$$

(IV)



Problem #3

(1) draw the small signal model, connect a test voltage $\tilde{v}_t$ to the output node ~~input~~



$$\begin{cases} \tilde{v}_a = \tilde{i}_t' \cdot R_D = \tilde{i}_t \cdot R_D \\ \tilde{i}_t = -g_{m1}\cdot\tilde{v}_{gs} + \dfrac{\tilde{v}_t - \tilde{v}_a}{r_{o1}} + g_{mb1}\cdot\tilde{v}_{sb} \\ \tilde{v}_{gs} = -\tilde{v}_t \\ \tilde{v}_{sb} = \tilde{v}_s - 0 = \tilde{v}_s \end{cases} \Rightarrow$$

$$\tilde{i}_t = g_{m1}\tilde{v}_t + \frac{\tilde{v}_t - \tilde{i}_t R_D}{r_{o1}} + g_{mb1}\tilde{v}_t$$

$$\boxed{R_{out} = \frac{\tilde{v}_t}{\tilde{i}_t} = \frac{r_{o1} + R_D}{1 + (g_{m1} + g_{mb1}) r_{o1}}}$$

(2) $R_D \Rightarrow \left(\frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}\right) \leftarrow$ $r_{o2} \leftarrow r_{o3} \parallel r_{o4}$

$g_{m2} \leftarrow g_{m3} + g_{m4}$

$g_{mb2} \leftarrow g_{mb3} + g_{mb4}$.

$$R_{out} = \frac{r_{o1} + \left(\frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}\right)}{1 + (g_{m1} + g_{mb1}) \cdot r_{o1}}$$

(3) $R_D \Rightarrow \left(\frac{1}{g_{mb3}} \parallel \frac{1}{g_{mb4}} \parallel \frac{1}{g_{m3}} \parallel \frac{1}{g_{m4}} \parallel r_{o3} \parallel r_{o4}\right)$

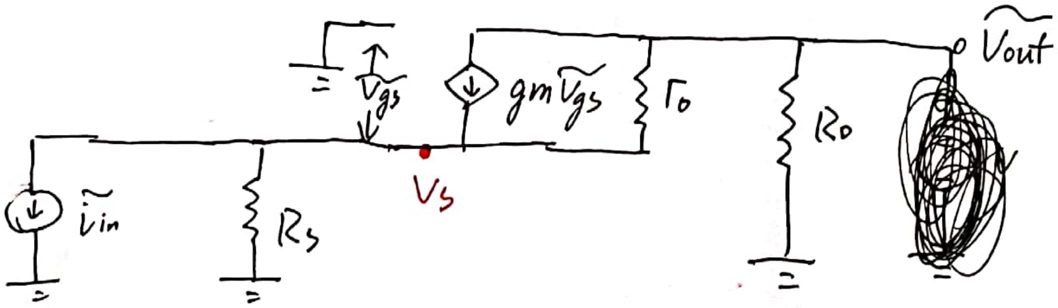
$g_{m1} + g_{mb1} \Rightarrow g_{m1} + g_{m2} + g_{mb1} + g_{mb2}$

$r_{o1} = r_{o1} \parallel r_{o2}$

$$R_{out} = \frac{(r_{o1} \parallel r_{o2}) + \left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{mb3}} \parallel \frac{1}{g_{m4}} \parallel \frac{1}{g_{mb4}} \parallel r_{o3} \parallel r_{o4}\right)}{1 + (g_{m1} + g_{m2} + g_{mb1} + g_{mb2})(r_{o1} \parallel r_{o2})}$$

Problem # 4.

(1) Draw the small signal model (since it's transimpedance, it's hard to use ~~go~~ models solve the small signal gain).

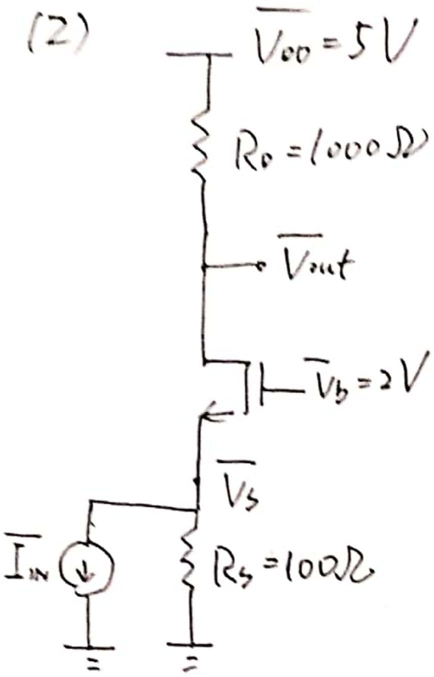


$$\begin{cases} \tilde{i}_{in} + \frac{\tilde{V}_S}{R_S} = g_m \cdot (-V_S) + \frac{\tilde{V}_{out} - \tilde{V}_S}{r_o} & \text{(KCL at source)} \\ g_m \cdot (0 - \tilde{V}_S) + \frac{\tilde{V}_{out} - \tilde{V}_S}{r_o} + \frac{\tilde{V}_{out} - 0}{R_D} = 0 & \text{(KCL at drain)} \end{cases}$$

$$\text{Transimpedance Gain} = \frac{\tilde{V}_{out}}{\tilde{i}_{in}}$$

$$\text{Gain} = \frac{-R_D R_S (g_m r_o + 1)}{R_D + R_S + g_m r_o R_S + r_o}$$

(2)



at saturation point: $\overline{V_{DS}} = \overline{V_{GS}} - \overline{V_{TH}}$

$\overline{V_{DS}} = \overline{V_{out}} - \overline{V_S}$, $\overline{V_{GS}} = 2 - \overline{V_S}$, $\overline{V_{TH}} = 0.7\ V$

$\Rightarrow \overline{V_{out}} - \cancel{\overline{V_S}} = 2 - \cancel{\overline{V_S}} - 0.7 \Rightarrow \overline{V_{OUT}} = 1.3\ V$

According to KCL, the current flows into the source of N-MOS is equal to the current flows out of the source,

Thus, ~~at~~ we have

$$\begin{cases} \dfrac{\overline{V_S}}{R_S} + \overline{I_{IN}} = \dfrac{\overline{V_{DD}} - \overline{V_{OUT}}}{R_D} & (1) \\[2ex] \dfrac{\overline{V_{DD}} - \overline{V_{OUT}}}{R_D} = \mu C_{ox} \dfrac{W}{L} \cdot \dfrac{1}{2} (\overline{V_{GS}} - \overline{V_{TH}})^2 [1 + \lambda \overline{V_{DS}}] & (2) \\[2ex] \overline{V_{GS}} = \overline{V_b} - \overline{V_S} & (3) \\[1ex] \overline{V_{DS}} = \overline{V_{OUT}} - \overline{V_S} & (4) \\[1ex] \mu = 350 \times 10^{-4}\ m^2/Vs,\ C_{ox} = \dfrac{\varepsilon_0 \varepsilon_r}{t_{ox}} = \dfrac{3.9 \times 8.85 \times 10^{-12}}{9 \times 10^{-9}} \\[2ex] \dfrac{W}{L_{effect}} = \dfrac{W}{L_{drawn} - 2L_D} = \dfrac{75 \times 10^{-6}}{(2 - 0.08 \times 2) \times 10^{-6}},\ \lambda = 0.1 \\[2ex] \overline{V_{OUT}} = 1.3V,\ \overline{V_{DD}} = 5V,\ \overline{V_b} = 2V, \end{cases}$$

In equation (2), The only unknown is V_S. so we first solve the equation #2

Bring all the numbers ~~and~~ and equation ①, ④ into equation ③, we have

$$\frac{5-1.3}{1000}=350\times10^{-4}\times\frac{3.5\times8.85\times10^{-12}}{9\times10^{-9}}\times\frac{75\times10^{-6}}{(2-2\times0.08)\times10^{-6}}\times\frac{1}{2}(2-V_s-0.7)^2[1+0.1(1.3-V_s)]$$

$\Rightarrow V_s=0.14V$

$\Rightarrow$ bring $V_s=0.14V$ into equation #①, we have

$$\frac{0.14}{100}+I_{in}=\frac{5-1.3}{1000}\Rightarrow I_{in}=0.0023A$$

Thus, the maximum I_{in} for the MOSFet stays in the saturation region is $0.0023A$.

For calculating the trans-impedance, we have

$$g_m=\frac{\partial I_{ds}}{\partial V_{gs}}=\frac{\partial\left[\mu C_{ox}\frac{W}{L_{effect}}(V_{GS}-V_{TH})^2\frac{1}{2}\cdot(1+\lambda V_{DS})\right]}{\partial V_{gs}}$$

$$=\mu C_{ox}\frac{W}{L_{effect}}(V_{GS}-V_{TH})(1+\lambda V_{DS})$$

$$=350\times10^{-4}\times\frac{3.5\times8.85\times10^{-12}}{9\times10^{-9}}\times\frac{75\times10^{-6}}{(2-2\times0.08)\times10^{-6}}\times\frac{1}{2}\times(2-0.14-0.7)\times[1+0.1\times(1.3-0.14)]$$

$$=3.52\times10^{-3}\ \mho$$

$$r_o=\frac{\partial V_{DS}}{\partial I_D}=\frac{1}{\frac{\partial I_D}{\partial V_{DS}}}=\frac{1}{\lambda\frac{1}{2}\mu C_{ox}\frac{W}{L_{effect}}(V_{GS}-V_{TH})^2}$$

$$=\frac{(1+\lambda V_{DS})}{\lambda I_D}\approx\frac{1}{\lambda I_D}=\frac{1}{\frac{5-1.3}{1000}\times0.1}=2703\,\Omega$$

$$\text{Trans-impedance}=\frac{-R_DR_S(g_mr_o+1)}{R_D+R_S+g_mr_oR_S+r_o}$$

$$=\frac{-1000\times100\times[3.52\times10^{-3}\times2703+1]}{1000+100+2703+[3.52\times10^{-3}\times100\times2703]}=-221$$

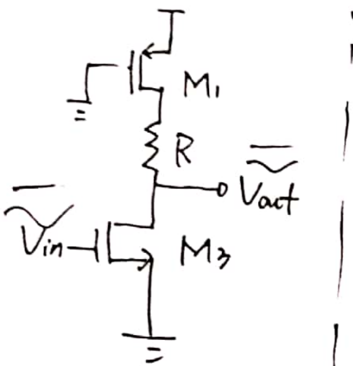
Prob #5

(1) if ~~R~~ $r_{o5} = \infty$, then the M_5 could be considered as an ideal DC current source. ~~Thus we have~~ Since the circuit is fully symmetric and M_5 is ideal DC. we have

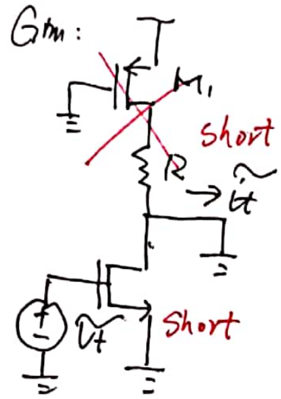
$A_{CM-CM} = 0$

$A_{CM-DM} = 0$

For A_{DM}: use the half-circuit method



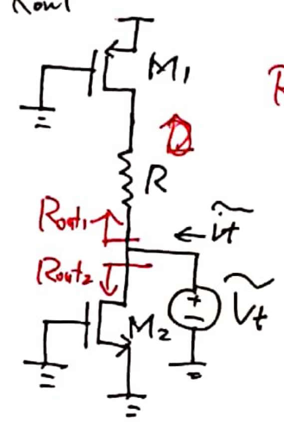
Gm:



$\tilde{i}_t = -g_{m1} \cdot \tilde{V}_t$

$G_m = \dfrac{\tilde{i}_t}{\tilde{V}_t} = -g_{m1}$

Rout



$R_{out} = R_{out1} \parallel R_{out2}$

$= (R + r_{o1}) \parallel r_{o2}$

Thus, $A_v = [r_{o2} \parallel (R + r_{o1})] \cdot (-g_{m1})$

$\tilde{V}_{out1} = -g_{m1} [r_{o2} \parallel (R + r_{o1})] \cdot \tilde{V}_{in1}$

$\tilde{V}_{out2} = -g_{m1} [r_{o2} \parallel (R + r_{o1})] \tilde{V}_{in2}$

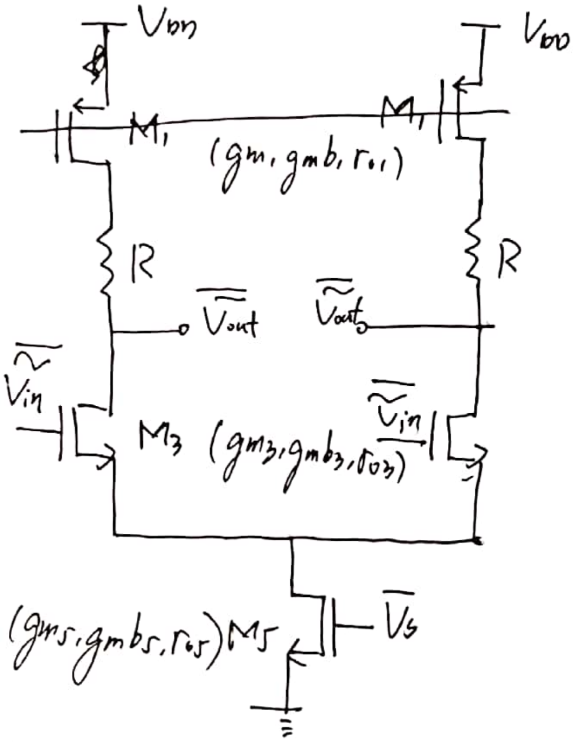
$A_v = \dfrac{\tilde{V}_{out1} - \tilde{V}_{out2}}{\tilde{V}_{in1} - \tilde{V}_{in2}} = -g_{m1} [r_{o2} \parallel (R + r_{o1})]$

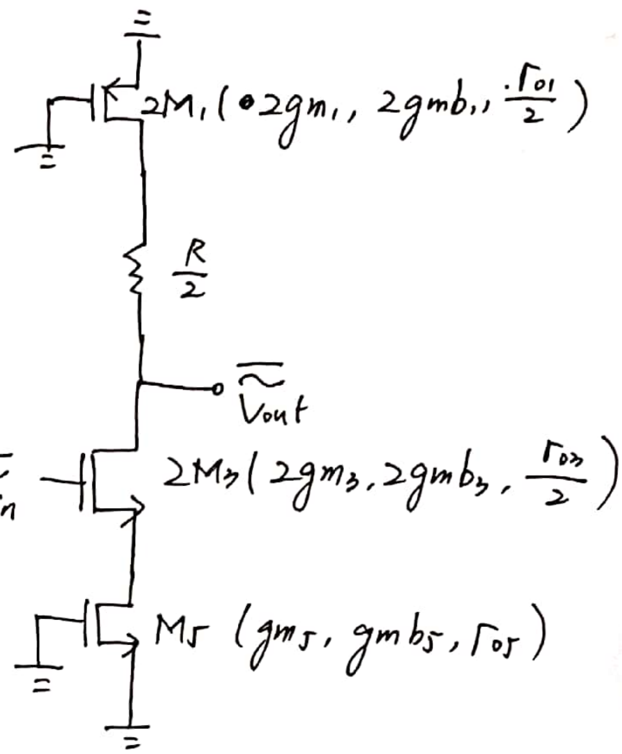
(2) If $r_{o5} \neq \infty$. M_5 is not an ideal ~~DC~~ current source.

$A_{CM-DM} = 0$, since it's fully symtric

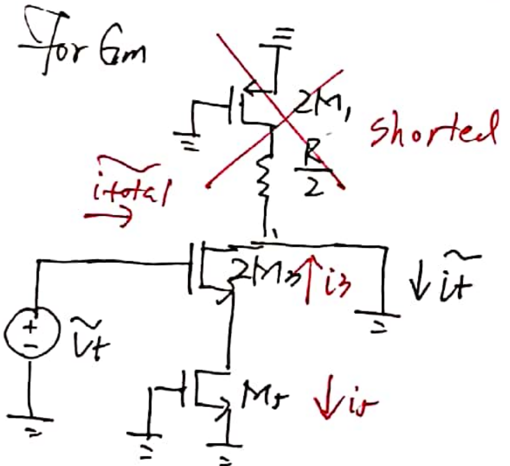
$A_{DM-DM} = -g_{m1} [r_{o2} \parallel (R + r_{o1})]$. since it's not influenced by the current source

For A_{CM}: (at the ~~back~~ next page)





For Gm



$$\tilde{i}_t = \tilde{i}_3$$

$$\tilde{i}_{total} = i_3 + i_5$$

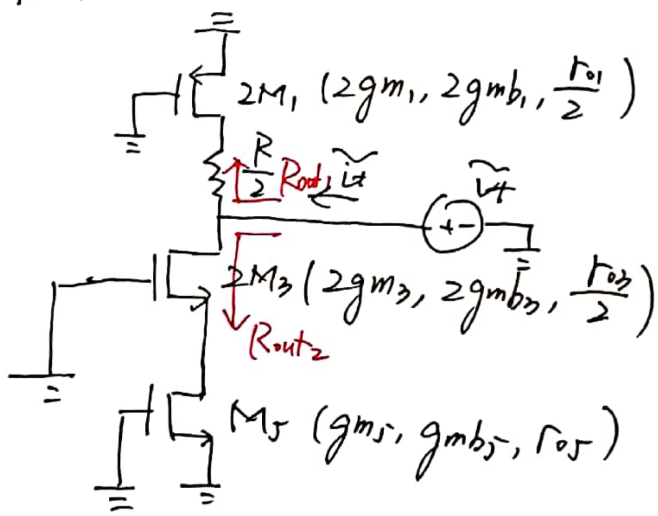
$$\tilde{i}_t = -2g_{m3} \cdot \frac{\tilde{i}_3}{i_3 + i_5} \cdot \tilde{v}_t$$

$$= -2g_{m3} \cdot \frac{Z_5}{Z_5 + Z_3} \cdot \tilde{v}_t$$

$$\frac{\tilde{i}_t}{\tilde{v}_t} = -2g_{m3} \cdot \frac{r_{o3}/2}{r_{o5} + r_{o3}/2 + r_{o5} r_{o3}(g_{m3} + g_{mb3})}$$

$$= \frac{-2g_{m3} r_{o3}}{2r_{o5} + r_{o3} + 2r_{o5} r_{o3}(g_{m3} + g_{mb3})}$$

For Rout



$$R_{out} = R_{out1} \parallel R_{out2}$$

$$= \left(\frac{R}{2} + \frac{r_{o1}}{2}\right) \parallel \left[\frac{r_{o3}}{2} + r_{o5} + (2g_{m3} + 2g_{mb3})\frac{r_{o3}}{2} r_{o5}\right]$$

$$= \frac{R + r_{o1}}{2} \parallel \left[\frac{r_{o3}}{2} + r_{o5} + (g_{m3} + g_{mb3}) r_{o3} r_{o5}\right]$$

$$A_{cm-cm} = G_m R_{out} = \frac{-2g_{m3} r_{o3} \cdot \left\{\frac{R + r_{o1}}{2} \parallel \left[\frac{r_{o3}}{2} + r_{o5} + (g_{m3} + g_{mb3}) r_{o3} r_{o5}\right]\right\}}{r_{o3} + 2r_{o5} + 2r_{o5} r_{o3}(g_{m3} + g_{mb3})}$$