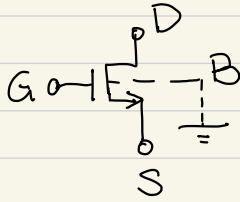
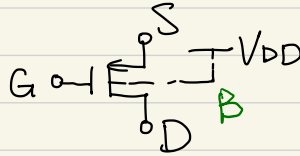


Single-Stage Amplifier RC

Recap from last RC:

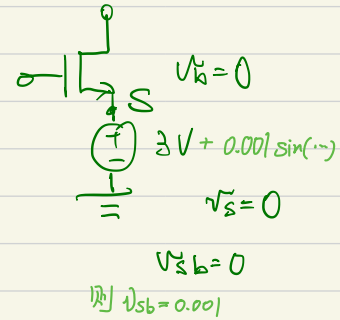
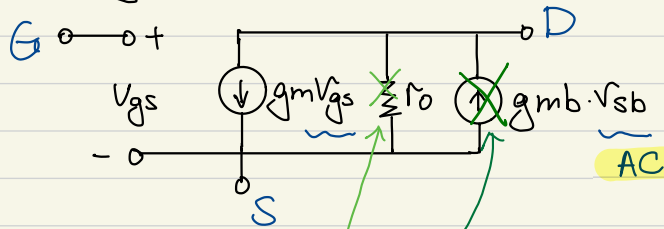


NMOS



PMOS

Small-signal model:



Note: ① No body effect: $1^\circ V_b = 0 \Rightarrow g_{mb} = 0$
 $2^\circ V_{sb} = 0$

② No channel length: $\lambda = 0 \Rightarrow r_o = \infty$

$$r_o = \frac{1}{\pm \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \lambda}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

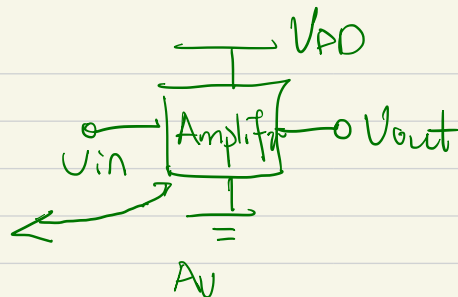
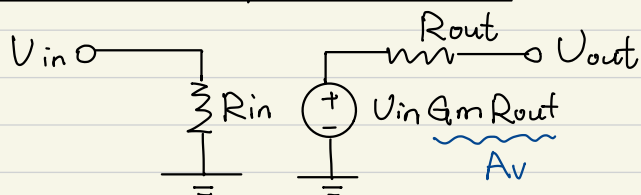
$$= \frac{2 I_D}{V_{GS} - V_{TH}}$$

$L: L_{eff}$

$L': L_{eff} (1 + \lambda V_{DS})$

$$g_{mb} = -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^{\frac{1}{2}} \cdot \frac{1}{\sqrt{12} \phi_F + |V_{SB}|}$$

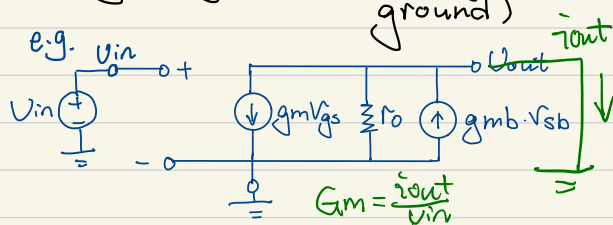
General Amplifier Model:



① To get G_m : $V_{out} = V_{in} G_m R_{out}$

1° short V_{out} to small-signal ground. (i_{out} goes into ground)

2° $G_m = \frac{i_{out}}{V_{in}}$

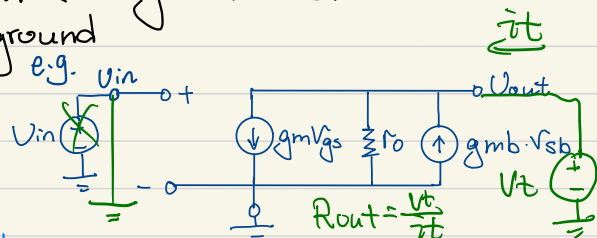


② To get R_{out} :

1° Put V_t at V_{out} (i_t goes into circuit)

2° Short V_{in} to ground

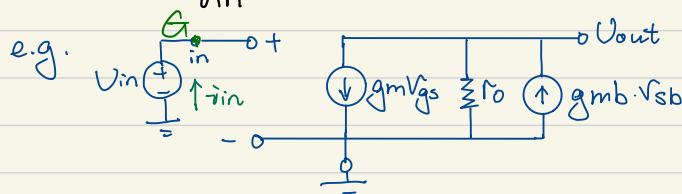
3° $R_{out} = \frac{V_t}{i_t}$



③ To get R_{in} :

1° Denote i_{in} goes into circuit (Do nothing else to current circuit)

2° $R_{in} = \frac{V_{in}}{i_{in}}$

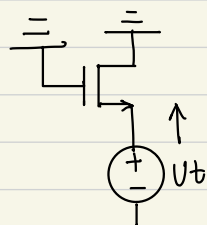


$R_{in} = \frac{V_{in}}{i_{in}} = \infty$

* How to obtain R_{out} for different circuit efficiently?

* 记住以下几个基本模型: (熟练运用!)

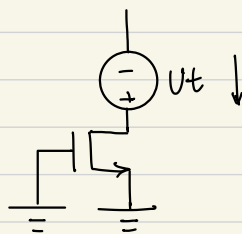
①



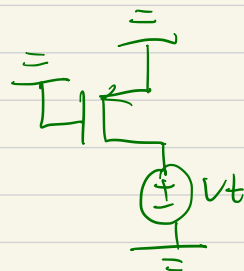
$$R_{out} = \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \parallel r_o$$



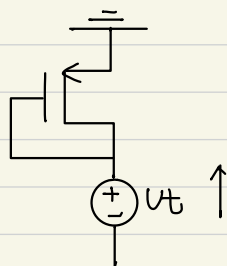
②



$$R_{out} = r_o$$



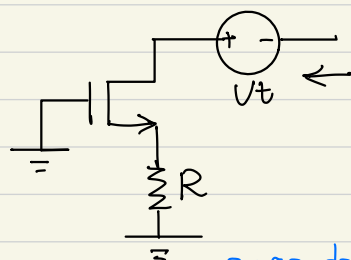
③



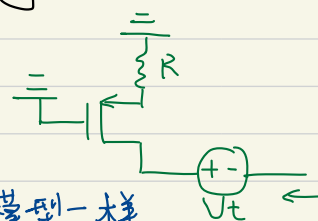
$$R_{out} = \frac{1}{g_m} \parallel r_o$$



④



$$R_{out} = (g_m + g_{mb}) r_o R + r_o + R$$

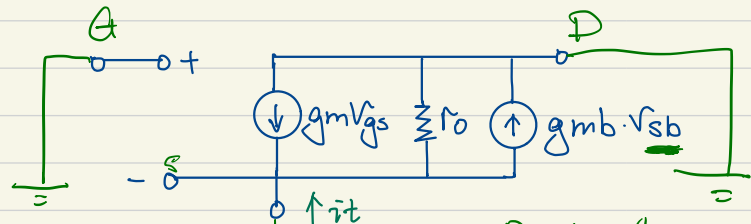
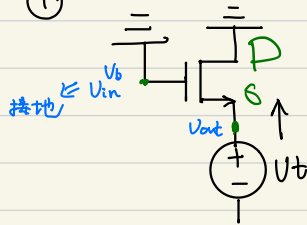


source degeneration: source有电阻

* PMOS 和 NMOS “上下翻转一样” 则模型一样

Analysis:

①



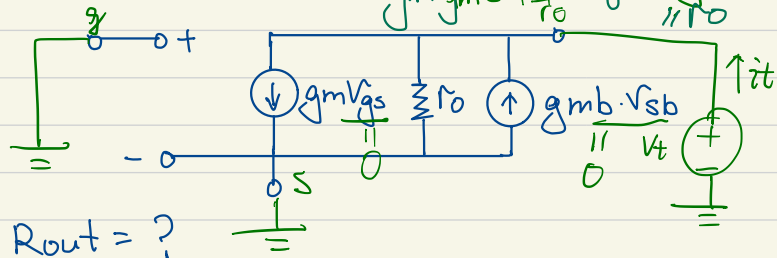
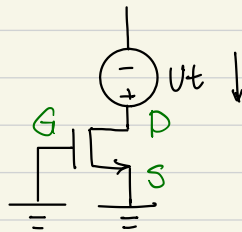
$$R_{out} = ?$$

$$R_{out} = \frac{V_t}{i_t}$$

$$i_t = g_m V_t + \frac{V_t}{r_o} + g_{mb} V_t$$

$$R_{out} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o}} = \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \parallel r_o$$

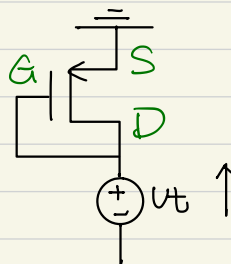
②



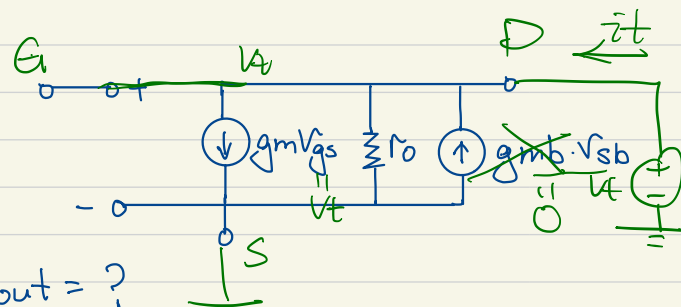
$$R_{out} = ?$$

$$R_{out} = \frac{V_t}{i_t} = \frac{V_t}{\frac{V_t}{r_o}} = r_o$$

③



PMOS 默认接 VDD
 信号中 $V_b = 0$.



$$R_{out} = ?$$

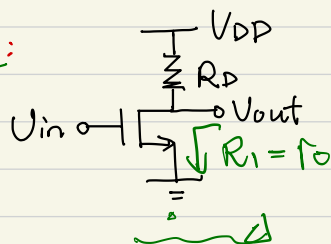
$$R_{out} = \frac{V_t}{i_t} = \frac{V_t}{g_m V_t + \frac{V_t}{r_o}} = \frac{1}{g_m} \parallel r_o$$

区为 Common Source, Source Follower, Common Gate :
看 signal path! (V_{in} 到 V_{out} 的 path)

	V_{in}	V_{out}	
CS	G	D	} $R_{in} = \infty$
SF	G	S	
CG	S	D	

Common Source: Gate 进 Drain 出

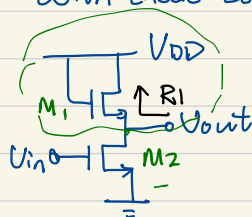
* 记:



$$\begin{aligned} R_{in} &= \infty \\ R_{out} &= R_D // r_o \\ G_m &= -g_m \end{aligned}$$

写的时候注意 V_{sb} .

* 变型: 2° with diode-connected load



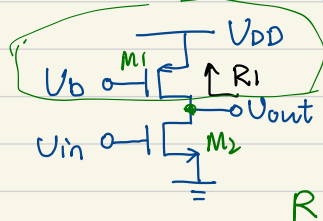
$$R_1 = \frac{1}{g_{m1}} // \frac{1}{g_{mb1}} // r_{o1}$$

$$G_m = ? -g_m$$

$$R_{out} = ? \frac{1}{g_{m1}} \left[\frac{1}{g_{mb1}} \right] \left[\cancel{r_{o1}} // \cancel{r_{o2}} \right]$$

$[\lambda=0] \quad [\gamma=0]$

3° with current source load

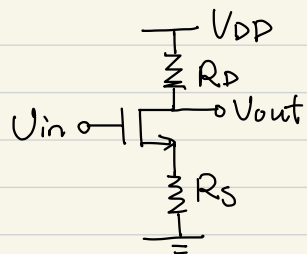


$$G_m = -g_m$$

$$R_{out} = ? r_{o1} // r_{o2}$$

$$R_1 = r_{o1}$$

* 记: 4° with source degradation

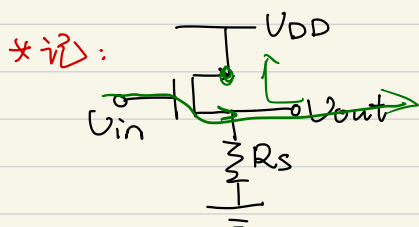


$$R_{in} = \infty$$

$$* G_m = \frac{-g_m r_o}{R_s + r_o + (g_m + g_{mb}) r_o R_s}$$

$$R_{out} = [R_s + r_o + (g_m + g_{mb}) r_o R_s] \parallel R_D$$

Source Follower: Gate in source out

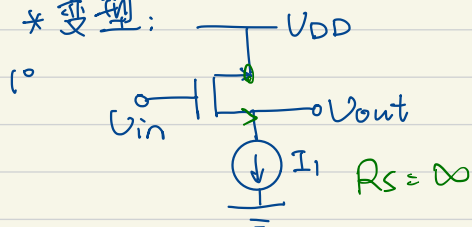


$$R_{in} = \infty$$

$$G_m = g_m$$

$$R_{out} = r_o \parallel R_s \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}}$$

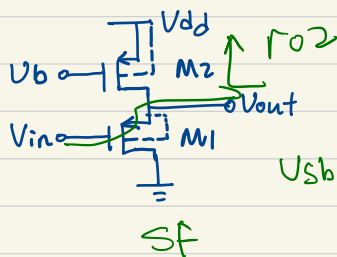
* 变型:



$$G_m = ? \quad g_m$$

$$R_{out} = ? \quad r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}}$$

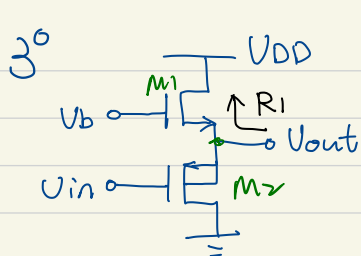
2° with current source load



$$R_{out} = ? \quad r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m1}}$$

$$V_{sb} = 0 \Rightarrow g_{mb} = 0$$

Sf

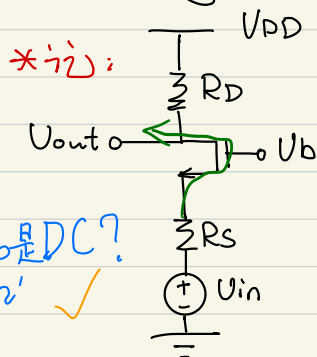


$$R_1 = \frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{o1}$$

$$R_{out} = ? \quad \left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{o1} \right) \parallel r_{o2} \parallel \frac{1}{g_{m2}} \quad \text{"Rs"}$$

Common-gate:

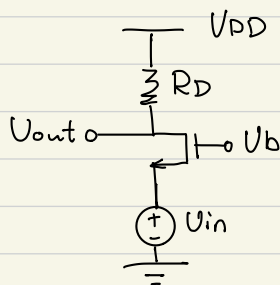
Source \uparrow Drain \uparrow



$$A_m = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_s + (g_m + g_{mb})r_o R_s}$$

$$R_{out} = R_D \parallel [r_o + R_s + (g_m + g_{mb})r_o R_s]$$

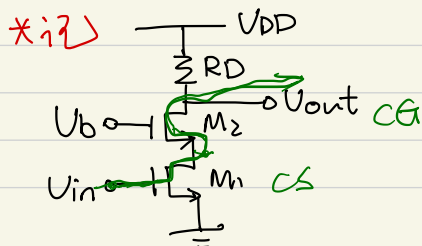
$$R_{in} = \frac{1 + R_s(g_m + g_{mb}) + \frac{1}{r_o}(R_s + R_D)}{g_m + g_{mb} + \frac{1}{r_o}}$$



$$* R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$$

P133?

Cascode CS + CG:



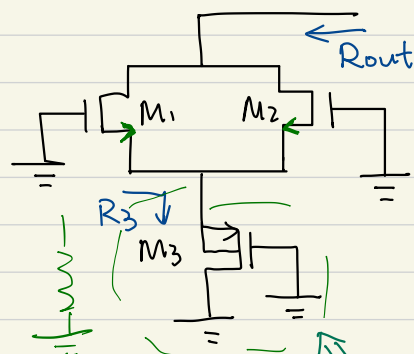
$$A_m = -g_{m1} \frac{r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}})}$$

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o1}r_{o2}] \parallel R_D$$

$$R_{in} = \infty$$

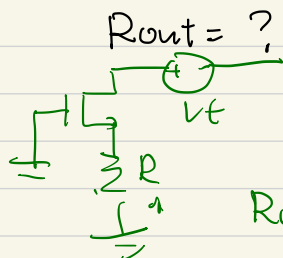
并联 MOS :

e.g.



* 两个 MOS 并联:

$$\begin{aligned} g_m &= g_{m1} + g_{m2} \\ g_{mb} &= g_{mb1} + g_{mb2} \\ r_o &= r_{o1} // r_{o2} \end{aligned}$$

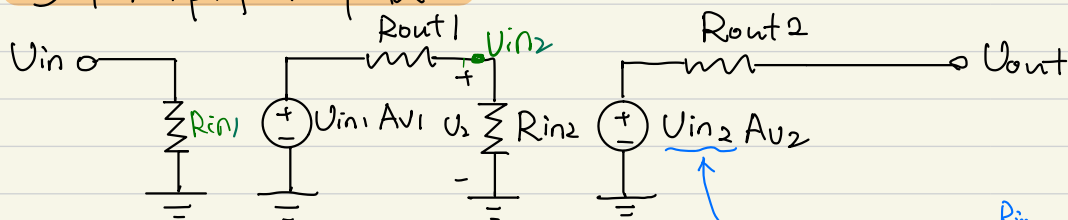


$$R_3 = \frac{1}{g_{m3}} // r_{o3}$$

$$R_{out} = (g_m + g_{mb}) r_o R + r_o + R$$

↑
 R_3

多个 Amplifier 串联:



$$A_v = \frac{V_{out}}{V_{in}} = A_{v1} \cdot \frac{R_{in2}}{R_{out1} + R_{in2}} \cdot A_{v2}$$

$$V_2 = V_{in1} \cdot A_{v1} \cdot \frac{R_{in2}}{R_{out1} + R_{in2}}$$

$$A_{v2} = \frac{V_{out}}{V_2}$$

其中 $A_{v1} = G_{m1} \cdot R_{out1}$

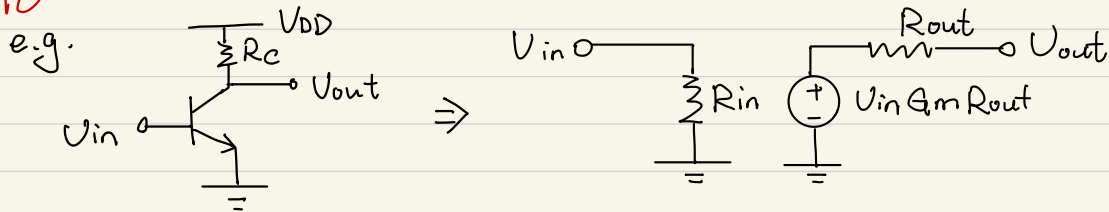
$$A_{v2} = G_{m2} \cdot R_{out2}$$

假设 100 个 Amplifier 连在一起, R_{in} , G_m , R_{out} 均相等

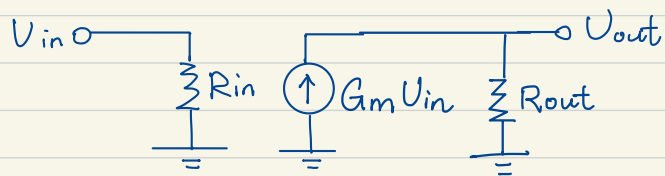
$$\begin{aligned}
 A_V &= A_{V1} \cdot \frac{R_{in}}{R_{out} + R_{in}} \cdot A_{V2} \cdot \frac{R_{in}}{R_{out} + R_{in}} \cdots A_{V100} \\
 &= G_m^{100} \frac{R_{out}^{100} \cdot R_{in}^{99}}{(R_{out} + R_{in})^{99}} \\
 &= G_m^{100} (R_{out} \parallel R_{in})^{99} R_{out}
 \end{aligned}$$

★ If $R_{in} = \infty$, $A_V = A_{V1} \cdot A_{V2} \cdots A_{V100}$

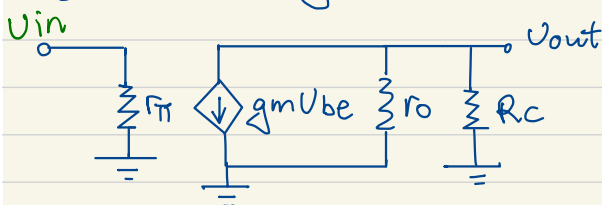
* BJT Amplifier
记



equivalent general amplifier model:



BJT's small-signal model:

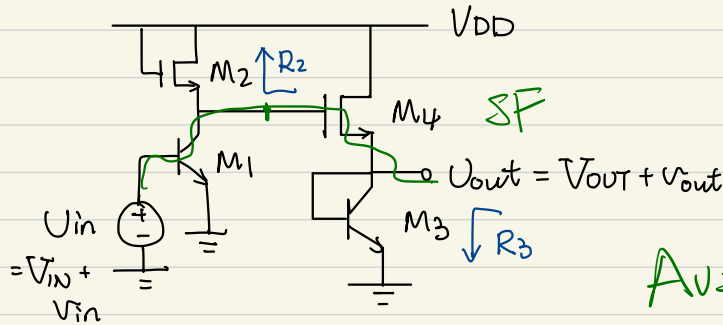


★

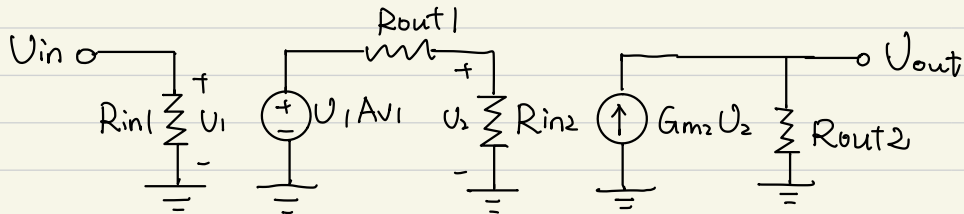
$$\begin{aligned}
 R_{in} &= r_{\pi} \\
 R_{out} &= r_o \parallel R_c \\
 G_m &= -g_m
 \end{aligned}$$

M_2 : Rout

Ex. (2020SU Final Exam)



$$A_v = A_{v1} \cdot \frac{R_{in2}}{R_{out1} + R_{in2}} \cdot A_{v2}$$



Derive analytical expressions for R_{in1} , R_{out1} , A_{v1} , R_{in2} , R_{out2} , G_{m2}

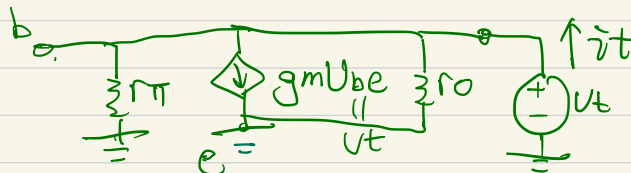
$$R_2 = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$

$$R_{out1} = r_{o1} \parallel R_2 = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}}$$

$$R_{in1} = r_{\pi 1}$$

$$G_{m1} = -g_{m1}$$

$$A_{v1} = G_{m1} \cdot R_{out1} = -g_{m1} (r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}})$$



$$R_3 = \frac{V_t}{i_t}$$

$$i_t = \frac{V_t}{r_{\pi}} + \frac{V_t}{r_o} + g_m V_t$$

$$R_3 = \frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{\pi 3}$$

SF: $R_{in2} = \infty$

$$G_{m2} = g_{m4}$$

$$R_{out2} = R_3 \parallel r_{o4} \parallel \frac{1}{g_{m4}} \parallel \frac{1}{g_{mb4}}$$

Solution:

① 1st stage (M_1)

$$R_2 = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$

$$R_{in1} = r_{\pi1}$$

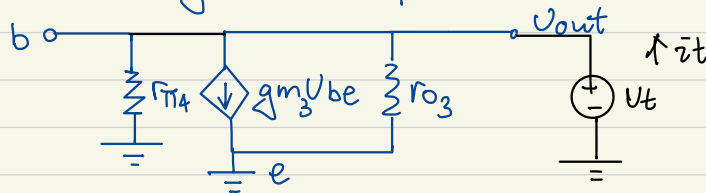
$$R_{out1} = r_{o1} \parallel R_2 = r_{o1} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$

$$G_{m1} = -g_{m1}$$

$$A_{v1} = G_{m1} R_{out1} = -g_{m1} \cdot (r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}})$$

② 2nd stage (M_4)

1° R_3 用 small-signal model 算 (沒有模型可以套):



$$R_3 = \frac{V_t}{i_t} \quad i_t = g_{m3} V_t + \frac{V_t}{r_{\pi3}} + \frac{V_t}{r_{o3}}$$

$$R_3 = \frac{1}{g_{m3}} \parallel r_{\pi3} \parallel r_{o3}$$

2° M_4 is Source Follower

$$R_{in2} = \infty$$

$$G_{m2} = g_{m4}$$

$$R_{out2} = r_{o4} \parallel \frac{1}{g_{m4}} \parallel \frac{1}{g_{mb4}} \parallel R_3$$

$$= r_{o4} \parallel \frac{1}{g_{m4}} \parallel \frac{1}{g_{mb4}} \parallel \frac{1}{g_{m3}} \parallel r_{\pi3} \parallel r_{o3}$$

$$A_v = ?$$