泰勒公式

$x \to 0$:

$\sin x = x - \frac{1}{6}x^3 + o(x^3)$

$\arcsin x = x + \frac{1}{6}x^3 + o(x^3)$

$\tan x = x + \frac{1}{3}x^3 + o(x^3)$

$\arctan x = x - \frac{1}{3}x^3 + o(x^3)$

$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$

$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + o(x^2)$ $\Rightarrow \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$

$\sin x - \tan x \sim -\frac{1}{2}x^3 \quad (x \to 0)$

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 \cdots$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + o(x^3)$

展开原则

1) $\frac{A}{B}$ 型 $\left(A \cdot B = \frac{A}{\frac{1}{B}}\right)$ 上下同阶

若分母 ( ) 是 x^k 则分子展开至 x^k.

$\lim\limits_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x} - 2}{x^2}$ $\left(\frac{0}{0}\right)$

$x \to 0$ $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$

$(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$ 后略.

2) $A-B$ 型 $(A+B = A-(-B))$ 幂次最低原则

将 A、B 分别展开至系数不相等的 x 的最低次幂为止.

已知 $x \to 0$ 时 $\cos x - e^{-\frac{x^2}{2}}$ 与 cx^k 为等价无穷小量. 求 c、k.

$x \to 0$ $\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$

$e^{-\frac{x^2}{2}} = 1 - \frac{x^2}{2} + \frac{1}{8}x^4$

$\cos x - e^{-\frac{x^2}{2}} = -\frac{1}{12}x^4 + o(x^4) \Rightarrow \cos x - e^{-\frac{x^2}{2}} \sim -\frac{1}{12}x^4$

$\Rightarrow c = -\frac{1}{12}$, $k = 4$.

$f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2 + f'''(a)(x-a)^3$

$$f(x) = \frac{\quad}{0!} + \frac{\quad}{1!}(x-a) + \frac{\quad}{2!}(x-a) + \frac{\quad}{3!}(x-a) + \cdots$$

$$+ \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$