解 non-homo

$A = (\quad)$ $\quad b(t) = (\quad)$

求 λ，求 $v_1, v_2 \cdots$

$y_1(t) = \cdots \quad y_2(t) = \cdots$

有 $\phi(t) = [y_1(t) \;\; y_2(t)]$ $\quad \det\phi = \checkmark$

求 $\phi^{-1}(t)$ $\quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \to A^{-1} = \frac{1}{|A|}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$ $\quad$ k列元素是 A k行元素代数余子式

$y(t) = \phi(t)\left(c + \int \phi^{-1}(t) b(t)\, dt\right)$ $\quad$ < 求 $\int \phi^{-1}(t) b(t)\, dt$，

Laplace. $\quad L[f(t) : t \to s] = \int_0^\infty e^{-st} f(t)\, dt = \bar{f}(s)$

$f(t)$	$\bar{f}(s)$	
1	$\frac{1}{s}$	
e^{at}	$\frac{1}{s-a}$	
t^n	$\frac{n!}{s^{n+1}}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sin at$	$\frac{1}{s^2+a^2}$	
$\cosh t$	$\frac{s}{s^2-a^2}$	
$\sinh t$	$\frac{1}{s^2-a^2}$	
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	
$e^{at}\sin bt$.	$\frac{b}{(s-a)^2+b^2}$	
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	
$H(t-a)$	$\frac{e^{-as}}{s}$	
$H(t-a) f(t-a)$	$e^{-as}\bar{f}(s)$	2 shift
$e^{at} f(t)$	$\bar{f}(s-a)$	1 shift
$f(at)$	$\frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$	
$\int_0^t f(t-w) g(w)\, dw$	$\bar{f}(s) \cdot \bar{g}(s)$	
$\delta(t-a)$	e^{-as}	
$f^{n}(t)$	$s^n \bar{f}(s) - \sum_{j=0}^{n-1} s^{n-j-1} f^{(j)}(0)$	

Linearity: $\quad L[a f(t) + b g(t) : t \to s] = a\bar{f}(s) + b\bar{g}(s)$

应用： $f'(t) \xrightarrow{L} s\bar{f}(s) - f(0)$

$f''(t) \xrightarrow{L} s^2 f(s) - s f(0) - f'(0)$

卷积 $(f * g)(t) = \int_0^{\infty} f(t-u)\, g(u)\, du$

$L[(f * g)(t) : t \to s] = \bar{f}(s)\, \bar{g}(s)$

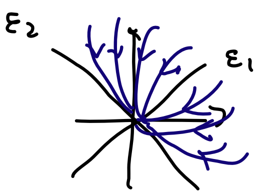
unit step function： $H(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geqslant a. \end{cases}$

unit impulse function $\delta_\varepsilon(t-a) = \begin{cases} \frac{1}{2\varepsilon} & |t-a| < \varepsilon \\ 0 & |t-a| \geqslant \varepsilon. \end{cases}$

$\int_{-\infty}^{+\infty} \delta(t-a)\, f(t) = f(a)$

- case 1 实 不同 ei. 同号.

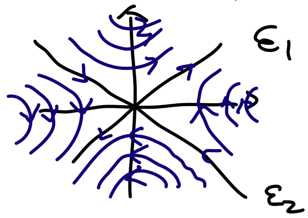
  $r_1 < r_2 < 0.$



node. 渐近 |r| 小的 vector.

- case 2 实 异号 ei.

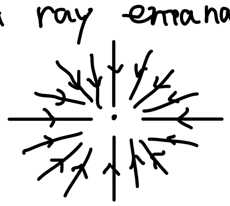
  $r_2 < 0 < r_1$

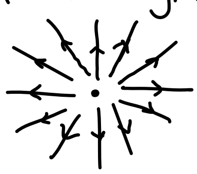


- case 3 等 ei

  ① 2 independent e.v.

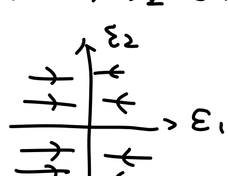
A ray emanating from origin： $\lambda < 0$ toward， $\lambda > 0$ away



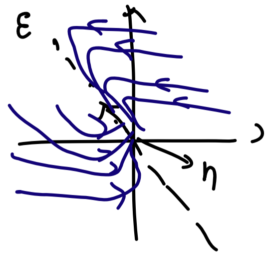


proper node.

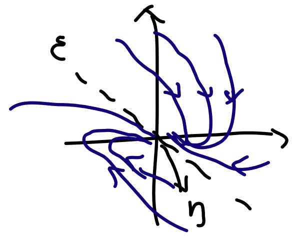
$\lambda_1 \neq 0, \lambda_2 = 0.$



② 1 independent.

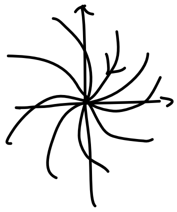


η到ε



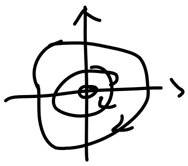
case 4. complex ei.

实部非0 $x=\lambda\pm i\mu$, spiral sink



$\lambda<0$.

实部为0.



$\mu>0$.

两 $r\leq 0$ $(\lambda\leq 0)$ stable.

有 $r>0$ $(\lambda>0)$ unstable.

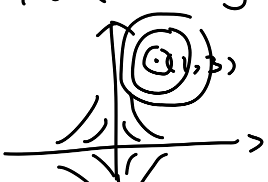
Ø. $\begin{cases} y_1' = \frac{3}{2}y_1 - \frac{1}{2}y_1y_2 = \phi_1 \\ y_2' = -\frac{1}{2}y_2 + \frac{1}{2}y_1y_2 = \phi_2 \end{cases}$

$\begin{cases} \frac{3}{2}y_1 - \frac{1}{2}y_1y_2 = 0 \\ -\frac{1}{2}y_2 + \frac{1}{2}y_1y_2 = 0. \end{cases} \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = 0 \end{cases} \quad \begin{cases} y_1 = 1 \\ y_2 = 3 \end{cases}$

Jacobian $J = \begin{vmatrix} \frac{\partial\phi_1}{\partial y_1} & \frac{\partial\phi_1}{\partial y_2} \\ \frac{\partial\phi_2}{\partial y_1} & \frac{\partial\phi_2}{\partial y_2} \end{vmatrix} = \begin{pmatrix} \frac{3}{2}-\frac{1}{2}y_2 & -\frac{1}{2}y_1 \\ \frac{1}{2}y_2 & -\frac{1}{2}+\frac{1}{2}y_1 \end{pmatrix}$

$(0,0)$ 代入J $\Rightarrow J = \begin{pmatrix} \frac{3}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \Rightarrow \lambda = -\frac{1}{2}, \frac{3}{2}$ saddle.

$(1,3)$ 代入 $J = \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{3}{2} & 0 \end{pmatrix} \Rightarrow \lambda = \pm\frac{\sqrt{3}}{2}i$ center



$$\ddot{x} - x + x^2 = 0.$$

变 $\longrightarrow \begin{cases} x' = y \\ y' = x - x^2 \end{cases}$ $\qquad J = \begin{pmatrix} 0 & 1 \\ 1-2x & 0 \end{pmatrix}$

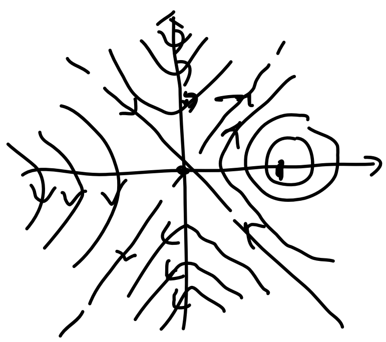
$y=0$, $x=0, 1.$

$(1,0)$ $\quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad \lambda = \pm i.$ $\quad$ circular

$\underline{(0,0)}$ $\quad J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \lambda = \pm 1.$

$\underline{\lambda = 1}$ $\quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $\quad -v_1 + v_2 = 0.$ $\quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -1}$ $\quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $\quad v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$



$$y = c_1 e^{t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$t \to -\infty$

$t \to \infty$