振动

external force: $F(t)$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$kx(t)$: shear force　viscous damping force: $c\dot{x}(t)$

地位移 $x_0(t)$，则　$m\ddot{x}(t) = -k(x(t) - x_0(t)) - c(\dot{x}(t) - \dot{x}_0(t))$

$$y(t) = x(t) - x_0(t)$$

$$\therefore\ m\ddot{y}(t) + c\dot{y}(t) + ky(t) = -m\ddot{x}_0(t)$$

ground acceleration　$-m\ddot{x}_0(t) = F(t)$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

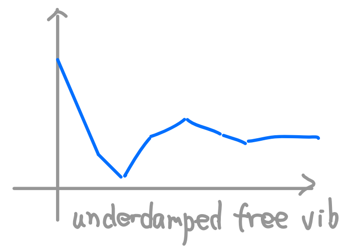
$\frac{k}{m} = \omega_0^2$,　$\frac{c}{m} = 2\varepsilon\omega_0$

$$\therefore\ \ddot{x} + 2\varepsilon\omega_0\dot{x} + \omega_0^2 x = \frac{F(t)}{m}$$

$x(t) = x_c(t) + x_p(t)$　root　$\lambda_{1,2} = \omega_0(-\varepsilon \pm \sqrt{\varepsilon^2 - 1})$

① underdamped system　欠阻尼　$\lambda_{1,2} = \omega_0(-\varepsilon \pm \sqrt{\varepsilon^2 - 1}) = -\varepsilon\omega_0 \pm i\omega_d$

$\omega_d = \omega_0\sqrt{1 - \varepsilon^2}$　(damped natural circular frequency)



$$x_c = e^{-\varepsilon\omega_0 t}(A\cos\omega_d t + B\sin\omega_d t)$$

determine A、B　with i.c.　$x(0) - x_0$, $\dot{x}(0) = V_0$

$$\Rightarrow x_c(t) = e^{-\varepsilon\omega_0 t}\left(x_0\cos\omega_d t + \frac{V_0 + \varepsilon\omega_0 x_0}{\omega_d}\sin\omega_d t\right),\quad 0 < \varepsilon < 1.$$

[special case] 无阻尼，$\varepsilon = 0$,　$x_c = a\cos(\omega_0 t - \varphi)$　$a = \sqrt{x_0^2 + \left(\frac{V_0}{\omega_0}\right)^2}$, $\varphi = \tan^{-1}\left(\frac{V_0}{\omega_0 x_0}\right)$

受阻振动 damped free vibration　$x_c(t) = a e^{-\varepsilon\omega_0 t}\cos(\omega_d t - \varphi)$

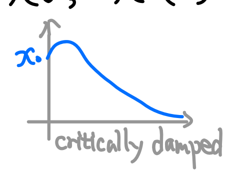
$$a = \sqrt{x_0^2 + \left(\frac{V_0 + \varepsilon x_0\omega_0}{\omega_d}\right)^2},\quad \varphi = \tan^{-1}\left(\frac{V_0 + \varepsilon\omega_0 x_0}{\omega_d x_0}\right)$$

② 临界阻尼

critically damped system

$\varepsilon = 1$.　$x_c(t) = e^{-\omega_0 t}(C_0 + C_1 t)$　with i.c.　$x(0) = x_0$, $\dot{x}(0) = V_0$

$$\Rightarrow x_c(t) = e^{-\omega_0 t}(x_0 + (V_0 + \omega_0 x_0)t)$$



过阻尼

③ over damped　$\varepsilon > 1$.

$$x_c(t) = C_1 e^{-\omega_0(\varepsilon - \sqrt{\varepsilon^2 - 1})t} + C_2 e^{-\omega_0(\varepsilon + \sqrt{\varepsilon^2 - 1})t}$$

- Forced Viberation

$$F(t) = F_0 \sin\Omega t$$

$$x_p(t) = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (2\varepsilon\omega_0\Omega)^2}}\sin(\Omega t - \varphi)$$

where, $\cos\varphi = \dfrac{\omega_0^2 - \Omega^2}{\sqrt{(\omega_0^2-\Omega^2)^2 + (2\varepsilon\omega_0\Omega)^2}}$ $\qquad \sin\varphi = \dfrac{2\varepsilon\omega_0\Omega}{\sqrt{(\omega_0^2-\Omega^2)^2 + (2\varepsilon\omega_0\Omega)^2}}$

$r = \dfrac{\Omega}{\omega_0}$ = frequency ratio.

$k = m\omega_0^2$, $\quad |x_p(t)| = \dfrac{F_0}{k} \dfrac{1}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$

- Resonance

$\varepsilon = 0$. $\quad \Omega = \omega_0$.

$$\ddot{x}_p + \omega_0^2 x_p = \frac{F_0}{m} \sin\omega_0 t$$

$$\Rightarrow x_p = -\frac{F_0 t}{2m\omega_0} \cos\omega_0 t$$