

Vector potential \vec{A} in $d \ll \frac{w}{c} = \frac{L}{2\pi} \ll r$

(4)

$$\vec{A}(r, \theta, t) = \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \frac{q_0 w \sin\left(w\left(t + \frac{r}{c}\right)\right)}{r} \hat{z}$$

$$= \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \underbrace{q_0 w \frac{1}{r} \left(1 + \frac{d \cos \theta}{2r}\right)}_{\approx 0} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}$$

$$= \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \quad q_0 w \frac{1}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}$$

$$= \frac{-\mu_0}{4\pi} \left(z \Big|_{-\frac{d}{2}}^{\frac{d}{2}} \right) \left(\frac{q_0 w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \right) \hat{z}$$

$$= \frac{-\mu_0}{4\pi} \left[\frac{d}{2} + \frac{d}{2} \right] \left(\frac{q_0 w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \right) \hat{z}$$

$$= -\frac{\mu_0}{4\pi} d q_0 \frac{w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}$$

$$\boxed{\vec{A}(r, \theta, t) = -\frac{\mu_0}{4\pi} p_0 \frac{w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}}$$

