$\bar{E}(r,\theta,t)$ at $d \ll \frac{c}{\omega} = \frac{\lambda}{2\pi} \ll r$

$$\bar{E}(r,\theta,t) = -\nabla\phi(r,\theta,t) - \frac{\partial \bar{A}(r,\theta,t)}{\partial t}$$

$$= -\frac{p_0}{4\pi\varepsilon_0}\frac{\omega^2}{c^2}\frac{\cos\theta}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\hat{r} - \left[-\frac{\mu_0}{4\pi} p_0 \frac{\omega^2}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\left(\cos\theta(t)\,\hat{r} - \sin\theta(t)\,\hat{\theta}\right)\right]$$

$$= \frac{p_0\omega^2}{4\pi}\left\{-\frac{1}{\varepsilon_0 c^2}\frac{\cos\theta}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\hat{r} + \frac{\mu_0}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\cos\theta\,\hat{r} - \frac{\mu_0}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\sin\theta\,\hat{\theta}\right\}$$

$$= \frac{p_0\omega^2}{4\pi}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\left\{\left(-\frac{1}{\varepsilon_0 c^2}\frac{\cos\theta}{r} + \frac{\mu_0}{r}\cos\theta\right)\hat{r} - \frac{\mu_0}{r}\sin\theta\,\hat{\theta}\right\}$$

$$= \frac{p_0\omega^2}{4\pi}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\left\{\frac{\cos\theta}{r}\left(-\frac{1}{\varepsilon_0 c^2} + \mu_0\right)\hat{r} - \frac{\mu_0}{r}\sin\theta\,\hat{\theta}\right\}$$

$$= \frac{p_0\omega^2}{4\pi}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\left\{\frac{\cos\theta}{r}\underbrace{\left(-\frac{\cancel{\varepsilon_0}\mu_0}{\cancel{\varepsilon_0}} + \mu_0\right)}_{=0}\hat{r} - \frac{\mu_0}{r}\sin\theta\,\hat{\theta}\right\}$$

$$= \boxed{-\frac{\mu_0}{4\pi} p_0\omega^2 \cos\left(\omega\left(t+\frac{r}{c}\right)\right)\frac{\sin\theta}{r}\,\hat{\theta} = \bar{E}(r,\theta,t)}$$

When $\frac{\partial \bar{A}}{\partial t}$ turnes on, $\bar{E}(r,\theta,t)$ rotates from $\hat{r}$ to $\hat{\theta}$, a 90° rotation in the Π plane!

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