Vector potential $\bar{A}$ in $d \ll \frac{w}{c} = \frac{\lambda}{2\pi} \ll r$

$$r_+ = \sqrt{r^2 + \frac{d^2}{4} + rd\cos\theta} \simeq r\left(1 - \frac{d}{2r}\cos\theta\right)$$

$$\frac{1}{r_+} \simeq \frac{1}{r}\left(1 + \frac{d\cos\theta}{2r}\right)$$

$$\sin\left(w\left(t + \frac{r_+}{c}\right)\right) = \sin\left[w\left(t + \frac{1}{c}r\left(1 - \frac{d}{2r}\cos\theta\right)\right)\right]$$

$$= \sin\left[w\left(t + \frac{1}{c}\left(r - \frac{d}{2}\cos\theta\right)\right)\right]$$

$$= \sin\left[wt + \frac{w}{c}\left(r - \frac{d}{2}\cos\theta\right)\right]$$

$$= \sin\left[w\left(t + \frac{r}{c}\right) - \frac{w}{c}\frac{d}{2}\cos\theta\right]$$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$$= \sin\left(w\left(t + \frac{r}{c}\right)\right)\cos\left(\frac{w}{c}\frac{d}{2}\cos\theta\right) + \cos\left(w\left(t + \frac{r}{c}\right)\right)\sin\left(\frac{w}{c}\frac{d}{2}\cos\theta\right)$$

$d \ll \frac{w}{c} \ll r$

$$\simeq \sin\left(w\left(t + \frac{r}{c}\right)\right) \cdot 1 + \cos\left(w\left(t + \frac{r}{c}\right)\right)\underbrace{\sin\left(\frac{w}{c}\frac{d}{2}\cos\theta\right)}_{\simeq 0}$$

$$\simeq \sin\left(w\left(t + \frac{r}{c}\right)\right)$$