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a) $\bar{E}(r,\Theta,t) = E_\Theta$; $\bar{B}(r,\Theta,t) = B_\varphi$; $E_r = E_\varphi = B_r = B_\Theta = 0$

$$\bar{S}(r,\Theta,t) = \frac{1}{\mu_0}\,\bar{E}(r,\Theta,t)\times\bar{B}(r,\Theta,t) = \frac{1}{\mu_0} E_\Theta B_\varphi\, \hat{\Theta}\times\hat{\varphi} =$$

$$= -\frac{1}{\mu_0}\left(\frac{\mu_0}{4\pi} p_0 \omega^2 \frac{\sin\Theta}{r}\cos\left(\omega\left(t+\frac{r}{c}\right)\right)\right)^2 \frac{1}{c}\,\hat{\Theta}\times\hat{\varphi}$$

$$= \frac{1}{\cancel{\mu_0}} \frac{\mu_0^{\cancel{2}} p_0^2 \omega^4}{16\pi^2} \frac{\sin^2\Theta}{r^2} \cos^2\left(\omega\left(t+\frac{r}{c}\right)\right)\frac{1}{c}\,\hat{r}$$

$\hat{\Theta}\times\hat{\varphi} = -\hat{r}$

$$= \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi^2}\frac{\sin^2\Theta}{r^2}\cos^2\left(\omega\left(t+\frac{r}{c}\right)\right)\hat{r}$$

b) $$\langle \bar{S}(r,\Theta,t)\rangle = \frac{1}{2}\frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi^2}\frac{\sin^2\Theta}{r^2} = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{32\pi^2}\frac{\sin^2\Theta}{r^2}$$

c) $$P = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{32\pi^2}\int_{\Theta=0}^{\pi}\frac{\sin^2\Theta}{\cancel{r^2}}\cancel{r^2}\sin\Theta\, d\Theta \int_{\varphi=0}^{2\pi} d\varphi$$

$$= \frac{\mu_0}{c}\frac{p_0^2\omega^4}{32\pi^2}\int_{\Theta=0}^{\pi}\sin^3\Theta\, d\Theta\left[\varphi\Big|_0^{2\pi}\right]$$

$$= \frac{\mu_0}{c} p_0^2\omega^4 \frac{2\cancel{\pi}}{32\pi^{\cancel{2}}}\left[\left(\frac{\cos 3\Theta}{12} - \frac{3}{4}\cos\Theta\right)\Big|_0^{\pi}\right]$$

$$= \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi}\left[\frac{\cos 3\pi}{12} - \frac{3}{4}\cos\pi - \left(\frac{\cos(0)}{12} - \frac{3}{4}\cos(0)\right)\right]$$

$$= \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi}\left[-\frac{1}{12} - \frac{3}{4}(-1) - \left(\frac{1}{12} - \frac{3}{4}\right)\right]$$

$$= \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi}\left[-\frac{2}{12} + \frac{6}{4}\right] = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi}\left[\frac{-2+18}{12}\right] = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{16\pi}\frac{16}{12} =$$

$$= \frac{\mu_0}{c} \frac{p_0^2 \omega^4}{12\pi}$$

d) $\mu_0 = \frac{1}{\varepsilon_0 c^2}$; $\omega = 2\pi\nu$; $\nu = \frac{1}{\tau}$; $c = \lambda\nu$

$$P = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{12\pi} = \frac{1}{\varepsilon_0 c^2}\frac{1}{c}\frac{p_0^2(2\pi\nu)^4}{12\pi} = \frac{1}{\varepsilon_0}\frac{1}{\lambda^3\nu^3}\frac{p_0^2(2\pi)^4\nu^4}{12\pi} =$$

$$= \frac{p_0^2 (2\pi)^4 \cancel{\nu^4}}{\varepsilon_0\, 2\pi\, 6\, \lambda^3 \cancel{\nu^3}} = \frac{p_0^2(2\pi)^3\nu}{6\varepsilon_0\lambda^3} = \frac{(2\pi)^3}{6}\frac{p_0^2}{\varepsilon_0}\frac{1}{\lambda^3\tau}$$

e) $$[P]_{units} = \frac{C^{\cancel{2}}\cancel{m^{2}}}{\cancel{C}}\, V\cancel{m}\, \frac{1}{\cancel{m^3}\, s} = \frac{CV}{s} = \frac{J}{s} = \text{Watt}$$

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