

Vector potential  $\vec{A}$  in  $d \ll \frac{w}{c} = \frac{L}{2\pi} \ll r$  ④

$$\begin{aligned}
 \vec{A}(r, \theta, t) &= \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \frac{q_0 w \sin\left(w\left(t + \frac{r}{c}\right)\right)}{r} \hat{z} \\
 &= \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \underbrace{q_0 w * \frac{1}{r} * \left(1 + \frac{d \cos \theta}{2r}\right) * \sin\left(w\left(t + \frac{r}{c}\right)\right)}_{\approx 0} \hat{z} \\
 &= \frac{-\mu_0}{4\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz q_0 w \frac{1}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z} \\
 &= \frac{-\mu_0}{4\pi} \left(2 \int_{-\frac{d}{2}}^{\frac{d}{2}} dz\right) \left(\frac{q_0 w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right)\right) \hat{z} \\
 &= -\frac{\mu_0}{4\pi} \left[\frac{d}{2} + \frac{d}{2}\right] \left(\frac{q_0 w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right)\right) \hat{z} \\
 &= -\frac{\mu_0}{4\pi} d q_0 \frac{w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}
 \end{aligned}$$

$$\boxed{\vec{A}(r, \theta, t) = -\frac{\mu_0}{4\pi} \rho_0 \frac{w}{r} \sin\left(w\left(t + \frac{r}{c}\right)\right) \hat{z}}$$

