A CELLULAR AUTOMATON TRAFFIC FLOW MODEL OF SELF-DRIVING AND NON-SELF-DRIVING CARS

1. Summary

A new cellular automata model for traffic flow based on Nagel-Schreckenberg(NS) model is developed and studied in this paper. This new model simulates the traffic flow composed of ordinary cars and self-driving, cooperating cars, examining the influence of self-driving cars upon the current traffic system.

The model integrates multiple-lane changing rules derived from STCA-II model. The rules of interaction between self-driving cars and non-self-driving cars are also introduced in this paper: When close enough, self-driving cars can form a "cluster" and act as a whole. They accelerate and decelerate simultaneously to increase the traffic efficiency.

For simulation purpose, some traffic data in the Greater Seattle area are pre-processed and applied to the model. With careful analysis, the output shows that the self-driving cars generally increase mean velocity of cars and mean traffic flow. And the influence of self-driving cars has upon the traffic flow decreases as the number of lanes or the traffic volume increases.

Contents

2. Introduction

Traffic problems have become one of the major issues of our contemporary society. Since the traffic capacity is hard to enhance in most of the time, governments try to increase the traffic efficiency by studying traffic flow models. As the development of self-driving technologies, self-driving cars bring new challenges and opportunities to the study.

2.1. Background.

In Traffic Index 2016, published by TomTom, Seattle tied for fourth-worst among U.S. cities for overall congestion levels, behind Los Angeles, San Francisco, and New York. Specifically for rush-hour commutes, Seattle had the second-worst congestion in U.S. with a 75 percent increase in travel time. Seattle drivers, who typically spend one hour driving each day, are wasting 148 hours per year due to traffic jam.

Due to recent technology breakthrough, self-driving, cooperating cars become more applicable to real-life scenario, rather than only remain at the research lab. A cooperating car can communicate and exchange data with other cooperating self-driving cars as it decides what to do, under different traffic conditions. So, self-driving, cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads.

Our team has been asked by the Governor of the state of Washington to develop a model to analyze the effects of self-driving, cooperating cars on the roads: Interstates 5, 90, and 405, as well as State Route 520. This model should contain the effects on traffic flow of the number of lanes, peak and/or average traffic volume, and percentage of vehicles using self-driving, cooperating systems.

2.2. Existing Models.

There have been former research models used to describe traffic flow. They are classified by different scopes of measurement: Microscopic modeling, Mesoscopic modeling, and Macroscopic modeling.

The Microscopic models reflect the behaviors of individual drivers interacting with surrounding vehicles. These models typically conclude functions of position, velocity, and acceleration. Microscopic models are typically created using ordinary differential equations, with each vehicle having its own equation. Since these models mostly depend on the behavior of a lead vehicle, they are also termed car-following models.

The Macroscopic models are based on the idea that traffic flow has similar patterns and attribute to the flow of a fluid through a river or pipe system. These models attempt to classify the average behaviors of a traffic system instead of an individual vehicle. Macroscopic flow variables, such as flow, density, speed and occupancy, reflect the average state of the traffic, in contrast to the microscopic traffic flow variables, which focus on individual drivers.

The Mesoscopic models combine properties of both microscopic and macroscopic models to capture the interaction between individual vehicles and the whole system, while simulate vehicles separately.

All three models have one common ancestor: the fundamental relation (or fundamental diagram). After introduction of the fundamental relation in the 1930s, Microscopic and Macroscopic models were introduced simultaneously in the 1950s. The family of Mesoscopic models was then developed a decade later.

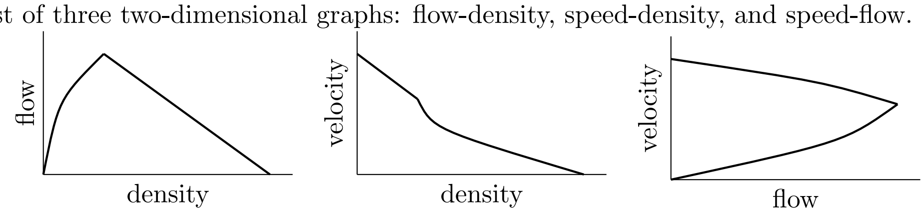
2.2.1. *Fundamental Diagram.*

All Traffic models are based on the assumptions that there is some relation between the distance between vehicles and their speed. Bruce Greenshields firstly studied the relation between distance and velocity, which was then called the fundamental relation (or fundamental diagram).

There are four basic statements of fundamental relation:

- There is a connection between traffic density and vehicle velocity: The more vehicles are on a road, the slower their velocity will be.
- To prevent congestion and to keep traffic flow stable, the number of vehicles entering the control zone has to be smaller or equal to the number of vehicles leaving the zone in the same time.
- At a critical traffic density and a corresponding critical velocity the state of flow will change from stable to unstable.
- If one of the vehicles brakes in unstable flow regime the flow will collapse.

To graphically display the relation between variables, fundamental diagrams consist of three two-dimensional graphs: flow-density, speed-density, and speed-flow.



2.2.2. *Microscopic Models.*

The most basic Microscopic Models used is Gipps Model, developed in 1970s. This model is given by:

$$\ddot{x}_n(t) = S(\frac{\dot{x}_n(t) - \dot{x}_{n-1}(t)}{x_n(t) - x_{n-1}(t)})$$

Where
C: Sensitivity Coefficient;
$x_n(t)$: the location of n^{th} car at time t;
$\dot{x}_n(t)$: the speed of n^{th} car at time t;
$\ddot{x}_n(t)$: the acceleration of n^{th} car at time t.

This model shows that the acceleration of current car should depend on the position and speed of the car in the front, with a sensitivity coefficient.

2.2.3. *Macroscopic Models.*

One of the classic macroscopic models is the LWR model, named after its creators Lighthill, Whitham, and Richards, developed in the 1950s. This model is considered a scalar, time-varying, non-linear, hyperbolic partial differential equation. LWR model is given by:

$$C_j(k+1) = C_j(k) + \frac{\Delta t}{l_j n_j}[q_{in,j}(k) - q_{out,j}(k)]$$

Where
$C_j(k)$:The average traffic density in space section j and in time period k
Δt:The time step

l_j:The length of the section j.
n_j:The number of lanes.
$q_{in,j}(k)$:The inflow in section j and in period k
$q_{out,j}(k)$:The outflow in section j and in period k

A more recent model is the AR model, named after its creators Aw and Rascle. There has been some criticism about LWR model that LWR model, like the fluid models show that disturbances can travel in all directions the same way. However, for traffic flow that is moving forward the driver behavior should be affected by what happens in the front and not in the back. AR model attempts to move away from a fluid-flow based model, and is given by:

$$p_t + (pv)_x = 0$$

$$(v + P(p))_t + v(v + P(p))_x = 0$$

2.2.4. *Mesoscopic Models.*
Prigogine and Herman have proposed the Boltzmann equation for the traffic flow, based on the idea of gas-kinetic, which is a classic mesoscopic model. The kinetic theory treats the vehicles as gas particles.

$$\frac{\partial f(x,v,t)}{\partial t} + v\frac{\partial f(x,v,t)}{\partial x} = -\frac{f(x,v,t) - \rho(x,t)F_{des}(v)}{\tau_{rel}} + (\frac{\partial f(x,v,t)}{\partial t})_{int}$$

$$(\frac{\partial f(x,v,t)}{\partial t})_{int} = \int_{w>v} dw[1 - \hat{p}(\rho)]|w - v|f(x,w,t)f(x,v,t)$$
$$- \int_{w<v} dw[1 - \hat{p}(\rho)]|v - w|f(x,v,t)f(x,w,t)$$

f(x,v,t):Velocity distribution function
τ_{rel}:Relaxation time
$F_{des}(v)$:Desired velocity distribution

2.2.5. *Cellular Automaton.*
Among different types of models mentioned above, we choose to use a Microscopic model, Cellular Automaton to fit into our analysis. Cellular Automaton was first derived by Ulam and von Neumann to calculate liquid motion in the late 1950s. It was originally introduced to consider a liquid as a group of discrete units and calculate the motion of each based on its neighbors behaviors. Until 1990s, Nagel and Schreckenberg conducted a research using Cellular Automaton model for traffics simulation, which was later called NS model.

A cellular automaton consists of a regular grid of cells, each in one of a finite number of states, such as on and off (in traffic flow, are vehicles and space). The fundamental concept is that the state of a chosen cell is determined by its neighborhood. For example, we use Rule 184, a one-dimensional binary cellular automaton rule, to describe traffic flow in a single lane. In this model, vehicles move in a single direction, stopping and starting depending on the cars in front of them. So, vehicles move when there is a space in front, and stop when there is a car in front.

3. Assumptions

3.1. **General Assumptions.**

a. Environment Conditions: Except for extreme weather, such as blizzard and tornado, slight weather changes will not influence our model significantly. Therefore, we consider our model under normal weather conditions.
b. Road Conditions: We assume that there is no construction undergoing on the road, and no obstacles on the way.
c. Types of Vehicles: The Cellular Automaton models divide a road into a series of regular grid of cells. Based on Cellular Automaton, our model considers each car has the same length, and occupies exactly one cell.
d. Car Accidents: No car accident will be considered in our model.

3.2. **Assumptions of Self-driving Cars.**

We make the following assumptions about self-driving, cooperating cars:

a. All self-driving cars fulfill the following cooperation rules with other self-driving cars.
b. All self-driving cars fulfill the following avoidance rules with other non-self-driving cars.
c. The behavior of self-driving cars are deterministic. In other words, their behaviors fulfill the above rules rigorously.

3.2.1. *Cooperating Rules.*

- A self-driving car will not surpass another self-driving car in front of it on the same lane. Instead, assuming that the safe distance between two cars on the same lane is d_{safe}, if car A is in front of car B and car B is v_δ faster than car A, then car B will slow down to the same speed as car A. We say that car A and car B form a cluster when they are on the same lane at the same speed with a distance less than $d_{cluster}$. Note that a single self-driving car is also a cluster, therefore two clusters are able to be merged.
- All self-driving cars within the same cluster accelerate simultaneously until to the top speed when there is no non-self-driving car right in front of them.

3.2.2. *Avoidance Rules.*

- If there is a non-self-driving car A in front of a self-driving car B that blocks car B, then car B will change a lane if certain conditions are met for the target lane.
- If there is no target lane available, the cluster will slow down to the speed of the car in front of it.

3.3. **Assumptions of Non-self-driving Cars.**

We make the following assumptions about the non-self-driving cars:

a. The behaviors of non-self-driving cars are independent. In other words, there is no cooperation between any two non-self-driving cars.
b. A non-self-driving car is able to surpass the car in front of it when certain conditions are met for the target lane, and make a lane change accordingly (refer to STCA model for multiple lanes). But it is not guaranteed to do so. There is a certain probability for non-self-driving cars to make lane change.

c. A non-self-driving car is possible to slow down at any moment. Therefore, the behaviors of non-self-driving cars are randomized, following a certain probabilistic distribution.

4. Our Model

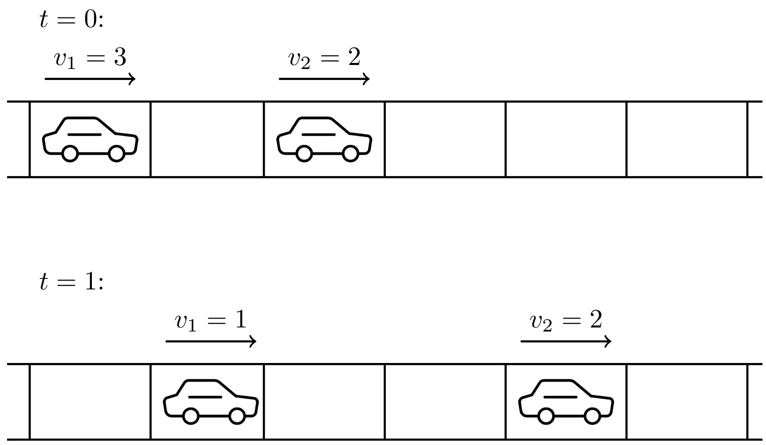
Based on the single-lane traffic cellular automaton model introduced by Negel and Schreckenberg, we developed a customized cellular automaton model that integrates lane change rule and interaction between self-driving, cooperating cars and ordinary cars.

4.1. **The Negel-SchreckenbergNS) Model.** In the Nagel-Schreckenberg model, a road is divided into cells. Each cell is either empty road or contains a single car. Each car has a velocity which is an integer between 0 and V_{max} ($V_{max} = 5$ in the original NS model).

Time is discretized into time steps. In each step, the following four actions are conducted one by one.

- Acceleration: All cars not at the maximum velocity have their velocity increased by one unit. For example, if the velocity is 4 it is increased to 5.
- Slowing down: All cars are checked to see if the distance between it and the car in front (in units of cells) is smaller than its current velocity (which has units of cells per time step). If the distance is smaller than the velocity, the velocity is reduced to the number of empty cells in front of the car to avoid a collision. For example, if the velocity of a car is now 5, but there are only 3 free cells in front of it, with the fourth cell occupied by another car, the car velocity is reduced to 3.
- Randomization: The speed of all cars that have a velocity of at least 1, is now reduced by one unit with a probability of p. For example, if p = 0.5, then if the velocity is 4, it is reduced to 3 50% of the time.
- Car motion: Finally, all cars are moved forward the number of cells equal to their velocity. For example, if the velocity is 3, the car is moved forward 3 cells.

These four actions are repeated many times, as long as is required to study any traffic jams that may form. The model is an example of a cellular automaton. The model is for a single lane where cars cannot pass each other; there is no overtaking.



The NS Model

4.2. **STCA-II Lane Changing Rules.**

The NS model explains "phantom traffic jam" phenomenon. However, it oversimplifies the real world situations: it can only simulate one lane traffic flow; no overtaking allowed. To improve the NS model, lane changing rules need to be defined.

Here we use the STCA-II model proposed by Chowdhury. This model defines the lane changing rules between two lanes:

$$C_n = \begin{cases} 1 - C_n & d_n < min\{V_n + 1, V_{max}\}, d_{n,other} > d_n, \\ & d_{n,back} > 1 + V_{max} - min\{V_n + 1, V_{max}\} \\ C_n & otherwise \end{cases}$$

Where

C_n is the lane the n^{th} car is in (C_n is either 1 or 0);

d_n is the distance between the n^{th} car and the car in front;

$d_{n,other}$ is the distance between the n^{th} car and the car in front in the adjacent lane;

$d_{n,back}$ is the distance between the n^{th} car and the car behind in the adjacent lane.

$d_n < min\{V_n + 1, V_{max}\}$ means the n^{th} car is blocked by the car in front; $d_{n,other} > d_n$ means the n^{th} car would run faster on the other lane; $d_{n,back} > 1 + V_{max} - min\{V_n + 1, V_{max}\}$ means this lane change is safe, assuming that the car behind on the other lane runs at top speed.

4.3. **Multiple Lanes Model with Self-Driving Cars.**

The model we developed can simulate the scenario where self-driving cars and non-self-driving cars travel in multiple lanes. It is a combination of conventional NS Model, STCA-II Lane Changing Rule and the interactions between self-driving cars and non-self-driving cars defined in 2.2 and 2.3.

4.3.1. *Modified Lane Changing Rule.*

The only difference between the lane changing rules in our model and the STCA-II rule is that our rules support left lane change in any number of lanes:

$$C_n = \begin{cases} (C_n - 1)\ mod\ k & d_n < min\{V_n + 1, V_{max}\}, d_{n,other} > d_n, \\ & d_{n,back} > 1 + V_{max} - min\{V_n + 1, V_{max}\} \\ C_n & otherwise \end{cases}$$

In our model, we treat the boundary of lanes as periodic so that all cars that turn left from the left-most lane will automatically appear in the right-most lane. The periodic condition also holds for the two ends of any road segment that we construct for simulation.

5. Simulation and Analysis

5.1. Data Input and Pre-processing.

We use the road network data in Thurston, Pierce, King, and Snohomish counties to run the simulation. We are interested in the following input data for each road segment:

L_0 - the length of a given road segment;

K - the number of lanes of a given road segment;

N - average daily traffic volume;

p_{down} - the probability that a non-self-driving-car slows down for 1 unit at any time;

p_{change} - the probability that a non-self-driving-car make a lane change when lane changing rules are satisfied at any time.

To adapt our model to the data, we need to normalize and rescale the data so that it can directly fit into our model. Below are some intermediate variables that derive from the original data:

v_{max} - the max number of units that each car can run, which is computed as following:

$$v_{max} = V_{max}/L_{car}$$

where L_{car} is the average length of cars in reality.

ρ - the average traffic volume per minutes, which is calculated as following:

$$\rho = N/(L_0 * 24 * 60)$$

The distribution of ρ is shown in Figure 1.

The data is left-skewed so we can take the log of it, resulted in the distribution shown in Figure 2:

For simplicity, we treat the average daily volume being uniformly distributed in each lane.

5.2. Simulation.

We simulate the interaction between self-driving cars and non-self-driving cars using the given data set. The chosen percentage of self-driving cars among all cars are 10%, 50% and 90%. We want to find the change of traffic flow with respect to the number of lanes K and the density of road ρ.

We fix the length the cellular automata grid $L_{fixed} = 100$, and randomly distribute a total of n cars into K lanes (within which the percentage of self-driving cars

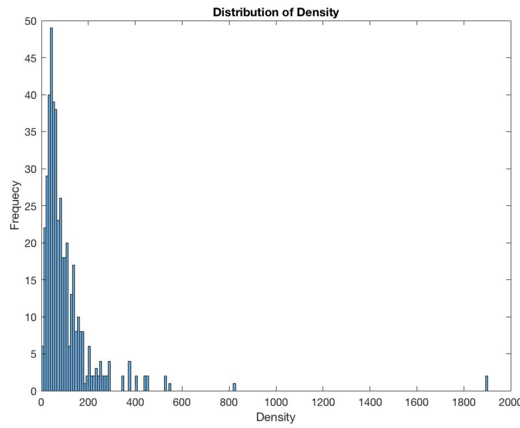


FIGURE 1. Distribution of density from input

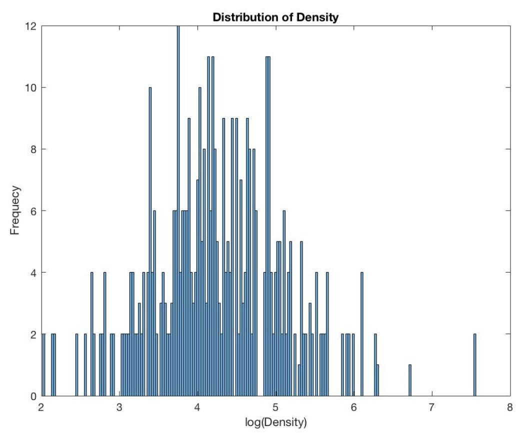


FIGURE 2. Distribution of log of density

and non-self-driving cars are specified by p_s) with velocity of 0. n is the average traffic volume in each round of simulation defined as following:

$$n = \rho * L_{fixed}$$

where ρ is the density of traffic in reality defined in the previous section.

For simplicity, we also arbitrarily determine

$$p_{down} = 0.5$$

and

$$p_{change} = 0.5$$

We simulate t_s steps of car movements within our multiple-lane cellular automata model, which contains two parts:

$$t_s = t_r + t_e$$

where t_s is the total steps, t_e is the stationary steps and t_r is the relaxation steps. Specifically, the program runs t_r additional time steps to form a relaxation interval before reaching the stationary state. The program retains only the last t_e time steps for measurement and analysis. Notice that In each step, a car can move at most v_{max} units within the grid.

5.3. **Output for Analysis.**

The output we are using to evaluate the traffic flow are:

v_{mean} - average velocity for all cars after certain simulation steps, which is calculated as following:

$$v_{mean} = \Sigma_{t=1}^{T} \Sigma_{i=1}^{N} v_{t,i}$$

where $v_{t,i}$ is the velocity of car i at time t;

f_{mean} - mean flow, the average number of for all cars at any time that simulated by our model, is calculated as following:

$$f_{mean} = v_{mean} * \rho$$

5.4. **Visualisation of Results.**

The results of our simulation upon different number of lanes with different percentage of self-driving cars on the given traffic data set is shown as following.

- Figure 3-5 displays results for $K = 2$
- Figure 6-8 displays results for $K = 3$
- Figure 9-11 displays results for $K = 4$
- Figure 12-14 displays results for $K = 5$

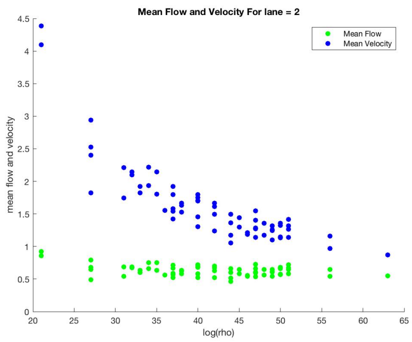


FIGURE 3. $K = 2, p = 0.1$

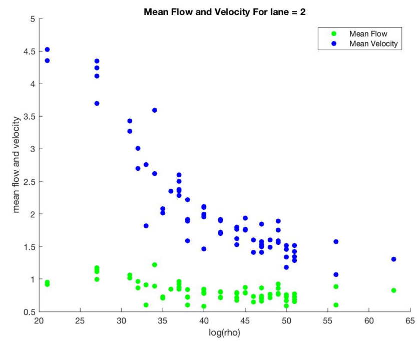


FIGURE 4. $K = 2, p = 0.5$

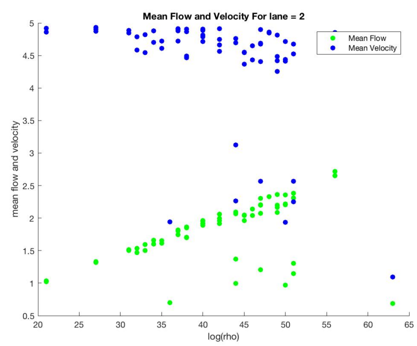


FIGURE 5. $K = 2, p = 0.9$

When $K = 2$, the percentage of self-driving cars has significant positive influence on the mean velocity and mean flow. The influence becomes less significant when the density grows.

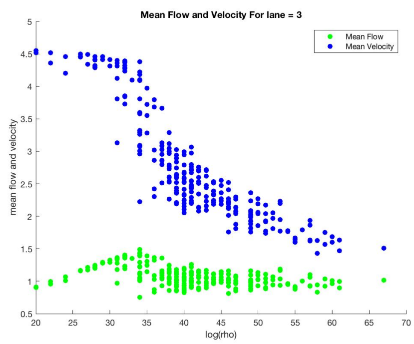


FIGURE 6. $K = 3, p = 0.1$

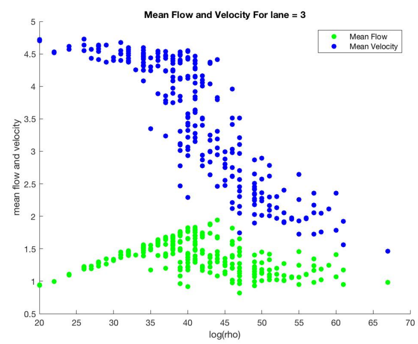


FIGURE 7. $K = 3, p = 0.5$

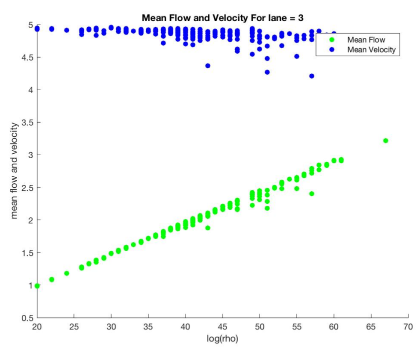


FIGURE 8. $K = 3, p = 0.9$

When $K = 3$, we observe similar pattern as $K = 2$, and that there is a threshold of density that can compromise the influence of self-driving cars for each percentage of self-driving cars.

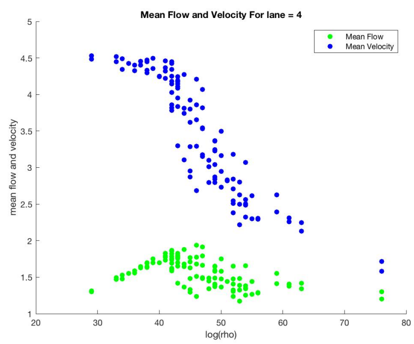


FIGURE 9. $K = 4, p = 0.1$

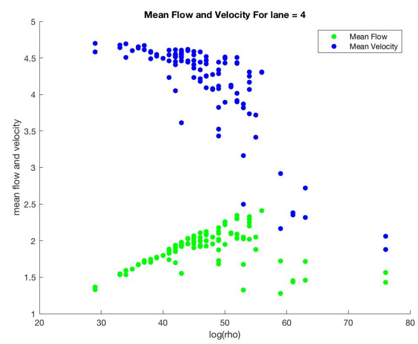


FIGURE 10. $K = 4, p = 0.5$

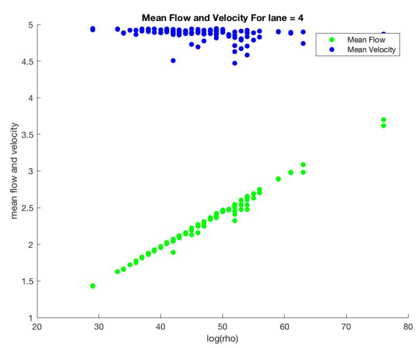


FIGURE 11. $K = 4, p = 0.9$

The $K = 4$ case has similar pattern as the previous cases. But the resulting data seems more disperse, implying that the influence of self-driving cars even less.

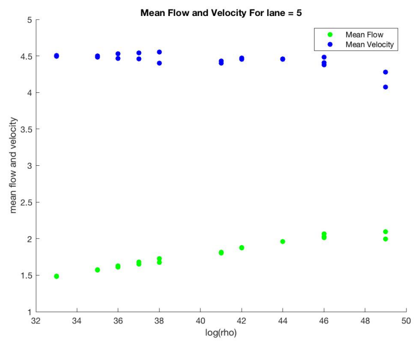


FIGURE 12. $K = 5, p = 0.1$

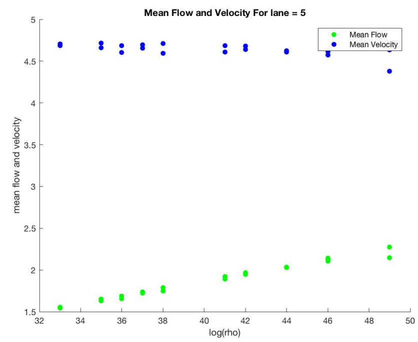


FIGURE 13. $K = 5, p = 0.5$

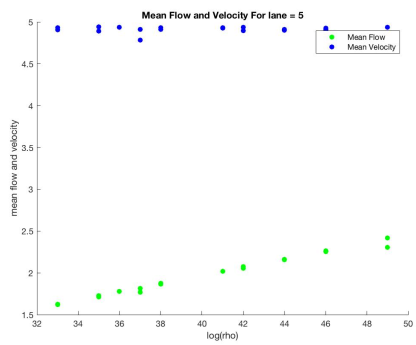


FIGURE 14. $K = 5, p = 0.9$

The $K = 5$ case implies that the the self-driving car has even less influence on both mean velocity and mean flow.

5.5. **Conclusion.**
We can easily make several observations from the plots above:

- Generally, adding more self-driving cars results in higher mean velocity and higher mean flow.
- The effects of self-driving cars have upon the traffic flow decrease as the number of lanes increase.
- When the traffic volume is extremely high, the number of self-driving cars has less influence on the mean flow and mean velocity of traffic.

Based on the simulation results from the multiple-lane cellular automaton model and the predefined interaction rules between self-driving, cooperating cars and ordinary non-self-driving cars, we can derive the following mathematical relation between several essential variables:

$$v_{mean}, f_{mean} \propto pK/\rho$$

Ideally, we hope all of the cars are self-driving cars so that the traffic efficiency would reach the maximum. But it would be implausible to abandon all non-self-driving cars in a short period of time. However, if we only add certain amount of self-driving cars, the traffic mean flow would be enhanced significantly.

For example, if we fix the number of lanes to 3 (mode of the number of lanes), we will have the following relations between the percentage of self-driving cars and average mean traffic flow for all densities:

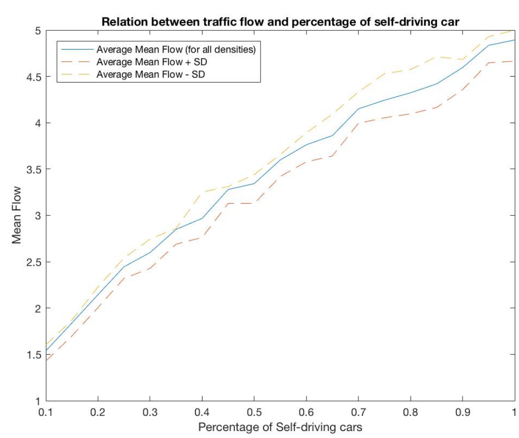


FIGURE 15. Average mean flow for different percentage of self-driving cars

We observe that the change of flow before $p = 0.45$ is slightly more significant than the change of flow after $p = 0.45$. And the standard deviation of the mean flow before $p = 0.45$ is relatively small. Therefore, we propose that 0.45% of self-driving cars is a good starting percentage in reality.

6. Evaluation

6.1. **Strength.** Our model is easy to implement and reproduce. Also, our model emerges the macroscopic behavior from microscopic model, extending several existing models and studies in both traffic fields. which can better capture the picture of traffic flow.

6.2. **Weakness.** The time complexity of our cellular automaton simulation is $O(T * n^2 * L)$, which makes it impossible to simulate a large amount of data. Besides, we can construct a model that takes the time between two consecutive cars entering the road segment into account to make a even more accurate model than the current one.

6.3. **Future Work.** Since we only have 200+ rows of information to explore the relation between flow/velocity, traffic volume, percentage of self-driving cars and lane numbers, we may want to predict the the flow given a set of inputs that doesn't show in the historical data. In this case, we can train a machine learning model, which may help us to find more interesting features about the traffic flow.

7. Letter to the Governor's Office

Dear Governor of the State of Washington:
Its a great honor to get the chance to present you our analysis of how self-driving, cooperating cars will impact the current traffic flow on the roads: Interstates 5, 90, and 405, as well as State Route 520. After defined the cooperating system in self-driving cars, we first adopt the microscopic Negel-Schreckenberg Model to study the four actions of vehicles: Acceleration, Deceleration, Randomization (non-self-driving cars has a certain chance to surpass the in front car), Car Motion (All cars are moved forward the number of cells equal to their velocity), to explain phantom traffic jam phenomenon. Secondly, we use the STCA-II model by Chowdhury to define the lane-change rule between two lanes, and add the scenario of interaction between self-driving and non self-driving cars to form our own model for new traffic flow situation. Thirdly, we programmed our model using Matlab, with input data such as number of lanes, average traffic volume per minute, the length of a given road segment, and etc to simulate interaction between self-driving cars and non-self-driving cars using the given data set. Through the observation on the statistical model: Flow vs Density, Flow vs Volume, we can make conclusion that generally adding more self-driving cars can result in higher mean velocity and higher mean flow, while the effects of self-driving cars upon the traffic flow is more significant when the number of lanes is relatively small. When applying to real life situation, however, it is impossible to have one hundred percent self-driving cars at once on the roads to improve the traffic flow. What we propose is to have an about 45 percent of self-driving car allowance on those roads, which can provide the largest marginal flow mean according to our statistical model.
Sincerely,
COMAP 63600

References

[1] WANG Yong-ming, ZHOU Lei-shan, LU Yong-bo. Lane Changing Rules Based on Cellular Automaton Traffic Flow Model[J]. Journal of Traffic and Transportation Engineering, 2008,21(1),89:93.

[2] J. P. May. A Concise Course in Algebraic Topology. University of Chicago Press. 1999.

[3] Tobias Oekiter, Hubert Partl, Irene Hyna and Elisabeth Schlegl. The Not So Short Introduction to $\LaTeX\,2_\varepsilon$. http://tobi.oetiker.ch/lshort/lshort.pdf.

[4] Femke van Wageningen-Kessels, Hans van Lint, Kees Vuik, Serge Hoogendoorn. Genealogy of Traffic Flow Models. http://ta.twi.tudelft.nl/nw/users/vuik/papers/Wag14LVH.pdf. Springer. 2014.

[5] Sven Maerivoet, Bart De Moor. Traffic Flow Theory. Department of Electrical Engineering ESAT-SCD (SISTA).2008.

[6] Serge Hoogendoorn, Victor Knoop. Traffic Flow Theory and Modeling.

[7] Geekwire by Taylor Soper. Study: Traffic in Seattle still horrible, ranks 2nd-worst in U.S. for evening rush hour congestion. March 22, 2016.

[8] City Metric by Lorna Wilson. Driverless cars plus mathematics could equal the end of traffic jams. November 17, 2016.

[9] David A. Rosenblueth, Carlos Gershenson. A Model of City Traffic Based on Elementary Cellular Automata. Complex Systems Publications.2011

[10] Pushkin Kachroo, Shankar Sastry. Traffic Flow Theory Mathematical Framework University of California Berkeley. http://faculty.unlv.edu/pushkin/TrafficFlowTheory.pdf

[11] Greenshields, B.D. (1933). The Photographic Method of studying Traffic Behaviour, Proceedings of the 13th Annual Meeting of the Highway Research Board.

[12] Greenshields, B.D. (1935). A study of highway capacity, Proceedings Highway Research, Record, Washington Volume 14, pp. 448-477.

[13] Lighthill, M.H., Whitham, G.B.,(1955). On kinematic waves II: A theory of traffic flow on long, crowded roads. Proceedings of The Royal Society of London Ser. A 229, 317-345.

[14] Richards, P.J., (1956), Shock waves on the highway, Chandler, R., R. Herman, and E. Montroll (1958).Traffic dynamics; studies in car-following.Operations Research 6, 165+.

[15] Herman R, Montroll E W, Potts R B, et al (1959). Traffic dynamic: Analysis of Stability in car-following [J]. Operations Research7, 86-106.

[16] Gazis, D., R. Herman, and R. Rothery (1961). Nonlinear follow-the-leader models of traffic flow, Operations Research 9, 545+.

[17] R. Barlovic, L. Santen, A. Schadschneider and M. Schreckenberg (1998), Metastable states in cellular automata for traffic flow, Eur. Phys. J. B 5, 793.

[18] D. Chowdhury, L. Santen and A. Schadschneider (2000). Statistical physics of vehicular traffic and some related systems, Physics Reports 329, 199. Transportation Research Board (2000), Highway Capacity Manual [M], Washington DC.National Research Council.

[19] Persaud B N (1998). Exploration of the Breakdown phenomenon in freeway traffic [J]. Transportation Research Record, 1634: 64-69.

[20] R. Barlovic, L.Santen, A.Schadschneider, M.Schreckenberg, (1998), Metastable states in cellular automata for traffic flow[J]. Eur. Phys.J.B, 5:793-800

[21] Xue Y, Dong L Y, Dai S Q. (2005), Effects of changing order in the update rules on traffic flow. Phys. Rev. E. 2005, 71:026123 1-6.