第二讲 计算流体力学概述

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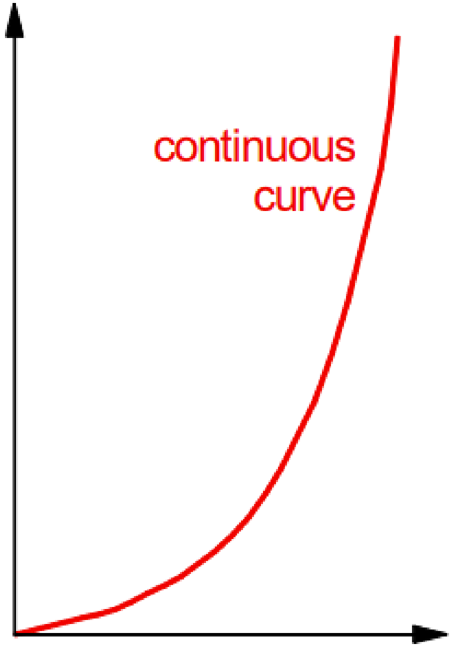
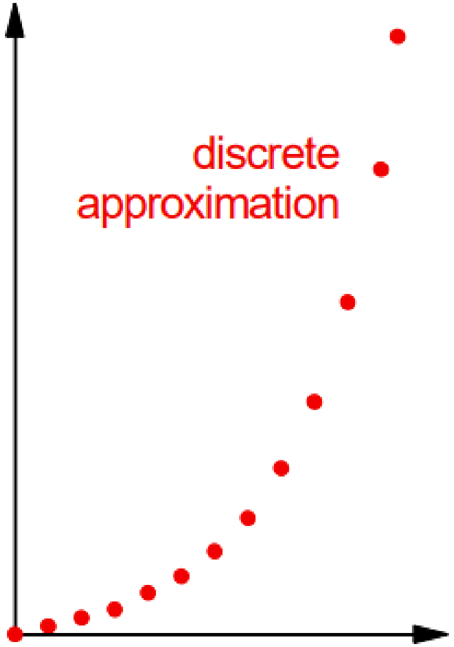
2 计算流体力学概述

计算流体力学（Computational Fluid Dynamics）是采用计算机通过特定的数值方法求解流体力学的控制方程，对流体力学问题进行模拟和分析的学科

求解过程：通过代数方程组来近似微分方程，将这些近似方法适用于空间和时间上的小区域（空间和时间离散），在计算机上进行求解。





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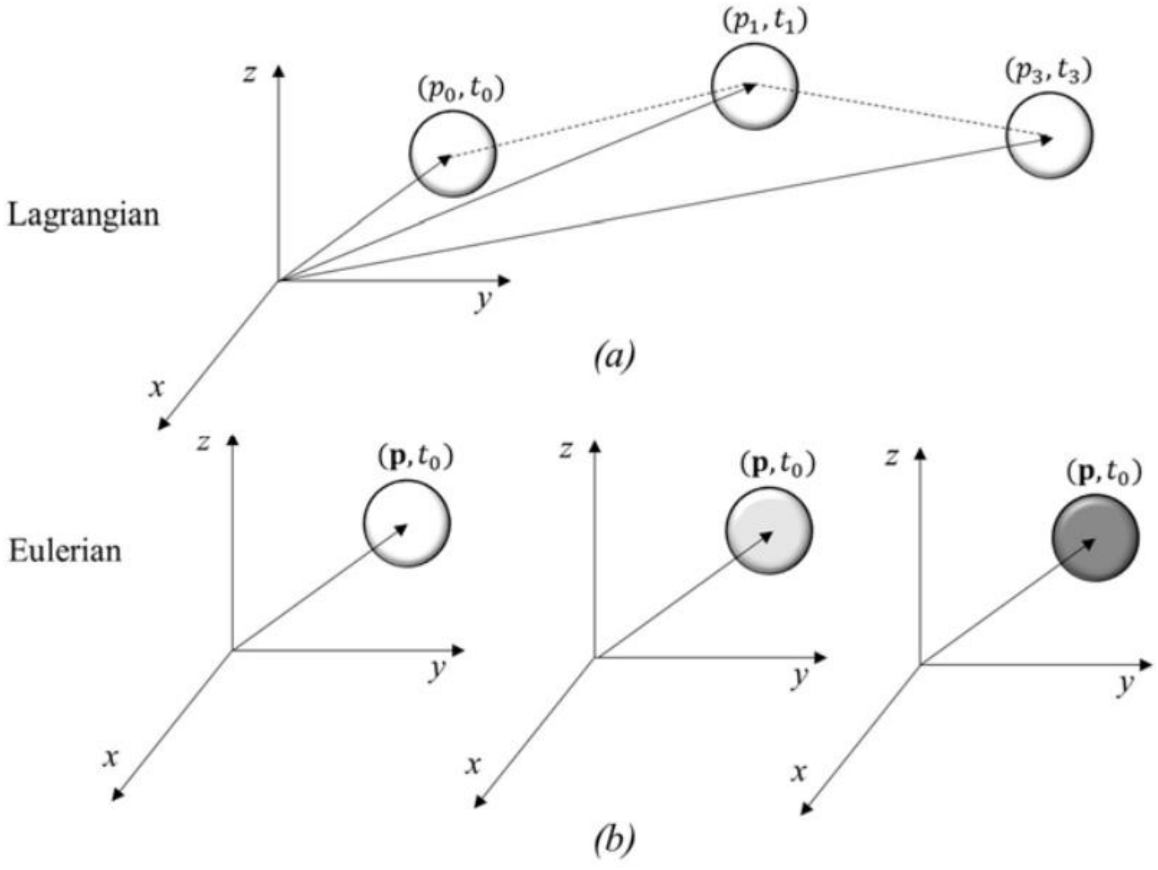
流场中物理量的描述方法

- 欧拉方法：将各个时刻流过空间的任一固定点的流体质点的某些物理量表示为该点位置和时间的函数

- 拉格朗日方法：描述每个流体质点自始至终的运动过程，即位置随时间的变化规律。



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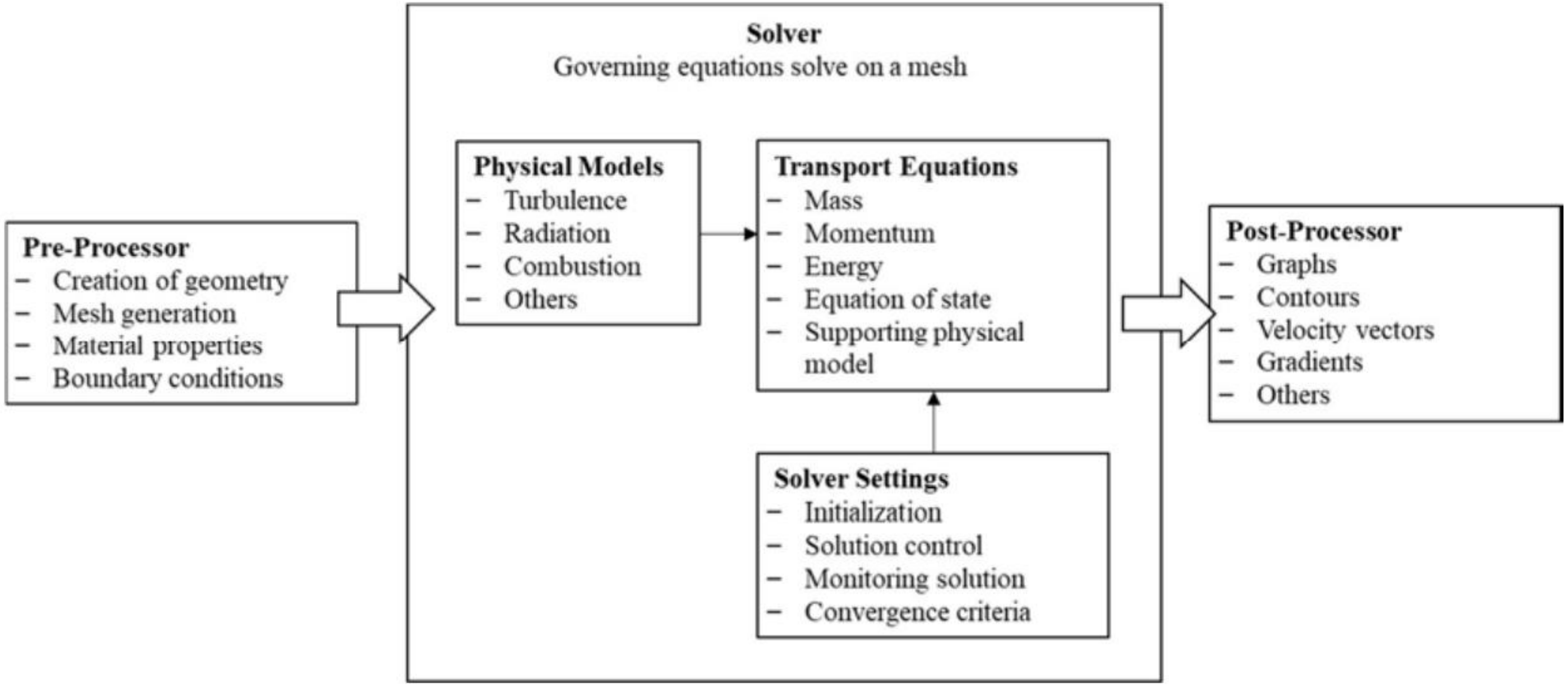
计算流体力学软件包括：

- 前处理：几何建模、网格生成和参数设定
- 求解器：根据给定条件求解流动控制方程
- 后处理：模拟结果的可视化和分析



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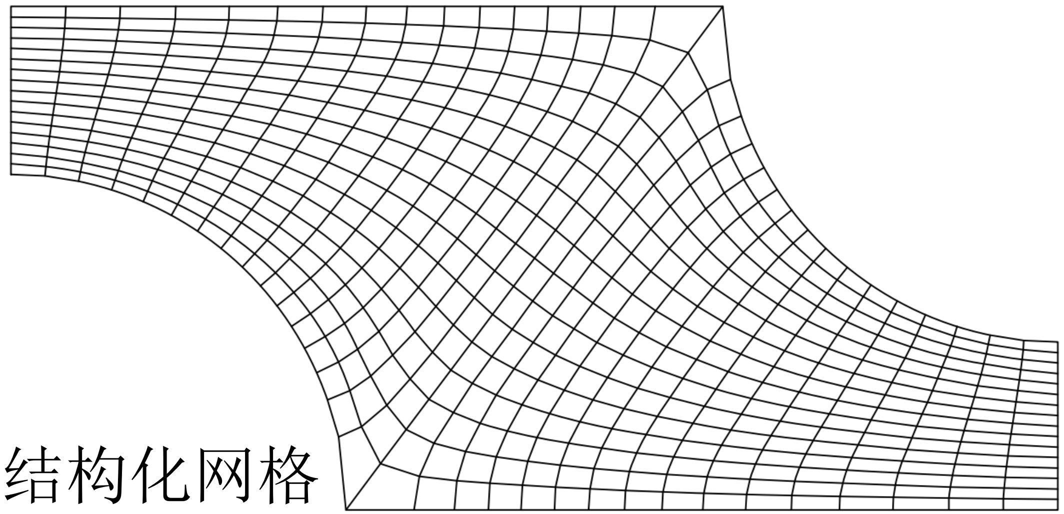

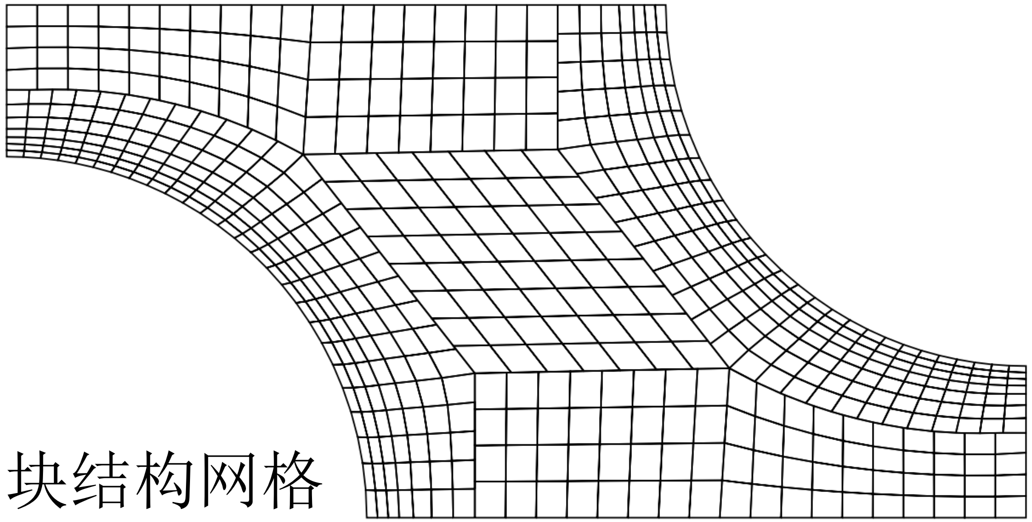

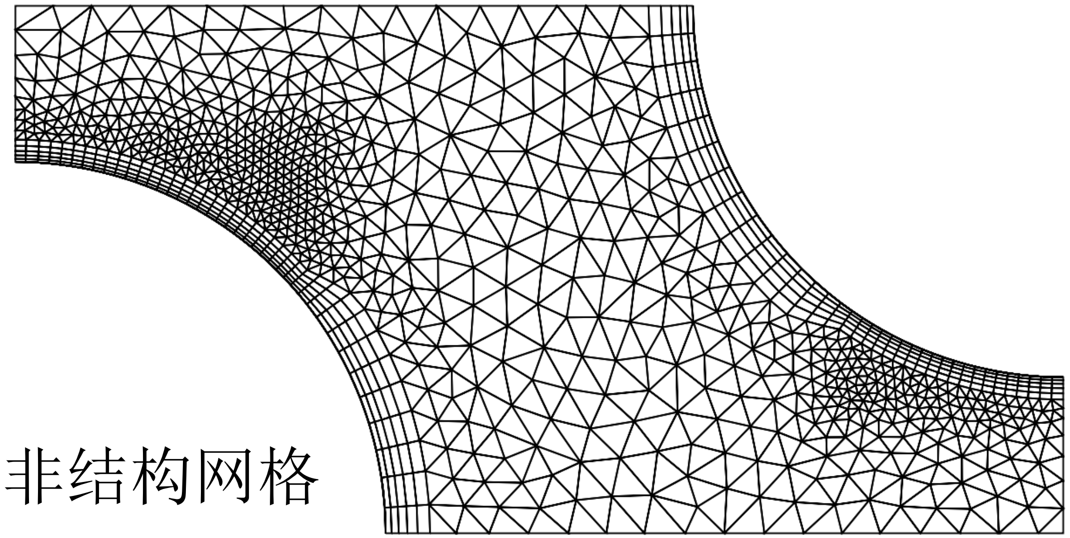

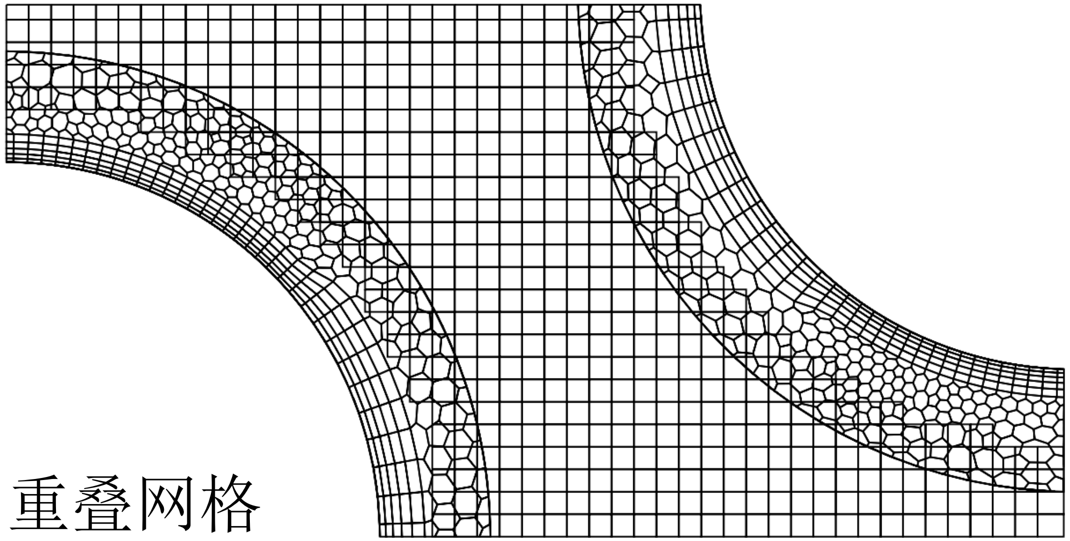

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连续方程：质量变化 = 0

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

动量方程：动量变化 = 力×时间

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot \left[\mu\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathsf{T}}\right)\right] + \nabla \cdot [\lambda (\nabla \cdot \mathbf{u}) \mathbf{I}] + \rho \mathbf{g}$$

能量方程：能量变化 = 做功 + 热量

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \nabla \cdot (k \nabla T) + \Phi$$

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不可压缩流体

连续方程： $$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

动量方程：

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

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无量纲参数

$$t^* = \frac{t}{t_0}\ ;\quad x_i^* = \frac{x_i}{L_0}\ ;\quad u_i^* = \frac{u_i}{v_0}\ ;\quad p^* = \frac{p}{\rho v_0^2}\ ;$$

连续方程：

$$\frac{\partial u_i^*}{\partial x_i^*} = 0\ ,$$

动量方程：

$$\mathrm{St}\frac{\partial u_i^*}{\partial t^*} + \frac{\partial (u_j^* u_i^*)}{\partial x_j^*} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u_i^*}{\partial x_j^{*2}} - \frac{\partial p^*}{\partial x_i^*} + \frac{1}{\mathrm{Fr}^2}\gamma_i\ ,$$

$$\mathrm{St} = \frac{L_0}{v_0 t_0}\ ;\qquad \mathrm{Re} = \frac{\rho v_0 L_0}{\mu}\ ;\qquad \mathrm{Fr} = \frac{v_0}{\sqrt{L_0 g}}$$

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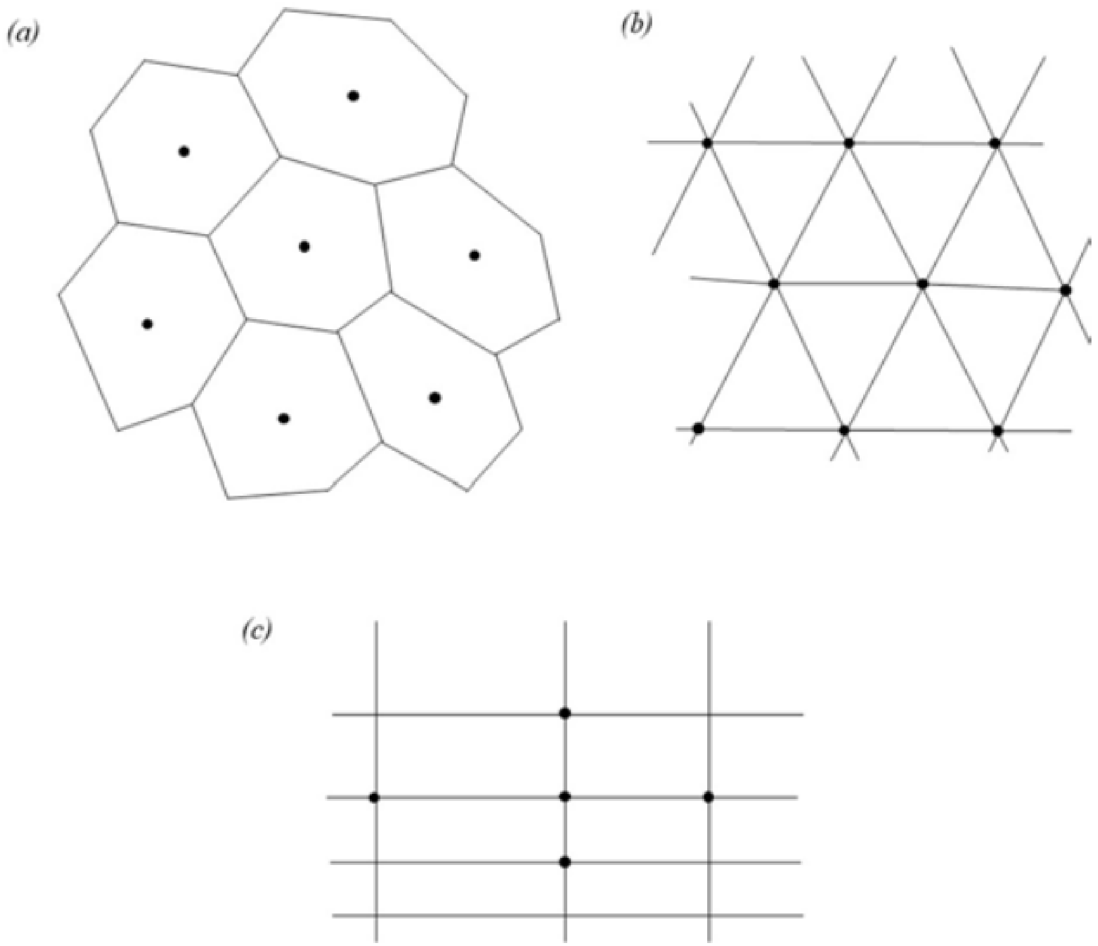
离散方法

a）有限体积法：守恒性好，可处理复杂网格，但高精度方法构建复杂

b）有限元法：适用于非结构网格，但求解复杂方程较为困难，多用于固体力学

c）有限差分法：简单，可构建高精度格式，但处理复杂网格不够灵活



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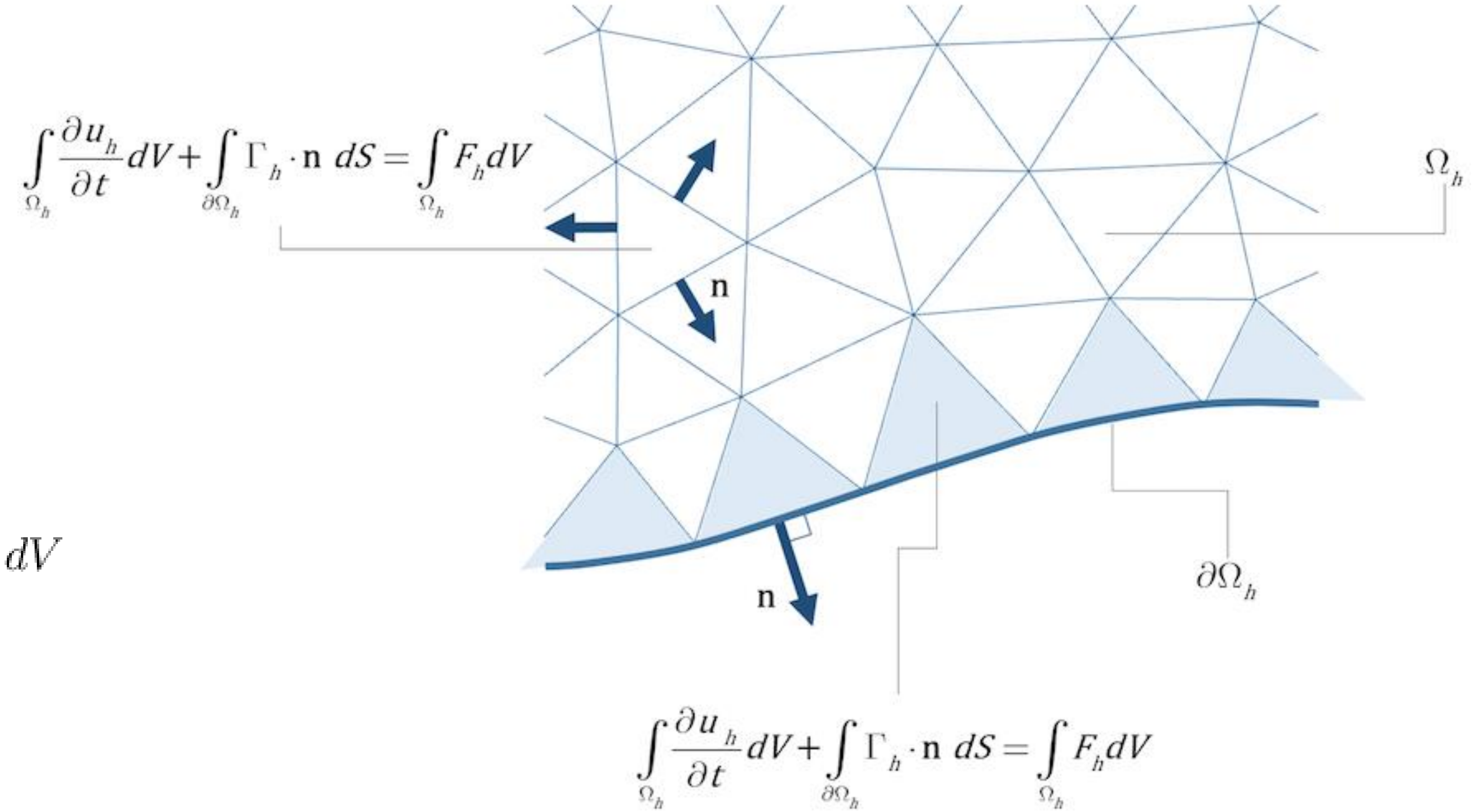
有限体积法：将计算域划分为不重复网格节点附近的控制体。然后对每个体积求解控制方程。控制体内物理量的随时间的变化量等于进入控制体界面的净流量。

$$\frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$

$$\int_{\Omega} \frac{\partial u}{\partial t} dV + \int_{\partial\Omega} \Gamma \cdot \mathbf{n}\, \mathbf{dS} = \int_{\Omega} F\, dV$$



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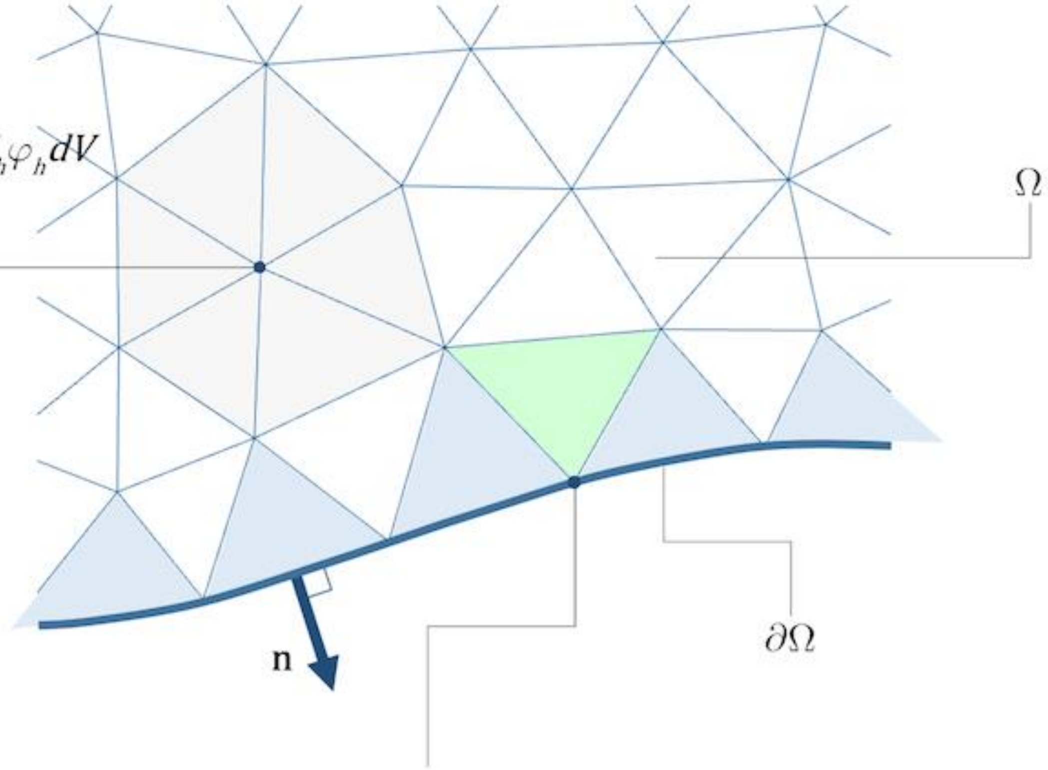
有限元法：与有限体积法类似，在有限元法计算域被分割成较小的域（有限元），有限元法与有限体积法最大的区别是在整个计算域上进行积分之前，方程会乘以一个权函数。

$$\int_{\Omega} \frac{\partial u_h}{\partial t} \varphi_h dV - \int_{\Omega} \Gamma_h \cdot \nabla \varphi_h dV = \int_{\Omega} F_h \varphi_h dV$$



$$\frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$

$$\int_{\Omega} \frac{\partial u}{\partial t} \varphi dV - \int_{\Omega} \Gamma \cdot \nabla \varphi dV + \int_{\partial\Omega} \Gamma \cdot \mathbf{n}\, \varphi dS = \int_{\Omega} F \varphi dV$$

$$\int_{\Omega} \frac{\partial u_h}{\partial t} \varphi_h dV - \int_{\Omega} \Gamma_h \cdot \nabla \varphi_h dV + \int_{\partial\Omega} \Gamma_h \cdot \mathrm{n}\, \varphi_h dS = \int_{\Omega} F_h \varphi_h dV$$

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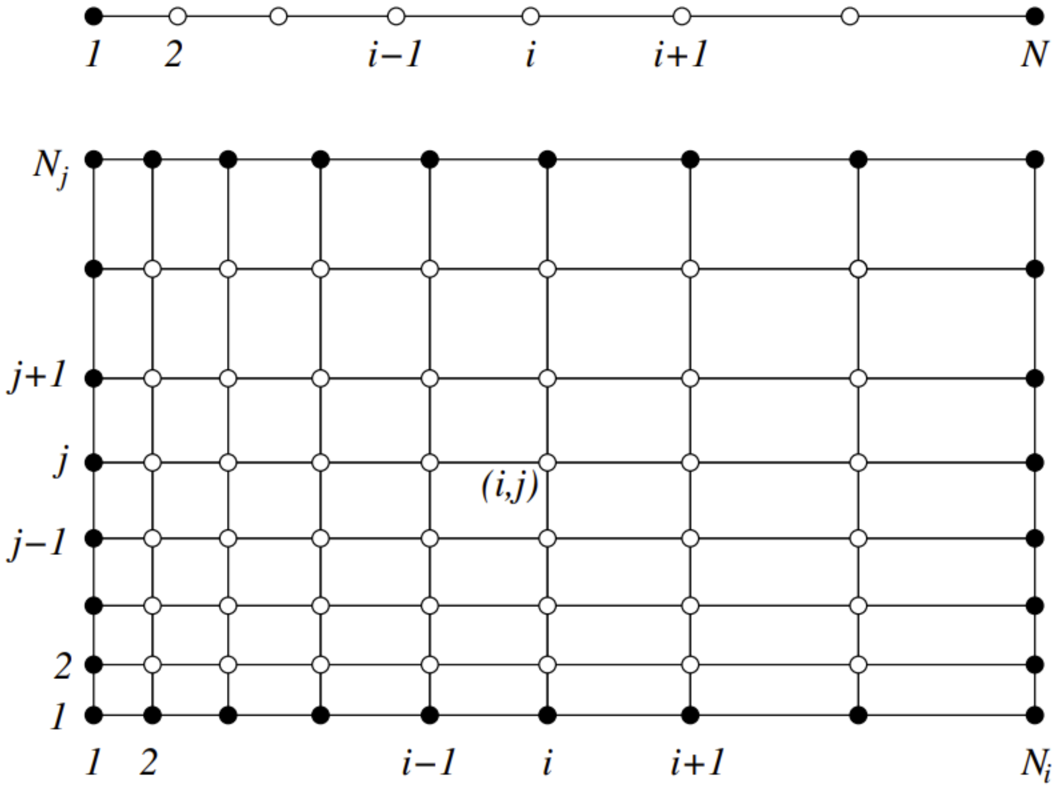
有限差分法：基于待求解偏微分方程的微分形式，通过有限差分来近似导数，从而寻求微分方程的近似解。

$$f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

$$\Downarrow$$

一维： $$f'(x_0)\approx\frac{f(x_0+h)-f(x_0)}{h}$$

多维：采用维数分裂的方式



差分格式构建方法： a）泰勒展开法
b）多项式逼近法

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泰勒展开：

$$\phi(x)=\phi(x_i)+(x-x_i)\left(\frac{\partial\phi}{\partial x}\right)_i+\frac{(x-x_i)^2}{2!}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i+\frac{(x-x_i)^3}{3!}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i+\cdots+\frac{(x-x_i)^n}{n!}\left(\frac{\partial^n\phi}{\partial x^n}\right)_i+H$$

在x_{i+1}处：

$$\left(\frac{\partial\phi}{\partial x}\right)_i=\frac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i}-\frac{x_{i+1}-x_i}{2}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i-\frac{(x_{i+1}-x_i)^2}{6}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i+H$$

前向差分：

$$\left(\frac{\partial\phi}{\partial x}\right)_i\approx\frac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i}$$

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泰勒展开：
$$\phi(x)=\phi(x_i)+(x-x_i)\left(\frac{\partial\phi}{\partial x}\right)_i+\frac{(x-x_i)^2}{2!}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i+\frac{(x-x_i)^3}{3!}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i+\cdots+\frac{(x-x_i)^n}{n!}\left(\frac{\partial^n\phi}{\partial x^n}\right)_i+H$$

在$x_{i\text{-}1}$处：
$$\left(\frac{\partial\phi}{\partial x}\right)_i=\frac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}}+\frac{x_i-x_{i-1}}{2}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i-\frac{(x_i-x_{i-1})^2}{6}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i+H$$

后向差分：
$$\left(\frac{\partial\phi}{\partial x}\right)_i\approx\frac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}}$$

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泰勒展开：

$$\phi(x) = \phi(x_i) + (x - x_i)\left(\frac{\partial \phi}{\partial x}\right)_i + \frac{(x - x_i)^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i + \frac{(x - x_i)^3}{3!}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i + \cdots + \frac{(x - x_i)^n}{n!}\left(\frac{\partial^n \phi}{\partial x^n}\right)_i + H$$

在x_{i+1}和$x_{i\text{-}1}$处：

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{x_{i+1} - x_{i-1}} - \frac{(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2}{2\,(x_{i+1} - x_{i-1})}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i - \frac{(x_{i+1} - x_i)^3 + (x_i - x_{i-1})^3}{6\,(x_{i+1} - x_{i-1})}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i + H\,.$$

中心差分：

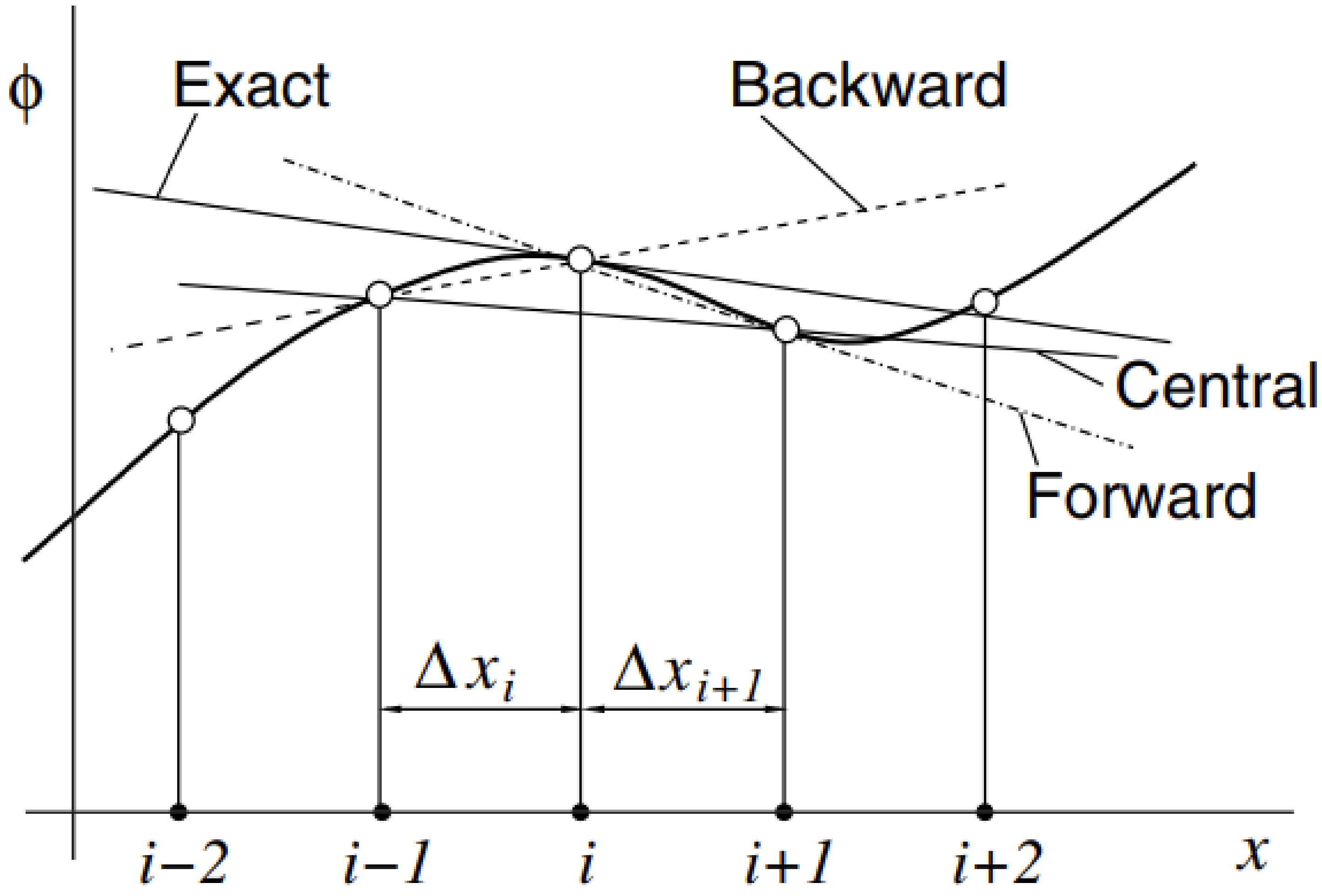
$$\left(\frac{\partial \phi}{\partial x}\right)_i \approx \frac{\phi_{i+1} - \phi_{i-1}}{x_{i+1} - x_{i-1}}$$

2 计算流体力学概述

截断误差：

$$\epsilon_\tau = (\Delta x)^m \alpha_{m+1} + (\Delta x)^{m+1} \alpha_{m+2}$$

$$\cdots + (\Delta x)^n \alpha_{n+1}$$



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多项式逼近：

1. 确定差分网格节点
2. 选择多项式函数分布
3. 计算多项式系数
4. 对多项式求导

选择x_{i+1}、x_i和x_{i-1}采用抛物线拟合得到差分格式：

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1}(\Delta x_i)^2 - \phi_{i-1}(\Delta x_{i+1})^2 + \phi_i[(\Delta x_{i+1})^2 - (\Delta x_i)^2]}{\Delta x_{i+1}\Delta x_i(\Delta x_i + \Delta x_{i+1})}$$

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二阶导数： $$\left(\frac{\partial^2\phi}{\partial x^2}\right)_i \approx \frac{\left(\frac{\partial\phi}{\partial x}\right)_{i+1}-\left(\frac{\partial\phi}{\partial x}\right)_i}{x_{i+1}-x_i}$$

扩散项： $$\left[\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)\right]_i$$

在中心点采用中心差分：

$$\left(\frac{\partial^2\phi}{\partial x^2}\right)_i \approx \frac{\phi_{i+1}(x_i-x_{i-1})+\phi_{i-1}(x_{i+1}-x_i)-\phi_i(x_{i+1}-x_{i-1})}{\frac{1}{2}(x_{i+1}-x_{i-1})(x_{i+1}-x_i)(x_i-x_{i-1})}$$

$$\left[\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)\right]_i \approx \frac{\Gamma_{i+\frac{1}{2}}\frac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i}-\Gamma_{i-\frac{1}{2}}\frac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}}}{\frac{1}{2}(x_{i+1}-x_{i-1})}$$

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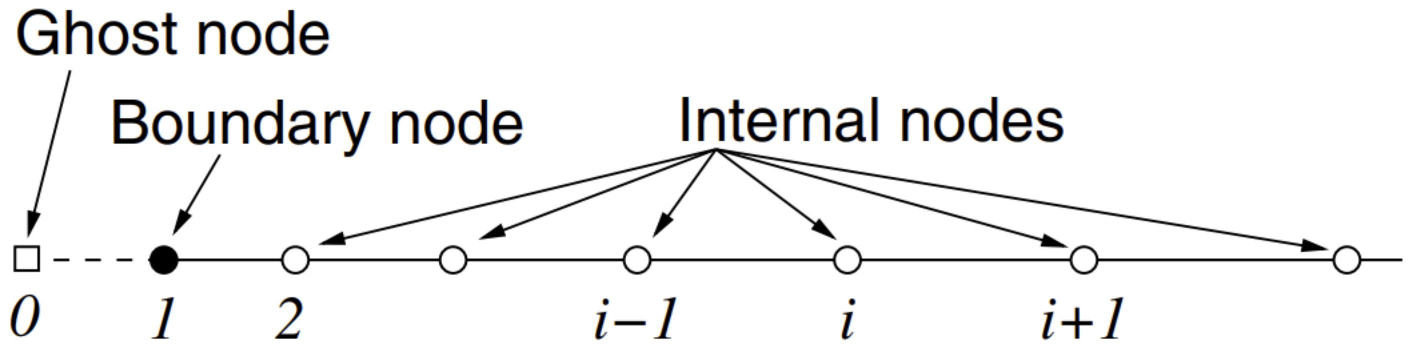
边界条件



使用内点的第二类边界条件：

$$\left(\frac{\partial \phi}{\partial x}\right)_1 \approx \frac{-\phi_3(x_2-x_1)^2+\phi_2(x_3-x_1)^2-\phi_1[(x_3-x_1)^2-(x_2-x_1)^2]}{(x_2-x_1)(x_3-x_1)(x_3-x_2)}$$

使用虚节点的第三类边界条件：

$$\left(\frac{\partial \phi}{\partial x}+c\phi\right)_1=0 \Rightarrow \frac{\phi_2-\phi_0}{2(x_2-x_1)}+c\phi_1=0 \Rightarrow \phi_0=\phi_2+2(x_2-x_1)c\phi_1$$

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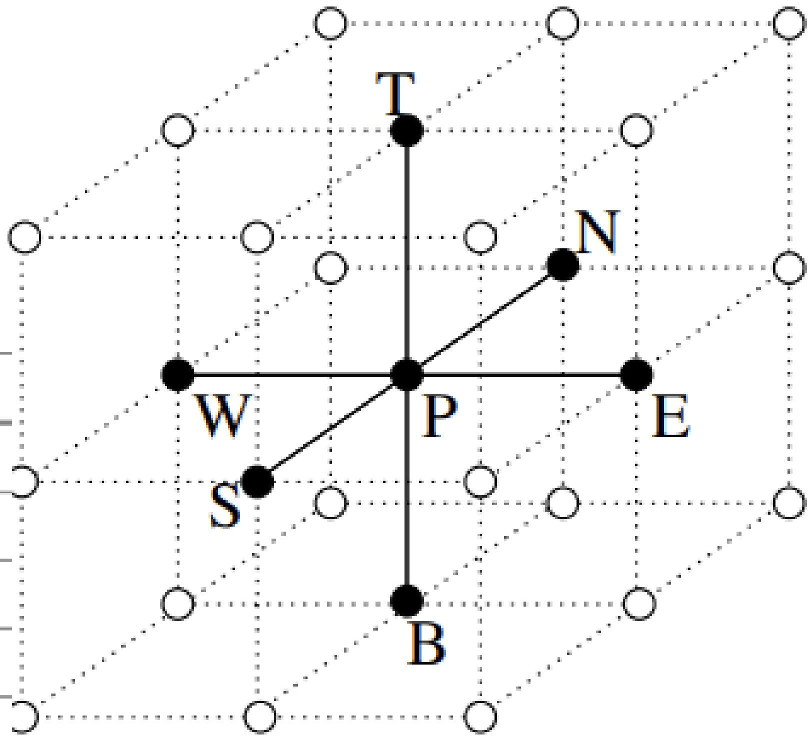
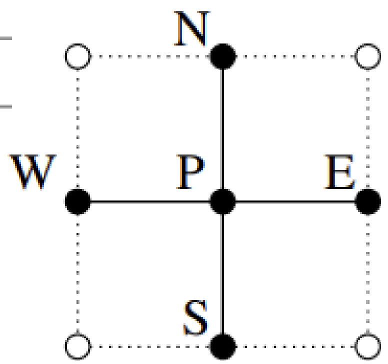
Grid location	Compass notation	Storage location
$i,\ j,\ k$	P	$l = (k-1)N_j N_i + (i-1)N_j + j$
$i-1,\ j,\ k$	W	$l - N_j$
$i,\ j-1,\ k$	S	$l - 1$
$i,\ j+1,\ k$	N	$l + 1$
$i+1,\ j,\ k$	E	$l + N_j$
$i,\ j,\ k-1$	B	$l - N_i N_j$
$i,\ j,\ k+1$	T	$l + N_i N_j$





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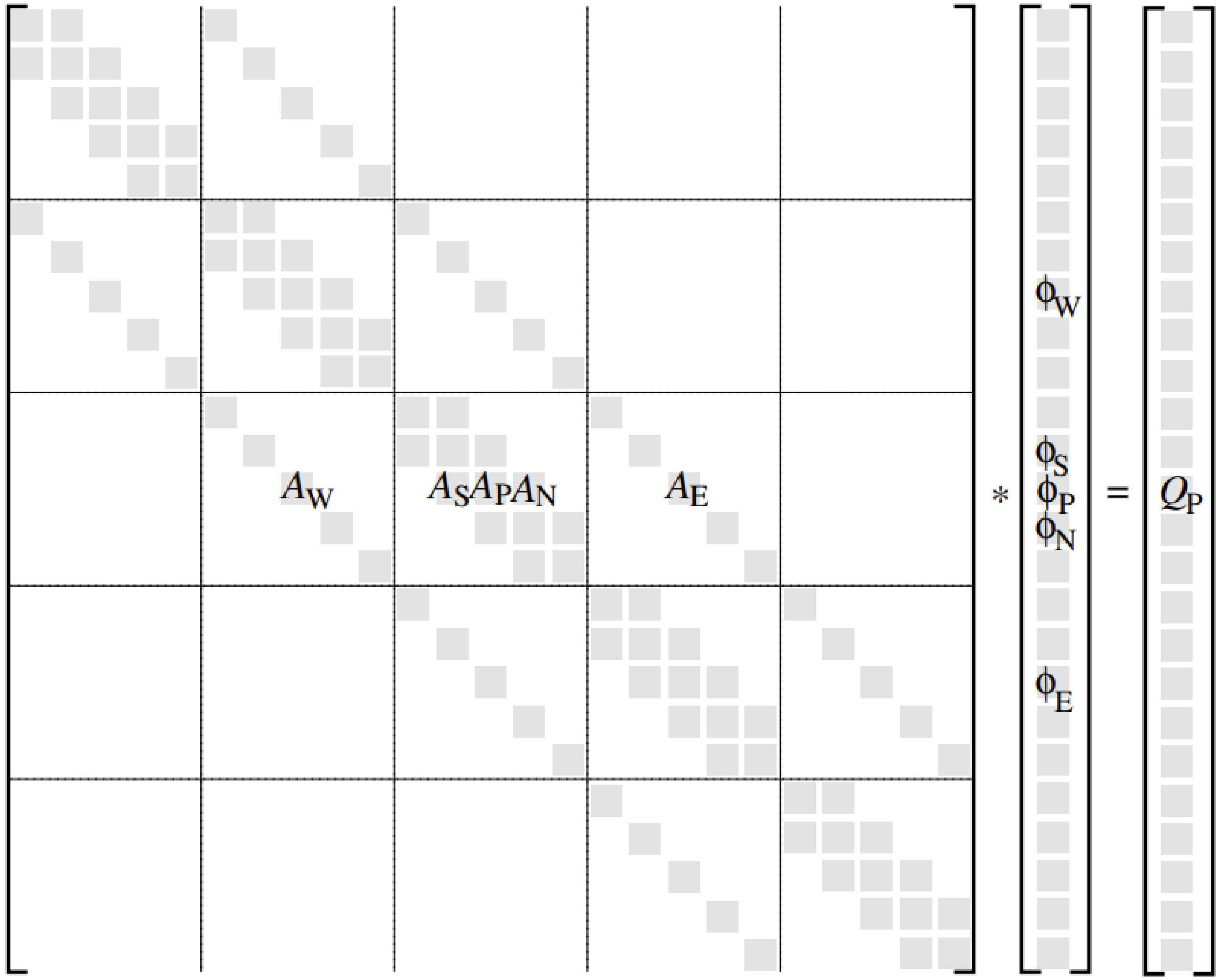
$$A_W\phi_W + A_S\phi_S + A_P\phi_P + A_N\phi_N + A_E\phi_E = Q_P$$



2 计算流体力学概述

一维定常对流扩散方程： $$\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)$$

采用三个网格节点差分： $$A_{\mathrm{P}}^{i}\phi_{i}+A_{\mathrm{E}}^{i}\phi_{i+1}+A_{\mathrm{W}}^{i}\phi_{i-1}=Q_{i}$$

扩散项：
中心差分

$$\left(\Gamma\frac{\partial\phi}{\partial x}\right)_{i+\frac{1}{2}}\approx\Gamma\frac{\phi_{i+1}-\phi_{i}}{x_{i+1}-x_{i}}\ ;\quad\left(\Gamma\frac{\partial\phi}{\partial x}\right)_{i-\frac{1}{2}}\approx\Gamma\frac{\phi_{i}-\phi_{i-1}}{x_{i}-x_{i-1}}$$

系数：

$$A_{\mathrm{E}}^{\mathrm{d}}=-\frac{2\,\Gamma}{(x_{i+1}-x_{i-1})(x_{i+1}-x_{i})}$$

$$A_{\mathrm{W}}^{\mathrm{d}}=-\frac{2\,\Gamma}{(x_{i+1}-x_{i-1})(x_{i}-x_{i-1})}$$

$$A_{\mathrm{P}}^{\mathrm{d}}=-(A_{\mathrm{E}}^{\mathrm{d}}+A_{\mathrm{W}}^{\mathrm{d}})\ .$$

2 计算流体力学概述

一维定常对流扩散方程： $$\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)$$

采用三个网格节点差分： $$A_{\mathrm{P}}^{i}\phi_i+A_{\mathrm{E}}^{i}\phi_{i+1}+A_{\mathrm{W}}^{i}\phi_{i-1}=Q_i$$

对流项：
前/后向差分

$$\left[\frac{\partial(\rho u\phi)}{\partial x}\right]_i\approx\begin{cases}\rho u\dfrac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}}\ , \text{ if } u>0\\[2ex] \rho u\dfrac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i}\ , \text{ if } u<0\end{cases}$$

系数： $$A_{\mathrm{E}}^{\mathrm{c}}=\frac{\min(\rho u,0)}{x_{i+1}-x_i}\ ;\qquad A_{\mathrm{W}}^{\mathrm{c}}=-\frac{\max(\rho u,0)}{x_i-x_{i-1}}$$

$$A_{\mathrm{P}}^{\mathrm{c}}=-(A_{\mathrm{E}}^{\mathrm{c}}+A_{\mathrm{W}}^{\mathrm{c}})\ .$$

2 计算流体力学概述

一维定常对流扩散方程： $$\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)$$

采用三个网格节点差分： $$A^i_{\mathrm{P}}\phi_i+A^i_{\mathrm{E}}\phi_{i+1}+A^i_{\mathrm{W}}\phi_{i-1}=Q_i$$

对流项：
中心差分

$$\left[\frac{\partial(\rho u\phi)}{\partial x}\right]_i\approx\rho u\frac{\phi_{i+1}-\phi_{i-1}}{x_{i+1}-x_{i-1}}$$

系数： $$A^{\mathrm{c}}_{\mathrm{E}}=\frac{\rho u}{x_{i+1}-x_{i-1}}\ ;\qquad A^{\mathrm{c}}_{\mathrm{W}}=-\frac{\rho u}{x_{i+1}-x_{i-1}}$$

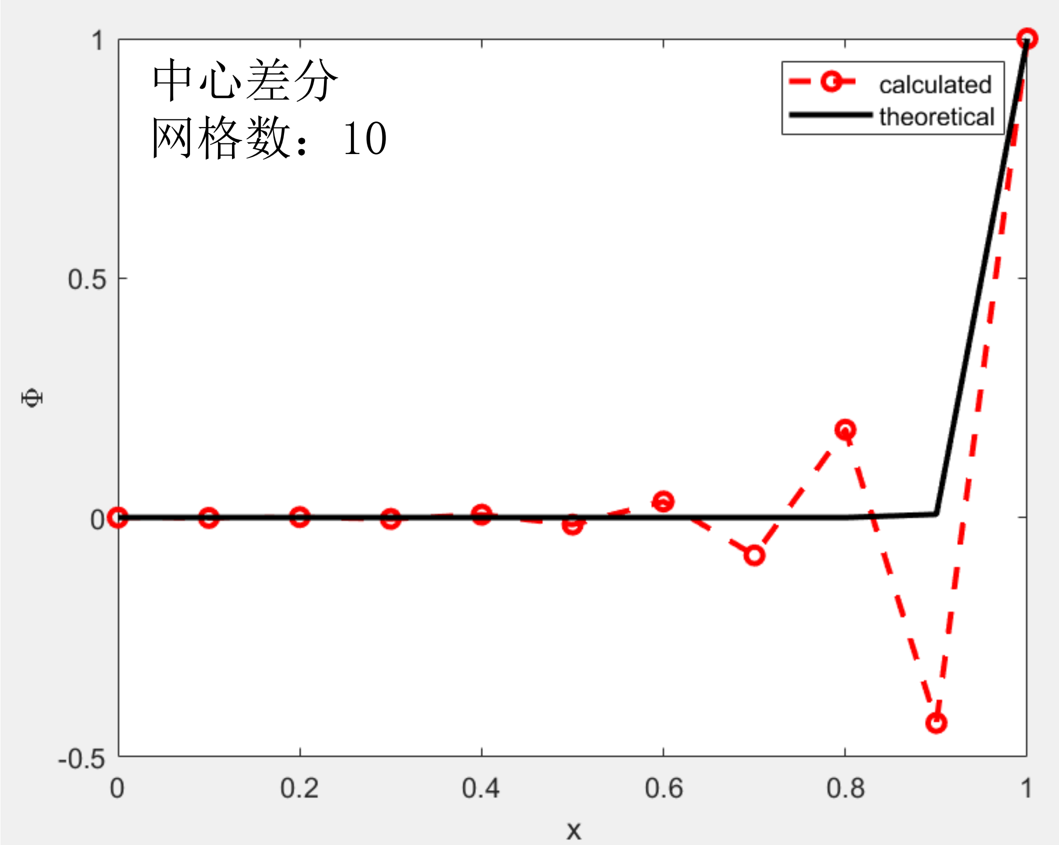
$$A^{\mathrm{c}}_{\mathrm{P}}=-(A^{\mathrm{c}}_{\mathrm{E}}+A^{\mathrm{c}}_{\mathrm{W}})=0\ .$$

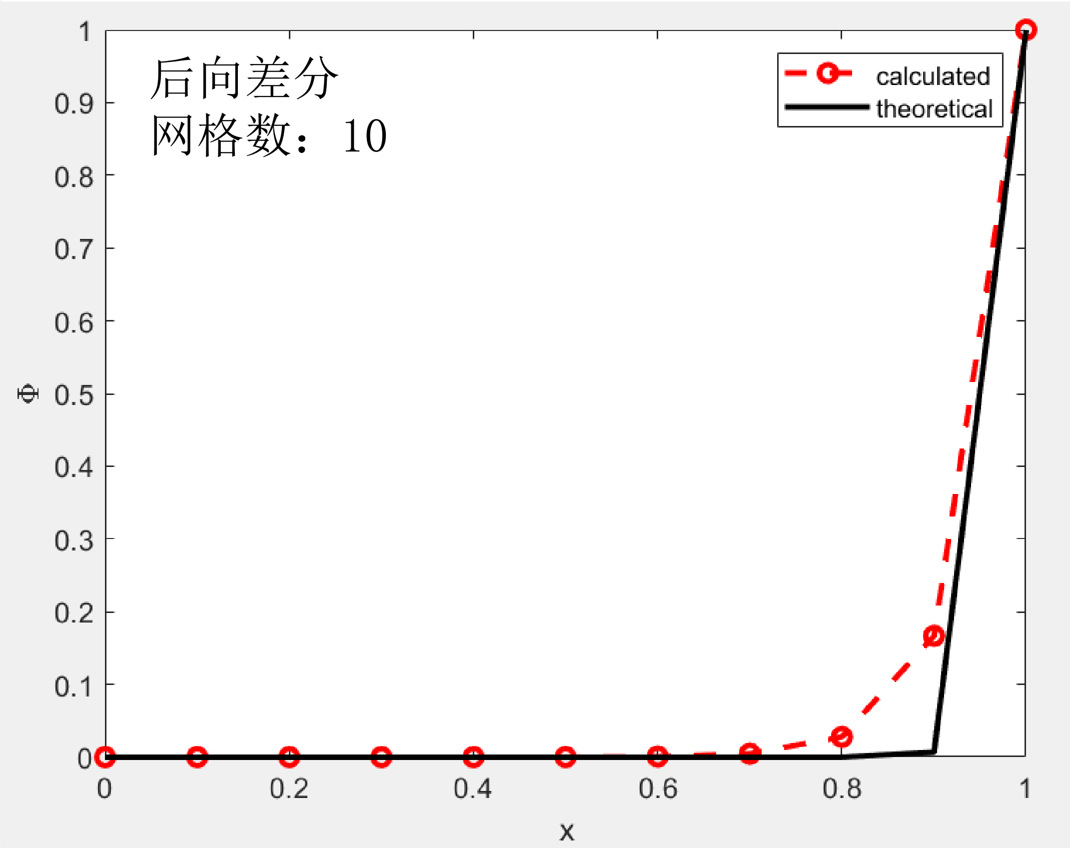
2 计算流体力学概述

理论解：

$$\phi = \phi_0 + \frac{e^{x\mathrm{Pe}/L} - 1}{e^{\mathrm{Pe}} - 1}(\phi_L - \phi_0)$$

参数： $L = 1.0\ \rho = 1.0,\ u = 1.0,\ \Gamma = 0.02,\ \phi_0 = 0 \text{ and } \phi_L = 1.0$





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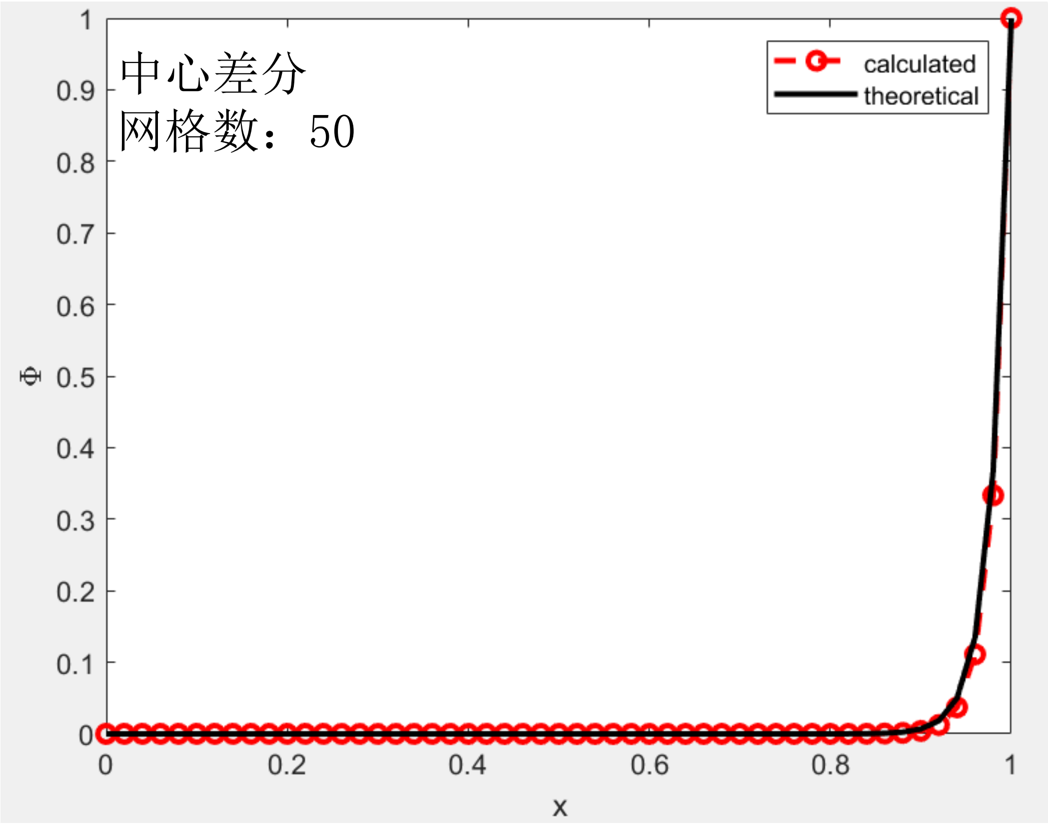
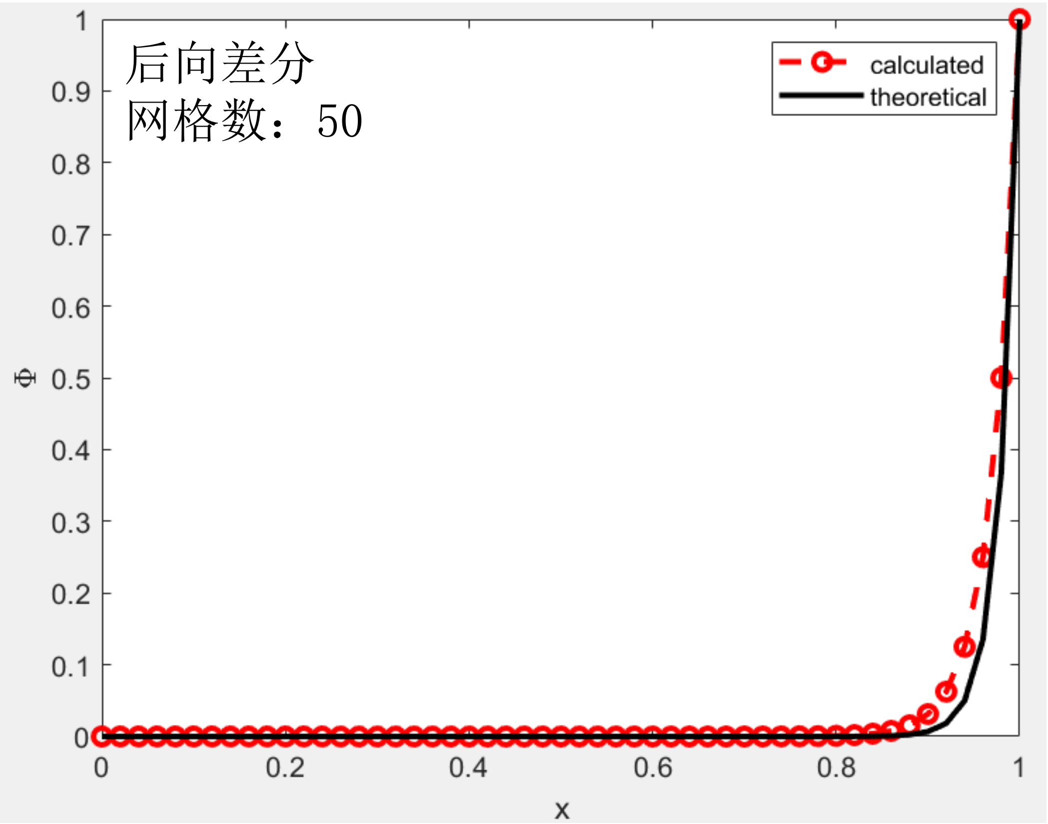
理论解： $$\phi = \phi_0 + \frac{e^{x\mathrm{Pe}/L} - 1}{e^{\mathrm{Pe}} - 1}(\phi_L - \phi_0)$$ ？前向差分？

参数： $L = 1.0\ \rho = 1.0,\ u = 1.0,\ \Gamma = 0.02,\ \phi_0 = 0 \text{ and } \phi_L = 1.0$





2 计算流体力学概述

初值问题： $$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = f(t, \phi(t))\ ; \quad \phi(t_0) = \phi^0$$

时间积分： $$\int_{t_n}^{t_{n+1}} \frac{\mathrm{d}\phi}{\mathrm{d}t}\,\mathrm{d}t = \phi^{n+1} - \phi^n = \int_{t_n}^{t_{n+1}} f(t, \phi(t))\,\mathrm{d}t$$

采用起点 $\Downarrow$ 　　　　 $\Downarrow$ 采用终点

显式欧拉法： $\phi^{n+1} = \phi^n + f(t_n, \phi^n)\,\Delta t$

隐式欧拉法： $\phi^{n+1} = \phi^n + f(t_{n+1}, \phi^{n+1})\,\Delta t$

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时间积分：

$$\int_{t_n}^{t_{n+1}} \frac{\mathrm{d}\phi}{\mathrm{d}t}\,\mathrm{d}t = \phi^{n+1} - \phi^n = \int_{t_n}^{t_{n+1}} f(t, \phi(t))\,\mathrm{d}t$$

利用中间参数

龙格-库塔法

二阶：

$$\phi^*_{n+\frac{1}{2}} = \phi^n + \frac{\Delta t}{2} f(t_n, \phi^n)\ ,$$

$$\phi^{n+1} = \phi^n + \Delta t\, f\left(t_{n+\frac{1}{2}}, \phi^*_{n+\frac{1}{2}}\right)$$

四阶：

$$\phi^*_{n+\frac{1}{2}} = \phi^n + \frac{\Delta t}{2} f(t_n, \phi^n)\ ,$$

$$\phi^{**}_{n+\frac{1}{2}} = \phi^n + \frac{\Delta t}{2} f\left(t_{n+\frac{1}{2}}, \phi^*_{n+\frac{1}{2}}\right)\ ,$$

$$\phi^*_{n+1} = \phi^n + \Delta t\, f\left(t_{n+\frac{1}{2}}, \phi^{**}_{n+\frac{1}{2}}\right)\ ,$$

$$\phi^{n+1} = \phi^n + \frac{\Delta t}{6}\Big[f(t_n, \phi^n) + 2f\left(t_{n+\frac{1}{2}}, \phi^*_{n+\frac{1}{2}}\right) + 2f\left(t_{n+\frac{1}{2}}, \phi^{**}_{n+\frac{1}{2}}\right) + f(t_{n+1}, \phi^*_{n+1})\Big]\,.$$

2 计算流体力学概述

一维输运方程：
$$\frac{\partial \phi}{\partial t} = -u\frac{\partial \phi}{\partial x} + \frac{\Gamma}{\rho}\frac{\partial^2 \phi}{\partial x^2}$$

显示欧拉法：　　空间离散　$\Downarrow$　中心差分

$$\phi_i^{n+1} = \phi_i^n + \left[-u\frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\,\Delta x} + \frac{\Gamma}{\rho}\frac{\phi_{i+1}^n + \phi_{i-1}^n - 2\,\phi_i^n}{(\Delta x)^2}\right]\Delta t$$

$$d = \frac{\Gamma\,\Delta t}{\rho(\Delta x)^2} \quad \Downarrow \quad c = \frac{u\,\Delta t}{\Delta x}$$

$$\phi_i^{n+1} = (1-2d)\,\phi_i^n + \left(d - \frac{c}{2}\right)\phi_{i+1}^n + \left(d + \frac{c}{2}\right)\phi_{i-1}^n$$

$$\Downarrow$$

$$\boldsymbol{\phi}^{n+1} = A\boldsymbol{\phi}^n$$

2 计算流体力学概述

一维输运方程：

$$\frac{\partial \phi}{\partial t} = -u\frac{\partial \phi}{\partial x} + \frac{\Gamma}{\rho}\frac{\partial^2 \phi}{\partial x^2}$$

隐式欧拉法：　　空间离散　　中心差分

$$\phi_i^{n+1} = \phi_i^n + \left[-u\frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1}}{2\,\Delta x} + \frac{\Gamma}{\rho}\frac{\phi_{i+1}^{n+1} + \phi_{i-1}^{n+1} - 2\,\phi_i^{n+1}}{(\Delta x)^2}\right]\Delta t$$

$$d = \frac{\Gamma\,\Delta t}{\rho(\Delta x)^2} \qquad c = \frac{u\,\Delta t}{\Delta x}$$

$$(1+2d)\,\phi_i^{n+1} + \left(\frac{c}{2} - d\right)\phi_{i+1}^{n+1} + \left(-\frac{c}{2} - d\right)\phi_{i-1}^{n+1} = \phi_i^n$$

$$A_E = \frac{c}{2} - d\,; \quad A_W = -\frac{c}{2} - d\,;$$

$$A_P = 1 + 2d = 1 - (A_E + A_W)\,; \quad Q_P = \phi_i^n \quad A_P\phi_i^{n+1} + A_E\phi_{i+1}^{n+1} + A_W\phi_{i-1}^{n+1} = Q_P$$