Experiment 81 - Design of a Feedback Control System

Lab Report

Department of Electrical Engineering and Electronics

April 2020

Xuhui Gong

Abstract

This report mainly studies the approximation of a second-order system by using first-order plus time delay method (FOPDT) and uses the Ziegler-Nichols method to design the P, PI, and PID controller, respectively. On this basis, for the above three different controllers, step disturbance is used to understand the approximate resistance of the system to disturbance.

Declaration

I confirm that I have read and understood the University's definitions of plagiarism and collusion from the Code of Practice on Assessment. I confirm that I have neither committed plagiarism in the completion of this work nor have I colluded with any other party in the preparation and production of this work. The work presented here is my own and in my own words except where I have clearly indicated and acknowledged that I have quoted or used figures from published or unpublished sources (including the web). I understand the consequences of engaging in plagiarism and collusion as described in the Code of Practice on Assessment (Appendix L).

Contents

1 Introduction

This experiment using the Ziegler-Nichols method to calculate the parameters of the proportional controller, proportional-integral controller, and proportional-integral-derivative controller based on the MATLAB Simulink [1]. Besides, the properties of the above controller are compared in the way of image and table. Finally, the ability of the controller to disturbance is tested.

2 Objectives

The objectives of the experiment are divided into two categories. The first is the skill objectives of the experiment. The second is the understanding and practice of theory.

For the skill of objectives [1]:

- Be able to use the MATLAB Simulink to build the control system [2]
- Be able to understand the process of proportional (P) and proportional-integral (PI) controllers
- Be able to describe the system performance by using the steady-state error, %OS (percentage overshoot), settling time and the rise time

For the understanding and practice of theory:

- Be able to select a suitable approximation of first-order plus time delay model
- Be able to understand the Taylor and Skogestad approximation in practice [3]
- Be able to use the Ziegler-Nichols method to estimate the parameter of proportional (P), proportional-integral (PI) and proportional-integral-derivative (PID) [4]

3 Theory

3.1 The Characteristic of PID Controller

3.2 First-Order Plus Delay Model

The first-order plus delay model (FOPDT) is used to approximate a higher-order system to a first-order system with delay [1]. The transfer function is [1]:

$$G(s) = \frac{K_p e^{-t_d s}}{(T_p s + 1)} \tag{1}$$

However, FOPDT is flawed because there is an increased time delay in this estimation, and all the differences between FOPDT and actual functions are significant at the beginning [5]. Although choosing the right parameter estimate will reduce the difference with the real function. There are two main methods to calculate time delay t_d and time constant T_p: Taylor approximation and Skogestad approximation.

3.2.1 Taylor Approximation

The particular case of the Taylor series, which is Maclaurin series in exponential function is defined in the formula [1], [6]:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots \tag{3}$$

The first two terms of the equation are used for approximation:

$$e^{x} \sim 1 + x \tag{4}$$

Replace x of equation (4) with θs:

$$e^{\theta s} \sim 1 + \theta s \tag{5}$$

The G(s) in the experiment is shown in the formula (6):

$$G(s) = K/(Ts + 1)^2 = \frac{K}{(Ts+1)(Ts+1)} \tag{6}$$

Substitute (5) into (6):

$$G(s) \sim \frac{K}{(Ts+1)\, e^{Ts}} = \frac{Ke^{-Ts}}{Ts+1} \tag{7}$$

3.2.2 Skogestad Approximation

The following preconditions exist for Skogestad approval [5]:

- The denominator should be in terms of τ_s
- A dominant time constant must exist $\tau_{Largest} \geq 1.5\, \tau_{Next\ Largest}$
- The system should be overdamped (Can be critically damped)

The method to calculate the Skogestad approximation is as follows [5]:

Based on the formula (1):

The original higher-order system shows in the formula (6):

$$G_s = \frac{K_p}{(\tau_1 S + 1)(\tau_2 S + 1)\ldots} \tag{8}$$

The parameter of FOPDT in Skogestad approximation is:

$$K_p = K_{Higher\ Order} \tag{9}$$

$$T_p = \tau_{Largest} + \frac{1}{2}\, \tau_{Next\ Largest} \tag{10}$$

$$t_d = \frac{1}{2}\tau_{Next\ Largest} + \sum\tau + \theta_{Higer\ Order} \tag{11}$$

3.3 Ziegler-Nichols Method

Ziegler-Nichols method can quickly adjust the value of the controller. Using this method, the parameter of proportional (P), proportional-integral (PI), and proportional-integral-derivative (PID) can be calculated [7]. However, this method is not mathematically rigorous, and it required a stable system. It has two methods to calculate the parameters, the procedure of the first method is shown below:

- Start a small K_P value and set the value of K_I and K_D to the 0.
- Then increase the value of the $K_P = K_U$ to let the output function until natural stability.
- The value of the K_P and the output period T_U are recorded.
- Based on Table 1, calculate the other parameters.

Table 1. Ziegler-Nichols Method Table (taken from [7])

Control Type	K_P	K_I	K_D
PID	$0.6K_U$	$1.2K_U/T_U$	$0.075K_UT_U$
P	$0.5K_U$	-	-
PI	$0.45K_U$	$0.54K_U/T_U$	-

However, the experiment provides the (6), which has twice the same poles in the negative half of the real axis. In Figure 1, the root locus shows the first method cannot have natural stability.

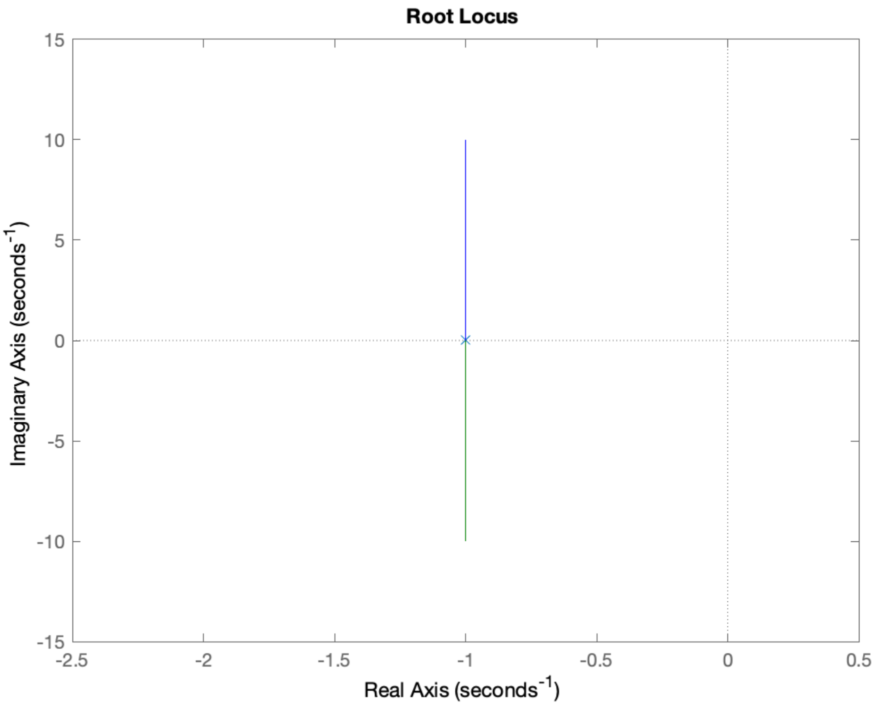


Figure 1. Root Locus in two poles in a negative one

The second method to calculate the parameters by using the Ziegler-Nichols method can apply to this experiment. However, the following preconditions exist for using this method [8], [9]:

- It only applied to the open-loop step response is shown in Figure 2
- It only applied to the FOPDT or second-order overdamped system

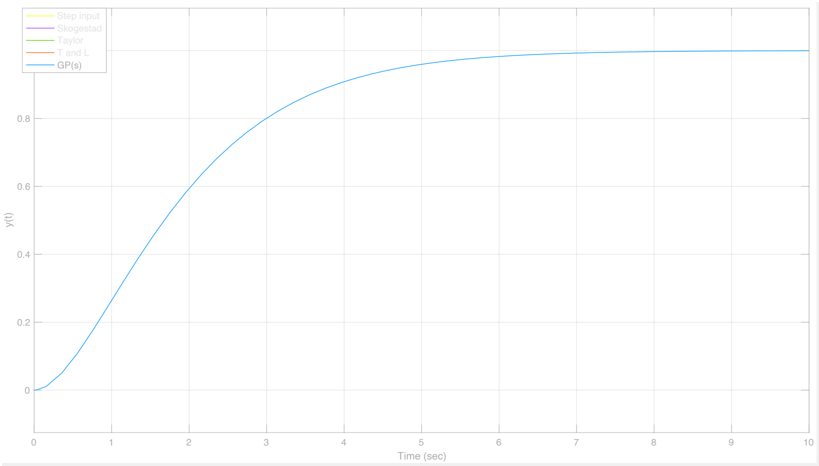


Figure 2. The method can be applied to the open-loop step response

In order to obtain PID parameters, the inflection point needs to be calculated. As shown in Figure 3, the fundamental parameters T and L can be obtained by solving the first derivative of the function in Figure 2 and obtaining the maximum value or solving the second derivative of the function by taking the intersection of the function and the horizontal axis [9].

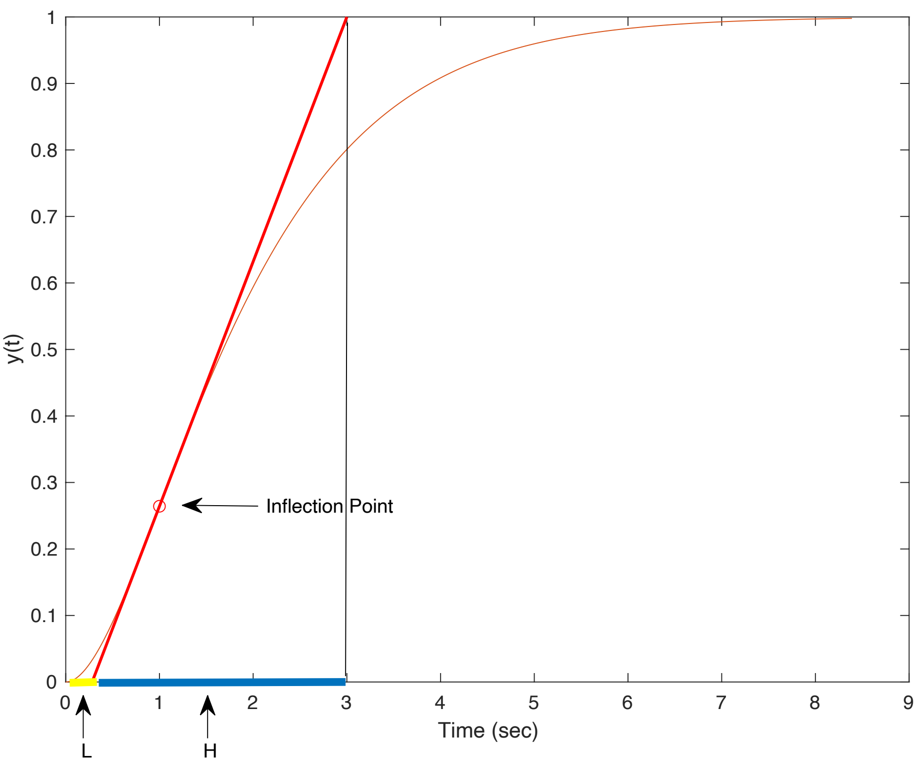


Figure 3. Intersection point in the y(t)

Table 2 shows the relationship between the T/L and controller parameters.

Table 2. Relationship between the T/L and Controller parameters (taken from [8])

Controller	K_P	K_I	K_D
P	T/L	-	-
PI	0.9 T/L	0.27 $\frac{T}{L^2}$	-
PID	1.2 T/L	0.6 $\frac{T}{L^2}$	0.6 T

4 Part I: First-order Plus Time Delay Approximation

In the experiment, the value of K and T is set for the one and one, so the transfer function is:

$$G(s) = 1/(s + 1)^2 \tag{12}$$

4.1 Experiment Method

4.1.1 Simulink Simulation Process

As shown in Figure 3, the MATLAB Simulink was built to process a unit-step input (start from 0s), in this Figure is an open-loop system and the gain is one, the GP(s) is the original transfer function (10). There are three functions used to approximate the G(s), T and L, Taylor, and Skogestad approximation. This report aims to find a good approximation to the G(s) by comparing the response and the steady-state error, %OS, settling time, and the rise time.

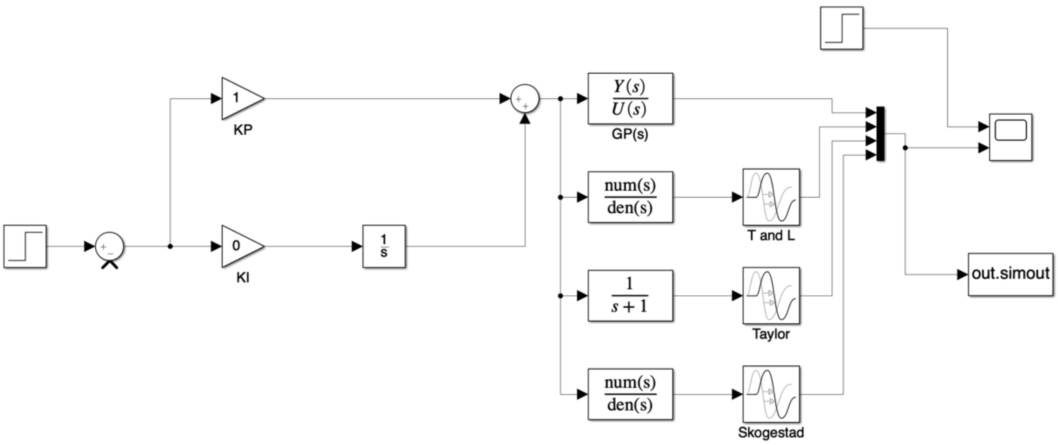


Figure 3. The Simulink Simulation Process in Part I

4.1.2 Taylor approximation

Based on the formula (7):

$$G(s) \sim \frac{K}{(Ts+1)\,e^{Ts}} = \frac{Ke^{-Ts}}{Ts+1} \tag{7}$$

The G(s) can be approximately to the:

$$G(s) \sim \frac{e^{-s}}{s+1} \tag{13}$$

4.1.3 Skogestad approximation

Based on the formula (8)(9)(10)(11):

The G(s) can be approximately to the:

$$G(s) \sim \frac{e^{-\frac{1}{2}s}}{\frac{3}{2}s+1} \tag{14}$$

Where K = 1; $T_p = {}^{3}\!/_{2}$; $t_d = {}^{1}\!/_{2}$

However, the preconditions for using skogestad approval need to be met:

- The denominator should be in terms of τ_s
- A dominant time constant must exist $\tau_{Largest} \geq 1.5\, \tau_{Next\ Largest}$
- The system should be overdamped (Can be critically damped)

In the second preconditions, it does not have the next largest time constant, so under such individual conditions, it is still satisfied. The advantage of Skogestad approximation in this situation is the time delay is smaller than the Taylor approximation.

4.1.4 T and L approximation

Based on the second method of Ziegler-Nichols, the T and L can be used to calculate the approximation of the second-order system. After calculation, as shown in Figure 3, the values of L and H are 0.2817 and 2.7183. Time delay corresponding to formula 1 is L, and the time constant is H.

The approximation of this function is as follows:

$$G(s) \sim \frac{e^{-0.2817s}}{2.7183s + 1} \tag{15}$$

4.2 Experiment Result

The result is shown in Figure 4. The GP(s) is the original transfer function G(s) with step input, and the line of the Skogestad, Taylor, and T/L approximation are shown in Figure 4.

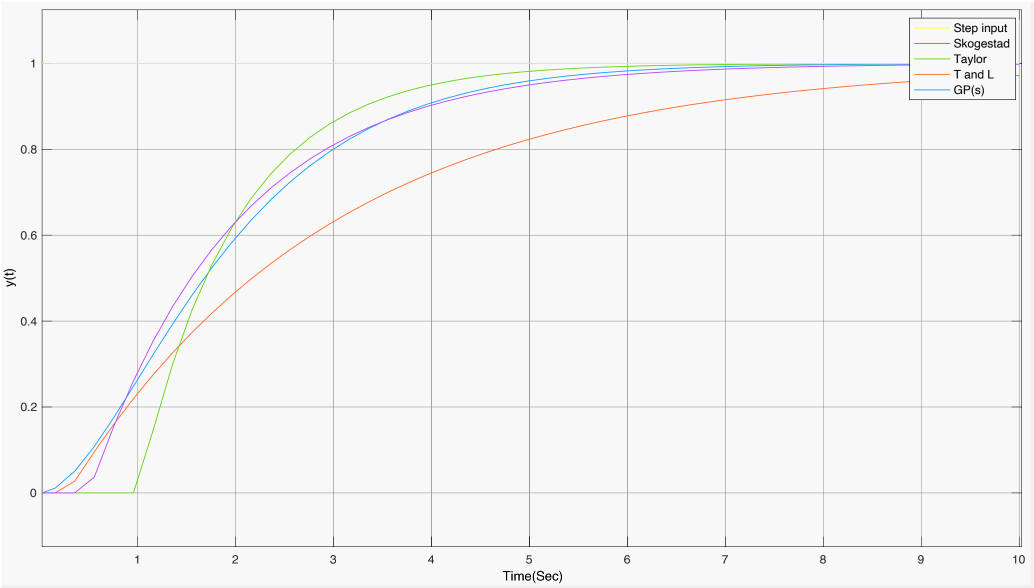


Figure 4. Experiment Result

From the image observation, the Skogestad has the best approximation to the original function. First of all, when time is almost to the 0.5s, only T and L approximation and Skogestad approximation have made a relatively fast response, and the slowest one is Taylor approximation, which also can be observed from the function (13)(14)(15). T and L approximation has a minimum delay time. However, when time after 2s, the T and L cannot catch the original function, the Taylor and Skogestad approximation has an excellent approximation to the original function. The Taylor approximation is higher than the original function. So, for the transfer function $G(s) = 1/(s+1)^2$, the Skogestad approximation is the best approximation for the G(s) in the step input.

The steady-state error, percentage overshoot, settling time, and the rise time of the approximation and are shown in Table 3. Note that the steady-state error is 0 because the numerator coefficient of G(s) is one, if this coefficient increases the steady-state error will increase.

Table 3. Experiment Data

Type	Steady-state error	%OS	Settling time	Rise Time

G(s)	0	0	5.80868463410876	3.35830867514247
T and L	0	0	8.55045837415739	5.34777429135351
Taylor	0	0	4.90970852656815	2.21079599134494
Skogestad	0	0	6.24927127759256	3.27158807252221

4.3 Experiment Analysis and Discussions

4.3.1 Theoretical calculation

Consider the step input u(t), Laplace transform is used to multiply the G(s), the formula is:

$$Y(s) = U(s) \times G(s) = \frac{1}{s\,(s+1)^2} \tag{16}$$

Applying Reverse Laplace transform:

$$y(t) = 1 - e^{-t} - e^{-t}t \tag{17}$$

4.3.2 Analysis and Discussion

The essence of the problem is transformed into a mathematical problem: which of the following transformation equations multiply the step input is more similar to the original function $y(t)$. Through the comparison of Table 3, Skogestad approximation in FOPDT is the closest to the original output. There are two reasons. Firstly, the time delay is more suitable. Although the time delay of this Skogestad approximation is more significant than that of T/L approximation, T/L approximation cannot fit the original function well after 2 seconds. After 2 seconds, compared with Taylor and Skogestad approximation, Skogestad approximation intersects the original function twice, so the

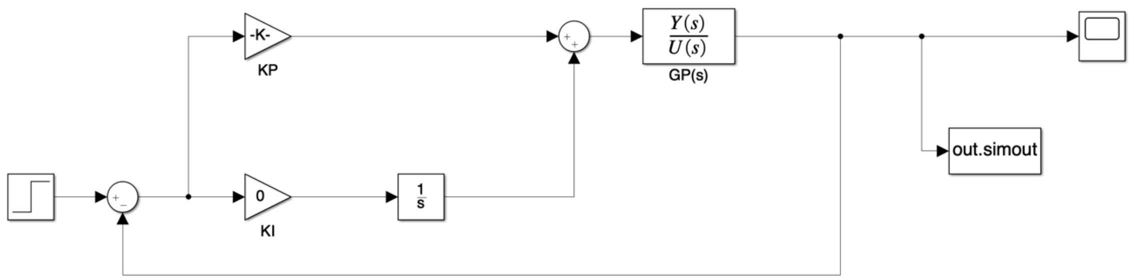
Skogestad approximation fits the original function better than other approximation methods. Table 3 can also provide evidence of this approximation. Because Skogestad approximation data is closer to the original function than other approximations.

5 Part II: The P Controller Design

5.1 Experiment Method

5.1.1 Simulink Simulation Process

The Simulink process is shown in Figure 5. In Figure 5, the close loop system is built, and the Ki is set to the 0, and the Kp is set to the T/L based on the Ziegler-Nichols method in Chapter 3. The value of the T and L is 2.7183 and 0.2817 based on the calculation of Figure 3. So, from Table 2, the Kp is set to the 9.6496 initially.



5.1.2 Adjust the Value of P Controller

In this experiment, KP was adjusted to 9.6496 ± 1, 9.6496 ± 2, 9.6496 ± 8, and 0.00001 to 10000. The purpose of adjusting the size of Kp can be used as the root locus to describe. As shown in Figure 1, the larger the gain is, the more poles move to the two sides of the real axis. Figure 5 shows the experiment in adjusting Kp.

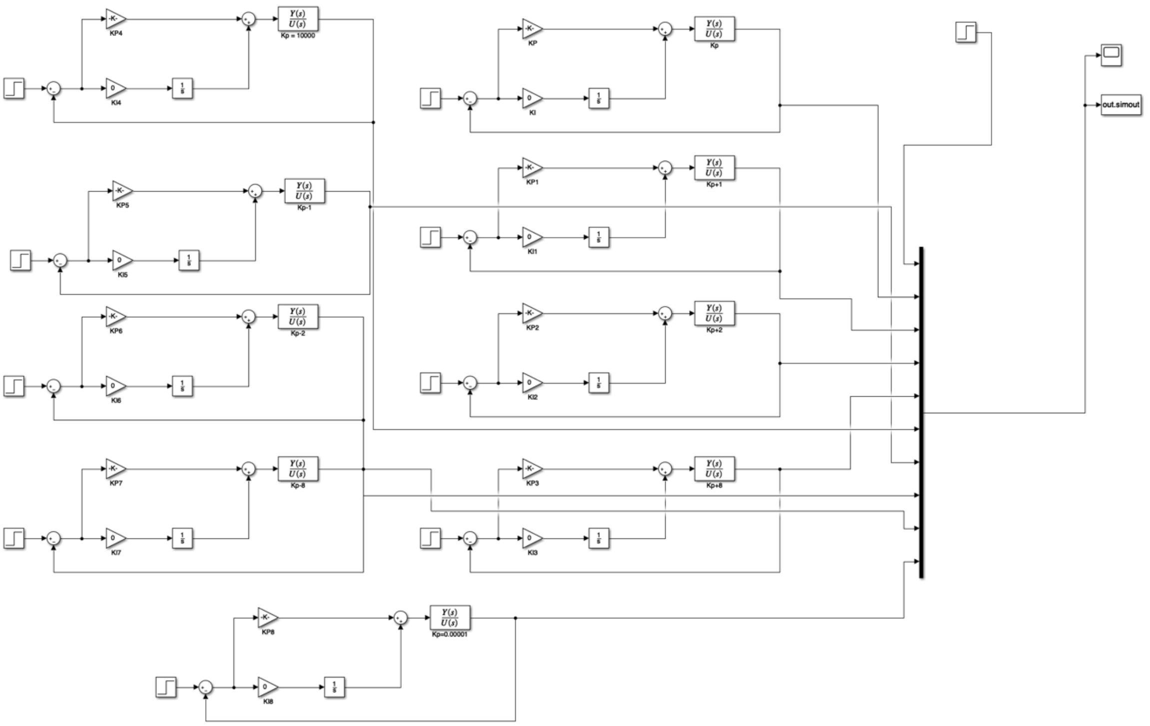


Figure 5. Adjust the value of the P controller

5.2 Experiment Result

The Part II result is shown in Figure 6 and Table 4. From the graph, it can be concluded that the larger the gain is, the larger the fluctuation of the function is, but the smaller the steady-state error is. By using the Ziegler-Nichols method, the Kp is 9.6496, using this value as gain has some advantages. The function fluctuates little and converges quickly, but the problem is that there are steady-state error equals to 0.0943 forms Table 4.

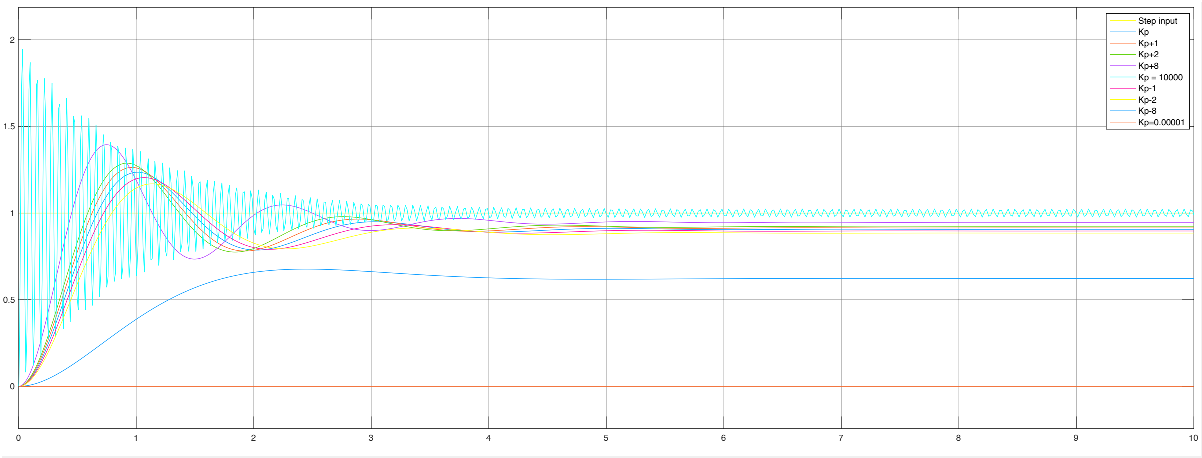


Figure 6. The result of Part II (x-axis: Time (sec), y-axis: y(t))

Table 4. Experiment Data Part II

Type	Steady-state error	%OS	Settling time	Rise Time
Kp = 9.6496	0.0943	36.375	3.436	0.407
Kp = 9.6496 + 1	0.0860	38.189	3.954	0.384
Kp = 9.6496 + 2	0.0791	39.828	3.878	0.364
Kp = 9.6496 + 8	0.0536	47.335	3.876	0.286
Kp = 10000	9.9530e-05	96.677	9.991	0.0109
Kp = 9.6496 - 1	0.1038	34.357	3.591	0.434
Kp = 9.6496 - 2	0.1156	32.110	3.760	0.467
Kp = 9.6496 - 8	0.3747	8.668	3.667	1.160
Kp = 0.00001	0.9979	0	5.805	3.352

5.3 Experiment Analysis and Discussions

5.3.1 Root Locus and Mathematical Analysis

Figure 7 shows the root locus when K from the 0 to the 9.64 in the close loop system. In the K to the 0, the system is critically damped, when the K to the 9.64, the system changes to the underdamped. The step input of inverse Laplace transfer in K equals to the 9.64 can be shown in the formula:

$$C(t) = Ae^{-\theta_d t} cos(\omega_d t - \phi) \tag{18}$$

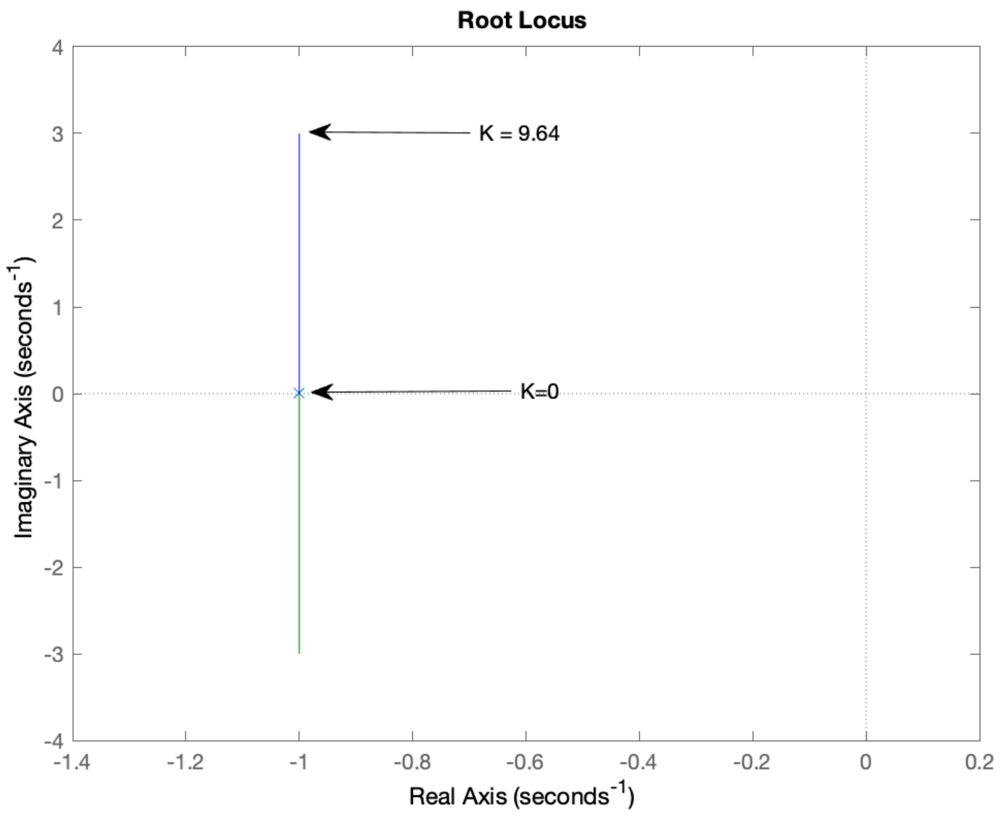


Figure 7. Root locus analysis

When k is large, the properties of the function change, it can be seen from formula (18) that the output function has a trigonometric function. At this time, the function is different from the traditional oscillator, because the exponential function also contributes to the stability of the whole output function. Hence, the amplitude of the output function decreases exponentially, and finally, the function is controlled to a stable range. However, the larger the gain, the larger the vibration amplitude of the output function. As shown in Figure 8, when the gain is close to 0, a closed-loop system is equivalent to no feedback. Therefore, the output characteristic of the output function is similar to that of the open-loop system except for the difference of unit size.

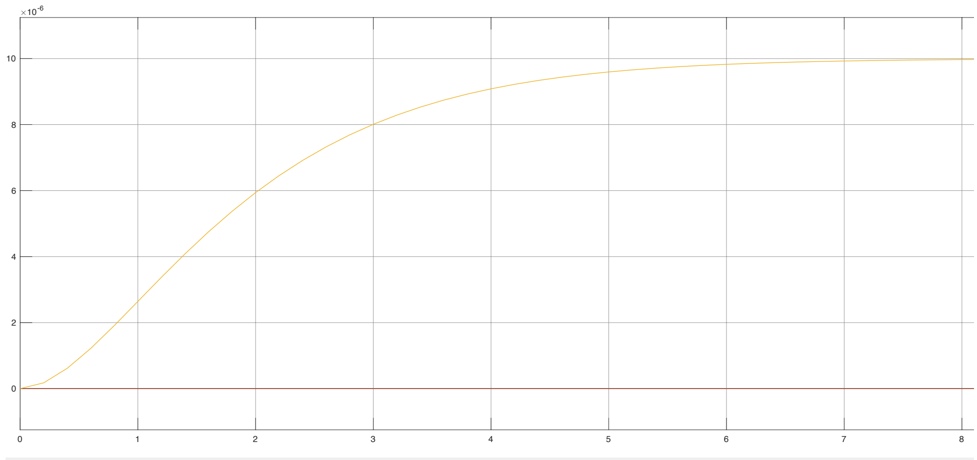


Figure 8. The gain almost to the 0 (x-axis: Time (sec), y-axis: y(t))

5.3.2 Steady-state Error Analysis

The steady-state error of the output function can be proved by the type-0 system. Table 5 shows the relationship between steady-state error and system type.

Table 5. Steady-state error and System type (taken from [10])

Input	Type-0	Type-1	Type-2
Step	1/1+kp	0	0
Ramp	∞	1/kv	0
Parabola	∞	∞	1/ka

The definition of kp is $kp \ = \lim_{s \to 0} G(s)$

However, the gain is 9.6496 by using the Ziegler-Nichols method. The G(s) now is $\frac{9.6496}{(s+1)^2}$. After calculation, the $kp = \lim_{s \to 0} G(s) = 9.6496$, so the type-0 error is 0.0939. So, this kind of relationship can be proved by using error relation.

5.3.3 Observation of other parameters

By observing the other parameters in Table 4, it can also conclude above: The larger the gain, the smaller the error of the output function, but the instability of the output function will be reflected. This is mainly because the type of the output function has changed significantly, and the trigonometric function is added to affect the performance of the OS%, settling time and rise time.

6 Part III: The PI Controller Design

6.1 Experiment Method

6.1.1 Simulink Simulation Process

As shown in Figure 8, the PI controller is used to build the closed-loop system. The value of the Kp and Ki is determined by the Ziegler-Nichols method. As shown in Table 6. T and L are calculated in chapter 3. After calculation, the values of these two parameters are respectively 8.68455(Kp) and 9.24884(Ki).

Table 6. Ziegler-Nichols Table with T and L (taken from [7])

Controller	K_P	K_I	K_D
PI	0.9 T/L	0.27 $\frac{T}{L^2}$	-

Figure 8 uses multiple P, and I parameters to make the image easier to compare.

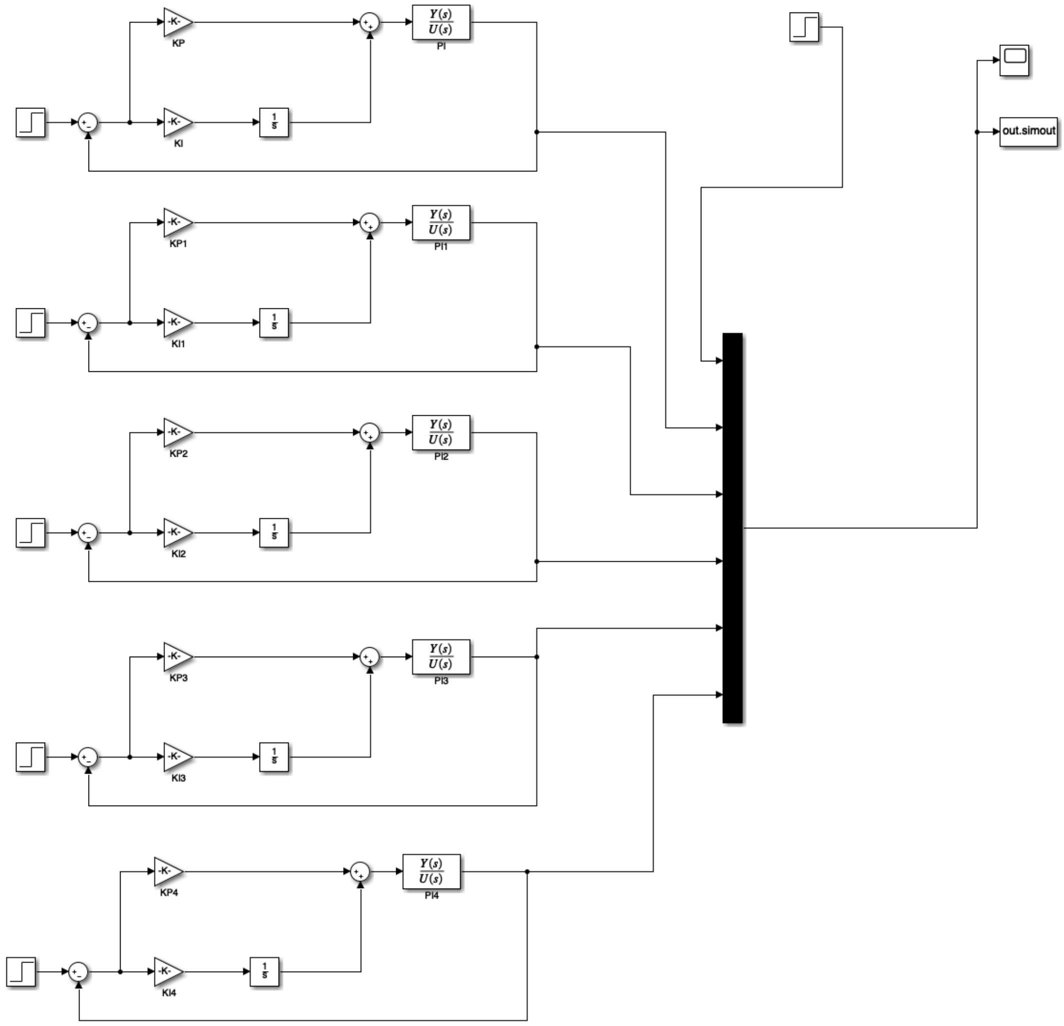


Figure 8. The Part III Simulink Design

6.1.2 Comparison of P and PI controller

Using the Ziegler-Nichols method, the parameter of the PI controller is calculated by using Table 6. Figure 9 shows the experiment setup in the Simulink. In the experiment, the output function of the P controller and PI controller will be used and compared. The parameters of the P controller are calculated by Part II.

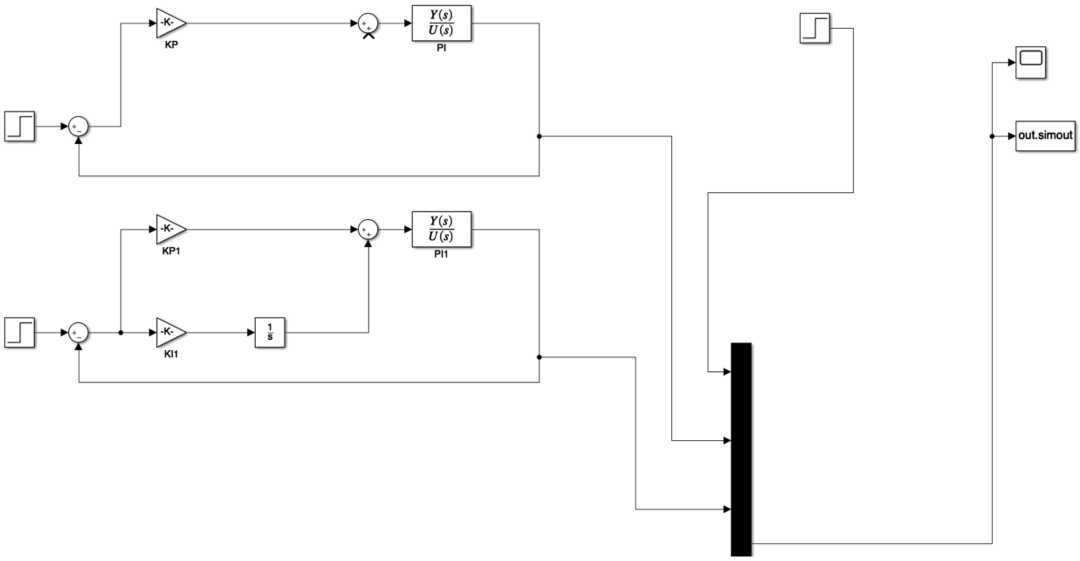


Figure 9. Experiment Setup in Simulink

6.1.3 Adjust the Value of P Controller

In this term, the value of Kp is adjusting from 0.9 T/L± 1, 0.9 T/L± 2, and the value of Ki is keeping to the 0.27 $\frac{T}{L^2}$. This variable helps analyze the effect of the P controller on the image output.

6.1.4 Adjust the Value of I Controller

In this term, the value of Ki is adjusted from 0.27 $\frac{T}{L^2} \pm 1$, 0.27 $\frac{T}{L^2} \pm 2$, and the value of Kp is keeping to the 0.9T/L. This variable helps analyze the effect of the I controller on the image output.

6.2 Experiment Result

6.2.1 Comparison between P and PI controller

As shown in Figure 10, the comparison between the PI controller and the P controller. It shows that the PI controller can let the steady-state error to 0, but the P controller has a steady-state error. The leading causes of this phenomenon can be explained in Table 5. The discussion section has a detailed explanation of this figure.

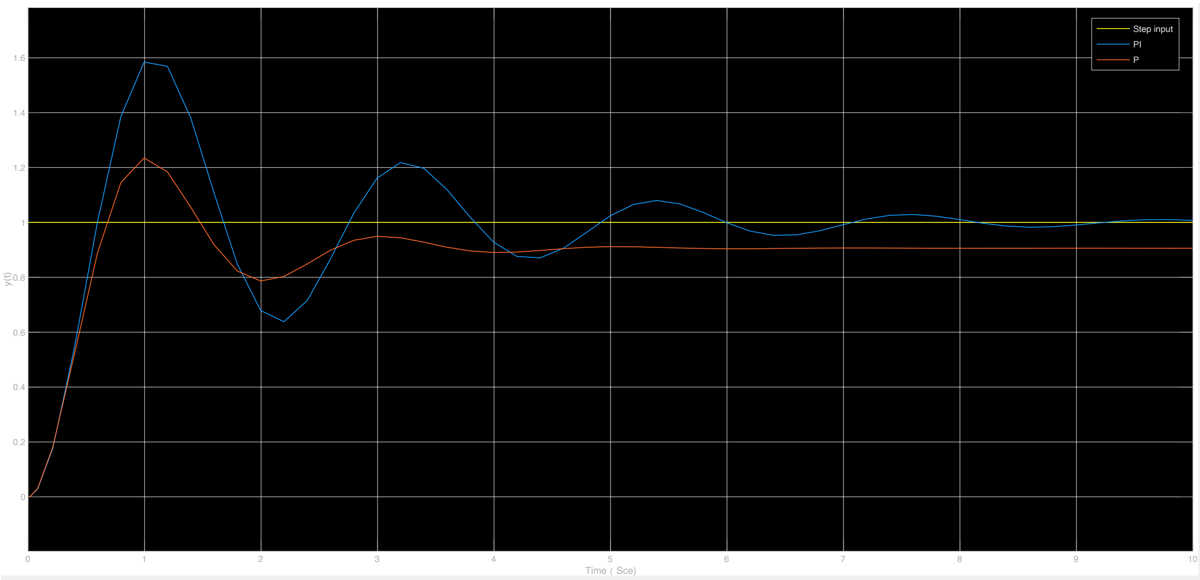


Figure 10. P and PI controller comparison

The experiment data is shown in Table 7.

Table 7. The Part III Comparison between P and PI controller

Type	Steady-state error	%OS	Settling time	Rise Time
PI	0	57.28	8.891	0.409
P	0.094	36.32	3.437	0.422

6.2.2 The Effect of adjusting the Kp

As shown in Figure 11 and Table 8, it can be seen that with the increase of the gain, the fluctuation of the output function increases, and the output function tends to shift to the right.

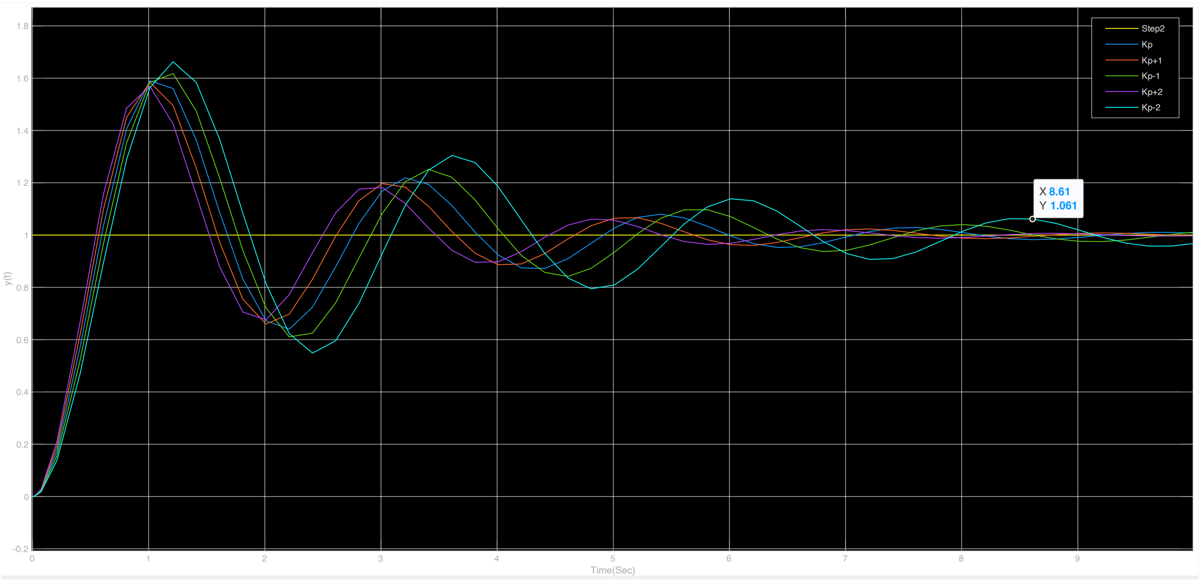


Figure 11. Increase the gain of Kp

Table 8. Kp adjusting data table

Type	Steady-state error	%OS	Settling time	Rise Time
Kp	0	57.8557564035074	8.89212079041936	0.411056462086315
Kp + 1	0	59.0156044506230	7.43135467692999	0.391679408822433
Kp - 1	0	59.9840835805890	9.60042158351390	0.429935451930562

Kp + 2	0	57.3654182171049	7.01250169285134	0.376883407492626
Kp - 2	0	71.8110550992092	9.25019245603786	0.436295244105381

6.2.3 The Effect of adjusting the Ki

From Figure 12, with the increase of Ki, the higher the vibration amplitude of the output function, the OS% data in Table 9 can reflect a trend.

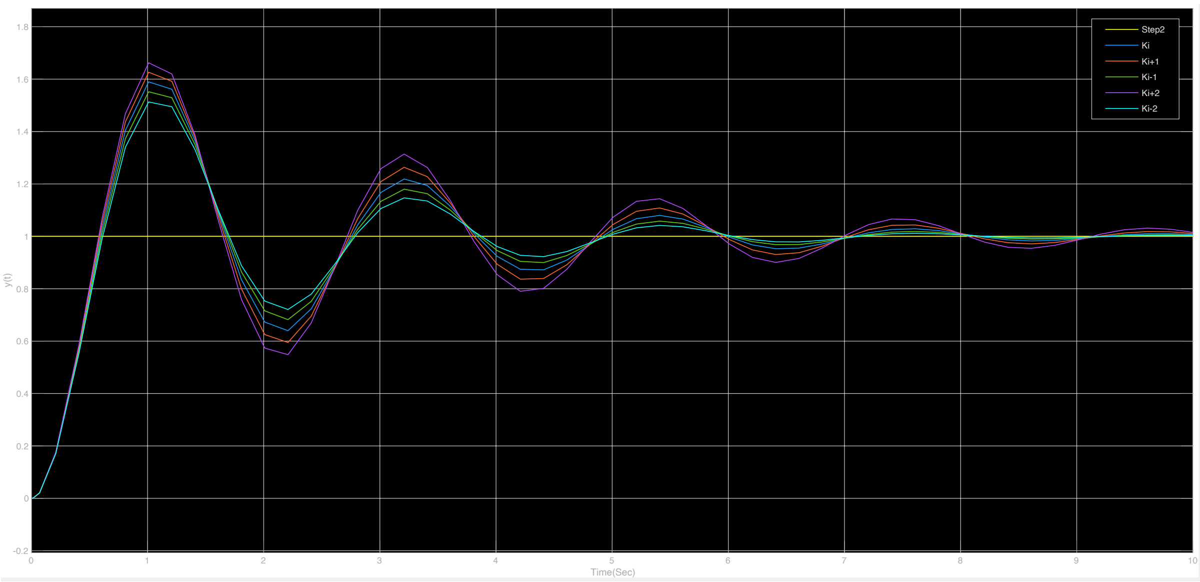


Figure 12. The Effect of adjusting the Ki

Table 9. The data for adjusting the Ki

Type	Steady-state error	%OS	Settling time	Rise Time
Ki	0	57.8070752634068	8.89201823583459	0.411533535662052

Ki + 1	0	60.8565784749104	9.05165417244967	0.406241531298206
Ki - 1	0	54.4876319673362	6.88933359701198	0.417437616144760
Ki + 2	0	63.7381396972224	9.08655689534297	0.401323669880817
Ki - 2	0	50.9039767434962	6.73691036052467	0.424013874170350

6.3 Experiment Analysis and Discussions

6.3.1 Root Locus and Mathematical Analysis

As shown in Figure 13, for the PI controller, one pole and one zero are added into the root locus. In the design of the PI controller, the trajectory can be controlled according to the two parameters of PI. In fact, in the actual design, there needs to be precise requirements to ensure that root locus achieves the actual design purpose.

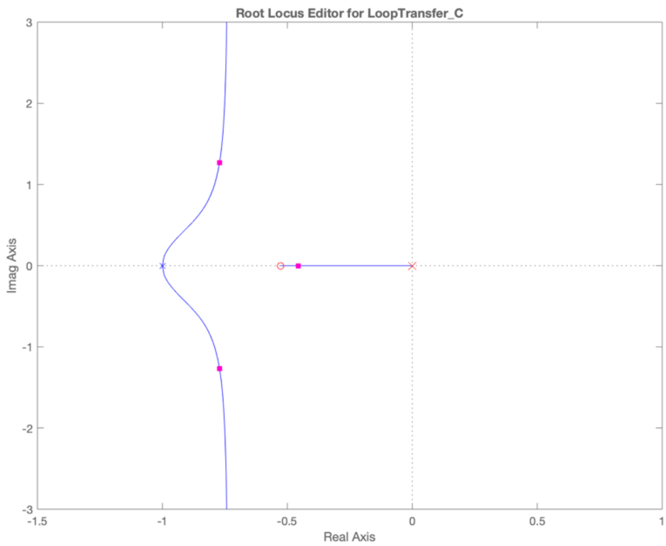


Figure 13. Root locus of PI Controller

6.3.2 Steady-State Error Analysis

As shown in Table 5, the steady-state error by using a PI controller in this system is 0. Because the integral controller adds one pole into the transfer function, the type-0 system becomes a type-1 system, for the step input, the type-1 system's steady-state error is 0 by definition. This proves that the steady-state errors in Table 9 under this condition are 0.

6.3.3 Observation of Effect

By observing the data and images, the following conclusions can be drawn:

- The proportional controller cannot deal with the steady-state error well
- The proportional controller can quickly control the scaling of the output function
- The proportional controller may make the output function unstable in a short time
- The integral controller can deal with the steady-state error well
- When the integral controller parameter increases, it will not change the intersection position of the original output function with step input, but only change the height of the output
- Because the integral controller has reasonable control of the steady-state error, it has a restoring force for the disturbance

7 Part IV: The Effect of Disturbance

7.1 Experiment Method

7.1.1 Simulink Simulation Process

Figure 14 shows that a disturbance of step input was added between the controller and the transfer function. In order to compare the anti-disturbance ability of the proportional controller and the proportional-integral controller, two groups of controllers are set up in the experiment, both of which have the same input and the same disturbance.

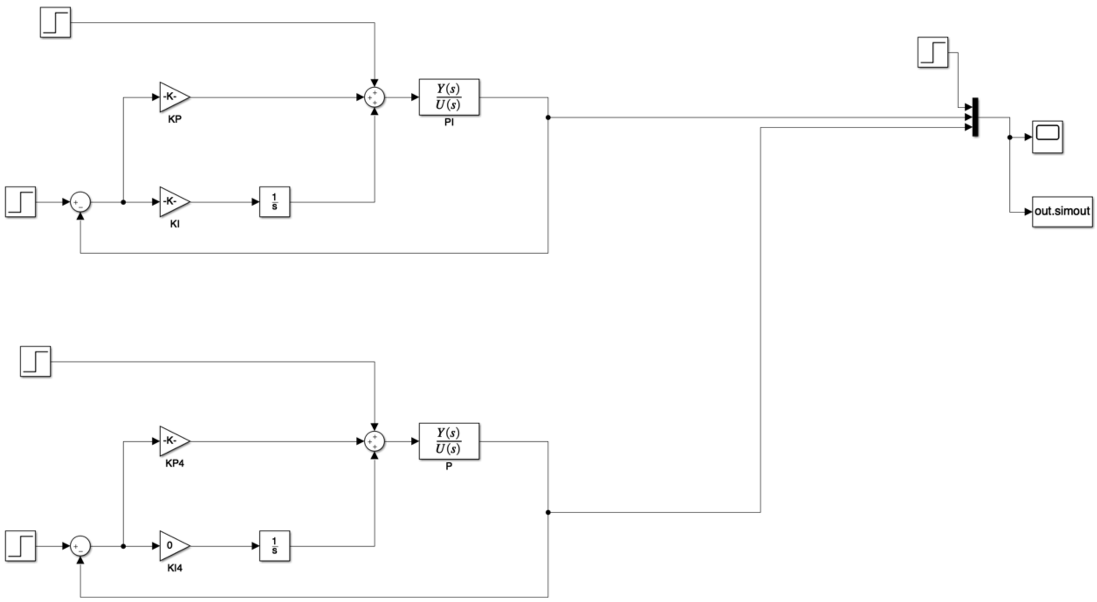


Figure 14. Part IV Experiment Setup

7.2 Experiment Result

Figure 15 shows the result of disturbance between P and PI controller, which parameter is based on Part III. The size of the step input set as a disturbance in the experiment is 10. It can be seen that there is a specific steady-state error between the proportional controller and the input value, but the error of the PI controller is 0. Table 10 shows the maximum output response, settling time, and steady-state error.

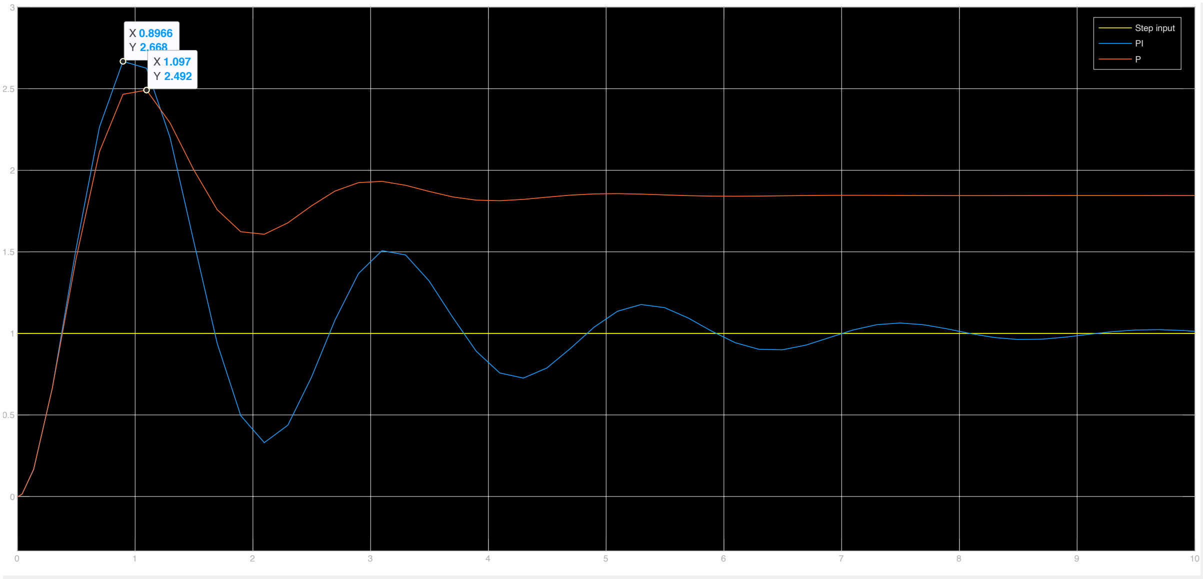


Figure 15. The effect of disturbance between P and Pi control

Table 10. The effect of disturbance

Type	Steady-state error	Settling time	Maximum output response
PI	0	8.931	2.668
P	0.845	3.435	2.492

7.3 Experiment Analysis and Discussions

7.3.1 The Ability of P and PI Controller

Based on the lecture note, the ability of the P and PI controller has shown in Table 11. It can be seen from Table 11, apart from the fast-transient response, the proportional control has no help to the stability and steady-state error of output and small overshoot. For the integral controller, it has the advantages of stability and zero steady-state error. However, both the integral controller and the proportional control exit significant overshoot. Therefore, the PI controller, which combines the disadvantages, will have a substantial overshoot. Figure 15 and Table 10 prove this viewpoint.

Table 11. The ability of P and PI Controller (taken from [10])

Component	Stability	Fast Transient Response	Zero Steady State Error	Small Overshoot
Proportional	_	+	_	_
Integral	+	_ _	+	_ _

7.3.2 Steady-state Error Analysis

According to the above point of view, the integral controller adds a pole, and the pole is at the origin. According to Table 5, the system type increased from the type-0 to the type-1. When the input is a step function, the system error is 0. Compared with it, the steady-state error of the proportional controller is in the controllable range, and the output function can be compensated properly by the compensator, o that the steady-state error of the proportional controller can be close to 0.

7.3.3 Effect of Disturbance

In summary, the proportion-integral controller can remove the effect of the disturbance in a steady-state, but the proportion controller cannot remove the effect because of the proportional controller in the mathematical aspects of its defects.

8 Part V: The PID Controller Design

8.1 Experiment Method

This section uses two methods as the design of the PID controller. The first method uses the Ziegler-Nichols method to calculate PID parameters, and the second method uses MATLAB control system designer to design.

8.1.1 Ziegler-Nichols Method and Simulink Simulation Process

Based on Table 2, the Kp is set to the 1.2 T/L, Ki is set to the 0.6 $\frac{T}{L^2}$ and Kd is set to the 0.6 T. The calculated parameters are as follows in Table 12:

Table 12: Parameters Setup In PID

Type	P	I	D
PID	11.579	20.552	1.630

Figure 16 shows the Simulink process. The PID controller is set up.

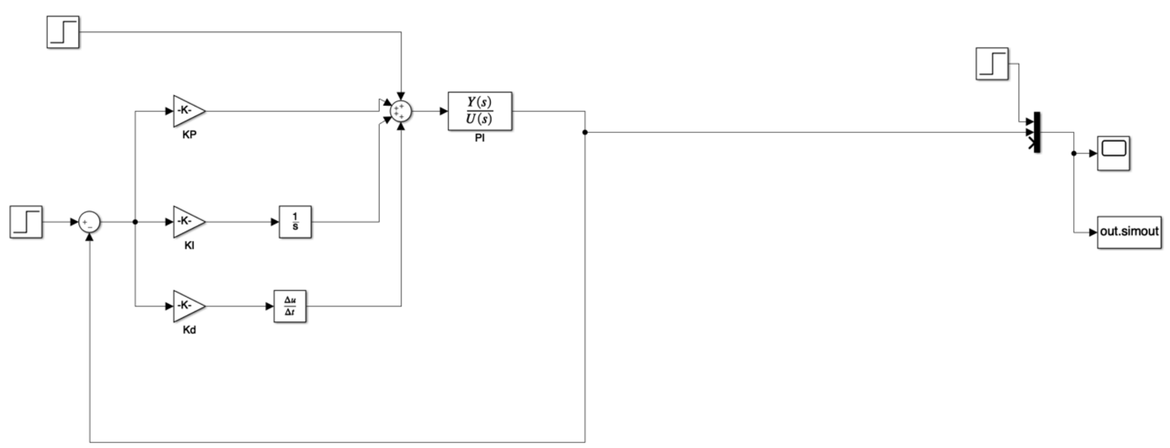


Figure 16. PID Controller Design

8.2 Experiment Result

The experimental results show that the PID controller has a reasonable control for step disturbance. Figure 17 shows the disturbance in P, PI, and PID controller.

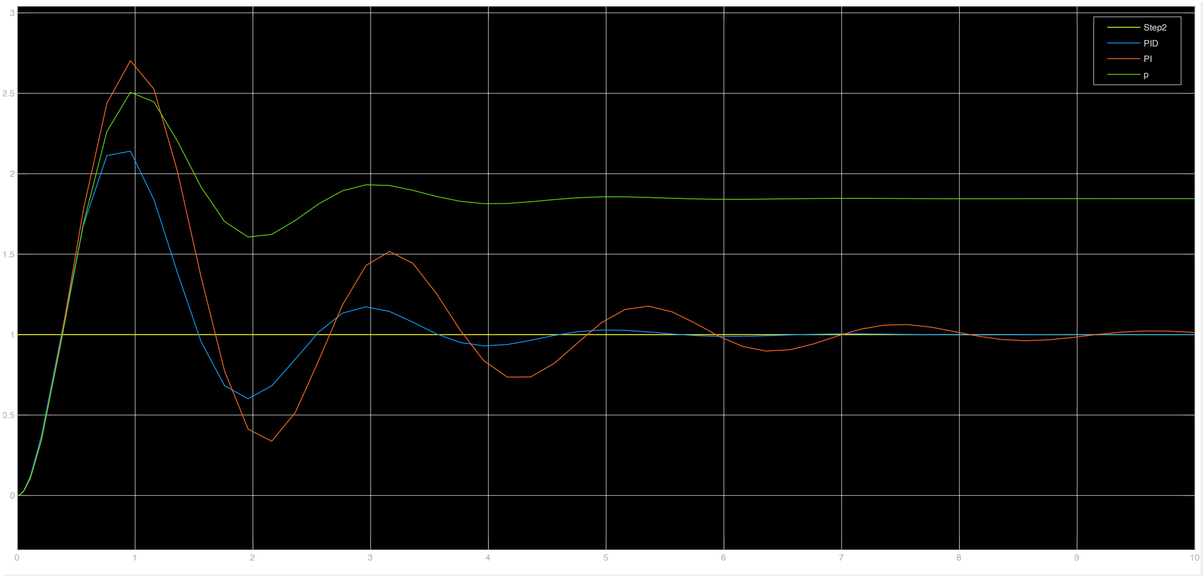


Figure 17. The P, PI and PID controller with disturbance

9 Conclusion

Based on the above experiments, it can be concluded that the following experiments: Lifting system type can effectively reduce the steady-state error. The FOPDT is a way to approximate the higher-order system. Using the Ziegler Nichols method is a quick way to select PID parameters, but the parameters may need to be further changed according to the requirements. Finally, the parameters of proportional control and proportional-integral control also discussed and observed the change of output function in detail, and the reason for this kind of phenomenon was explained by root locus and other methods.

References

[1] A. Al-Ataby, "Experiment 81 - Design of a Feedback Control System," 2015.

[2] J. B. Dabney and T. L. Harman, *Mastering simulink*. Pearson, 2004.

[3] M. Chaturvedi and P. Juneja, "Effect of dead time approximation on controller performance designed for a second order delayed model," in *2013 International Conference on Advanced Electronic Systems (ICAES)*, 2013: IEEE, pp. 313-315.

[4] C. C. Hang, K. J. Åström, and W. K. Ho, "Refinements of the Ziegler–Nichols tuning formula," in *IEE Proceedings D (Control Theory and Applications)*, 1991, vol. 138, no. 2: IET, pp. 111-118.

[5] M. Senra, "Approximating Higher-Order Transfer Functions as FOPDTs Lecture Note," 2014.

[6] M. Abramowitz and I. Stegun, "1970," *Handbook of mathematical functions: Dover, New York*.

[7] J. G. Ziegler and N. B. Nichols, "Optimum settings for automatic controllers," *trans. ASME,* vol. 64, no. 11, 1942.

[8] J. Zhong, "PID controller tuning: A short tutorial," *Mechanical Engineering, Purdue University,* pp. 1-10, 2006.

[9] s. sharma, "Matlab code for PID tuning using Ziegler-Nichols Method 1 Lecture Note."

[10] N. S. Nice, "Control System Engineering," *Addison-Westley Publishing Company,* pp. 240-260, 1995.