

PSTAT 126 HW4

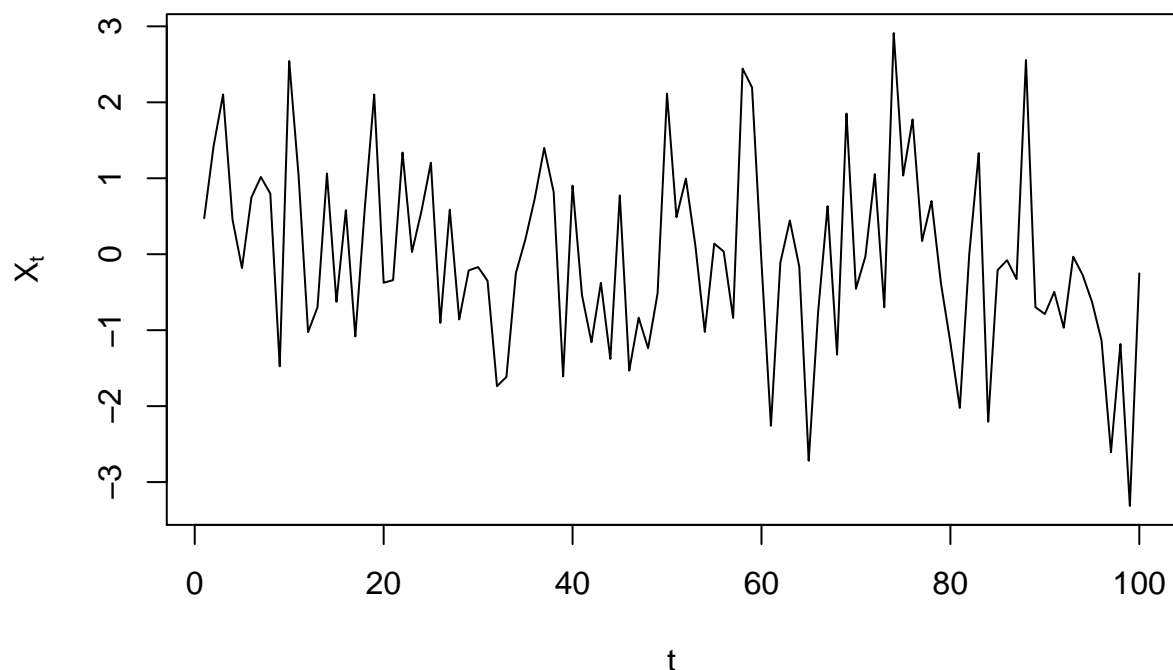
Mujie Wang

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ARMA(1,1)

```
x1 <- arima.sim(n = 100,model = list(ar = -0.6,ma = 1.2))
plot(x1, xlab = "t",ylab = expression(X[t]),
     main = expression(paste(ARMA(1,1),phi[1] == -0.6,theta[1] == 1.2,sep = " ")))
```

ARMA(1, 1) $\phi_1 = -0.6\theta_1 = 1.2$

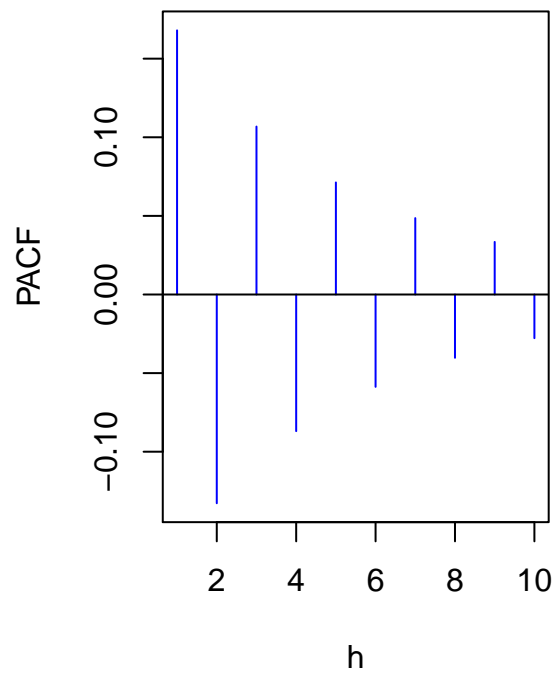
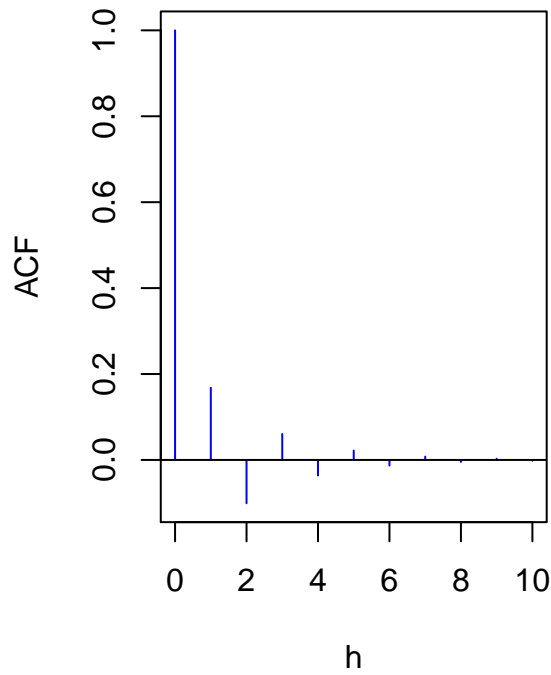


(ii)

compute and graph their theoretical ACF and PACF using R

```
# compute ACF and PACF
acf1 = ARMAacf(ar = -0.6,ma = 1.2,pacf = FALSE,lag.max = 10)
pacf1 = ARMAacf(ar = -0.6,ma = 1.2,pacf = TRUE,lag.max = 10)
# Plot
op <- par(mfrow = c(1,2))
# ACF
plot(x = 0:10,y = acf1,type = "h",col = "blue",ylab = "ACF",xlab = "h")
abline(h = 0)
#PACF (Theoretical PACF values are computed starting form lag = 0)
plot(x = 1:10,y = pacf1,type = "h",col = "blue",ylab = "PACF",xlab = "h")
abline(h = 0)
title(expression(paste(ARMA(1,1),phi[1] == -0.6,theta[1] == 1.2,sep = " ")),
      line = -1, outer=TRUE)
```

$$\text{ARMA}(1, 1)\phi_1 = -0.6\theta_1 = 1.2$$

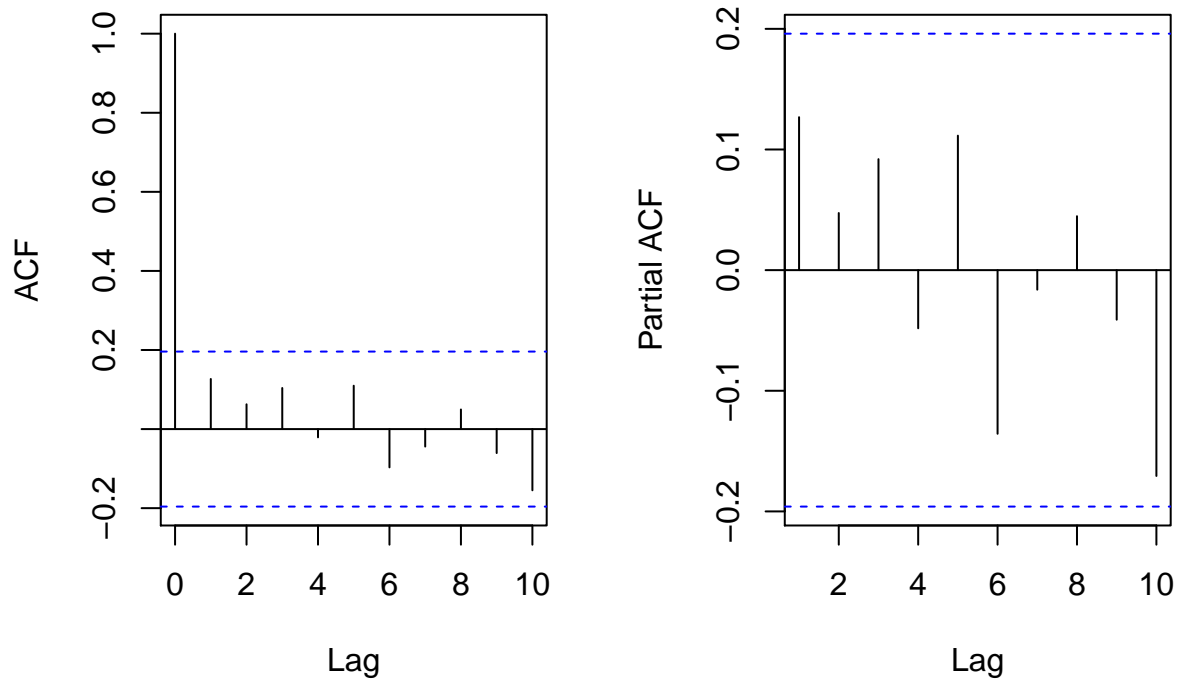


```
par(op)
```

(iii) compute and graph their sample ACF and PACF using R. How do sample functions compare to their theoretical counterparts?

```
op <- par(mfrow = c(1,2))
#ACF
acf(x1,lag.max = 10,main = "")
# PACF
pacf(x1,lag.max = 10,main = "")
title(expression(paste("Simulated ARMA(1,1): ",phi[1] == -0.6,theta[1] == 1.2,sep = " ")),
      line = -1, outer=TRUE)
```

Simulated ARMA(1,1): $\phi_1 = -0.6\theta_1 = 1.2$



```
par(op)
```

Both sample ACF and PACF cut off after lag 1, and remain in the confidence band (blue dashed lines), which matches with the theoretical ACF/PACF behavior; both spike at 1, and then decay exponentially.

- (iv) Analyze smoothness of the simulated processes using their ACF/PACFs. Please include the code with clear comments explaining the meaning of the code. Make sure to label the graphs.

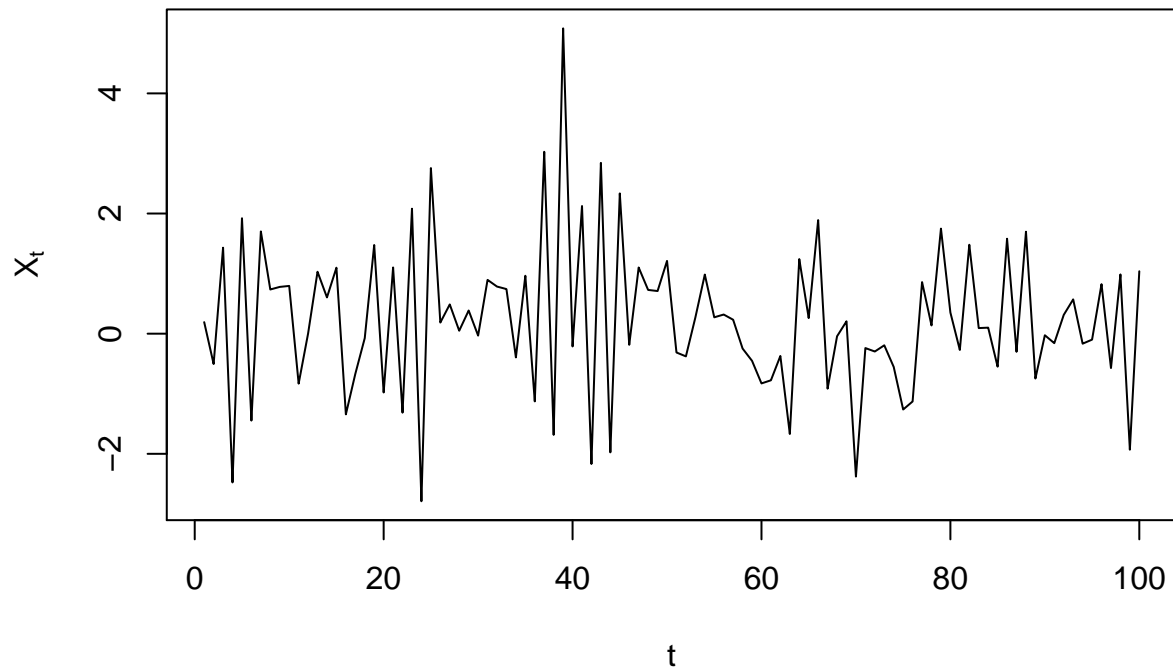
It is not smooth because the ACF oscillates between positive and negative values and it then decays very rapidly, which implies small dependence on the previous values.

(b)

- (c) Simulate and plot 100 values of the processes

```
# ARMA(2,0) = AR(2)
x2 <- arima.sim(n = 100,model = list(ar = c(-0.2,0.48)))
plot(x2, xlab = "t",ylab = expression(X[t]), main = expression(paste(AR(2),phi[1] == -0.2,phi[2] == 0.48)))
```

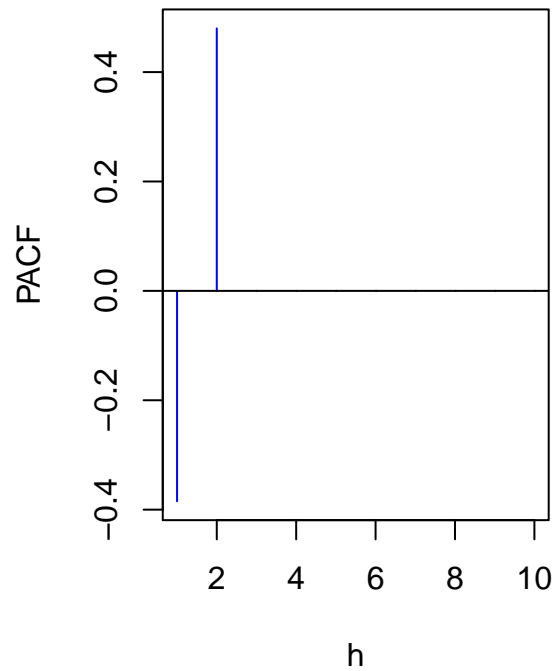
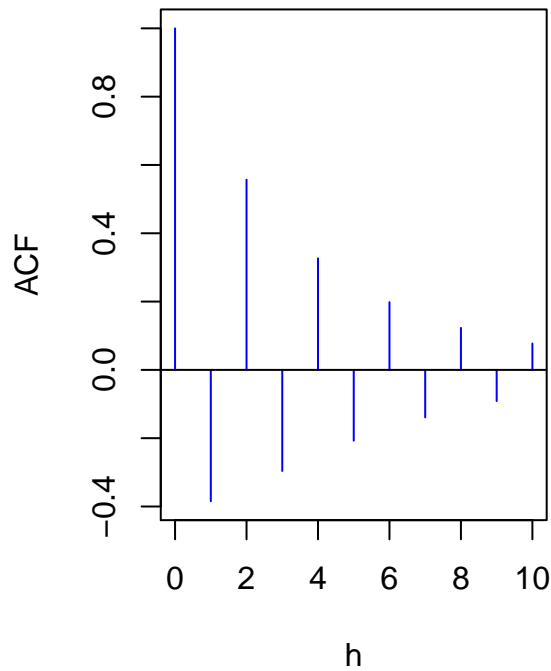
$$\text{AR}(2)\phi_1 = -0.2\phi_2 = 0.48$$



(ii) compute and graph their theoretical ACF and PACF using R

```
# Compute ACF/PACF
acf2 = ARMAacf(ar = c(-0.2,0.48),pacf = FALSE,lag.max = 10)
pacf2 = ARMAacf(ar = c(-0.2,0.48),pacf = TRUE,lag.max = 10)
op <- par(mfrow = c(1,2))
# ACF
plot(x = 0:10,y = acf2,type = "h",col = "blue",ylab = "ACF",xlab = "h")
abline(h = 0)
#PACF (Theoretical PACF values are computed starting form lag = 0)
plot(x = 1:10,y = pacf2,type = "h",col = "blue",ylab = "PACF",xlab = "h")
abline(h = 0)
title(main = expression(paste(AR(2),phi[1] == -0.2,phi[2] == 0.48,sep = " ")),
      line = -1, outer=TRUE)
```

$$\text{AR}(2)\phi_1 = -0.2\phi_2 = 0.48$$

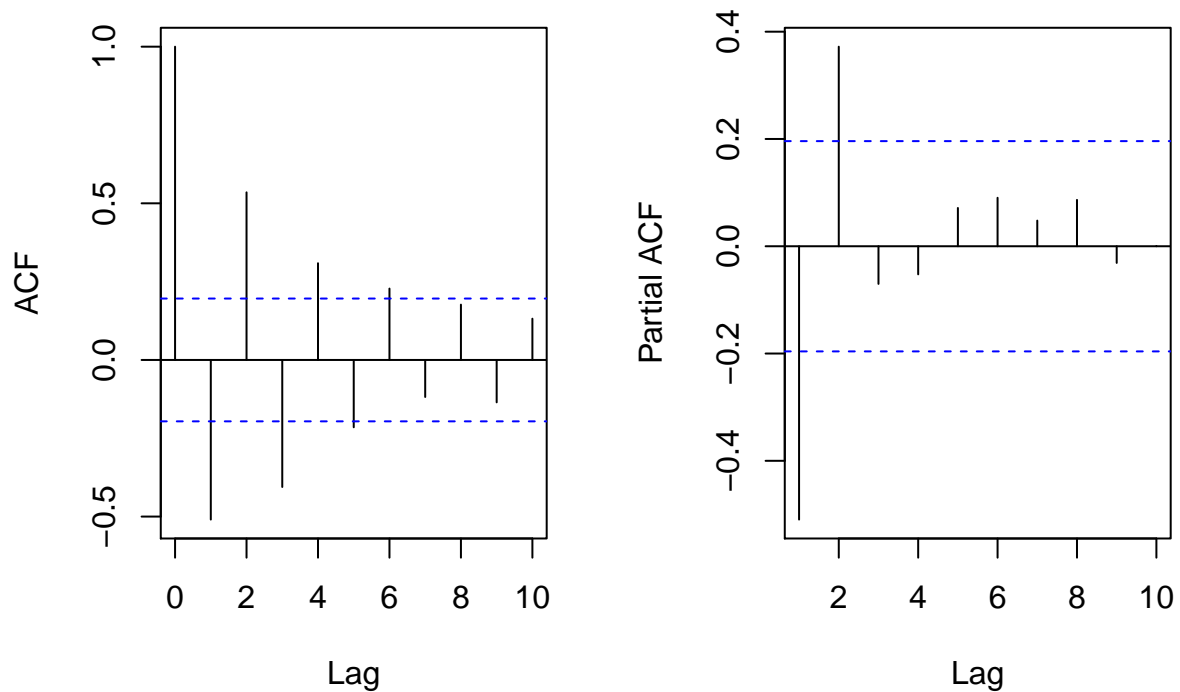


```
par(op)
```

(iii) compute and graph their sample ACF and PACF using R. How do sample functions compare to their theoretical counterparts?

```
op <- par(mfrow = c(1,2))
# ACF
acf(x2,lag.max = 10,main = "") # PACF
pacf(x2,lag.max = 10,main = "")
title(main = expression(paste("Simulated AR(2): ",phi[1] == -0.2,phi[2] == 0.48,sep = " ")),
      line = -1, outer=TRUE)
```

Simulated AR(2): $\phi_1 = -0.2, \phi_2 = 0.48$



```
par(op)
```

The overall behaviour of the sample and theoretical ACF/PACF is the same: the first 2 sample PACF values are significantly different than zero and its sign is the same. Moreover, the sample PACF lie within the estimated 95% confidence bounds after lag-2. On the otherhand, the ACF decays exponentially as the lag increases oscillating between positive and negative values (due to the change in sign of the AR coefficients).

- (iv) Analyze smoothness of the simulated processes using their ACF/PACFs. Please include the code with clear comments explaining the meaning of the code. Make sure to label the graphs.

Since the sign of the ACF oscillates and the magnitude of the ACF decays proportionally with respect to the lag, then this indicates that the time series will change vary rapidly so the time series is not smooth.