< Variational Autoencoder >

$$\int_z q(z|X) = 1$$

$$p(z,x) = p(z|x)\,p(x)$$

$$E[X] = \int x f(x)$$

$$D_{KL}(P\|Q) = \int p(x) \log \frac{p(x)}{q(x)}$$

$f_\theta(x|z)$

$$X \longrightarrow z \xrightarrow{p(x|z)} X$$

모든 z에 대해 Maximum Likelihood 인 θ를 구하고 싶다.
즉, $P(X)$를 최대로 하는 θ를 구하는 것

MLE (Maximum log likelihood)로 바꿔서 적어보면

$$\log P(X) = \int q(z|X) \log P(X)\, dz$$
$$= \int q(z|X) \log \frac{P(z,x)}{P(z|X)}\, dz$$
$$= \int q(z|x) \log \frac{P(z,x)}{q(z|X)} \frac{q(z|x)}{p(z|x)}\, dz$$
$$= \underbrace{\int q(z|x) \log \frac{P(z,x)}{q(z|x)}\, dz}_{\text{ELBO}} + \underbrace{\int q(z|x) \log \frac{q(z|x)}{p(z|x)}\, dz}_{\text{KL Divergence 정의}}$$

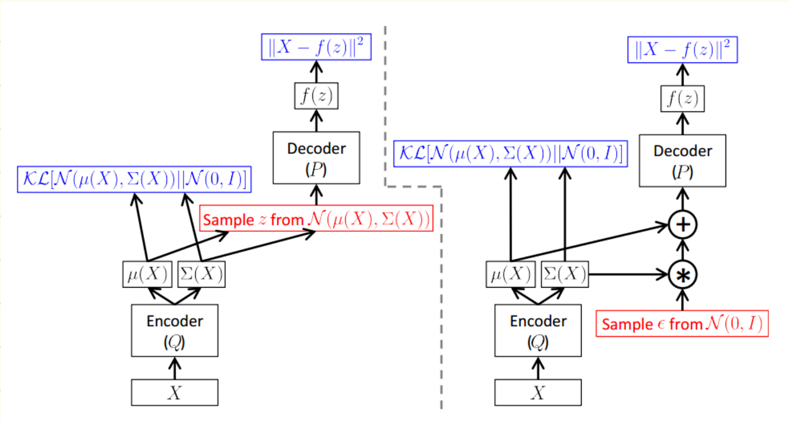
$$= \text{ELBO} + \underbrace{KL(q(z|x)\,\|\,p(z|x))}_{\geq 0}$$

$$\text{ELBO} = \int q(z|x) \log \frac{p(z,x)}{q(z|x)}\, dz = \int q(z|x) \log \frac{p(x|z)\,p(z)}{q(z|x)}\, dz$$
$$= -\int q(z|x) \log \frac{q(z|x)}{p(z)}\, dz + \int q(z|x) \log P(x|z)\, dz$$
$$= \underbrace{-KL(q(z|x)\,\|\,p(z))}_{\text{KL Divergence}} + \underbrace{E_{q(z|x)}[\log P(x|z)]}_{\text{Reconstruction}}$$

$$-KL(q(z|x)||P(z)) = \frac{1}{2}\sum_{j=1}^{J}(1+\log \sigma_j^2 - \mu_j^2 - \sigma_j^2)$$

Gaussian $\quad$ $N(0,I)$인 Gaussian $\quad$ 증명은 논문 Appendix 참고

- Reparameterization Trick



왼쪽처럼 Sampling하는 방식은 미분 불가능하기 때문에
같은 효과를 내면서 미분이 가능하도록 해주는 방법

$z \sim N(\mu, \sigma^2) \rightarrow z = \mu + \sigma \cdot \epsilon$ where $\epsilon \sim N(0,1)$