EN3330 Assignment 03

M. T. U. Sampath K. Perera

1. Consider the constrained optimization problem

$$\min_x x \quad \text{subject to } x \geq 5. \tag{1}$$

Using the quadratic penalty method, the unconstrained penalized objective becomes

$$f(x) = x + \rho\big(\max(5-x,0)\big)^2, \tag{2}$$

where ρ is the penalty parameter.

(a) The optimization problem given in eq. (1) is transformed to problem given in eq. (2). Analyze the suitability of this transform by graphically. [marks 60]

(b) What is the sign of ρ ($\rho > 0$ or $\rho < 0$). Decide with a justification. [marks 15]

2. Consider the constrained problem

$$\min_x (x-5)^3 \quad \text{s.t.} \quad x \geq 10.$$

Similar to question 1, the following transformation is used.

$$\min_x (x-5)^3 + \rho\big(\max\{0,\ 10-x\}\big)^2,$$

where $\rho > 0$ is the penalty parameter.

(a) Analyze the suitability of this transform by graphically and decide it is a correct transformation or not. If it is not correct propose a transformation. [marks 25]

3. Consider the Lagrangian function of a constrained optimization problem:

$$L(\mu, \boldsymbol{x}) = f(\boldsymbol{x}) + \mu h(\boldsymbol{x}),$$

where the constraint is $h(\boldsymbol{x}) \leq 0$. The Lagrange multiplier is denoted by μ. The Lagrangian dual function is defined as

$$g(\boldsymbol{x}) = \max_\mu L(\mu, \boldsymbol{x}) = \max_\mu \big(f(x) + \mu h(\boldsymbol{x})\big).$$

(a) Assume that $h(\boldsymbol{x}) > 0$ and $\mu > 0$. Graphically show what happens to $\max_{\mu \geq 0} L(\mu, \boldsymbol{x})$ as $\mu \to \infty$. [marks 40]

(b) Now, suppose $h(\boldsymbol{x}) > 0$ and allow $\mu < 0$. Explain why this situation cannot be part of the Lagrangian dual maximization. [marks 30]

(c) Show why $\mu \geq 0$ is required in order for the Lagrangian dual problem to be well-defined. Further, show that this condition guarantees constrain is satisfied. [marks 30]

Submission

- Upload a report as a pdf file named as "Yourindexno_EN3330_A03.pdf". Include the index number and the name within the report as well.
- The interpretation of results and the discussion are important in the report.
- Pay careful attention to formatting such as font size, spacing, and margins.
- Include a title page with necessary information (e.g., title, author, date, index no).
- Use consistent and professional formatting throughout the document.
- Plagiarism will be checked and in cases of plagiarism, an extra penalty of 50% will be applied. In case of copying from each other, both parties involved will receive a grade of zero for the assignment. Academic integrity is of utmost importance, and any form of plagiarism[1] or cheating will not be tolerated.
- An extra penalty of 20% is applied for late submission.

[1] https://en.wikipedia.org/wiki/Plagiarism