Controllo di un UAV con traiettoria elicoidale intorno ad un oggetto

UNIVERSITÀ
DEGLI STUDI
DI PALERMO

Mobile and distribuited robotics
Corso magistrale in ingegneria elettronica

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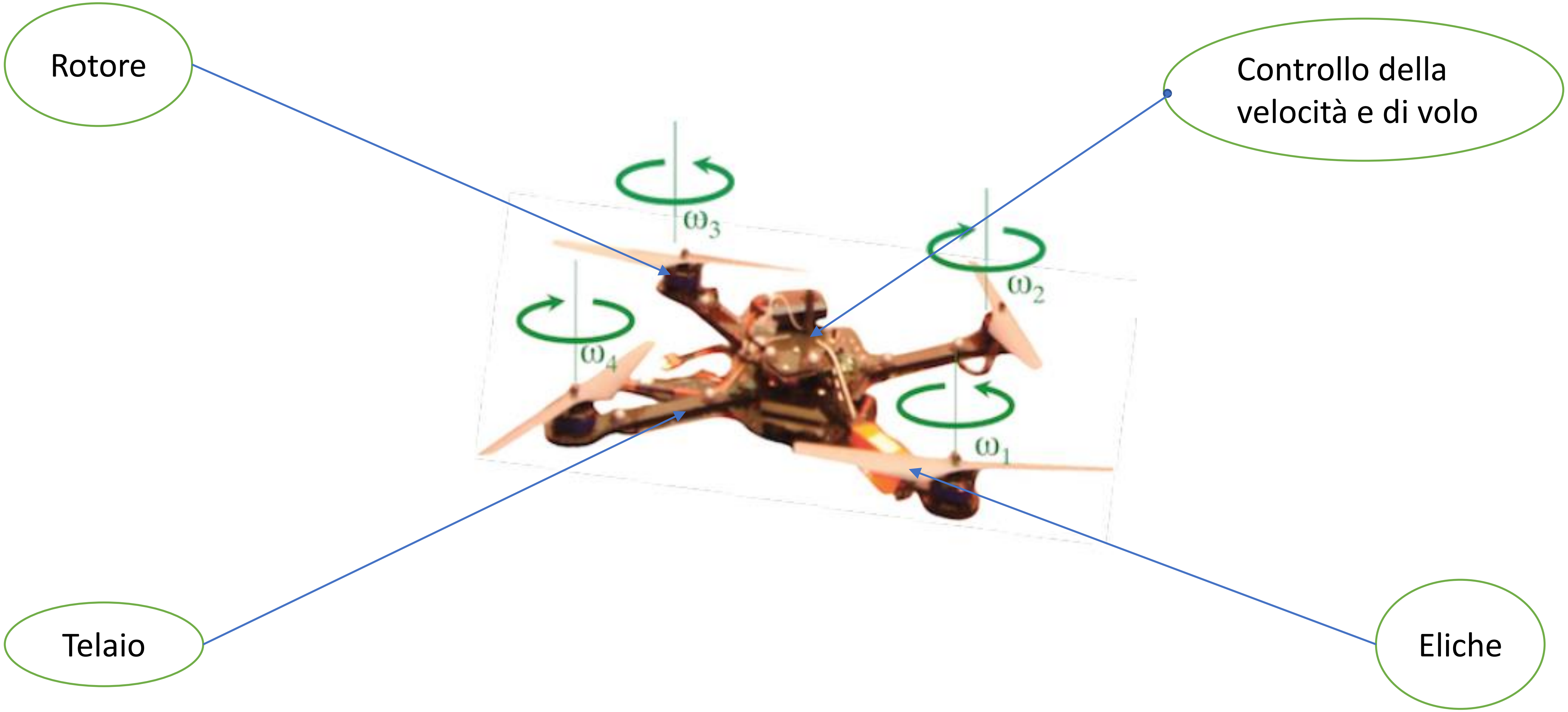
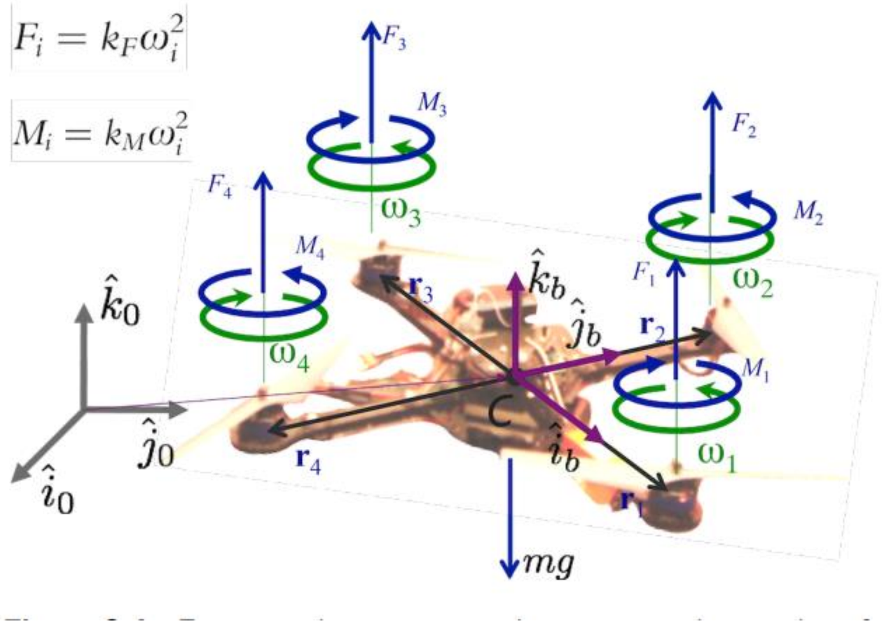
Struttura del quadricottero



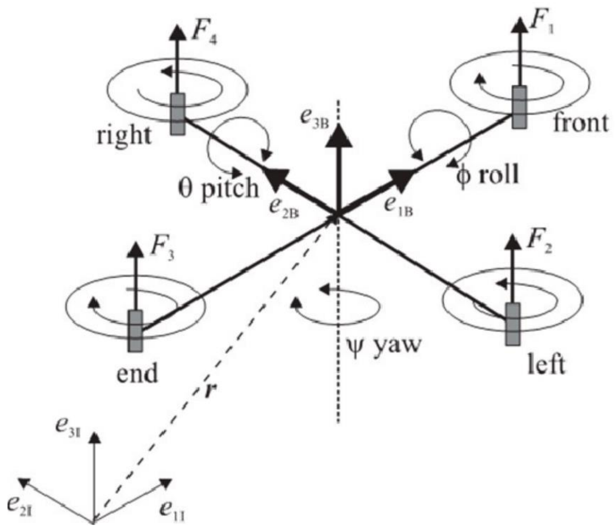
Introduzione teorica

- Modello generale 3D del quadricottero
- Controllo dell’orientamento
- Controllo della posizione



Modello completo 3D del quadricottero

$$\begin{pmatrix} f \\ \tau_\varphi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ 0 & \frac{l}{I_{xx}} & 0 & -\frac{l}{I_{xx}} \\ -\frac{l}{I_{yy}} & 0 & \frac{l}{I_{yy}} & 0 \\ \frac{k_M}{k_F I_{zz}} & -\frac{k_M}{k_F I_{zz}} & \frac{k_M}{k_F I_{zz}} & -\frac{k_M}{k_F I_{zz}} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$



$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} (s_\theta c_\psi + s_\varphi c_\theta s_\psi) f \\ (s_\theta s_\psi - s_\varphi c_\theta c_\psi) f \\ c_\varphi c_\theta f - g \end{pmatrix},$$

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} c_\theta & 0 & -c_\varphi s_\theta \\ 0 & 1 & s_\varphi \\ s_\theta & 0 & c_\varphi c_\theta \end{pmatrix}^{-1} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \tau_\varphi - \frac{I_{zz}-I_{yy}}{I_{xx}} q\, r \\ \tau_\theta - \frac{I_{xx}-I_{zz}}{I_{yy}} p\, r \\ \tau_\psi - \frac{I_{zz}-I_{yy}}{I_{zz}} p\, q \end{pmatrix}.$$

$$\Omega \stackrel{\text{def}}{=} (p, q, r)^T$$

Velocità angolari riferite al corpo fisso

$$\dot{\varphi},\ \dot{\theta},\ \text{and}\ \dot{\psi}$$

Velocità angolari degli angoli di Eulero per Roll,Pitch e Yaw

Controllo dell'orientamento del quadricottero

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} c_\theta & 0 & -c_\varphi s_\theta \\ 0 & 1 & s_\varphi \\ s_\theta & 0 & c_\varphi c_\theta \end{pmatrix}^{-1} \begin{pmatrix} p \\ q \\ r \end{pmatrix},$$

$$\begin{pmatrix} I_{xx}\,\dot{p} \\ I_{yy}\,\dot{q} \\ I_{zz}\,\dot{r} \end{pmatrix} = -\begin{pmatrix} (I_{zz}-I_{yy})\,q\,r \\ (I_{xx}-I_{zz})\,p\,r \\ (I_{xx}-I_{yy})\,p\,q \end{pmatrix} + \begin{pmatrix} l\,k_F(\Omega_2^2-\Omega_4^2) \\ l\,k_F(\Omega_3^2-\Omega_1^2) \\ k_M(\Omega_1^2-\Omega_2^2+\Omega_3^2-\Omega_4^2) \end{pmatrix}$$

In bilico ipotizzo $\varphi, \theta \approx 0$

$\dot{\varphi} \approx p$, $\dot{\theta} \approx q$, and $\dot{\psi} \approx r$.

$$\begin{pmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \begin{pmatrix} \frac{l\,k_F}{I_{xx}}(\Omega_2^2-\Omega_4^2) - \frac{I_{zz}-I_{yy}}{I_{xx}}\,\dot{\theta}\,\dot{\psi} \\ \frac{l\,k_F}{I_{yy}}(\Omega_3^2-\Omega_1^2) - \frac{I_{xx}-I_{zz}}{I_{yy}}\,\dot{\varphi}\,\dot{\psi} \\ \frac{k_M}{I_{zz}}(\Omega_1^2-\Omega_2^2+\Omega_3^2-\Omega_4^2) - \frac{I_{xx}-I_{yy}}{I_{zz}}\,\dot{\varphi}\,\dot{\theta} \end{pmatrix}$$

$$\delta\psi = \psi - \psi_d,\ \delta\Omega_i = \Omega_i - \Omega_0,$$

$$C_\varphi = \frac{l\sqrt{mgk_F}}{I_{xx}},\ C_\theta = \frac{l\sqrt{mgk_F}}{I_{yy}} \text{ and } C_\psi = \frac{k_M\sqrt{mgk_F}}{I_{zz}}$$

$$\begin{pmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \delta\ddot{\psi} \end{pmatrix} = \begin{pmatrix} C_\varphi(\delta\Omega_2 - \delta\Omega_4) \\ C_\theta(\delta\Omega_3 - \delta\Omega_1) \\ C_\psi(\delta\Omega_1 - \delta\Omega_2 + \delta\Omega_3 - \delta\Omega_4) \end{pmatrix}$$

Ponendo un controllo PD

$$\begin{pmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \begin{pmatrix} -k_{v,\varphi}\dot{\varphi} - k_{p,\varphi}\delta\varphi \\ -k_{v,\theta}\dot{\theta} - k_{p,\theta}\delta\theta \\ -k_{v,\psi}\dot{\psi} - k_{p,\psi}\delta\psi \end{pmatrix}$$

$$k^*_{v,\varphi} = \frac{k_{v,\varphi}}{C_\varphi} = \frac{I_{xx}k_{v,\varphi}}{l\sqrt{mgk_F}},\quad k^*_{p,\varphi} = \frac{I_{xx}k_{p,\varphi}}{l\sqrt{mgk_F}}$$

$$k^*_{v,\theta} = \frac{I_{yy}k_{v,\theta}}{l\sqrt{mgk_F}},\quad k^*_{p,\theta} = \frac{I_{yy}k_{p,\theta}}{l\sqrt{mgk_F}},$$

$$k^*_{v,\psi} = \frac{I_{zz}k_{v,\psi}}{k_M\sqrt{mgk_F}},\quad k^*_{p,\psi} = \frac{I_{zz}k_{p,\psi}}{k_M\sqrt{mgk_F}}.$$

$$\delta\varphi \stackrel{\text{def}}{=} \varphi - \varphi_c,\ \delta\theta \stackrel{\text{def}}{=} \theta - \theta_c,$$

Legge di controllo per l'orientamento del quadricottero

$$\begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{pmatrix} = \frac{1}{\sqrt{mgk_F}} \begin{pmatrix} \frac{1}{4} & 0 & \frac{I_{yy}}{2l} & -\frac{I_{zz}}{k_M} \\ \frac{1}{4} & -\frac{I_{xx}}{2l} & 0 & \frac{I_{zz}}{k_M} \\ \frac{1}{4} & 0 & -\frac{I_{yy}}{2l} & -\frac{I_{zz}}{k_M} \\ \frac{1}{4} & \frac{I_{xx}}{2l} & 0 & \frac{I_{zz}}{k_M} \end{pmatrix} \begin{pmatrix} 2mg + \sqrt{mgk_F}\,\delta\Omega_z \\ k_{v,\varphi}\dot{\varphi} + k_{p,\varphi}(\varphi - \varphi_c) \\ k_{v,\theta}\dot{\theta} + k_{p,\theta}(\theta - \theta_c) \\ k_{v,\psi}\dot{\psi} + k_{p,\psi}(\psi - \bar{\psi}_d) \end{pmatrix}$$

Controllo della posizione

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} (s_\theta c_\psi + s_\varphi c_\theta s_\psi) f \\ (s_\theta s_\psi - s_\varphi c_\theta c_\psi) f \\ c_\varphi c_\theta f - g \end{pmatrix}$$

$$f \stackrel{\text{def}}{=} \frac{k_F}{m}(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$

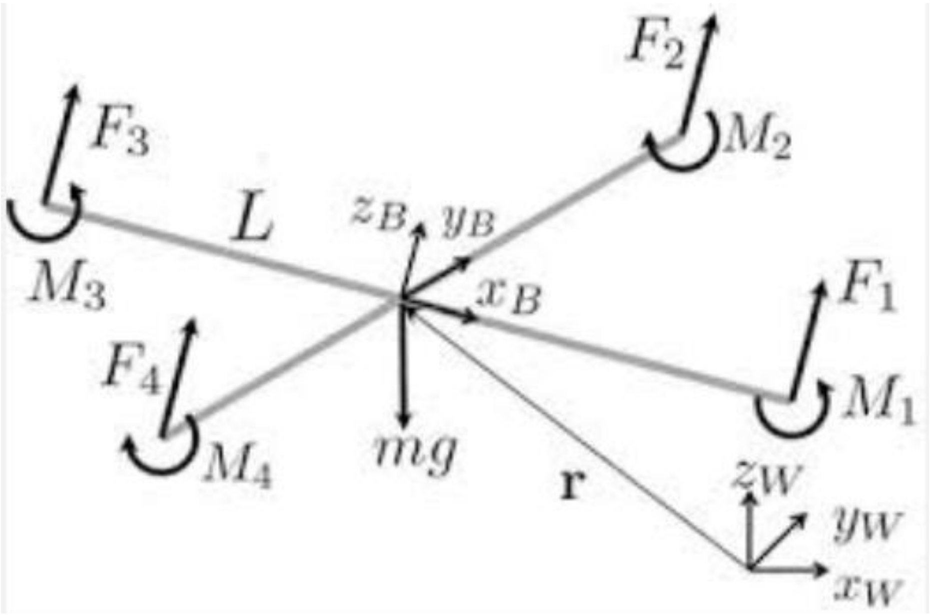
Considero per il controllo di tipo PD gli errori riferiti alle posizioni desiderate $(\bar{x}_d, \bar{y}_d, \bar{z}_d, \bar{\psi}_d)$

$$\delta x = x - \bar{x}_d$$

$$\delta y = y - \bar{y}_d$$

$$\delta z = z - \bar{z}_d \text{ and } \delta\psi = \psi - \bar{\psi}_d$$

$$\delta f = f - g,\ \delta\varphi = \varphi_c \text{ and } \delta\theta = \theta_c$$



Considerando le condizioni precedenti linearizzo il modello e ottengo :

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 0 & g s_{\bar{\psi}_d} & g c_{\bar{\psi}_d} \\ 0 & -g c_{\bar{\psi}_d} & g s_{\bar{\psi}_d} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta f \\ \delta \varphi \\ \delta \theta \end{pmatrix} = \begin{pmatrix} g \begin{pmatrix} s_{\bar{\psi}_d} & c_{\bar{\psi}_d} \\ -c_{\bar{\psi}_d} & s_{\bar{\psi}_d} \end{pmatrix} \begin{pmatrix} \delta \varphi \\ \delta \theta \end{pmatrix} \\ \delta f \end{pmatrix}$$

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} -k_{v,x}\dot{x} - k_{p,x}\delta x \\ -k_{v,y}\dot{y} - k_{p,y}\delta y \\ -k_{v,z}\dot{z} - k_{p,z}\delta z \end{pmatrix}$$

$$\delta\Omega_z \stackrel{\text{def}}{=} \delta\Omega_1 + \delta\Omega_2 + \delta\Omega_3 + \delta\Omega_4$$

Eguagliando entrambe ottengo la legge di controllo per la posizione con roll e pitch e la forza totale applicata dai rotori :

$$\begin{pmatrix} \varphi_c \\ \theta_c \\ f \end{pmatrix} = \begin{pmatrix} \frac{1}{g} \begin{pmatrix} s_{\bar{\psi}_d} & -c_{\bar{\psi}_d} \\ c_{\bar{\psi}_d} & s_{\bar{\psi}_d} \end{pmatrix} \begin{pmatrix} -k_{v,x}\dot{x} - k_{p,x}(x - \bar{x}_d) \\ -k_{v,y}\dot{y} - k_{p,y}(y - \bar{y}_d) \end{pmatrix} \\ \frac{k_F}{m}\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) + 2\sqrt{\frac{k_F g}{m}}\,\delta\Omega_z \end{pmatrix}$$

$$\delta\Omega_z = -k_{v,z}\dot{z} - k_{p,z}(z - \bar{z}_d) \,.$$