THE UNIVERSITY OF HONG KONG
DEPARTMENT OF PHYSICS

Assignment 1

Course: *Computational Physics (PHYS4150/8150)* – Professor: *Prof. Zi Yang Meng*
Tutor: *Mr. Tim Lok Chau, Mr. Min Long*
Due date: *4:20pm 26th September*

We have learned that the planet move around the sun in a elliptic orbit and the gravitational force between a planet and the sun is governed by the Newton's law of gravitation $\overrightarrow{F} = -\frac{G_N M_{sun} m}{r^2}\frac{\overrightarrow{r}}{r}$. In this assignment, let's discuss the Mercury's motion around the sun.

1. With the definition of Schwarzschild radius $r_s = \frac{2G_N M_{sun}}{c^2}$, rewrite the Newton's law of gravitation in the following form,

$$\ddot{\vec{r}} = -\frac{c^2}{2}\left(\frac{r_s}{r^2}\right)\frac{\vec{r}}{r} \tag{1}$$

   where c is the speed of light, G_N is the Newtonian constant of gravitation and M_{sun} is the mass of sun. Please show your steps.

2. Use Euler method, Leap-Frog method and 4th order Runge-Kutta method to computationally solve the equation of motion for the orbit of Mercury. For each method, please use **different value of Δt (Euler: 5×10^{-4}, 5×10^{-1}, 1 ; Leap Frog: 5×10^{-4}, 5×10^{-1}, 1 ; 4th order Runge Kutta: 5×10^{-1}, 1)** (Δt is in the unit of earth day). We express distance in unit $R_0 = 10^{10}m$ and time in $T_0 = 1$ earth day. We consider the system are in a two-dimensional space. The initial condition and constant for Mercury are set as $x = 6R_0, y = 0, v_x = 0, v_y = 0.510\frac{R_0}{T_0}, \frac{c^2 r_s}{2} = 0.99\frac{R_0^3}{T_0^2}$. Please show the orbit of Mercury in **4 Mercury year** (i.e. 1 Mercury year is equal to 88.0 earth days). **Plot the orbit in the same graph and compare the performance of different methods and the same method at different Δt.**

3. Kepler's second law [1], as shown in Fig. 1 states that a planet moves in its ellipse so that the line between it and the Sun placed at a focus sweeps out equal areas in equal times. **Use the numerical results by 4th order Runge-Kutta method implemented in former question to prove this law**. Complete code and necessary explanation are required.

4. In reality, Mercury orbit is not a static ellipse and the perihelion of Mercury would precess with a very small amount every year. It is shown in Fig 2. [2].

[1] In astronomy, Kepler's laws of planetary motion, published by Johannes Kepler between 1609 and 1619, describe the orbits of planets around the Sun. The laws modified the heliocentric theory of Nicolaus Copernicus, replacing its circular orbits and epicycles with elliptical trajectories, and explaining how planetary velocities vary.

[2] Image source: "https://en.wikipedia.org/wiki/Tests_of_general_relativity#/media/File:Apsidendrehung.png"

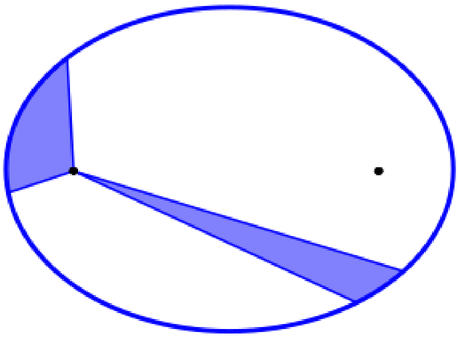

Figure 1: Illustration of Kepler's second law, the area of two colored are is the same.

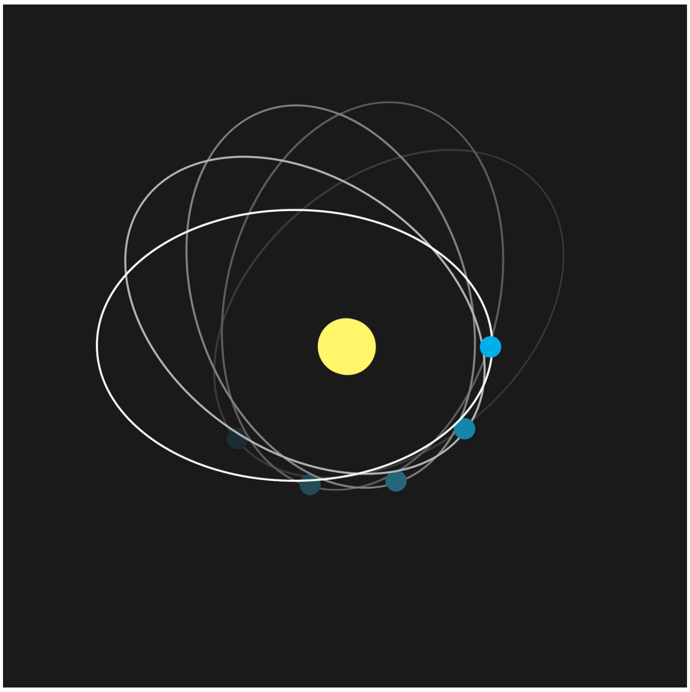

Figure 2: Mercury's perihelion precession

In order to characterize the correction to the Mercury's orbit due to gravitational effect from other planets and the effect of General Relativity, we introduce the constant α and β to the gravitational equation and the acceleration of Mercury is described as,

$$\ddot{\vec{r}} = -\frac{c^2}{2}\frac{r_s}{r^2}\left(1 + \alpha\frac{r_s}{r} + \beta\frac{r_L^2}{r^2}\right)\frac{\vec{r}}{r} \tag{2}$$

Please use Leap Frog and 4th order Runge Kutta method to computationally solve the equation of motion for Mercury again. Set $\Delta t = 0.05$, $\alpha = \beta = 10^6$, $r_L^2 = 8.19 \times 10^{-7}$, time for update to be 200,000 earth days and the same initial condition as $x = 6R_0, y = 0, v_x = 0, v_y = 0.510\frac{R_0}{T_0}$, $\frac{c^2 r_s}{2} = 0.99\frac{R_0^3}{T_0^2}$. **Draw the trajectory of the Mercury and show that the orbit indeed exhibit precession as in Fig. 2, compare the the performance of Leap Frog and 4th order Runge Kutta in the same plot.**