PHYS2160 Introductory Computational Physics
2023/24 Project
(Due date: 17 Apr 2024, 5:00 pm)

Each student should work on **any one** of the following problems. You should submit a project report which includes the following parts: (1) theory, (2) algorithm, (3) listing of your program, (4) results, and (5) discussion. Your program will be marked based on the use of Python's modules and packages, programming style, and capability of explaining physical phenomena.

1. An object of mass m is launched from level ground with initial speed v_0 at angle θ above the horizontal. When the object moves through the air, the gravity exerts a downward force on the object with magnitude mg where $g = 9.807\,\mathrm{m/s^2}$ is the gravitational acceleration near Earth's surface. On the other hand, the air resistance causes a drag force $\vec{F}_d$ on the object that is always opposite to its velocity $\vec{v}$. We choose the origin be the object's initial position with x-axis pointing along the object's initial direction of horizontal motion and y-axis pointing vertically upward. Applying the Newton's second law to the object, we will obtain a pair of second-order differential equations for its x and y coordinates at time t, i. e. $x(t)$ and $y(t)$. Consider the motion of the object in the xy-plane for the following cases.

   <u>Case I</u>

   Suppose the object is an an oil drop which experiences a linear drag force with magnitude

   $$F_d = -bv$$

   where b is a constant that depends on both the properties of the object and the air. As a result, the motion of the projectile is governed by the pair of second-order differential equations:

   $$\frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt}, \qquad \frac{d^2y}{dt^2} = -g - \frac{b}{m}\frac{dy}{dt}$$

   Assume $b/m = 1.28\times 10^5\,\mathrm{s^{-1}}$ for the oil drop.

   <u>Case II</u>

   Suppose the object is a cannon shell which experiences a quadratic drag force with magnitude

   $$F_d = -cv^2$$

   where c is the quadratic drag parameter that is directly proportional to the air density. As a result, the motion of the object is governed by the pair of second-order differential equations:

   $$\frac{d^2x}{dt^2} = -\frac{c}{m}\frac{dx}{dt}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}, \qquad \frac{d^2y}{dt^2} = -g - \frac{c}{m}\frac{dy}{dt}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Consider the scenario that the cannon shell does not travel to a very high altitude and thus the variation of the air density with its altitude is negligible. For such case, the quadratic drag parameter c has a constant value c_0. Assume $c_0/m = 1.460\times10^{-4}\,\mathrm{m}^{-1}$ for the cannon shell.

Case III

Consider the scenario that the cannon shell in case II travels to a very high altitude and thus the variation of the air density with its altitude is no longer negligible. Using the adiabatic model of the air density, the air density at altitude h above the sea level is given by

$$\rho(h) = \rho_0 \left(1 - \frac{Lh}{T_0}\right)^{4.256}$$

where $\rho_0 = 1.225\,\mathrm{kg/m^3}$ is the air density at sea level, $L = 0.0065\,\mathrm{K/m}$ is the temperature lapse rate, and $T_0 = 288.2\,\mathrm{K}$ is the sea level temperature. For such case, the quadratic drag parameter c is given by

$$c = c_0 \left(1 - \frac{Lh}{T_0}\right)^{4.256}$$

where c_0 is the value of the quadratic drag parameter at sea level. Assume again $c_0/m = 1.460\times10^{-4}\,\mathrm{m}^{-1}$ for the cannon shell.

To study the two-dimensional motion of the object through the air for the above cases, write Python programs to perform the following tasks:

(a) Use SciPy function `odeint` to numerically solve the differential equations in each case to find the position of the object $(x(t), y(t))$ as a function of time t until hitting the ground for different combination of initial speed v_0 and launching angle θ. Compare the results in each case by plotting the trajectory of the object with different launching angle θ on the same graph for each initial speed v_0.

(b) Using the `matplotlib` module and the data obtained in part (a), plot the range of the object versus the launching angle θ in each case for different initial speed v_0 on the same graph.

(c) Using the `matplotlib` module and the data obtained in part (a), plot the maximum height of the object versus the launching angle θ in each case for different initial speed v_0 on the same graph.

(d) Using the `matplotlib` module and the data obtained in part (a), plot the time of flight of the object versus the launching angle θ in each case for different initial speed v_0 on the same graph.

For case I, you should consider the oil drop to have an initial speed of the order of $10^{-4}\,\mathrm{m/s}$. For cases II and II, you should consider the cannon shell to have an initial speed greater than $500\,\mathrm{m/s}$.

2. A particle is in a one-dimensional finite potential well with potential energy given by:

$$U(x) = \begin{cases} 0 & -L < x < L \\ U_0 & x < -L \text{ or } x > L \end{cases}$$

The particle is in a bound state when it has energy $E < U_0$. Since the potential energy is symmetric, the bound-state wave function of the particle must be either even or odd. It can be shown that the even bound-state wave functions are given by:

$$\psi_e(x) = \begin{cases} A\cos\xi\exp[\sqrt{\xi_0^2 - \xi^2}(1 + x/L)] & x < -L \\ A\cos(\xi x/L) & -L < x < L \\ A\cos\xi\exp[\sqrt{\xi_0^2 - \xi^2}(1 - x/L)] & x > L \end{cases}$$

and the odd bound-state wave functions are given by:

$$\psi_o(x) = \begin{cases} -B\sin\xi\exp[\sqrt{\xi_0^2 - \xi^2}(1 + x/L)] & x < -L \\ B\sin(\xi x/L) & -L < x < L \\ B\sin\xi\exp[\sqrt{\xi_0^2 - \xi^2}(1 - x/L)] & x > L \end{cases}$$

where $\xi_0 = \sqrt{2mU_0}L/\hbar$, $\xi = \sqrt{2mE}L/\hbar$, and A and B are normalization constants. Moreover, the allowed values of the energy E for the even and odd bound states satisfy the equations:

$$\sqrt{\xi_0^2 - \xi^2} = \xi\tan\xi \qquad \text{(for even states)}$$

$$\sqrt{\xi_0^2 - \xi^2} = -\xi\cot\xi \qquad \text{(for odd states)}$$

To investigate the properties of this finite potential well, write Python programs to perform the following tasks for the case that the particle is an electron of mass $m_e = 0.511\,\mathrm{MeV}/c^2$ with $U_0 = 10\,\mathrm{eV}$ and $L = 1\,\mathrm{nm}$:

(a) Use `matplotlib` module to plot the quantities $\sqrt{\xi_0^2 - \xi^2}$, $\xi\tan\xi$, and $-\xi\cot\xi$ as a function of ξ from $\xi = 0$ to $\xi = 18$ on the same graph. From your plot, estimate the energies of the five lowest energy levels of the particle in electron volts.

(b) Use the SciPy function `bisect` to solve the above equations to find the energies of the five lowest energy levels of the particle in electron volts to an accuracy of $0.01\,\mathrm{eV}$ and compare with your results in part (a).

(c) Use the SciPy function `integrate` to determine the normalization constants A and B and then use the `matplotlib` module to plot the even-state and odd-state wave functions $\psi_e(x)$ and $\psi_o(x)$ as well as the corresponding probability densities $|\psi_e(x)|^2$ and $|\psi_o(x)|^2$ as a function of x/L from $x = -1.5L$ to $x = 1.5L$ on two graphs for the five lowest energy levels of the particle.