The University of New South Wales
Session 1 Examination 2012

COMP3121/3821
Algorithms and Programming Techniques

and

COMP9101/9801
Design and Analysis of Algorithms

- Time allowed: **3 hours**
- 3121/9101 students: Answer questions 1-4
- 3821/9801 students: Answer questions 1-5
- Each problem is worth 10 points
- You **can** keep the examination paper
- Write **LEGIBLY**
- You **MUST JUSTIFY** each answer
- **No** study material or textbooks are allowed
- UNSW approved calculators are **permitted**

1	2	3	4	**5 (extended only)**	**total**

ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK.

1. We are given a checkerboard which has 4 rows and n columns, and has an integer written in each square. We are also given a set of $2n$ pebbles, and we want to place some or all of these on the checkerboard (each pebble can be placed on exactly one square) so as to maximize the sum of the integers in the squares that are covered by pebbles. There is one constraint: for a placement of pebbles to be legal, no two of them can be on horizontally or vertically adjacent squares (diagonal adjacency is fine).

   (a) Determine the number of legal *patterns* that can occur in any column (in isolation, ignoring the pebbles in adjacent columns) and describe these patterns.

   Call two patterns *compatible* if they can be placed on adjacent columns to form a legal placement. Let us consider sub-problems consisting of the first k columns $1 \leq k \leq n$. Each sub-problem can be assigned a type, which is the pattern occurring in the last column.

   (b) Using the notions of compatibility and type, give an O(n)-time algorithm for computing an optimal placement.

2. Skiers go fastest with skis whose length is about their height. Your team consists of n members, with heights $h_1, h_2, \ldots, h_n$. Your team gets a delivery of n pairs of skis, with lengths $l_1, l_2, \ldots, l_n$. Your goal is to write an algorithm to assign to each skier one pair of skis to minimize the sum of the absolute differences between the height h_i of the skier and the length of the corresponding ski he got, i.e., to minimize

$$\sum_{1 \leq i \leq n} |h_i - l_{j(i)}|$$

   where $l_{j(i)}$ is the length of the ski assigned to the skier of height h_i.

3. You work for a new private university which wants to keep the sizes of classes small. Each class is assigned its maximal capacity - the largest number of students which can enroll in it. Students pay the same tuition fee for each class they get enrolled in. Students can apply to be enrolled in as many classes as they wish, but each of them will eventually be enrolled to at most 5 classes at any given semester. You are given the wish lists of all students, containing for each student the list of all classes they would like to enroll this particular semester and you have to chose from the classes they have put on their wish lists in which classes you will enroll them, without exceeding the maximal enrolment of any of the classes and without enrolling any student into more than 5 classes. Your goal is, surprisingly, to maximize the income from the tuition fees for your university. Design an efficient algorithm for such a task.

4. You are given two DNA sequences s_1 and s_2 of lengths l_1 and l_2 respectively, and a third sequence S of length $l_1 + l_2$. (Recall that DNA sequences are sequences consisting of four letters A, T, G, C). Your task is to find out if sequence S can be obtained by merging the two sequences s_1 and s_2 in a way which preserves the order of letters in the initial sequences s_1, s_2. For example, $ATGC$ is a correct merging of sequences AC and TG, but the sequence $AGTC$ is not, because the letters G, T are not in the same order as they in the sequence from which they are taken. Thus, you are essentially allowed to take any number of consecutive letters from each sequence, starting from their beginnings, alternating between the two sequences and joining them in the same order to see if you can get in this way the sequence S.

5. **(EXTENDED CLASSES 3821 and 9801 ONLY)**

   Solve problem 2 if you have n skiers, but have m pairs of skis, where $m > n$.