COMP3821
Maximal Flow Practice

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1. You are running a dating agency and have m guys and f girls as customers. Each guy and each girl have reviewed all profiles of the candidates of opposite sex and have sent you their corresponding lists of people whose profiles they liked. Your task is to organise a largest possible number of first dates so that everyone meets at most one person of opposite sex and so that both parties liked each others profile (i.e., have included the other party on their list of people whose profile they liked).

2. Assume that you are the administrator of a network of computers; each computer is connected by unidirectional fiberoptic cables to a few other computers on the same network (so the network can be modeled by a directed graph). You noticed that computers P1, P2, ...Pn are mounting an attack on computers Q_1, Q_2, ..., Q_m. The total number of computers on the network is $N > m + n$. Since it is a real emergency, you must disconnect some of the optical cables of the network so that none of computers P_1, P2, ..., P_n can send packets to any of Q_1, Q_2, ..., Q_m. Since you must send crews to disconnect some of the fiberoptic cables, for each cable c_{ij} for traffic from a computer X_i to a computer X_j there is an associated cost c_{ij} for disconnecting it. Your task is to design an algorithm for determining which cables to disconnect to isolate computers Q_1, Q_2, ..., Q_m from all of the computers P_1, ..., P_n so that the total cost incurred is minimal.

3. You work for a new private university which wants to keep the sizes of classes small. Each class is assigned its maximal capacity - the largest number of students which can enrol in it. Students pay the same tuition fee for each class they get enrolled in. Students can apply to be enrolled in as many classes as they wish, but each of them will eventually be enrolled to at most 5 classes at any given semester. You are given the wish lists of all students, containing for each student the list of all classes they would like to enrol this particular semester and you have to chose from the classes they have put on their wish lists in which classes you will enrol them, without exceeding the maximal enrollment of any of the classes and without enrolling any student into more than 5 classes. Your goal is, surprisingly, to maximise the income from the tuition fees for your university. Design an efficient algorithm for such a task.

4. Assume each student can borrow at most 10 books from the library, and the library has three copies of each title in its inventory. Each student submits a list of books he wishes to borrow. You have to assign books to students, so that a maximal number of volumes is checked out.

5. The emergency services are responding to a major earthquake that has hit a wide region, and left n people injured who need to be sent to a hospital. Let P be the set of n people and H be the set of k hospitals. Several hospitals are available to treat these people, but there are some constraints:

   (a) Each injured person needs to be sent to a hospital no further than one hour drive away. Let H_p be the set of hospitals that are within range for person p.

   (b) Each hospital h has a capacity c_h, the maximum number of people that the hospital can receive.

   Design an efficient algorithm that determines whether it is possible to assign each person to a hospital in a way that satisfies these constraints, and returns such an assignment if so.