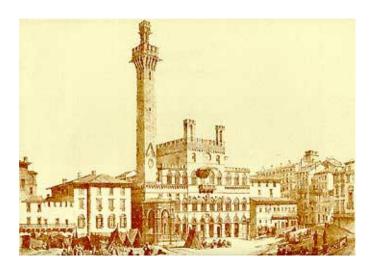
# Manual for RSiena

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#### Abstract

SIENA (for Simulation Investigation for Empirical Network Analysis) is a computer program that carries out the statistical estimation of models for the evolution of social networks according to the dynamic actor-oriented model of which an overview is in Snijders (2017), and also the random coefficient multilevel network model of Koskinen and Snijders (2023). This is the manual for RSiena, a contributed package to the statistical system R. It complements, but does not replace the help pages for the RSiena functions! It also contains contributions written by Mark Huisman, Michael Schweinberger, and Christian Steglich.

This manual is frequently updated, mostly only in a minor way. This version was renewed for RSiena version 1.4.14.

# Contents

1	Ger	General information							
	1.1	Acknowledgements							
	1.2	Giving references to RSiena							
2	Get	ting started with SIENA							
	2.1	The logic of Stochastic Actor-Oriented Models							
		2.1.1 Types of Stochastic Actor-Oriented Models							
		2.1.2 Data, variables and effects							
		2.1.3 Outline of estimation procedure							
		2.1.4 Design issues for longitudinal network modeling							
		2.1.5 Identifiability							
		2.1.6 Further useful options in RSiena							
	2.2	Installing R and RSiena							
	$\frac{2.2}{2.3}$	Using RSiena within R							
	$\frac{2.3}{2.4}$	Example R scripts for getting started							
	2.5	Steps for looking at results: Executing SIENA							
	2.6	Getting help with problems							
3	Ste	ps of modelling 22							
4	Inn	ut data 2							
•	4.1								
	4.1	4.1.1 Network data							
		4.1.3 Behavioral data							
		4.1.4 Continuous behavioral data							
		4.1.5 Individual covariates							
		4.1.6 Dyadic covariates							
	4.2	Internal data treatment							
		4.2.1 Interactions and dyadic transformations of covariates							
		4.2.2 Centering							
		4.2.3 Centering of behavioral variables							
		4.2.4 Monotonic dependent variables							
		4.2.5 Multivariate dependent variables with constraints							
	4.3	Further data specification options							
		4.3.1 Structurally determined values							
		4.3.2 Missing data							
		4.3.3 Composition change: joiners and leavers							
5	Mo	del specification 3							
9	5.1	Definition of the model							
	0.1	5.1.1 Elementary effects							
		The state of the s							
	F 0	5.1.3 Mathematical specification							
	5.2	r							
		one-mode networks							
		5.2.1 Rate function							

	5.3	Important structural effects for network dynamics:
		two-mode networks
	5.4	Effects for network dynamics associated with covariates
	5.5	Cross-network effects for dynamics of multiple networks
	5.6	Hierarchy requirements
	5.7	Constraints between networks
	5.8	Effects on behavior evolution
	5.9	Model Type: non-directed networks
	5.10	Model Type: directed networks
		Model Type: behavior
		Interaction effects
		5.12.1 Internal effect parameters for interaction effects
		5.12.2 The interactionType of interaction effects
		5.12.3 Interaction effects for network dynamics
		5.12.4 Interaction effects for behavior dynamics
	5 13	Time heterogeneity in model parameters
		Limiting the maximum outdegree
		Goodness of fit with auxiliary statistics: sienaGOF
	0.10	5.15.1 Plots
		5.15.2 <i>p</i> -values
		5.15.3 Composition change, missing data, and structural values in sienaGOF 6
		5.15.5 Composition change, missing data, and structural values in sicilador
6	Esti	mation 6
	6.1	An overview of the estimation procedure
	6.2	Parameters of the algorithm
	6.3	Results of the estimation
		6.3.1 Initial Values
		6.3.2 Convergence Check
		6.3.3 Continued estimation to obtain convergence
	6.4	Use of the algorithm parameters
	6.5	What to do if there are convergence problems
	6.6	Some important components of the sienaFit object
	6.7	Algorithm
	6.8	Output
	0.0	6.8.1 Convergence check
		6.8.2 Parameter values and standard errors
		6.8.3 Collinearity check
	6.9	Other estimation procedures
		Generalized Method of Moments estimation
	0.10	6.10.1 Using the function siena07
		· · · · · · · · · · · · · · · · · · ·
	<i>C</i> 11	
		Maximum Likelihood and Bayesian estimation
	0.12	Other remarks about the estimation algorithm
		6.12.1 Conditional and unconditional estimation
		6.12.2 Fixing parameters
		6.12.3 Automatic fixing of parameters
	0.10	6.12.4 Required changes from conditional to unconditional estimation 9
	6.13	Using multiple processes

7	Standard errors								
	7.1	Multicollinearity	95						
	7.2	Precision of the finite differences method	97						
8	Test	Tests							
	8.1	Wald-type tests	98						
		8.1.1 Multi-parameter Wald tests	99						
		8.1.2 Tests of linear combinations	101						
	8.2	Score-type tests	103						
	8.3	Example: one-sided tests, two-sided tests, and one-step estimates	104						
		8.3.1 Multi-parameter tests	105						
	8.4	Alternative application: convergence problems	107						
	8.5	Testing differences between independent groups or periods	107						
	8.6	Testing time heterogeneity in parameters	108						
	0.0	resume time necessity in parameters	100						
9	Sim	ulation	110						
	9.1	Accessing the generated networks	111						
	9.2	Conditional and unconditional simulation	112						
	9.3	Simulating with variable parameter values	113						
10	Get	ting started	114						
-0		Model choice	115						
11		tilevel network analysis	116						
		Function sienaGroupCreate	117						
		Multi-group Siena analysis	118						
	11.3	Meta-analysis of Siena results	119						
		11.3.1 Meta-analysis directed at the mean and variance of the parameters	120						
		11.3.2 Meta-analysis directed at testing the parameters	122						
		11.3.3 Contrast between the two kinds of meta-analysis	123						
	11.4	Random coefficient multilevel Siena analysis	123						
		11.4.1 Which data sets to use for sienaBayes	124						
		11.4.2 Model specification	125						
		11.4.3 How to enter your data in sienaBayes	127						
		11.4.4 How to choose the parameter settings for sienaBayes	127						
		11.4.5 Prior distributions	128						
		11.4.6 Operation of sienaBayes	131						
		11.4.7 Assessing convergence	131						
		11.4.8 Interpreting results of sienaBayes	132						
19	For	mulas for effects	134						
		Network evolution	134						
	12.1	12.1.1 Network evaluation function	135						
		12.1.2 Special effects for non-directed networks	164						
		12.1.3 Multiple network effects	166						
		12.1.4 Network creation and endowment functions	190						
		12.1.5 Network rate function	190						
	19 9	Behavioral evolution	$191 \\ 193$						
	14.4	12.2.1 Behavioral evaluation function	193						
		12.2.2 Behavioral creation function	$\frac{195}{209}$						
		14.4.4 Denaviolal decation function	409						

		12.2.3 Behavioral endowment function	209
		12.2.4 Behavioral rate function	210
	12.3	Effects for estimation by Generalized Method of Moments	212
<b>13</b>	Para	ameter interpretation	213
	13.1	Networks	213
	13.2	Behavior	214
	13.3	Ego – alter selection tables	216
	13.4	Ego – alter influence tables	222
		13.4.1 Average alter	222
		13.4.2 Average similarity	225
	13.5	Effect sizes and measures of fit	229
		13.5.1 Relative importance of effects	229
		13.5.2 Entropy / relative certainty	229
		13.5.3 Standard deviation of change statistics	230
14	Erro	or messages	231
		After updating	231
		During estimation	232
	14.3	As the result of a score-type test (including time test)	234
		In sienaGOF	235
<b>15</b>	For	programmers: Get the source code	236
16	For	programmers: Other tools you need	236
17	For	programmers: Building, installing and checking the package	236
18		programmers: Understanding and adding an effect The use of the hash (#) sign	237
		in names of effects and estimation statistics	240
	18.2	Example: adding the truncated out-degree effect	240
	18.3	Notes on effectGroups and two-mode networks	244
A	App	pendix: List of Functions in Order of Execution	247
В	$\mathbf{App}$	pendix: Changes compared to earlier versions	259
$\mathbf{C}$	Refe	prences	306

## 1 General information

SIENA<sup>1</sup>, shorthand for Simulation Investigation for Empirical Network Analysis, is a set of methods implemented in a computer program that carries out the statistical estimation of models for repeated measures of social networks according to the Stochastic Actor-oriented Model ('SAOM') of Snijders and van Duijn (1997), Snijders (2001), Snijders et al. (2007), Snijders et al. (2010a), Snijders et al. (2013), Greenan (2015a), and Koskinen and Snijders (2023); also see Steglich et al. (2010). A tutorial for these models is in Snijders et al. (2010b). A review article is Snijders (2017). Books are in preparation (Snijders and Steglich, 2021, 2023).

A website for SIENA is maintained at https://www.stats.ox.ac.uk/~snijders/siena/. At this website ('publications' tab) you shall also find references to introductions in various other languages, as well as the file Siena\_algorithms.pdf which gives a sketch of the main algorithms used in RSiena. The website further contains references to many published examples, example scripts illustrating various possibilities of the package, course announcements, etc.

RSiena is a contributed package for the R statistical system (R Core Team, 2023) and can be installed in the usual way from CRAN; see http://cran.r-project.org. Development versions of RSiena are at https://github.com/stocnet/rsiena/. Here also the most recent version is available at https://github.com/stocnet/rsiena/releases; it is recommended to use this version.

This is a manual for RSiena; the manual is provisional in the sense that it is continually being updated, taking account of updates in the package. For the operation of R, the reader is referred to this manual, the help pages, the website, and the corresponding literature.

For working with RSiena, please use jointly the following information:

- 1. The help pages for the functions in RSiena, which can be consulted in the usual R way by entering? followed by the function name; note that by clicking on the version mentioned at the bottom of each help page, you are brought to a directory of all help pages for the package;
- 2. this manual (of which the web version is updated quite frequently);
- 3. the website http://www.stats.ox.ac.uk/~snijders/siena/, e.g., its News page and its RSiena scripts page;
- 4. the users' group for SIENA meant to exchange information and technical advice; the address is https://groups.io/g/RSiena.

RSiena was originally programmed by Ruth Ripley and Krists Boitmanis, in collaboration with Tom Snijders. Since May 2012 the maintainer is Tom Snijders. Major contributions were made by Josh Lospinoso, Charlotte Greenan, Michael Schweinberger, Felix Schönenberger, Johan Koskinen, Mark Ortmann, Nynke Niezink, Viviana Amati, and

<sup>&</sup>lt;sup>1</sup>This program was first presented at the International Conference for Computer Simulation and the Social Sciences, Cortona (Italy), September 1997, which originally was scheduled to be held in Siena. See Snijders and van Duijn (1997).

Mark Huisman. Other contributors include Christian Steglich, Christoph Stadtfeld, Per Block, Robert Krause, Marion Hoffman, Alvaro Uzaheta, Robert Hellpap, James Hollway, and Steffen Triebel. Currently, RSiena is maintained by a research group at the University of Groningen, seconded by researchers in Zürich, Stockholm, Geneva, and elsewhere.

In addition to the 'official' R distribution of RSiena and the GitHub development version, there is the old distribution RSienaTest at R-Forge, which includes some additional options, but of which the maintenance is discontinued. A few options described in this manual may apply to RSienaTest only.

#### 1.1 Acknowledgements

The development of RSiena was carried out by researchers of the Universities of Oxford, Groningen, Konstanz, Zürich, Manchester, and Melbourne. We are grateful to NIH (National Institutes of Health, USA) for their funding of programming RSiena during 2008-2014. This was done as part of the project Adolescent Peer Social Network Dynamics and Problem Behavior, funded by NIH (Grant Number 1R01HD052887-01A2), Principal Investigator John M. Light (Oregon Research Institute).

For earlier work on SIENA, we are grateful to NWO (Netherlands Organisation for Scientific Research) for their support to the project *Models for the Evolution of Networks and Behavior* (project number 461-05-690), the integrated research program *The dynamics of networks and behavior* (project number 401-01-550), the project *Statistical methods for the joint development of individual behavior and peer networks* (project number 575-28-012), the project *An open software system for the statistical analysis of social networks* (project number 405-20-20), and to the foundation ProGAMMA, which all contributed to the work on SIENA.

#### 1.2 Giving references to RSiena

When using SIENA, it is appreciated that you refer to the package as such or this manual; and to one or more relevant references of the methods implemented in the program. The reference to this manual and to the package can be obtained in R from

#### citation("RSiena")

For the methods implemented in RSiena, a standard general reference is Snijders (2017). Tutorials are Snijders et al. (2010b) and Kalish (2020). A basic reference for the network dynamics model is Snijders (2001) or Snijders (2005). Basic references for the model of network-behavior co-evolution are Snijders et al. (2007) and Steglich et al. (2010). Review articles are Snijders (2017) and Snijders and Pickup (2016).

More specific references are Koskinen and Edling (2012) for the dynamics of two-mode networks and Snijders et al. (2013) for the co-evolution of multiple networks – two mode or one-mode or both. For the model for diffusion of innovations in dynamic networks, please refer to Greenan (2015a). More technical references are Schweinberger (2012) for the score-type tests and Schweinberger and Snijders (2007b) for the calculation of standard errors of the Method of Moments estimators. For assessing and correcting time heterogeneity, refer to Lospinoso et al. (2011). For goodness of fit assessment and associated model selection considerations, refer to Lospinoso and Snijders (2019). A basic reference for the Bayesian estimation is Koskinen and Snijders (2007) and for the maximum likelihood estimation Snijders et al. (2010a). For Generalized Method of Moments estimators, references are Amati et al. (2015, 2019). For the integrated multilevel network analysis implemented in function sienaBayes, the reference is Koskinen and Snijders (2023).

A general reference for R is R Core Team (2023).

# 2 Getting started with SIENA

There may be various strategies for getting acquainted with RSiena. In any case, it is a good idea to study the tutorial Snijders et al. (2010b). Two recommended options for learning the 'how to' are the following:

- 1. One excellent option is to read the User's Manual from start to finish (leaving aside the Programmer's Manual).
- 2. A second option is to read this Minimal Introduction, to get a sense of the rest by looking at the table of contents, and then follow the references to specific sections of your interest. The searchable pdf file makes it easy to look for the relevant words.

This Minimal Introduction explains the basics of Stochastic Actor-Oriented Models and gives practical information on running RSiena. We start with section 2.1 which gives a brief and non-technical introduction to the types of Stochastic Actor-Oriented Models, to the most important concepts related to them, to the data required to apply SIENA, and to further features of the program. In Section 2.2 we explain how to install and run SIENA as the package RSiena from within R. Section 2.4 and Section 2.5 provide example R scripts and guidance for understanding the results. If you are looking for help with a specific problem, read Section 2.6.

#### 2.1 The logic of Stochastic Actor-Oriented Models

SIENA (Simulation Investigation for Empirical Network Analysis) is a statistical tool developed for the analysis of longitudinal network data, collected in a network panel study with two or more 'waves' of observations. It incorporates different variants of a dynamic network model family: the Stochastic Actor-Oriented Model (SAOM). In this section, we give a very concise introduction to how these models work in principle and what type of data they are suitable to analyze. For sake of simplicity, SAOMs implemented in SIENA are often referred to as 'SIENA models'. In this subsection, we only consider the case of network evolution; see below for the more complex cases of coevolution. For a further introduction, consult Snijders et al. (2010b). An introduction for applications in the context of adolescent development is Veenstra et al. (2013).

The defining characteristic of Stochastic Actor-Oriented Models is their 'actor-oriented' nature which means that they model change from the perspective of the actors (nodes). That is, Stochastic Actor-Oriented Models always "imagine" network evolution as individual actors creating, maintaining or terminating ties to other actors. When thinking about network dynamics, researchers usually assume that these decisions (conscious or subconscious) of actors are influenced by the structure of the network itself and the characteristics and behaviors of the focal actor (ego) who is making a decision and those of other actors in the network (alters). Stochastic Actor-Oriented Models provide a means to quantify the ways, the extent and the uncertainty with which these factors are associated with network evolution between observations.

The Stochastic Actor-Oriented Model can be regarded as an agent-based ('actor-based') simulation model of the network evolution; where all network changes are decomposed into very small steps, so-called *ministeps*, in which one actor creates or terminates one outgoing tie. These ministeps are probabilistic and made sequentially. The transition from the observation at one wave to the next is done by means of normally a large number of ministeps. The actors respond to the network in the sense that the probabilities of these changes depend on the current (unobserved) state of the network. Each further ministep changes the network state and therefore the actors are each others' ever changing context (Zeggelink, 1994). This allows the model to represent the feedback process that is typical for network dynamics. These changes are not individually observed, but they are simulated; what is observed is the state obtained at the next observation wave.

This simulation model implements the statistical model for the network dynamics. The statistical procedures utilize a large number of repeated simulations of the network evolution from each wave to the next. They estimate and test the parameters producing a probabilistic network evolution that 'could have' brought these observations to follow one another.

To avoid misunderstandings, two notes have to be made about the meaning of actor "decisions" and the role of Stochastic Actor-Oriented Models in causal inference. First, the fact that SIENA models are actor-oriented does not imply the assumption that the actors take decisions in any real sense. It means that the changes in the network are organized, so to say, by the nodes in the network. This aligns very well with a substantive standpoint where the nodes have agency (Snijders, 1996) but it does not necessarily reflect a commitment to or belief in any particular theory of action elaborated in the scientific disciplines. In fact, the purpose of SIENA in this matter is to assist substantive researchers in further developing their theories of action by e.g. exploring the relative importance of individual, contextual, and social factors in network change. The second, and related, point is that, like other generalized regression models, SIENA does not by itself solve all causal questions. When inferring causality from model results, one has to face difficulties very similar to those with other statistical methods; see, e.g., Lomi et al. (2011) and Goldthorpe (2001). In any case, causal interpretations should be supported by further results from the discipline the explanations originate in. However, Stochastic Actor-Oriented Models do allow research to profit from a longitudinal design – therefore, they may be helpful in tackling some issues related to causality, like the selection-influence problem (Steglich et al., 2010; Lomi et al., 2011).

# 2.1.1 Types of Stochastic Actor-Oriented Models: evolution of one-mode networks, two-mode networks and behaviors

So far, we have mostly talked about SIENA as a tool to analyze the evolution of a single network. However, there are different variants of Stochastic Actor-Oriented Models that can be applied to more complex data structures. The availability of these options depends on the research question and the quantity and type of data one has. In this section, we briefly discuss the currently implemented model types, which will help researchers determine what kind of analyses they are able to carry out with Stochastic Actor-Oriented

Models given the data at hand.

A minimal dataset suitable for analysis with RSiena consists of two observations of a single network defined on the same set of nodes. In this case, one is able to test how the structure of the network contributes to its own evolution. However, depending on the data available, further modeling options may be applicable. Currently, the implemented Stochastic Actor-Oriented Models are suitable for the analysis of

- 1. the evolution of a directed or non-directed one-mode network (e.g., friendships in a classroom) (Snijders, 2001);
- 2. the evolution of a two-mode network (e.g., club memberships in a classroom: the first mode is constituted by the students, the second mode by the clubs) (Koskinen and Edling, 2012);
- 3. the evolution of an individual behavior (e.g., smoking), and
- 4. the co-evolution of one-mode networks, two-mode networks and individual behaviors (e.g., the joint evolution friendship and smoking; or of friendship and club membership) (Steglich et al., 2010; Snijders et al., 2013).

In all these cases, the data can also include covariates: observed variables that influence the dynamics, but of which the values are not themselves modeled.

In the first two cases, one can assess with RSiena the ways and the extent to which changes in a given one- or two-mode network depend on the network structure itself and on covariates. The third option, modeling changes in an individual behavior on its own, without reference to its embeddedness in a network, is rarely used. For this type of data numerous alternative longitudinal modeling techniques exist.

Accordingly, the fourth model type has been becoming widely used. Analyzing the joint evolution of networks and behavior allows researchers to address questions related to selection and influence processes, for example, whether smokers tend to become friends with each other or friends tend to become similar in their smoking habits. The strength of the SIENA co-evolution models is that one can simultaneously take into account the impact of network structure on network evolution, the actual level of a behavior on behavior change, the network structure on behavior change, and the actual level of behavior on network evolution. Besides network and behavior co-evolution, this class of Stochastic Actor-Oriented Models also allow for the joint analysis of multiple networks (e.g. friendship and advice, friendship and dislike, or all three of them), and the analysis of ordered multiple networks (where the presence of a tie in one network presumes the existence of a tie in the other network, like in the case of friendships and best friend relations).

#### 2.1.2 Data, variables and effects

Now that we have discussed some core features of Stochastic Actor-Oriented Models and introduced the different implemented model types, we turn our attention, still just presenting an outline, to data types and the specification of a model. In general, the number of waves must be at least two in order to analyze a data set with Stochastic Actor-Oriented

Models. In case of modeling evolution across more than two observations in time, estimated parameter values are assumed to be equal in all periods (unless time heterogeneity is specifically represented by changing parameters – see Section 5.13 for further details).

This section focuses on three related topics: the type of network and behavioral data SIENA works with, the meaning of explanatory variables, or so called effects, in Stochastic Actor-Oriented Models, and the different dependent variables with which SIENA captures network and behavior evolution.

#### Network data

Stochastic Actor-Oriented Models operate on binary networks, that is, on relations on a given set of actors, where tie variables between actors have two states: existent (1) or non-existent (0). Weighted networks are not allowed, but as mentioned above, it is possible to define multiple networks representing discrete levels of relationships. It is possible to specify that some ties in the network are impossible ("structural zeros") or necessary ("structural ones") (see Section 4.3.1 for more details). For the network evolution, Stochastic Actor-Oriented Models how ties are being created, maintained or terminated by actors.

#### Behavioral data

Behavioral variables in Stochastic Actor-Oriented Models can be thought of as indicating the presence or intensity of a behavior. For example, behavioral data can represent whether an actor is a smoker or not, as well as a number of ordered categories expressing the number of cigarettes usually smoked. The term "behavior" should not be taken literally here, it is possible to model changes in attitudes or other actor attributes. In the models, behavioral variables can be binary, ordinal discrete, or continuous. For the ordinal discrete case, the number of categories should be small (mostly 2 to 5; larger ranges are possible). In the case of behaviors, Stochastic Actor-Oriented Models express how actors increase, decrease, or maintain the level of their behavior.

A special case of the fourth type is the diffusion of innovations in dynamic networks (Greenan, 2015a): here the behavior variable representing having adopted the innovation is binary, coded 0 or 1, and once an actor has the value 1 s/he is stuck with it. The only possible transitions are  $0 \Rightarrow 1$ , representing that the actor adopts the innovation. See Section 12.2.4.

#### Covariates

In every model type, it is possible to define and use covariates, which are variables that are exogenous in the sense that their values are not modeled, but used to explain network or behavior change. Covariates can be dummy variables (e.g., sex) or continuous (e.g., attitudes or age). Also, they may have constant values across all observations or their value may change across time periods – this is the distinction between constant and varying covariates (e.g., sex and salary). Finally, there are individual (monadic) and

dyadic covariates that refer, respectively, to characteristics of individual actors (e.g., sex) and to attributes of pairs of actors (e.g., living in the same neighborhood or kinship).

# Missing data and composition change

Stochastic Actor-Oriented Models distinguish between two types of missing values: absence of actors from the network and random missingness. The first case refers to changing composition: it is possible to specify that some actors leave or join the network between two observations (during the simulation process). This then applies to all dependent variables (networks, behaviors) simultaneously (see Section 4.3.3 for more details). In the second case, missing values are treated as randomly missing (see Section 4.3.2 for more details). Stochastic Actor-Oriented Models can deal with some, but not too much, randomly missing data (as a rule of thumb, more than 20% is considered to be too much). With too many missing values, the simulation can become unstable, and also the estimated parameters may not be substantively reliable anymore. And of course, missing data are likely to are caused by processes that are not totally random, and therefore risk to bias the resuls.

#### Explanatory variables: the effects

When defining Stochastic Actor-Oriented Models, we have to specify the exact ways in which current network structure or covariates may affect network or behavior change. This is defined by combinations of configurations (or situations) which are called "effects" in Stochastic Actor-Oriented Models. Effects can be treated as the explanatory variables of the models. Effects can be structural (depending on the network structure itself, also called endogenous), or covariate-related; also various combinations between structure and covariates are possible. Some examples for effects:

- structural effects: reciprocity, transitivity;
- covariate effects: sex of the tie sender, sex of the receiver, same sex, similarity in salary;
- combinations: average level of smoking of friends, interaction between sex of the sender and reciprocity.

#### Dependent variables: network evaluation, creation and endowment functions

As we discussed earlier, RSiena is capable of analyzing and modeling the evolution of networks and behavior, jointly or separately. Consequently, a model may have more than one dependent variable. Here we introduce the ways network and behavior dependent variables can be defined in Stochastic Actor-Oriented Models. We start with network evolution.

Given two observations of a binary network, a single network tie variable can follow four patterns, as shown in Table 1. In Stochastic Actor-Oriented Models, however, tie change

Table 1: Possible tie change patterns for two observations  $(t_1 \text{ and } t_2)$ 

$t_1$	$t_2$	
i $j$	$i \rightarrow j$	creation of a tie
$i \to j$	$i \rightarrow j$	maintenance of a tie
$i \to j$	i $j$	termination of a tie
i $j$	i $j$	maintenance of a 'no-tie'

can be defined in three ways: we can model the creation of previously not existing ties (creation), the maintenance of existing ties (endowment), or the presence of ties regardless of whether they were newly created or maintained (evaluation). These are the three possible values of the change in tie variables, constituting the dependent variables of the network evolution model. The effects model the probabilities (more precisely: they are components of the linear predictor for the log-probability) for the creation, maintenance or presence of network ties. Table 2 helps to imagine what the probabilities refer to in each case: we compare the probability of green cases to that of blue cases.

Table 2: Tie changes considered by the evaluation, creation and endowment functions

a) eval	luation	b) creation			c) $endowment$		
$t_1$	$t_2$	$t_1$	$t_2$		$t_1$	$t_2$	
i $j$	$i \rightarrow j$	i $j$	$i \rightarrow j$		i $j$	$i \rightarrow j$	
$i \rightarrow j$	$i \rightarrow j$	$i \rightarrow j$	$i \rightarrow j$		$i \rightarrow j$	$i \rightarrow j$	
$i \rightarrow j$	$\mid i \mid j$	$i \rightarrow j$	i $j$		$i \rightarrow j$	i  j	
i $j$	$\mid i \mid j$	i $j$	$\mid i \mid j$		i $j$	i $j$	

According to this distinction, network evolution may be modeled in RSiena by three functions: the evaluation, creation and endowment functions. Effects can appear as components of one or two of these functions in a single model, but never in all three (this would lead to perfect collinearity). Using only the evaluation effect assumes that the creation and endowment effects are equal (and equal to the evaluation effect). The estimated parameters for each effect should be interpreted as log-probability ratios (comparable to the log-odds ratios known from logistic regression; since the choice here is multinomial rather than binary, the correct term is probability ratios rather than odds ratios). From a practical point of view, it is meaningful to start modeling with evaluation effects, unless one has a clear idea about how tie creation and endowment may be different in the analyzed data set. Separating the contribution of an effect into two functions requires more of the data, and if a given effect is similarly strong for the creation and maintenance of ties the statistical power will decrease by this split. For these reasons, most SI studies limit their attention to evaluation effects. However, if there is enough data, the distinction between creation and maintenance of ties can produce powerful insights (e.g., Cheadle et al., 2013).

#### Dependent variables: behavior evaluation, creation and endowment functions

The distinction between the different behavior evolution functions follows a logic similar to the case of network evolution. The three possibilities for change in behavior are increasing or decreasing the level of behavior by one unit, or maintaining its actual level. In case of the evaluation function, the model does not distinguish between upward and downward changes, only looks at the resulting level of behavior. By using the creation and endowment functions, we can obtain separate parameters (and assess the different impact) of effects for the increase and the decrease of behavior.

#### 2.1.3 Outline of estimation procedure

RSiena estimates parameters by the function siena07 and (alternatively) sienacpp, using the following procedure:

- 1. Certain statistics are used that reflect the parameter values; the finally obtained parameters should be such that the *expected values* of the statistics are equal to the *observed values*.

  Expected values are approximated as the averages over a lot of simulated networks.
  - Expected values are approximated as the averages over a lot of simulated networks. Observed values are calculated from the data set. These are also called the *target values*.
- 2. To find these parameter values, an *iterative stochastic simulation algorithm* is applied. This works as follows:
  - (a) In Phase 1, the sensitivity of the statistics to the parameters is roughly determined.
  - (b) In Phase 2, provisional parameter values are updated iteratively: this is done by simulating a network according to the provisional parameter values, calculating the statistics and the deviations between these simulated statistics and the target values, and making a little change (the 'update') in the parameter values that hopefully goes into the right direction. A lot of such updating steps are taken, each using the parameter that was produced in the preceding step.
    - (Only a 'hopefully' good update is possible, because the simulated network is only a random draw from the distribution of networks, and not the expected value itself.)
  - (c) In Phase 3, the final result of Phase 2 is used, and it is checked if the average statistics of many simulated networks are indeed close to the target values. This is reflected in the so-called overall maximum convergence ratio and the t statistics for deviations from targets. If some of these are too high (a threshold of 0.25 is used for the overall maximum convergence ratio, and a threshold of 0.1 for the absolute value of the t statistics for deviations from targets), the estimation must be repeated. Standard errors for the parameters are also estimated in this phase.

If the estimation has to be repeated, this can be done by employing the argument prevAns in the call of siena07 (or sienacpp). See the help page for siena07.

#### 2.1.4 Design issues for longitudinal network modeling

Issues concerning the design of longitudinal network models according to the Stochastic Actor-Oriented Model are discussed in Stadtfeld et al. (2020).

#### 2.1.5 Identifiability

Identifiability of a statistical model means that different parameter vectors imply different probability distributions. This is a basic requirement for any statistical model.

The Stochastic Actor-Oriented Model is so complicated mathematically that it is hard to give proofs of any properties. For this type of model, which comes to terms with strongly dependent random variables, mathematical proofs of desirable properties are so hard that they should not be prerequisites for the use of the model. Identifiability depends, of course, on the model specification. The Stochastic Actor-Oriented Model is a combination of generalized linear models (one for the timing and another one for the choices of tie or behavior changes) with very much missing data (the ministeps); the basis in generalized linear models suggests that – unless there are redundant effects, e.g., effects of covariates that are constant across actors and across waves – the model is identifiable indeed. The estimation algorithm is such that the fact that there is convergence (see Section 6.3.2) strongly suggests that there is identifiability.

#### 2.1.6 Further useful options in RSiena

- Checking for time heterogeneity (Sections 5.13 and 8.6)
- Goodness of fit (Section 5.15)
- Meta-analysis of RSiena results (Section 11.3)
- Simulation without estimation (Section 9)

#### 2.2 Installing R and RSiena

This and the next section give an overview of steps one needs to go through from installing R to running models in RSiena. Installing needs to be done only once (but should be repeated when next versions of the software appear).

#### 1. Install R.

This can be done from http://cran.r-project.org/.

Many users prefer some kind of additional environment, such as RStudio, or the combination of Notepad++ with NppToR.

For Mac, you may also need X11, which is used for tcltk; but this is not a necessity. See the FAQ list for R on Mac.

2. Install the package RSiena, with dependencies. The other packages used are parallel and tools (all included in the basic R distribution); Matrix, MASS, lattice, codetools ('recommended' packages included in most R distributions), network and xtable. tcltk is optional. For goodness of fit testing it will be useful also to install sna and igraph.

Currently (mid February 2024), RSiena is archived on CRAN, which means that it is not directly available. This hopefully will be restored soon.

You can download the latest version of RSiena from GitHub or the SIENA website. To do this, go to

https://github.com/stocnet/rsiena/releases

http://www.stats.ox.ac.uk/~snijders/siena/siena\_downloads.htm

and there download the appropriate version of the package appropriate for your operation system (Windows, Mac, Unix).

Installation from GitHub can be done as follows. Download the latest release from <a href="https://github.com/stocnet/rsiena/releases/">https://github.com/stocnet/rsiena/releases/</a>. Once the file has been downloaded, unzip it, and install the binary as appropriate for your Operating System. This can be done in Windows using the dropdown menu under Packages, and for all Operating Systems using function install.packages with repos = NULL.

In RStudio, go to Tools  $\rightarrow$  Install packages and choose

Install From: Package archive file (zip; tar.gz).

Alternatively, install the remotes package in R, and run

remotes::install\_github("stocnet/rsiena", build=FALSE)

## 2.3 Using RSiena within R

- 1. Load data (networks, behavior, covariates) into R (see Section 4.1):
  - (a) Network data should be in objects of class matrix or sparse matrix ("TsparseMatrix-class", package Matrix).
  - (b) Behavioral data should be in objects of class matrix;
  - (c) Individual constant covariates should be in objects of class vector or should be in columns or rows of a matrix;
  - (d) Individual varying covariates should be in objects of class matrix;
  - (e) Dyadic covariates should be in objects of class matrix or "TsparseMatrix-class".
- 2. All missing data should be set to NA (see Section 4.3.2).
- 3. Check whether your data objects meet the following criteria:
  - (a) Each object contains the same nodes/actors;
  - (b) Nodes are in the same order in each object;

(c) Nodes are in the same order in rows and columns of matrix objects (in case of one-mode networks)<sup>2</sup>.

If a two-mode network is studied, then of course there will be two node sets.

- 4. Create data objects suitable for RSiena using the appropriate functions (see Section 4.1):
  - (a) sienaDependent for networks and behavior variables;
  - (b) only for two-mode networks, sienaNodeSet for defining nodesets;
  - (c) coCovar and varCovar for constant and changing/varying individual covariates respectively;
  - (d) coDyadCovar and varDyadCovar for constant and changing/varying dyadic covariates respectively;
  - (e) In case of two-mode networks, for each object it should be specified which nodeset it is defined on, using the nodeSets argument in the above functions.
- 5. Create a siena data object containing all the objects specified above using the function sienaDataCreate (see Section 4.1).
- 6. Use getEffects to create an effects object. This already gives a very simple model specification containing the outdegree and a reciprocity effects (see Section 5.2 for two-mode networks see Section 5.3).
- 7. Use sienaAlgorithmCreate to create an algorithm object (see Section 5).
- 8. Use print01Report to produce an output file presenting some descriptive statistics for the objects included in the model.
- 9. Use functions include Effects, set Effect and include Interaction to further specify the model (see Sections 5.2 5.8).
- 10. Use siena07 or sienacpp to run the estimation procedure.<sup>3</sup>
- 11. Basic output is written to a log file in the actual working directory. The filename is the project name specified in the sienaAlgorithmCreate function. Results can also be inspected in R using various functions.

#### 2.4 Example R scripts for getting started

The following scripts on the RSiena website go through the steps outlined in the previous section, providing additional details and options:

<sup>&</sup>lt;sup>2</sup>For directions on how to handle composition change, see Section 4.3.3

 $<sup>^{3}</sup>$ The use of multiple processes can speed up the estimation. For directions on how to utilize multiple processors, see Section 6.13.

- basicRSiena.r: a minimal example of a basic sequence of commands for estimating a model by function siena07 of RSiena.
- Rscript01DataFormat.R: gives a brief overview of R functions and data formats that are essential for using RSiena.
- Rscript02SienaVariableFormat.R: shows how to prepare data for a RSiena analysis, including the creation of RSiena objects; and how to specify effects for RSiena models.
- Rscript03SienaRunModel.R: shows how to carry out the estimation and look at the results;
- Rscript04SienaBehaviour.R: illustrates how to specify models for dynamics of networks and behaviour.

The website contains a lot of other scripts illustrating other functionalities of RSiena.

#### 2.5 Steps for looking at results: Executing SIENA.

1. Look at the start of the output file obtained from print01Report for general data description (degrees, etc.), to check your data input and get a general overview of the data set.

In this file, there is a section 'Change in networks' which contains some basic descriptives. Some of these refer to the *periods*: these are the combinations of two successive waves. For example, a two-wave data set has one period, and a three-wave data sets has periods  $1 \Rightarrow 2$  and  $2 \Rightarrow 3$ . The Distance mentioned there is the Hamming distance between successively observed networks, i.e., the number of tie variables that differ. The Jaccard index is the Jaccard similarity index between the successive networks:

$$\frac{N_{11}}{N_{01}+N_{10}+N_{11}}\ ,$$

where  $N_{hk}$  is the number of tie variables with value h in one wave and value k in the next wave. The Jaccard index is a measure for stability; see Snijders et al. (2010b). Both for the Hamming distance and the Jaccard index, only those cells in the adjacency matrix are counted that have available data in the wave at the start and the wave at the end of the period concerned.

If Jaccard indices are very low while the average degree is not strongly increasing, this indicates that the turnover in the network may be too high to consider the data as an evolving network, and perhaps the SIENA method is not suitable for the data set. For networks of the type that are mostly used for this method (sparse but not too sparse, with average degrees not too different from wave to wave and between 2 and 15 for all waves), Jaccard values of .3 and higher are good; values lower than .2 indicate that there might be difficulties in estimation; values lower than .1 are quite low indeed. Using the SIENA method for two waves with an extremely low Jaccard

index and average degrees that remain more or less constant will mean that the first wave hardly plays a role in the results, and for non-conditional estimation it will be close to treating the second wave as a sample from the stationary distribution of the network dynamics.

If Jaccard indices are low because the network is mainly increasing (creation of new ties) or decreasing (termination of ties), this is no problem for the SIENA method. Very sparse networks (with most degrees less than 2) also may have lower Jaccard values without negative consequences for estimation.

- 2. When parameters have been estimated, first look at the overall maximum convergence ratio and the t statistics for deviations from targets. We say that the algorithm has converged if the former is less than 0.25, and the latter all are smaller than 0.1 in absolute value; and that it has nearly converged if the former is less than 0.35, and the latter are all smaller than 0.15. Results obtained for non-converged estimation runs may be misleading. (Very small deviations from these values are of course immaterial.) See Section 6.3.2.
- 3. In rare circumstances, when the data set leads to instability of the algorithm, the following may be of use. The Initial value of the gain parameter determines the step sizes in the parameter updates in the iterative algorithm. This is the parameter called firstg in function sienaAlgorithmCreate. A too low value implies that it takes very long to attain a reasonable parameter estimate when starting from an initial parameter value that is far from the 'true' parameter estimate. A too high value implies that the algorithm will be unstable, and may be thrown off course into a region of unreasonable (e.g., hopelessly large) parameter values. It usually is unnecessary to change this, but in some cases it may be useful.
- 4. If all this is to no avail, then the conclusion may be that the model specification is incorrect for the given data set.
- 5. Further help in interpreting output is in Section 6.8 of this manual.

#### 2.6 Getting help with problems

For methodological help, consult the tutorial Snijders et al. (2010b) or this manual. The website, http://www.stats.ox.ac.uk/~snijders/siena/, contains various further publications (also in other languages than English) that may be helpful, as well as example scripts. There is a users' group for SIENA to exchange information and seek technical advice; the address is https://groups.io/g/RSiena.

For technical problems running RSiena, follow the following points.

**Help pages** Study the R help page for the function you are using and that seems to give the problems. This manual complements the help pages, but does not replace them!

Check your version of RSiena The 'News' page of the SIENA website gives information about new versions of RSiena. Details of the latest version available can be found

at the 'Downloads' page. The version is identified by a version number (e.g. 1.4.14). You can find the number of your current installed version by opening R , and typing packageVersion("RSiena")

or

packageDescription("RSiena"). The version is near the top. It is also displayed at the start of RSiena output files.

- Check your version of R When there is a new version or revision of RSiena it will only be available to you automatically if you are running the most recent major version of R. (You can force an installation if necessary by downloading the tarball or binary and installing from that, but it is better to update your R.)
- Check both repositories We have two repositories in use for RSiena: CRAN and GitHub. The latest version will always be available from GitHub. (Frequent updates are discouraged on CRAN, so bug-fixes are likely to appear first on GitHub.)
- **Installation** When using the repository at GitHub, *install* the package rather than updating it. Then check the version and revision numbers.
- Users' group Consult the archives of the Users' Group mentioned above, or post a message to the Users' Group. In your message, please tell which operating system, which version of R, and which version of RSiena you are using.

# 3 Steps of modelling

The operation of the RSiena program is comprised of five main parts:

- 1. input of basic data description (see Section 4);
- 2. model specification (see Section 5);
- 3. estimation of parameter values using stochastic simulation (see Section 6); the main output of the estimation procedure is written to a text file named *pname*.txt, where *pname* is the name specified in the call of sienaAlgorithmCreate;
- 4. testing parameters and assessing goodness of fit (see Sections 7 and 8);
- 5. simulation of the model with given and fixed parameter values (see Section 9).

The normal operation is to start with data input, then specify a model and estimate its parameters, assess goodness of fit and the significance of the parameters, and then possibly continue with new model specifications followed by estimation or simulation.



# 4 Input data

RSiena is a program for the statistical analysis of repeated measures of social networks, and requires, at the very least, network data collected at two or more time points. It is also possible to include other types of variables in the models – these are discussed in Section 4.1. Section 4.2 describes the most commonly occurring data transformations that are done internally by SIENA. Finally, Section 4.3 shows further options for users to define their data.

# 4.1 Data types

As we discussed in Section 2.1.2, dependent variables in Stochastic Actor-Oriented Models are defined from network or behavioral data. Independent variables (effects) are defined from individual or dyadic covariate data, which can be constant or varying. RSiena requires each of these data types to have a specific format – this is presented in the current section.

In general, data specification in RSiena consists of two steps. First, the role of each variable to be used must be defined using the functions sienaDependent, coCovar, varCovar, coDyadCovar, varDyadCovar, or sienaCompositionChange. Second, the variables must be combined into one siena data set by the function sienaDataCreate. This function puts together the data set and carries out some preliminary calculations.

It is advisable to use names of variables consisting of at most 12 characters. This is because they are used as parts of the names of effects which can be included in the model, and the effect names should not be too long.

RSiena does not work with case numbers. The correspondence between cases in the different components of the data set is by the order of the rows in the data matrices. For a data set with n actors, each data matrix should have n rows and always the i'th row should correspond to the i'th actor.

It is also useful to note here that in case of co-evolution models (those with more than one dependent networks and/or behaviors), data for all dependent variables must be available for the same set of time points.

#### 4.1.1 Network data

There can be one or more networks in the data set. Networks can be one-mode or two-mode. Modes are identified with node sets. All one-mode networks must have the same node set, also referred to as 'actors'. If there are some one-mode networks and any two-mode networks, the first node set of all two-mode networks should be the node set (actor set) of the one-mode networks. The choice between one- and two-mode is specified by the argument type in the sienaDependent function, which for networks can be oneMode or bipartite; the latter signifies a two-mode network. If there are one-mode as well as two-mode dependent networks, the one-mode networks should be mentioned first in the call of sienaDependent.

For data specification by sienaDependent, the network must be specified as a matrix or array or list of sparse matrix of triples. The help page for sienaDependent has an example of input by sparse matrices.

For data specification by the function sienaDataCreateFromSession, edge list formats are also allowed. This can be either the format of the Pajek program, or a raw edge list, here called Siena format. For large number of nodes (say, larger than 100), the edge list format is more efficient in use of computer memory.

Note that sienaDataCreateFromSession is available only in RSienaTest.

Sparse matrices, which can be used by input via sienaDependent, have for running the estimations the same efficiency as Pajek or Siena format. The three possible formats for digraph input are as follows.

#### 1. Adjacency matrices.

These can be used in sienaDependent and in sienaDataCreateFromSession.

In the usual case of a one-mode network the adjacency matrix is given in a matrix of n rows and n columns containing integer numbers. The diagonal values are meaningless but must be present. In the case of a two-mode network (which is a network with two node sets, and all ties are between the first and the second node set) the matrix does not have to be square, as usually the number of nodes in the first set will not be equal to the number of nodes in the second set; and if it would be square, the diagonal still would be meaningful.

Although this section talks only about digraphs (directed graphs), for one-mode networks it is also possible that all observed adjacency matrices are symmetric. This will be automatically detected by SIENA, and the program will then utilize methods for non-directed networks.

The values of the ties must be 0, 1, or NA (not available = missing); or 10 or 11 for structurally determined values (see below).

The help file for sienaDependent shows by examples how the specification can be given by sparse matrices.

#### 2. Pajek format.

These can be used in sienaDataCreateFromSession (only available in RSienaTest). If the digraph data file has extension name .net, then the program assumes that the data file has Pajek format. The file should relate to one observation only, and should contain a list of vertices (using the keyword \*Vertices, together with (currently) a list of arcs, using the keyword \*Arcs followed by data lines according to the Pajek rules. These keywords must be in lines that contain no further characters. An example of such input files is given in the s50 data set that is distributed in the examples directory of the source code.

#### 3. Siena format.

These can be used in sienaDataCreateFromSession (only available in RSienaTest). An edge list is a matrix containing three or four columns: from, to, value, wave (optional).

Like the Pajek format, this has the advantage that absent ties (tie variables with the value 0) do not need to be mentioned in the data matrix. By specifying the waves

in the fourth column in the Siena format, one matrix can be used to contain data for all the waves.

Missing values must be indicated in the way usual for R, by NA.

If the data set is such that it is never observed that ties are terminated, then the network dynamics is automatically specified internally in such a way that termination of ties is impossible. (In other words, in the simulations of the actor-based model the actors have only the option to create new ties or to retain the status quo, not to delete existing ties.) Similarly if ties never are created (but only terminated), then this will be respected in the simulations. See Section 4.2.4 and note the possibility of using allowOnly=TRUE.

#### 4.1.2 Transformation between matrix and edge list formats

The following R commands can be used for transforming an adjacency matrix to an edge list, and back again. If a is an adjacency matrix, then the following commands can be used to create the corresponding edge list, called edges here.

Some notes on the commands used here:

These commands can be used not only if the adjacency matrix contains only 0 and 1 entries, but also if it contains values NA, 10, or 11. The possibility of NA entries requires special attention; %in% does just what we need, as it quietly says that NA's are not %in% anything, returning FALSE, which is transformed to TRUE by the ! function. The edge list is created having all 0 values and at the end should have no 0 values at all.

It is more efficient, however, to work with sparse matrices; this also is done internally in RSiena. Using the Matrix package for sparse matrix manipulations, the same results can be obtained as follows.

```
library(Matrix)
tmp <- as(a, "dgTMatrix")
edges2 <- cbind(tmp@i + 1, tmp@j + 1, tmp@x)</pre>
```

Conversely, if edges is an edge list, then the following commands can be used to create the corresponding adjacency matrix, called adj, with n nodes. (For a bipartite network the two dimensions will normally be distinct numbers.)

```
# create empty adjacency matrix
adj <- matrix(0, n, n)
# put edge values in desired places
adj[edges[, 1:2, drop = FALSE]] <- edges[, 3, drop = FALSE]</pre>
```

Note that this starts with a matrix having all 0 entries, and results in a matrix with no 0 entries at all. To check the results, after doing these two operations, the command

```
length(which(a != adj))
```

should return the value 0.

Note that the basic edge list, edges, lacks information as to the size of the adjacency matrix. tmp above is a sparse matrix which is in edge list format but includes information on the size of the adjacency matrix, and can be used in a similar way to the original matrix a while saving memory space.

For some insight into how this is done with RSiena, you could request

#### RSiena:::sparseMatrixExtraction0

to see how this function is defined, using either the data set (for i == NULL) or the simulated data from siena07 (for i != NULL).

#### 4.1.3 Behavioral data

RSiena also allows dependent behavior variables. This can be used in studies of the co-evolution of networks and behavior, as described in Snijders et al. (2007) and Steglich et al. (2010). These behavior (or 'action') variables represent the actors' behavior, attitudes, beliefs, etc. The difference between dependent behavior variables and changing actor covariates (see below) is that the latter have values determined by the input data and are assumed to change exogenously, i.e., according to mechanisms not included in the model, while the dependent action variables change endogenously, i.e., depending on their own values and on the changing network. Unlike the changing individual covariates, the values of dependent action variables are not assumed to be constant between observations.

Dependent behavioral variables can be ordinal discrete or continuous. This is indicated by the values behavior or continuous, respectively, for keyword type in sienaDependent.

Ordinal discrete behavioral variables must have nonnegative integer values; e.g., 0 and 1, or a range of integers like 0,1,2 or 1,2,3,4,5. The number of different values should not be too high: ten values is on the high side, but can still be totally adequate. For categorization of variables that originally are continuous or have many values, it often is advisable to take approximately equal intervals on the original scale, and to achieve a rather smooth distribution of values, which might resemble a discretized normal or skew distribution, as the case may be. Using a small number of values may amount to a loss of information. If the original variable is continuous then 8 to 10 categories may be a good number. This is studied in Niezink (2018, Chapter 6).

Each dependent action variable must be given in one matrix, containing M columns, corresponding to the M observation moments.

If any values are not integers but type=behavior, a warning will be printed on the initial report given by print01Report and the values will be truncated towards zero.

A special case of behavioral data can be used for diffusion of innovations (Greenan, 2015a): here the behavior variable representing having adopted the innovation is binary, coded 0 or 1, and changes  $1 \Rightarrow 0$  are impossible. Model specifications that are especially useful for this data type are presented in Section 12.2.4.

#### 4.1.4 Continuous behavioral data

For continuous behavioral data, see https://www.stat.cmu.edu/~nynke/code.html.

#### 4.1.5 Individual covariates

Individual (i.e., actor-bound, or monadic) variables are defined by the functions coCovar in the case they are constant over time, and varCovar if they are changing over time.

Each constant actor covariate has one value per actor valid for all observation moments, and has the role of an independent variable.

Changing variables can change between observation moments; then they are called 'changing individual covariates', and have the role of independent variables.

The internal operation of RSiena is not scale-independent. It is advisable to scale the covariates so that they have standard deviations in the range between 0.1 and 10.

Changing individual covariates are assumed to have constant values from one observation moment to the next. If observation moments for the network are  $t_1, t_2, ..., t_M$ , then the changing covariates should refer to the M-1 moments  $t_1$  through  $t_{M-1}$ , and the m-th value of the changing covariates is assumed to be valid for the period from moment  $t_m$  to moment  $t_{m+1}$ . The value at  $t_M$ , the last moment, does not play a role. Changing covariates, as independent variables, are meaningful only if there are 3 or more observation moments, because for 2 observation moments the distinction between constant and changing covariates is not meaningful.

Each changing individual covariate must be specified in a separate call of varCovar, using for input an  $n \times (M-1)$  matrix where the columns correspond to the M-1 periods between observations.

By default, the mean is subtracted from the covariates. This can be turned off by constructing the covariate with the keyword centered=FALSE. Centering of covariates improves convergence of the estimation algorithm. Therefore the advice is to use the default (centering) unless there are reasons to use non-centered covariates, such as when they are used as weights in effects (e.g., for inPopX). It is possible to include a given actor variable in the Siena data set as a covariate in the centered as well as the non-centered version. See Section 4.2.2 on centering.

#### 4.1.6 Dyadic covariates

Like the digraph data, also each measurement of a dyadic covariate must be contained in a separate matrix. For one-mode data this is a square data matrix, and the diagonal values are meaningless. A distinction is made between constant and changing dyadic covariates, where change refers to changes over time. Each constant covariate has one value for each pair of actors, which is valid for all observation moments, and has the role of an independent variable. Changing covariates, on the other hand, have one such value for each period between measurement points. If there are M waves (i.e., observation moments) of network data, this covers M-1 periods, and accordingly, for specifying a single changing one-mode dyadic covariate, a  $n \times n \times (M-1)$  array is needed.

Two-mode dyadic covariates also are possible; see the help pages for coDyadCovar and coDyadCovar,

The internal operation of RSiena is not scale-independent. It is advisable to scale the covariates so that they have standard deviations in the range between 0.1 and 10.

Like is the case for monadic covariates, changing dyadic covariates are assumed to have constant values from one observation moment to the next. If observation moments for the network are  $t_1, t_2, ..., t_M$ , then the changing covariates refer to the M-1 moments  $t_1$  through  $t_{M-1}$ , and the m-th value of the changing covariates is assumed to be valid for the period from moment  $t_m$  to moment  $t_{m+1}$ . The value at  $t_M$ , the last moment, does not play a role.

Constant dyadic covariates are specified using function coDyadCovar, and changing dyadic covariates by varDyadCovar.

The mean is always subtracted from the covariates. See Section 4.2.2 on centering.

#### 4.2 Internal data treatment

#### 4.2.1 Interactions and dyadic transformations of covariates

For actor covariates (also called monadic covariates), two kinds of transformations to dyadic covariates are made internally in SIENA, by the definition of the corresponding effects. Denote the actor covariate by  $v_i$ , and the two actors in the dyad by i and j. Suppose that the range of  $v_i$  (i.e., the difference between the highest and the lowest values) is given by  $r_V$ . The two transformations are the following:

1. dyadic similarity is defined by

$$\sin_{ij}^{V} = 1 - \frac{|v_i - v_j|}{r_V} \ . \tag{1}$$

This similarity measure is 1 if the two actors have the same value, and 0 if one has the highest and the other the lowest possible value. For the various similarity effects, this measure is centered by subtracting the mean over all dyads, so the mean of this similarity variable becomes 0.

The advantage of this similarity measure is that its coefficients are comparable in the sense that the measure always ranges between 0 and 1.

The mean of (1) is calculated by function sienaDataCreate and stored as the simMean attribute of mydata\$cCovars\$myvar, where mydata is the name of the object created by sienaDataCreate, and myvar is the name of the variable used as the argument for sienaDataCreate, while the name cCovars applies for constant monadic covariates,

and is to be replaced by vCovars for changing (varying) monadic covariates; for centering issues, further see Section 4.2.2.

2. same V is defined by

$$same_{ij}^{V} = \begin{cases} 1 & \text{if } v_i = v_j \\ 0 & \text{if } v_i \neq v_j \end{cases}$$
 (2)

This can also be referred to as dyadic identity with respect to V.

Dyadic similarity is relevant for variables that can be treated as interval-level variables; dyadic identity is relevant for categorical variables.

In addition, RSiena offers the possibility of user-defined two- and three-variable interactions between covariates; see Section 5.12.

#### 4.2.2 Centering

Individual as well as dyadic covariates can be centered by the program in the following way.

For individual covariates, the mean value is subtracted by function sienaDataCreate. The centered values then are stored (see below), and all calculations use these centered variables. For the changing covariates, the mean value used is the global mean (averaged over all periods). The values of these subtracted means are reported in the output of print01Report. For the multi-group option (section 11.2), the subtracted values are the global means across all groups.

Centering is the default. Centering of covariates can be turned off by specifying centered=FALSE in the call of coCovar, varCovar, coDyadCovar, or varDyadCovar, respectively. Centering of covariates improves convergence of the estimation algorithm. Therefore the advice is to use the default (centering) unless there are reasons to use non-centered covariates, such as when they are used as weights in effects (as is the case, e.g., for effect inPopX). If some actor variable is to be used as a non-centered covariate, it is recommended to include it as a covariate in the centered as well as the non-centered version in the Siena data set, and use the non-centered version only when it is necessary to get the appropriate effect definition.

For the dyadic covariates and the similarity variables derived from the individual covariates, the grand mean is calculated and stored by function SienaDataCreate; the stored values of the variables are not centered, but the means are subtracted during the program calculations. (Thus, dyadic covariates are treated internally by the program differently than individual covariates in the sense that the mean is subtracted at a different moment, but the effect is the same; except for multi-group data sets, see below.) Unlike the 'covariate similarity' effect, the 'same covariate' effect is not centered but keeps its 0-1 values.

For the multi-group option (section 11.2), dyadic covariates are treated <u>differently</u> from individual covariates: for dyadic covariates in multi-group data sets, centering is done by the within-group mean; actor covariates in multi-group data sets created by sienaGroupCreate are centered by the overall mean.

For dependent behavioral variables, the effects are defined in Section 12.2 as functions of centered variables.

The means of covariates are stored as attributes on the object created by sienaData-Create. If you wish to access them, the following steps can show where these means can be found. For example, suppose that the command given was

```
mydata <- sienaDataCreate( friendship, smoke1, alcohol )</pre>
```

The structure of this object is obtained by requesting

```
str(mydata, 1)
```

Looking at the response, you will see that this object contains (among other things):

- 1. the constant actor covariates as mydata\$cCovars
- 2. the varying actor covariates as mydata\$vCovars
- 3. the constant dyadic covariates as mydata\$dycCovars
- 4. the varying dyadic covariates as mydata\$dyvCovars
- 5. the dependent behavioral variables as mydata\$depvars

Since smoke1 is a constant covariate and alcohol a changing covariate, their means can be requested by

```
attr(mydata$cCovars$smoke1, "mean")
attr(mydata$vCovars$alcohol, "mean")
```

and the centered values for, e.g, the variable alcohol by

```
mydata$vCovars$alcohol
```

The mean of the similarity variable is stored as the simMean attribute, and is obtained by, e.g.,

```
attr(mydata$cCovars$smoke1, "simMean")
```

The formula for balance is a kind of dissimilarity between rows of the adjacency matrix. The mean dissimilarity is subtracted in this formula, having been calculated according to a formula given in Chapter 12. It is also reported in the output file produced by print01Report and available - e.g., for the first dependent variable - as

```
attr(mydata\$depvars[[1]], "balmean")
```

Instead of [[1]] you can request a different number or the name of the variable.

For multi-group data sets constructed by function sienaGroupCreate, the attribute "vCovarMean" contains the overall means of the actor covariates.

#### 4.2.3 Centering of behavioral variables

Dependent behavior variables are not centered by sienaDataCreate, but internally during the simulations. Therefore their mean is not stored in the Siena data set. However, the values and their frequencies are stored in (using mydat and smoke as example names)

```
attr(mydata$depvars$smoke, "vals")
```

From this, the mean can be calculated by using the functions

```
theMean <- function(x){
# Mean of a named vector of frequencies, where the names are the values
    sum(as.integer(names(x)) * x, na.rm = TRUE)/sum(x, na.rm = TRUE)
}
meanByWave <- function(x, vari, w){
    theMean(attr(x$depvars[[vari]], "vals")[[w]])
}
grandMean <- function(x, vari){
    xxx <- attr(x$depvars[[vari]], "vals")
    mean(sapply(xxx, theMean))
}</pre>
```

For comprehensibility, these three functions are given separately here. The first is a general utility function, the second and third specific for RSiena. They can be applied, e.g., as

```
grandMean(mydata, "smoke")
sapply(1:mydata$observations, meanByWave, x = mydata, vari = "smoke")
and for multi-group data as
sapply(myGroupData, grandMean, vari = "smoke")
```

#### 4.2.4 Monotonic dependent variables

In some data sets, a dependent variable only increases, or only decreases. For a network, this means that ties can be created but not terminated, or the other way around. This may be the case for all periods (a period is defined by the two consecutive observation waves at its start and end points) or just in some of the periods. RSiena will note when a dependent variable only increases or only decreases in any given period, and mention this in the output file generated by print01Report. This constraint then is also respected in the simulations, in the periods where it is observed. This is represented internally by a variable called uponly indicating that the dependent variable cannot decrease, and a variable downonly indicating that the dependent variable cannot increase. The constraints signaled by the uponly and downonly variables can be lifted by using allow0nly = FALSE in the call of sienaDependent (see the help file for this function).

If a dependent variable is only increasing or only decreasing for all periods and siena-Dependent was called with allowOnly=TRUE (the default), then two basic effects are not identified. These are the outdegree effect for a dependent network variable, and the linear shape effect for a dependent behavior variable; these effects define the balance between the probabilities of going up and going down. These effects then are dropped automatically from the effects object. If this is not desired, this can be prevented by calling sienaDependent with allowOnly=FALSE.

### 4.2.5 Multivariate dependent variables with constraints

When analysing multiple dependent networks, it is possible that they are related in some deterministic way. For example, two networks might be mutually exclusive (this could be the case for negative and positive ties – although there are many cases where positive and negative ties may coexist!), or networks could be ordered, i.e., one network could be a sub-network of the other (networks with ordinal tie values represented as a multivariate network). This is treated in Section 5.7.

# 4.3 Further data specification options

#### 4.3.1 Structurally determined values

It is allowed that some of the values in the digraph are structurally determined, i.e., deterministic rather than random. This is analogous to the phenomenon of 'structural zeros' in contingency tables, but in RSiena not only structural zeros but also structural ones are allowed. A structural zero means that it is certain that there is no tie from actor i to actor j; a structural one means that it is certain that there is a tie. This can be, e.g., because the tie is impossible or formally imposed, respectively.

For estimation by the Method of Moments, structural zeros provide an easy way to deal with actors leaving or joining the network between the start and the end of the observations: specify all their incoming and outgoing tie variables, at the moment that they are not present, as structural zeros. Note that actors having all values specified as structural zeros in this way take part of the simulations only starting at the observation moment where they are not totally structurally zero; therefore, this way of representing partially absent actors is not meaningful for actors who are present only at the very last wave. In particular, this includes the case where there are two waves only for actors who join the network after the first wave.

Another way (more complicated but more flexible, because it gives possibilities to represent actors entering or leaving at specified moments between observations) is the method of joiners and leavers, described in Section 4.3.3. For actors present only at the last wave, the method of joiners and leavers is preferable.

When endowment or creation effects are to be included in the model specification, changing structural values should not be used, and the method of joiners and leavers then also is preferable.

It should be noted that if the model also contains behavioral dependent variables and structural zeros are used to represent changing composition of the network (or networks), the behavioral dependent variables still will be possible to change. To keep them from changing for the absent actors, a non-centered dummy actor variable can be used which is

1 for the absent actors and 0 for all others, and which is given a fixed RateX effect for the behavioral variables of a large negative value, e.g., -100; this effect should be fixed and not tested.

Structurally determined values are defined by reserved codes in the input data: the value 10 indicates a structural zero, the value 11 indicates a structural one. Structurally determined values can be different for the different time points. (The diagonal of the data matrix for a one-mode network always is composed of structural zeros, but this does not have to be indicated in the data matrix by special codes.) The correct definition of the structurally determined values can be checked from the brief report of this in the output file of print01Report — which is given only, however, if the diagonal entries of the one-mode networks are also set to 10.

If there are a lot of structurally determined values then unconditional estimation (see Section 6.12.1) is preferable.

When RSiena simulates networks including some structurally determined values, if these values are constant across all observations then the simulated tie values are likewise constant. If the structural fixation varies over time, the situation is more complicated. Consider the case of two consecutive observations m and m+1, and let  $X_{ij}^{\text{sim}}$  be the simulated value at the end of the period from  $t_m$  to  $t_{m+1}$ . If the tie variable  $X_{ij}$  is structurally fixed at time  $t_m$  at a value  $x_{ij}(t_m)$ , then  $X_{ij}^{\text{sim}}$  also is equal to  $x_{ij}(t_m)$ , independently of whether this tie variable is structurally fixed at time  $t_{m+1}$  at the same or a different value or not at all. This is the direct consequence of the structural fixation. On the other hand, the following rule is also used. If  $X_{ij}$  is not structurally fixed at time  $t_m$  but it is structurally fixed at time  $t_{m+1}$  at some value  $x_{ij}(t_{m+1})$ , then in the course of the simulation process from  $t_m$  to  $t_{m+1}$  this tie variable can be changed as part of the process in the usual way, but after the simulation is over and before the statistics are calculated it will be fixed to the value  $x_{ij}(t_{m+1})$ .

The target values for the algorithm of the Method of Moments estimation procedure are calculated for all observed digraphs  $x(t_{m+1})$ . However, for the variables  $X_{ij}$  that are structurally fixed at time  $t_m$ , the observed value  $x_{ij}(t_{m+1})$  is replaced by the structurally fixed value  $x_{ij}(t_m)$ . This gives the best possible correspondence between target values and simulated values in the case of changing structural fixation.

For estimation by Maximum Likelihood, treated in Section 6.11, structural zeros operate differently than for the Method of Moments, and probably it is not a good idea to use changing structural zeros; see Section 6.11.

Structural zeros offer the possibility of analyzing several networks simultaneously under the assumption that the parameters are identical. However, a preferable option to do this is the multi-group option of Section 11.

The following explanation of the option of using structural zeros to analyze several networks simultaneously is given mainly for historical reasons (to understand earlier publications); this is not a recommended option.

E.g., if there are three networks with 12, 20 and 15 actors, respectively, then these can be integrated into one network of 12 + 20 + 15 = 47 actors, by specifying that ties

between actors in different networks are structurally impossible. This means that the three adjacency matrices are combined in one  $47 \times 47$  data matrix, with values 10 for all entries that refer to the tie from an actor in one network to an actor in a different network. In other words, the adjacency matrices will be composed of three diagonal blocks, and the off-diagonal blocks will have all entries equal to 10. In this example, the number of actors per network (12 to 20) is rather small to obtain good parameter estimates, but if the additional assumption of identical parameter values for the three networks is reasonable, then the combined analysis may give good estimates.

In such a case where K networks (in the preceding paragraph, the example had K=3) are combined artificially into one bigger network, it will often be helpful to define K-1 dummy variables at the actor level to distinguish between the K components. These dummy variables can be given effects in the rate function and in the evaluation function (for 'ego"), which then will represent that the rate of change and the out-degree effect are different between the components, while all other parameters are the same.

It will be automatically discovered by RSiena when monadic covariates depend only on these components defined by structural zeros, between which tie values are not allowed. For such variables, only the ego effects are defined and not the other effects defined for the regular actor covariates and described in Section 5.4. This is because the other effects then are meaningless. If at least one case is missing, then the other covariate effects are made available.

#### 4.3.2 Missing data

RSiena allows that there are some missing data on network variables, on covariates, and on dependent action variables. Missing data must be indicated by the usual missing data code for R, NA.

Missingness of data is treated as non-informative. One should be aware that having many missing data can seriously impair the analyses: technically, because estimation will be less stable; substantively, because the assumption of non-informative missingness often is not quite justified. Up to 10% missing data will usually not give many difficulties or distortions, provided missingness is indeed non-informative (Huisman and Steglich, 2008). When one has more than 20% missing data on any variable, however, one may expect problems in getting good estimates.

In the current implementation of SIENA, missing data are treated in a simple way, trying to minimize their influence on the estimation results. This method is further explained in Huisman and Steglich (2008), where comparisons are also made with other ways of dealings with the missing information. The default method in RSiena for handling missings in the dependent variable is the fourth method as described in Huisman and Steglich (2008).

The basic idea is the following.

A brief sketch of the procedure is that missing values are imputed to allow meaningful simulations; for the calculation of the target statistics in the Method of Moments, tie variables and actor variables with missings are not used. More in detail, the procedure is as follows.

The simulations are carried out over all variables, as if they were complete. To allow this, missing data are imputed. In the initial observation, missing entries in the adjacency matrix are set to 0, i.e., it is assumed that there is no tie; this is done because normally data are sparse, so 'no tie' almost always is the modal value of the tie variable. In the further observations, for any tie variable, if there is an earlier observed value of this variable then the last observed value is used to impute the current value (the 'last observation carry forward' option, cf. Lepkowski (1989)); if there is no earlier observed value, the value 0 is imputed. For the dependent behavior variables a similar principle is used: if there is a previous observation of the same variable then this value is imputed, if there is none but there is a next observation then this is imputed, if this also is absent then the observationwise mode of the variable is imputed. Missing covariate data are, by default, replaced by the variable's global mean; but in the definition of actor covariates, by the functions coCovar or varCovar, the user has the option to supply other values for imputation of the missings. In the course of the simulations, however, the imputed values of the dependent behavior variables and of the network variables are allowed to change.

In order to ensure a minimal impact of missing data treatment on the results of parameter estimation (Method of Moments estimation) and/or simulation runs, the calculation of the target statistics used for estimation by the Method of Moments, and reporting in these procedures uses only non-missing data. When for an actor in a given period, any variable is missing that is required for calculating a contribution to such a statistic, this actor in this period does not contribute to the statistic in question. For network and dependent behavior variables, the tie variable or the actor variable, respectively, must provide valid data both at the beginning and at the end of a period for being counted in the respective statistics. This is implemented as follows: if a tie variable is missing in wave m, for the calculation of observed as well as simulated target statistics for periods m-1, m, and m+1 it is replaced by the value 0; if a dependent behavior variable is missing in wave m, for the calculation of observed as well as simulated target statistics, where always the centered values are used, for periods m-1, m, and m+1 the value is replaced by 0 (which, given the centering, is equivalent to the overall mean).

For non-centered covariates, the treatment of missing data is less well thought out; this may cause problems in the analysis of data with a lot of missing values for non-centered covariates.

By using the argument imputationValues in coCovar and varCovar, other values (i.e., values different from the mean that is used by default for imputation) can be given for imputation of missings in monadic covariates. These are then used for the simulations; since they were indicated as missings (NA) in the data themselves, they will not be used for the calculation of target statistics in the Method of Moments.

For estimation by Maximum Likelihood, missing values in the dependent variables at the end of a period are treated in a model-based way. For missings at the start of a period, independent priors are used. However, periods are treated separately: simulations in period m do not help provide information for period m+1. See Siena\_algorithms.pdf for a further description.

The default method in RSiena for treating missing behavioral data in RSiena was compared with several alternative methods in Zandberg and Huisman (2019), and was found to perform quite well.

There is ongoing work by Robert Krause and others about using multiple imputation for RSiena modeling; see Krause et al. (2018). A script illustrating this for longitudinal network analysis is at the Siena scripts webpage, http://www.stats.ox.ac.uk/~snijders/siena/multipleImputation\_for\_RSiena.R Methods for co-evolution studies are in preparation.

#### 4.3.3 Composition change: joiners and leavers

RSiena can also be used to analyze networks of which the composition changes over time, because actors join or leave the network between the observations. This can be done in two ways: using the method of Huisman and Snijders (2003), or using structural zeros. (For the maximum likelihood estimation option, the Huisman-Snijders method is not implemented, and only the structural zeros method can be used.) Structural zeros can specified for all elements of the tie variables toward and from actors who are absent at a given observation moment. How to do this is described in subsection 4.3.1. This is straightforward and not further explained here. This subsection explains the method of Huisman and Snijders (2003), also called the method of joiners and leavers, which uses the information about composition change in a somewhat more efficient way.

Network composition change, due to actors joining or leaving the network, is handled separately from the treatment of missing data. The data matrices must contain all actors who are part of the network at any observation time. If adjacency matrices are used as data input, they must therefore all have the same number of n rows, each actor having a separate (and fixed) line in these matrices, even for observation times where the actor is not a part of the network (e.g., when the actor did not yet join or the actor already left the network).

The times of composition change can be given either in a data file or in a list available in the R session. For networks with constant composition (no entering or leaving actors), this file or list is omitted and the current subsection can be disregarded.

If there is composition change, estimation by the Method of Moments is forced to be unconditional (see Section 6.12.1).

For these waves, where the actor is not in the network, the entries of the adjacency matrix can be specified in two ways. First as missing values using missing value code NA. In the estimation procedure, these missing values of the joiners before they joined the network are regarded as 0 entries, and the missing entries of the leavers after they left the network are fixed at the last observed values. This is different from the regular missing data treatment. Note that in the initial data description the missing values of the joiners and leavers are treated as regular missing observations. This will increase the fractions of missing data and influence the initial values of the density parameter.

A second way is by giving the entries a regular observed code, representing the absence or presence of a tie (as if the actor was a part of the network). In this case, additional information on relations between joiners and other actors in the network before joining,

or leavers and other actors after leaving can be used if available. Note that this second option of specifying entries always supersedes the first specification: if a valid code number is specified this will always be used.

The functions used to specify the times actors join or leave the network (i.e., the times of composition change) are sienaCompositionChangeFromFile in case a file is used, and sienaCompositionChange in case a list is used. How to use a separate input file, called the exogenous events file, is described in the help page for sienaCompositionChangeFromFile.

In the second case, a list must be given of length n, where n is the number of actors in the node set. The i'th element of this list must be a vector of numbers (characters are also allowed), composed of an even number of elements, indicating the intervals during which actor i was present. For example, 1 4 indicates that the actor was present from wave 1 to wave 4 (end points included) and 1 3.2 5.01 7 indicates that the actor was present from wave 1 to 20% of the time between waves 3 and 4, and then again from just after wave 5 to wave 7.

As an example, suppose we have 50 actors and 6 waves; almost all actors were present all the time, but actor 11 was present from wave 3 onward, actor 20 was present until wave 4, and actor 33 was present from mid-way between waves 1 and 2 until wave 3, and then again from just after wave 4 to wave 6. Then the list can be created by the following commands.

```
comp <- rep(list(c(1,6)), 50)
comp[[11]] <- c(3,6)
comp[[20]] <- c(1,4)
comp[[33]] <- c(1.5,3, 4.01,6)
changes <- sienaCompositionChange(comp)</pre>
```

(The use of blanks in the line for comp[[33]] is only for visually keeping the pairs of start-end times together.)

The first line, creating a list with the (default) first and last end point for everybody, could also be replaced by

```
comp <- vector("list", 50)
comp[] <- list(c(1,6))</pre>
```

Here it may be noted that [] keeps structures etc. unchanged while replicating the expression to fit.

The object changes created by the functions sienaCompositionChangeFromFile or siena-CompositionChange is of class compositionChange and can be used in the function siena-DataCreate.

The method of joiners and leavers for representing composition change does not combine properly with the sienaGOF function (Section 5.15). But if you use NA codes for the tie variables of absent actors, and these are indicated as such by a compositionChange object, then sienaGOF will work properly.

The implementation of endowment and creation effects does not agree with changing structural zeros or ones. Therefore, if there is changing composition in a data set and the model should contain endowment and/or creation effects, the changing composition should be reflected by a changing composition and not by structural zeros.

# 5 Model specification

## 5.1 Definition of the model

After defining the data, the next step is to specify a model. The model specification consists of a selection of 'effects' for the evolution of each dependent variable (network or behavior). To understand this, first a brief review of the definition of the actor-oriented model is given (for further explanations see Snijders, 2001, 2005; Snijders et al., 2007, 2010b).

The model is based on four functions, which first are explained in an intuitive way. They are defined specifically for all dependent variables (network, behavior, or more of these if included in the model). These functions depend on the actor (hence the name 'actor-oriented') and on the state of the network, behavior, and covariates. All these functions are constituted by a weighted sum of so-called *effects*, which define the characteristics of the network (and behavior, if this is included as a dependent variable) that determine the probabilities of changes.

# • rate function

The rate function models the speed by which the dependent variable changes; more precisely: the speed by which each network actor gets an opportunity for changing her score on the dependent variable.

Advice: in most cases, start modeling with a constant rate function without additional rate function effects.

When there are important differences between actors in some measure of size or importance, it is possible that different advice must be given. This can be the case, e.g., when the network has a clear core-periphery structure; or when the outdegree distribution is very skewed. Then it may be necessary to let the rate function depend on the outdegrees or on an individual covariate indicating this size.

#### • evaluation function

The evaluation function<sup>4</sup> is the primary determinant of the probabilities of changes. Probabilities are higher for moving towards states with a higher value of the evaluation function. One way of representing this is that the evaluation function models the actor's 'satisfaction'<sup>5</sup> with her/his local network neighborhood configuration. It is assumed that actors change their scores on the dependent variable such that they improve their total satisfaction – with a random element to represent the limited predictability of behavior. In contrast to the creation and endowment functions (described below), the evaluation function evaluates only the local network neighborhood configuration that results from the change under consideration, without considering 'where you come from'. In most applications, the evaluation function will be the main focus of model selection.

<sup>&</sup>lt;sup>4</sup>The evaluation function was called *objective function* in Snijders (2001).

<sup>&</sup>lt;sup>5</sup>The term 'satisfaction' should be interpreted here in a very loose sense; the satisfaction interpretation is not necessary at all, but it does give a convenient intuitive way of thinking about the model.

#### • creation function

The creation function<sup>6</sup> distinguishes between new and old network ties (when evaluating possible network changes) and between increasing or decreasing behavioral scores (when evaluating possible behavioral changes). It is a component of the probabilities of change only for changes in an upward direction: creation of new ties, augmentation of values of the behavior dependent variable. Creation effects can be the creation parts of an evaluation effect, or elementary effects (see below).

In the interpretation using satisfaction, the creation function models the gain in satisfaction incurred when network ties are created or behavioral scores are increased.

#### • endowment or maintenance function

The endowment function<sup>7</sup>, which also may be called *maintenance function*, also distinguishes between new and old network ties (when evaluating possible network changes) and between increasing or decreasing behavioral scores (when evaluating possible behavioral changes). It is a component of the probabilities of change only for changes in a downward direction: maintenance vs. termination of existing ties, decrease of values of the behavior dependent variable.

Again, endowment effects can be the maintenance parts of an evaluation effect, or elementary effects (see below).

In the interpretation using satisfaction, the endowment function models the loss in satisfaction incurred when network ties are dissolved or behavioral scores are decreased (hence the label 'endowment').

Leaving aside the rate effects, a given effect can normally be included in the model in any of the three 'types' or 'roles' of evaluation, creation, or endowment effect. In almost all cases, the advice is to start modeling without any creation or endowment effects, and add them perhaps at a later stage. For example, if the network dynamics in a given data set is such that ties mainly are created, and they are dissolved rather rarely, then the data will contain little information about the question whether creating ties follows different rules than dissolving ties, and if one would try to include creation or endowment effects for effects already included in the evaluation function, this would lead to large standard errors. Creation and endowment effects for behavior for behavior variables with more than 2 values are still under investigation, and their interpretation for practical research still is uncertain.

A model specification with only evaluation effects and without creation and endowment effects leads to exactly the same network dynamics as a specification where these effects are turned into creation and endowment effects, if the same parameters would have exactly the same values. For any given effect, it makes no sense to include the effect in all three roles: evaluation, creation, endowment. If one wishes to go beyond evaluation effects, then the user has to choose between adding an effect in either the creation or the endowment role.

<sup>&</sup>lt;sup>6</sup>A special case of the *gratification function* in Snijders (2001).

<sup>&</sup>lt;sup>7</sup>The endowment function also is a special case of the *gratification function* in Snijders (2001).

For any effect, the model specifications of (evaluation & creation), (evaluation & endowment), and (creation & endowment) are equivalent, and pairs of parameters can be directly transformed into each other (cf. Section 12.1.4), but the estimates by the Method of Moments will not have the same correspondence because different pairs of target statistics are used. Each of the pair specifications could be tried out, in principle. Perhaps the first two pair specifications, containing the evaluation effect and one of the other two, give the most precise estimates; but this is not necessarily always the case.

# 5.1.1 Elementary effects

Not all contributions to the probability of change can be written as the change in some basic function (evaluation function). Therefore we sometimes need to directly represent contributions to a tie change or behavior change, without invoking an evaluation function. This can be done by using elementary effects. (In Snijders (2001) this was called a gratification function; as a more neutral term, we now use the word 'elementary effects'.)

The basic example here is transitive closure, which can be represented by the tendency toward forming closed triplets as in this figure. When the focal actor is i, ties that lead to the closure are  $i \to j$  and  $i \to h$ ; but the first of these ties means the closing of a two-path  $i \to h \to j$ , while the second means forming a tie to an actor h who made the same outgoing choice to the third actor j, a sign of structural equivalence; so these are distinct processes. The evaluation effect corresponding to the tendency toward forming closed triples is the transTrip effect, which is composed of the two distinct elementary effects transTrip1, contributing to creating or maintaining the  $i \to j$  tie, and transTrip2, contributing to the  $i \to h$  tie; see Section 12.



An elementary effect is a contribution to the creation or maintenance of a tie, defined directly, i.e., without expressing it based on the change in some evaluation function. This means that elementary effects are more general than evaluation effects, and all effects could be represented as elementary effects. For the sake of interpretation, however, the evaluation function formulation is used whenever possible.

Elementary effects can apply similarly to the creation and maintenance of a tie; or they can apply exclusively to tie creation, or exclusively to tie maintenance. In RSiena the difference between elementary effects and evaluation effects is only in the internal programming code, and the possible values of the type of effect as specified in the effects object and the functions include Effects and set Effect are only eval, creat, and endow. In Chapter 12 almost all effects are evaluation effects, and the effects that are elementary (and not evaluation) effects are mentioned as such.

## 5.1.2 Specification in RSiena

The model specification is defined in RSiena by the so-called *effects object*, which formally is an object of class sienaEffects or, for multiple groups as discussed in Chapter 11 of class sienaGroupEffects. It is a special kind of data.frame. The effects object is originally created

by the function getEffects and subsequently modified by functions includeEffects, setEffects, and others. The scripts on the SIENA website give examples. An important ingredient here is the so-called shortName of each effect, a column of the data frame, which is used to identify it. For the practical use of SIENA, the shortNames are very important. Effects of covariates (and of other dependent variables used in the role of explanatory variable for the effect in question) need, in addition, the name of the covariate because the shortName does not specify the covariate. If there are several dependent variables (networks and/or behavioral variables), the name of the dependent variable (name) also is required to specify the effect, and will be given in the column interaction1 (etc.) of the effects object.

The distinction between evaluation, endowment, and creation effects is specified by the type of the effect; this can be eval, endow, or creation. The default is eval. Only for rate effects this distinction does not exist; their type is rate. The type can be set in the functions includeEffects, setEffect, and includeInteraction.

A list of all effects available in the current version of RSiena with their shortNames can be displayed in a browser by using the function:

```
effectsDocumentation()
```

A list of all effects available for the given effects object, e.g., myeff – which depends on the set of variables and the number of periods in the data object – can be displayed in a browser by requesting

```
effectsDocumentation(myeff)
```

```
For example, the command
```

```
cbind(myeff$effectName, myeff$type, myeff$shortName)[1:20,]
```

gives a list of the first 20 effects in the myeff object. As another example,

cbind(myeff\$effectName, myeff\$type, myeff\$shortName)[myeff\$type == "eval",]

lists all evaluation effects in myeff, and

unique(myeff\$shortName)

gives the set of all shortNames.

# 5.1.3 Mathematical specification

To attach precise meaning to the intuitive explanations above, the mathematical definition of the model is given as follows. To keep notation simple, we leave all statistical parameters out of the formulae. To keep the section short, we do not give a lot of explanation, but refer to the mentioned literature for that purpose.

As explained in Snijders et al. (2010b), the model is a continuous-time Markov chain, and represents how the network (and behavior) has changed in small steps (the so-called *ministeps*) from one observed to a later observed value. Each ministep entails a change in only one tie value, or one behavioral variable, and is modeled as follows.

First consider the network dynamics. At any given moment, let the network be denoted  $x^0$ . The rate function for actor i is denoted  $\lambda_i(x)$ ; the evaluation function is  $f_i(x)$ ; the creation function is  $c_i(x)$ ; and the endowment function is  $e_i(x)$ .

At any given moment, let the current network be denoted  $x^0$ . The time duration until the next opportunity of change is exponentially distributed with parameter

$$\lambda_+(x^0) = \sum_i \lambda_i(x^0) .$$

This means that the expected time duration is

$$\frac{1}{\lambda_+(x^0)} \ .$$

The probability that actor i will be the next to have an opportunity for change is

$$\frac{\lambda_i(x^0)}{\lambda_+(x^0)}$$

Now suppose that actor i is the one who has the next opportunity for change; one could say, this is the focal actor. Actor i then has the possibility to change one network tie, or to keep the network as it is. Denote by  $\mathcal{C}$  the set of all networks that can be obtained as a result. Then the probability of the network obtained from this step depends on something called the objective function  $u_i(x^0, x)$  which will be defined in a moment. Let  $x \in \mathcal{C}$  be some network that could be obtained as a result of the ministep; then x will be the same as  $x^0$ , except perhaps for the tie  $i \to j$  for some j; this tie might exist in x but not in  $x^0$ , or vice versa. The probability that the next network is x is given by

$$\frac{\exp(u_i(x^0, x))}{\sum_{x' \in \mathcal{C}} \exp(u_i(x^0, x'))} . \tag{3}$$

The numerator is required to make all probabilities for this step sum to 1.

The objective function is defined as follows. If there is only an evaluation function (mathematically, this means that the creation and endowment functions are 0), then the objective function is equal to the evaluation function for the new state,

$$u_i(x^0, x) = f_i(x) .$$

Because of the properties of the exponential function one can just as well define the objective function as the gain in evaluation function,

$$u_i(x^0, x) = f_i(x) - f_i(x^0) . (4)$$

To define the general case, note that if  $x^0$  and x are not the same, then they differ in only one tie variable  $x_{ij}$ . Define  $\Delta^+(x^0, x) = 1$  if x has one tie more than  $x^0$ , meaning that a tie is created by this change, and  $\Delta^+(x^0, x) = 0$  otherwise. Similarly, define  $\Delta^-(x^0, x) = 1$  if

x has one tie less than  $x^0$ , meaning that a tie is dissolved by this change, and  $\Delta^-(x^0, x) = 0$  otherwise. Then the general definition of the objective function is

$$u_i(x^0, x) = (f_i(x) - f_i(x^0)) + \Delta^+(x^0, x) (c_i(x) - c_i(x^0)) + \Delta^-(x^0, x) (e_i(x) - e_i(x^0)).$$
(5)

This shows that the change in creation function plays a role only if a tie is created  $(\Delta^+(x^0, x) = 1)$ , and the change in endowment function plays a role only if a tie is dissolved  $(\Delta^-(x^0, x) = 1)$ .

If also elementary effects are included, then denote the linear combination for a tie variable  $x_{ij}$  for general (evaluation-type) elementary effects by  $f_{ij}^{el}(x)$ , for creation elementary effects by  $c_{ij}^{el}(x)$ , and for endowment elementary effects by  $e_{ij}^{el}(x)$ . To the objective function  $u_i(x^0, x)$  we then still have to add

$$f_{ij}^{\rm el}(x) + \Delta^+(x^0, x) c_{ij}^{\rm el}(x) + \Delta^-(x^0, x) e_{ij}^{\rm el}(x)$$
.

For behavior dynamics the definitions are analogous. Here a basic assumption is that, when there is an opportunity for change, the possible new values for the behavior variable are the current value, this value +1, and this value -1, as long as these changes do not take the value out of the permitted range. More elaborate explanations are in (Snijders et al., 2007, 2010b; Steglich et al., 2010; Veenstra et al., 2013).

The evaluation, creation, and endowment functions are constructed as a linear combination of the effects:

$$f_i(x) = \sum_k \beta_k \, s_{ik}(x) \tag{6}$$

where  $\beta_k$  are parameters and  $s_{ik}(x)$  are effects. For models with more than one dependent variable (networks, behaviors), this is specific to each dependent variable; e.g., in a network and behavior study with one network and one behavioral variable, there will be two evaluation functions. Because of formula (4), what is essential for the effects for networks is the difference

$$\Delta_{kij}(x) = s_{ki}(x^{+ij}) - s_{ki}(x^{-ij}) \tag{7}$$

where  $x^{+ij}$  is network x with the tie  $i \to j$  and  $x^{-ij}$  is network x without this tie. This is called the *change statistic*.

Elementary effects are defined by

$$s_{ijk}^{\rm el}(x) = x_{ij} \, s_{ijk}^{\rm el0}(x) \; ,$$

for a statistic  $s_{ijk}^{el0}(x)$  which does not depend on  $x_{ij}$ . Therefore, the corresponding change statistic is

$$\Delta^{\rm el}_{kij}(x)\,=\,s^{\rm el0}_{ijk}(x)\ .$$

# 5.2 Important structural effects for network dynamics: one-mode networks

For the structural part of the model for network dynamics, for one-mode (or unipartite) networks, the most important effects are as follows. The mathematical formulae for these and other effects are given in Chapter 12. Here we give a more qualitative description.

A default model choice could consist of (1) the out-degree (density) and reciprocity effects; (2) one network closure effect, e.g. transitive triplets, transitive ties, or gwesp; the transitive reciprocated triplets effect and/or the 3-cycles effect; (3) the in-degree popularity effect (raw or square root version); the out-degree activity effect (raw or square root version); and either the in-degree activity effect or the out-degree popularity effect (raw or square root function). The two effects (1) are so basic they cannot be left out. The effects selected under (2) represent the dynamics in local (triadic) structure (also see Block, 2015, for the transitive reciprocated triplets effect); and the three effects selected under (3) represent the dynamics in in- and out-degrees (the first for the dispersion of in-degrees, the second for the dispersion of out-degrees, and the third for the covariance between in- and out-degrees) and also should offer some protection, albeit imperfect, for potential ego- and alter-effects of omitted actor-level variables.

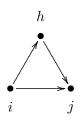
Information about basic contact opportunities, if available, should also be taken into account. This can be, e.g., membership of the same classroom or department, or some measure of proximity used as a dyadic covariate. Especially for large networks (more than 100 nodes) this is important.

The basic list of these and other effects is as follows.

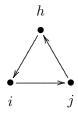
- 1. The *out-degree* (*density*) *effect* which always must be included, to give the balance between creation and termination of ties (conditionally on all other effects), and has the role of an intercept.
- 2. The reciprocity effect which practically always must be included.
- 3. There is a choice among several network closure effects. Usually it will be sufficient to express the tendency to network closure by including one or two of these. They can be selected by theoretical considerations and/or by their empirical statistical significance and contribution to a good fit.

In older publications, when the gwesp effect was not available, it was sometimes advised to include the transitive triplets together with the transitive ties effect. Currently, the impression is that often the gwesp effect is at least as good as a combination of transitive triplets and transitive ties. For many data sets, a model containing either the transitive triplets effect or the gwesp effect (not both) is adequate to represent transitivity, and the user may try out which of these yields the best fit.

a. The transitive triplets effect, which is the classical representation of network closure by the number of transitive triplets. For this effect the contribution of the tie  $i \to j$  is proportional to the total number of transitive triplets that it forms – which can be transitive triplets of the type  $\{i \to j \to h; i \to h\}$  as well as  $\{i \to h \to j; i \to j\}$ ;



- b. The transitive ties effect is similar to the transitive triplets effect, but instead of considering for each other actor j how many two-paths  $i \to h \to j$  there are, it is only considered whether there is at least one such indirect connection. Thus, one indirect tie suffices for the network embeddedness.
- c. The gwesp effect; see later in this manual. Figure 3 shows that the contribution of the number of two-paths for the gwesp effect is between that for the transitive ties effect ( $\alpha = 0$ ) and for the transitive triplets effect ( $\alpha = \infty$ ). The default for gwesp is  $\alpha = 0.69$ , corresponding to an internal effect parameter of 69.
- d. There are various other effects that may express network closure, such as the balance effect, the Jaccard similarity effects for incoming and for outgoing ties, and the number of actors at distance two effect. The balance and Jaccard similarity effects are ways to express tendencies to structural equivalence. The number of actors at distance two effect expresses network closure inversely: stronger network closure (when the total number of ties is fixed) will lead to fewer geodesic distances equal to 2. When this effect has a negative parameter, actors will have a preference for having few others at a geodesic distance of 2 (given their out-degree, which is the number of others at distance 1); this is one of the ways for expressing network closure. However, it is theoretically ambiguous, because the number of others at a geodesic distance of 2 can be decreased in two quite different ways: dropping ties to those who have many connections 'further out', or making ties to those who have many connections to others to whom one is already directly connected.
- 4. The three-cycles effect, which can be regarded as generalized reciprocity (in an exchange interpretation of the network) but also as the opposite of hierarchy (in a partial order interpretation of the network). A negative three-cycles effect, together with a positive transitive triplets or transitive ties effect, may be interpreted as a tendency toward local hierarchy. The three-cycles effect also contributes to network closure.



Block (2015) has argued convincingly that instead of the three-cycles effect, it is often advisable to use the transitive reciprocated triplets effect.

In a non-directed network, the three-cycles effect is identical to the transitive triplets effect.

5. Another triadic effect is the betweenness effect, which represents brokerage: the ten-

dency for actors to position themselves between not directly connected others, i.e., a preference of i for ties  $i \to j$  to those j for which there are many h with  $h \to i$  and  $h \not\to j$ . This can, however, not be combined with the transitive triplets effect because it is a different mechanism leading to the same structures.

The betweenness effect has the property that it is defined using configurations characterized by the <u>absence</u> of certain ties, not only by their presence. This leads to difficulties in interpretation, and may be undesirable.

- The following eight degree-related effects may be important especially for networks where degrees are theoretically important and represent social status or other features important for network dynamics; and/or for networks with high dispersion in in- or out-degrees (which may be an empirical reflection of the theoretical importance of the degrees).
  - The out-degree popularity and in-degree activity effects are closely related; their raw (not-sqrt) versions are exactly collinear in the Method of Moments.
  - Degree assortativity are higher-order effects, and will be included mainly if there are strong theoretical reasons for them.
- 6. The *in-degree popularity effect* (again, with or without 'sqrt', with the same considerations applying) reflects tendencies to dispersion in in-degrees of the actors; or, tendencies for actors with high in-degrees to attract extra incoming ties 'because' of their high current in-degrees.
- 7. The out-degree popularity effect (again, with or without 'sqrt', with the same considerations applying) reflects tendencies for actors with high out-degrees to attract extra incoming ties 'because' of their high current out-degrees. This leads to a higher correlation between in-degrees and out-degrees.
- 8. The *in-degree activity effect* (with or without 'sqrt') reflects tendencies for actors with high in-degrees to send out extra outgoing ties 'because' of their high current in-degrees. This leads to a higher correlation between in-degrees and out-degrees. The in-degree activity and out-degree popularity effects are not distinguishable in Method of Moments estimation; then the choice between them must be made on theoretical grounds.
- 9. The *out-degree activity effect* (with or without 'sqrt') reflects tendencies for actors with high out-degrees to send out extra outgoing ties 'because' of their high current out-degrees. This also leads to dispersion in out-degrees of the actors.
- 10. The *in-in degree assortativity effect* (where parameter 2 is the same as the sqrt version, while parameter 1 is the non-sqrt version) reflects tendencies for actors with high in-degrees to preferably be tied to other actors with high in-degrees.
- 11. The *in-out degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies for actors with high in-degrees to preferably be tied to other actors with high out-degrees.

- 12. The *out-in degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies for actors with high out-degrees to preferably be tied to other actors with high in-degrees.
- 13. The *out-out degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies for actors with high out-degrees to preferably be tied to other actors with high out-degrees.

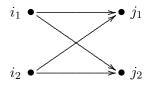
#### 5.2.1 Rate function

In some cases the rate function needs special attention. This can be the case, e.g., when the network has a clear core-periphery structure; or when the outdegree distribution is very skewed. Then it may be necessary to let the rate function depend on the outdegrees, using one of the three effects outRateLog, outRateInv, or outRate; or on an individual covariate indicating this size. Experience shows that often a rate dependence on outdegrees gives best results, and that outRateLog often is preferable. See Section 12.1.5.

# 5.3 Important structural effects for network dynamics: two-mode networks

The Stochastic Actor-Oriented Model for two-mode (or bipartite) networks is treated in Koskinen and Edling (2012). The co-evolution of one-mode and two-mode networks is treated in Snijders et al. (2013). The most important effects are as follows. The mathematical formulae for these and other effects are given in Chapter 12. Here we give a more qualitative description.

- 1. The out-degree effect which always must be included.
- 2. Transitivity in two-mode networks is expressed in the first place by the number of *four-cycles* (Robins and Alexander, 2004). This reflects the extent to which actors who make one choice in common also make other choices in common.



- The following three degree-related effects may be important especially for networks where degrees are theoretically important and represent social status or other features important for network dynamics; and/or for networks with high dispersion in in- or out-degrees (which may be an empirical reflection of the theoretical importance of the degrees). Include them if there are theoretical reasons for doing so, but only in such cases.
- 3. The *out-degree activity effect* (with or without 'sqrt'; often the sqrt version, which transforms the degrees in the explanatory role by the square root, works better) reflects tendencies to dispersion in out-degrees of the actors.

- 4. The *in-degree popularity effect* (again, with or without 'sqrt', with the same considerations applying) reflects tendencies to dispersion in in-degrees of the column units.
- 5. The *out-in degree assortativity effect* (where parameter 2 is the same as the sqrt version, while parameter 1 is the non-sqrt version) reflects tendencies for actors with high out-degrees to preferably be tied to column units with high in-degrees.

# 5.4 Effects for network dynamics associated with covariates

For each individual covariate, there are several effects which can be included in a model specification, both in the network evolution part and in the behavioral evolution part (should there be dependent behavior variables in the data). Of course for two-mode networks, the covariates must be compatible with the network with respect to number of units (rows/columns).

- network rate function
  - 1. the covariate's effect on the rate of network change of the actor;
- network evaluation, creation, and endowment functions
  - 1. the covariate-similarity effect, which is suitable for variables measured on an interval scale (or at least an ordinal scale where it is meaningful to use the absolute difference between the numerical values to express dissimilarity); a positive parameter implies that actors prefer ties to others with similar values on this variable thus contributing to the network-autocorrelation of this variable not by changing the variable but by changing the network; for categorical variables, see the 'same covariate' effect below;
  - 2. the effect on the actor's activity (covariate-ego); a positive parameter will imply the tendency that actors with higher values on this covariate increase their out-degrees more rapidly;
  - 3. the effect on the actor's popularity to other actors (covariate-alter); a positive parameter will imply the tendency that the in-degrees of actors with higher values on this covariate increase more rapidly;
  - 4. the effect of the squared variable on the actor's popularity to other actors (squared covariate-alter) (included only if the range of the variable is at least 2). This normally makes sense only if the covariate-alter effect itself also is included in the model. A negative parameter implies a unimodal preference function with respect to alters' values on this covariate;
  - 5. the interaction between the value of the covariate of ego and of the other actor (covariate ego × covariate alter); a positive effect here means, just like a positive similarity effect, that actors with a higher value on the covariate will prefer ties to others who likewise have a relatively high value; when used together with the alter effect of the squared variable this effect is quite analogous to

- the similarity effect, and for dichotomous covariates, in models where the ego and alter effects are also included, it even is equivalent to the similarity effect (although expressed differently), and then the squared alter effect is superfluous;
- 6. the 'same covariate', or covariate identity, effect, which expresses the tendency of the actors to be tied to others with exactly the same value on the covariate; whereas the preceding four effects are appropriate for interval scaled covariates (and mostly also for ordinal variables), the identity effect is suitable for categorical variables;
- 7. the interaction effect of covariate-similarity with reciprocity;
- 8. the effect of the covariate of those to whom the actor is indirectly connected, i.e., through one intermediary but not with a direct tie; this value-at-a-distance can represent effects of indirectly accessed social capital.

The usual order of importance of these covariate effects on network evolution is: evaluation effects are most important, followed by creation, endowment and rate effects. Inside the group of evaluation effects, for variables measured on an interval scale (or ordinal scale with reasonable numerical values), it is the covariate-similarity effect that is most important, followed by the effects of covariate-ego and covariate-alter.

When the network dynamics is not smooth over the observation waves — meaning that the pattern of ties created and terminated, as reported in the initial part of the output file produced by print01Report under the heading *Initial data description – Change in networks – Tie changes between subsequent observations*, is very irregular across the observation periods — it can be important to include effects of time variables on the network. Time variables are changing actor covariates that depend only on the observation number and not on the actors. E.g., they could be dummy variables, being 1 for one or some observations, and 0 for the other observations.

For actor covariates that have the same value for all actors within observation waves, or – in the case that there are structurally determined values – that are constant for all actors within the same connected components, only the ego effects are defined, because only those effects are meaningful. This exclusion of the alter, similarity and other effects for such actor variables applies only to variables without any missing values.

For each dyadic covariate, the following network evaluation effects can be included in the model for network evolution:

- network evaluation, creation, and endowment functions
  - 1. main effect of the dyadic covariate;
  - 2. the interaction effect of the dyadic covariate with reciprocity.

The main evaluation effect is usually the most important. But dyadic covariates can also be transformed, e.g., to patterns of indirect connections, which then are again usable as dyadic covariates. Dyadic covariates can also be used in various ways as weights for behavioral evolution.

## 5.5 Cross-network effects for dynamics of multiple networks

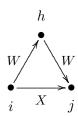
If there are multiple dependent network variables, these can be one-mode networks, two-mode networks, or a combination of these. The co-evolution of one-mode and two-mode networks is treated in Snijders et al. (2013), but this paper can also be used as an introduction to the dynamics of multiple one-mode networks. For multiple dependent network variables, the following effects may be important. This is explained here jointly for the case of one-mode and two-mode networks. The number of columns is defined as the number of actors for one-mode networks, and as the number of units/nodes/... in the second node set for two-mode networks. For cross-network effects the network in the role of dependent variable is denoted by X and the network in the role of explanatory variable by W; thus, effects go from W to X. All these effects are regarded as effects determining the dynamics of network X.

- 1. If both networks have the same number of columns, then the basic effect is the entrainment of X by W, i.e., the extent to which the existence of a tie  $i \xrightarrow{W} j$  promotes the creation or maintenance of a tie  $i \xrightarrow{X} j$ .
- 2. If both networks are one-mode, then a next effect is the reciprocity effect with W on X, representing the extent to which the existence of a tie  $j \xrightarrow{W} i$  promotes the creation or maintenance of a tie, in the reverse direction,  $i \xrightarrow{X} j$ .
- 3. If both networks are one-mode, then a next effect is the mutuality effect with W on X, representing the extent to which the existence of a mutual tie  $i \stackrel{W}{\leftrightarrow} j$  promotes the creation or maintenance of a tie  $i \stackrel{X}{\rightarrow} j$ .
- 4. The outdegree W activity effect (where parameter 2 is the sqrt version, while parameter 1 is the non-sqrt version see above for explanations of this) reflects the extent to which actors with high outdegrees on W will make more choices in the X network.
- Several mixed transitivity effects can be important.
- 5. If X is a one-mode network, the from W agreement effect represents the extent to which agreement between i and j with respect to outgoing W-ties promotes the creation or maintenance of a tie  $i \stackrel{X}{\to} j$ .
- W i X j h W X

X

6. If W is a one-mode network, the W to agreement effect represents the extent to which a W tie  $i \stackrel{W}{\rightarrow} h$  leads to agreement between i and h with respect to outgoing X-ties to others, i.e., X-ties to the same third actors j,  $i \stackrel{X}{\rightarrow} j$  and  $k \stackrel{X}{\rightarrow} j$ .

7. If X and W both are one-mode networks, the *closure* of W effect represents the tendency closure of W-W two-paths  $i \stackrel{W}{\rightarrow} h \stackrel{W}{\rightarrow} j$  by an X tie  $i \stackrel{X}{\rightarrow} j$ .



## 5.6 Hierarchy requirements

From linear regression we know that when a model contains an interaction effect, usually also the corresponding main effects should be included. Similarly for higher-order interactions, their lower-order terms will usually be included. This is called the *hierarchy* principle in model building.

The situation for structural effects in Stochastic Actor-Oriented Models is similar but more complex.

For covariate effects the considerations are similar to those in regression analysis and do not need reiteration. The hierarchy principle for the Stochastic Actor-Oriented Model is that when an effect represented by some graphical structure is included in the model, the effects for the <u>sub-structures</u> that are part of the <u>structure</u> should also be included. In other words, testing of higher-order structures implies that the corresponding lower-order structures should also be included.

An example is the transitive triplets effect *transTrip*.

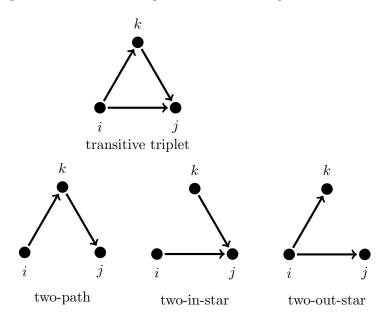


Figure 1: Transitive triplets and included subgraphs

In Figure 1 we see the transitive triplet together with the three two-arc configurations

that it contains as subgraphs. These correspond, respectively, to the indegree-activity or outdegree-popularity effect, the indegree-popularity effect, and the outdegree-activity effect. To use the test of the transitive triplets effect as evidence for or against transitivity it is necessary that the model includes these three effects (for the first it does not matter whether inAct or outPop is chosen). Note the condition "to use the test of the transitive triplets effect as evidence for or against transitivity"; if the purpose is just to obtain a well-fitting model, then it might be superfluous to include all three lower-order effects.

This principle also applies to multivariate networks as discussed in the previous section.

## 5.7 Constraints between networks

When analysing multiple dependent networks, the following constraints between networks can be considered:

**disjoint:** for any given pair of actors (i, j), it is impossible that ties exists in both networks;

**higher:** the tie variables for the first network are higher than or equal to those for the second network: for any given pair of actors (i, j), it is impossible that a tie exists in the second but not in the first network;

**atleastone:** for any given pair of actors (i, j), there must be a tie in at least one of the networks.

Whether this is so, is indicated in the attributes of the data set (of class siena). These constraints are observed and set during the creation of the siena object in sienaDataCreate. When one of the constraints is true, this will be reported in the output of print01Report. These constraints will continue to be respected during the simulations (and by implication for the estimations). The constraints for the simulations can be changed (e.g., turned off) by function sienaDataConstraint.

Script http://www.stats.ox.ac.uk/~snijders/siena/RscriptSienaOrdered.R gives a demonstration of how this feature can be used for analyzing networks with ordered values.

Constraints will work for all pairs of networks of the same kind: both networks directed one-mode, both symmetric (non-directed) one-mode, or both two-mode with the same node sets. For the combination of a one-mode directed and a symmetric non-directed network, the constraint **disjoint** is possible; and the constraint **higher** is possible, if the symmetric network is higher than the directed network.

Constraints do not work for multi-group data sets! By implication, they do not work for sienaBayes.

#### 5.8 Effects on behavior evolution

For models with one or more dependent behavior variables, i.e., models for the co-evolution of networks and behavior, the most important effects for the behavior dynamics are the

following; see Steglich et al. (2010). In these descriptions, with the 'alters' of an actor we refer to the other actors to whom the focal actor has an outgoing tie. The dependent behavior variable is referred to as Z.

- 1. The shape effect, expressing the basic drive toward high values on Z. A zero value for the shape will imply a drift toward the midpoint of the range of the behavior variable.
- 2. The effect of the behavior Z on itself, or quadratic shape effect, which is relevant only if the number of behavioral categories is 3 or more. This can be interpreted as giving a quadratic preference function for the behavior. When the coefficient for the shape effect is  $\beta_1^Z$  and for the effect of Z on itself, or quadratic shape effect, is  $\beta_2^Z$ , then the contributions of these two effects are jointly  $\beta_1^Z$   $(z_i \bar{z}) + \beta_2^Z$   $(z_i \bar{z})^2$ . With a negative coefficient  $\beta_2^Z$ , this is a unimodal preference function, with the maximum attained for  $z_i = \bar{z} 2\beta_1^Z/\beta_2^Z$ . (Of course additional effects will lead to a different picture; but as long as the additional effects are linear in  $z_i$  which is not the case for similarity effects! —, this will change the location of the maximum but not the unimodal shape of the function.) This can also be regarded as negative feedback, or a self-correcting mechanism: when  $z_i$  increases, the further push toward lower values of  $z_i$  will become smaller and when  $z_i$  decreases, the further push toward lower values of  $z_i$  will become smaller. On the other hand, when the coefficient  $\beta_2^Z$  is positive, the feedback will be positive, so that changes in  $z_i$  are self-reinforcing. This can be an indication of addictive behavior.
- 3. The average similarity effect, expressing the preference of actors to being similar with respect to Z to their alters, where the total influence of the alters is the same regardless of the number of alters.
- 4. The total similarity effect, expressing the preference of actors to being similar to their alters, where the total influence of the alters is proportional to the number of alters.
- 5. The average alter effect, expressing that actors whose alters have a higher average value of the behavior Z, also have themselves a stronger tendency toward high values on the behavior.
- 6. The total alter effect, expressing that actors whose alters have a higher total value of the behavior Z, also have themselves a stronger tendency toward high values on the behavior.
- 7. The indegree effect, expressing that actors with a higher indegree (more 'popular' actors) have a stronger tendency toward high values on the behavior.
- 8. The outdegree effect, expressing that actors with a higher outdegree (more 'active' actors) have a stronger tendency toward high values on the behavior.

Effects 1 and 2 will practically always have to be included as control variables. (For dependent behavior variables with 2 categories, this applies only to effect 1.) When the behavior dynamics is not smooth over the observation waves — meaning that the pattern of steps up and down, as reported in the initial part of the output file produced by print01Report under the heading *Initial data description — Dependent actor variables — Changes*, is very irregular across the observation periods — it can be important to include effects of time variables on the behavior. Time variables are changing actor covariates that depend only on the observation number and not on the actors. E.g., they could be dummy variables, being 1 for one or some observations, and 0 for the other observations.

The average similarity, total similarity, average alter, and total effects are different specifications of social influence. The choice between them will be made on theoretical grounds and/or on the basis of statistical significance. Do not include them all together in one model, as this would most likely lead to multicollinearity and non-convergence.

For each actor-dependent covariate as well as for each of the other dependent behavior variables, the effects on Z which can be included is the following.

- 1. The main effect: a positive value implies that actors with a higher value on the covariate will have a stronger tendency toward high Z values.
- 2. Various effects of the combination of covariate values for members of the personal network of the focal actor (outgoing ties, incoming ties, distance-two ties): search in this manual for avXAlt, avXInAlt, avXAltDist2, avXInAltDist2 and their manifold variations.
- 3. Interactions between two or three actor variables, see Section 5.12.

# 5.9 Model Type: non-directed networks

Dynamics for non-directed networks is explained in Snijders and Pickup (2016).

RSiena detects automatically when the networks all are non-directed, and then employs a model for this special case. For non-directed networks, the Model Type has five possible values, as described in Snijders and Pickup (2016). This is specified by the parameter modelType in function sienaAlgorithmCreate. Value modelType = 1 is for directed networks, values 2-6 for non-directed networks.

- 1. Directed networks option modelType = 1 is not used for non-directed networks.
- Forcing model, modelType = 2: one actor takes the initiative and unilaterally imposes that a tie is created or dissolved.
- 3. Unilateral initiative and reciprocal confirmation, modelType = 3: one actor takes the initiative and proposes a new tie or dissolves an existing tie; if the actor proposes a new tie, the other has to confirm, otherwise the tie is not created; for dissolution, confirmation is not required.

For the confirmation decision, an offset is added to the objective function, representing the difference between a binary yes/no choice and the multinomial choice of one among n-1 tie variables to change. This offset is a fixed parameter, not estimated. The higher the offset, the more likely it is that the confirmation will be given. It is determined by the value of Offset in function sienaAlgorithmCreate. This parameter must be given as a named vector; see the help page for sienaAlgorithmCreate. Plausible values for this offset are in the range from 0 to 3, and should slowly increase with n (roughly proportionally to  $\log(n)$ ).

- 4. Pairwise disjunctive (forcing) model, modelType = 4:
  a pair of actors is chosen and reconsider whether a tie will exist between them; the
  tie will exist if at least one of them chooses for the tie, it will not exist if both do
  not want it.
- 5. Pairwise conjunctive model, modelType = 5: a pair of actors is chosen and reconsider whether a tie will exist between them; the tie will exist if both agree, it will not exist if at least one does not choose for it.
- 6. Pairwise compensatory (additive) model, modelType = 6:
  a pair of actors is chosen and reconsider whether a tie will exist between them; this
  is based on the sum of their objective functions for the existence of this tie.
  If the evaluation function is symmetric in ego and alter, and there are no creation
  and/or maintenance functions, this is the Longitudinal Exponential Random Graph
  Model ('LERGM') of Koskinen and Snijders (2013).

In the first two of these models, where the initiative is one-sided, the rate function is comparable to the rate function in directed models. In the last three models, however, the pair of actors is chosen at a rate which is the *product* of the rate functions  $\lambda_i$  and  $\lambda_j$  for the two actors. This means that opportunities for change of the single tie variable  $x_{ij}$  occur at the rate  $\lambda_i \times \lambda_j$ . The numerical interpretation is different from that in the first two models.

Some effects specially designed for non-directed networks are presented in Section 12.1.2.

# 5.10 Model Type: directed networks

As of version 1.3-22, there are additional model options for dynamics of directed one-mode networks, specified by the parameter modelType in function sienaAlgorithmCreate. The value modelType = 1 is the default for directed networks, values modelType = 7, 8, 9, 10 are options included for the purpose of investigating the consequences of details of the specification of the ministep.

These values implement the so-called *double step*. The ministeps are explained above in Section 2.1 and more in detail in Section 5.1.3. A double step is defined by a ministep taken by some focal actor i; suppose this means the creation or termination of the tie  $i \to j$  for some j; then a following ministep is taken by actor j. In these model specifications the 'pure' ministeps and the double steps are mixed randomly. For the respective values

modelType = 7, 8, 9, 10, the probability that a double step is taken is, respectively, 0.25, 0.50, 0.75, and 1.00.

These model types are not intended for use in regular applications.

# 5.11 Model Type: behavior

As of version 1.1-306, there are two model options for dynamics of behavior, specified by the parameter behModelType in function sienaAlgorithmCreate. They differ in the treatment of the boundaries of the range of the behavioral variable. Recall that the behavioral microsteps consist of the addition of -1, 0, or +1 to the dependent variable, under the condition of remaining within the integer range from the minimum  $z^-$  to the maximum  $z^+$ .

If the current value of the behavior is at the boundary,  $z^-$  or  $z^+$ , then only two possible outcomes remain. The default is the option available before, with behModelType = 1. This is also called the *restricting* model, because the probabilities of the two options are given by (3), where the set of possible new states  $\mathcal{C}$  is just the restricted set  $\{z^-, z^- + 1\}$  in the case of the minimum, and  $\{z^+ - 1, z^+\}$  in the case of the maximum.

The second option, for behModelType = 2, is called the absorbing model, because the probabilities of the two options are calculated as if the variable could go outside of the range, but when such an excursion occurs it is absorbed into the range again. Thus, probabilities are calculated according to (3), with  $\mathcal{C} = \{z^- - 1, z^-, z^- + 1\}$  or  $\mathcal{C} = \{z^+ - 1, z^+, z^+ + 1\}$ , respectively; but the outcome  $z^- - 1$  when it occurs is replaced by  $z^-$ , and  $z^+ + 1$  when it occurs is replaced by  $z^+$ . The objective function for the virtual outcome  $z^- - 1$  is taken to be the same as for  $z^-$ , and similarly for  $z^+ + 1$  it is taken to be the same as for  $z^+$ .

# 5.12 Interaction effects

For the Stochastic Actor-Oriented Model and other network models, the definition of interaction effects is not evident. For example, what is the interaction of reciprocity and transitive triplets? One possibility is the transitive reciprocated triplets effect (transRecTrip) effect defined in Section 12.1.1. This definition makes perfect sense; nevertheless, the reciprocal tie could have been put at a different place than in dyad (i,j) in the picture there — which would have led to a different type of interaction 'reciprocity  $\times$  transitivity'. Also for interactions between covariates and structural effects, when the structural effect is expressed by some configuration of ties, there will usually be several possibilities to choose the actor in the configuration for which the covariate should be considered, and these will often lead to different effects.

This leads to a distinction between ad hoc interaction effects and standard interaction effects. Ad hoc interaction effects have to be defined specifically in each case, like transRecTrip; Section 12.1.1 contains many other examples. Standard interaction effects can be defined directly. The change statistic of a standard interaction effect is the product of the change statistics of the interacting effects. This does not always make sense; whether it is permitted is defined by the 'interactionType' of the effects.

Interactions can be specified by the function includeInteraction. The basis of the implementation is provided by the definition, by SIENA, of 'unspecified interaction effects'. The interacting effects are given by the columns effect1 and effect2, and for three-way interaction effects, effect3, in the effects object; they contain the effectNumber (sequence number in the effects object) of the effects that are interacting. The interaction effect must be 'included' to be part of the model, but the underlying effects need only be 'included' if they are also required individually. (In most cases this is advisable.) The number of possible user-defined interaction effects is limited, and is set in the call of getEffects.

The information necessary for working with interaction effects – the interaction types, short names, and sequence numbers of the effects – is contained in the document produced for a given effects object, say myeff, by the function call

```
effectsDocumentation(myeff)
```

Further see the help page for the function effectsDocumentation. Chapter 12 of this manual also gives the short names of all effects. The short name of all unspecified interaction effects is unspInt for network effects, and behUnspInt for behaviour effects.

It should be mentioned that the keywords interaction1, interaction2, and interaction3 in functions such as includeEffects do not point to interactions between effects. These keywords are used only to exactly specify an effect. They point to the covariate or other dependent variable used in the effect; this name was chosen for the keyword because this other variable may be said to be interacting with the dependent variable (network or behavior).

#### 5.12.1 Internal effect parameters for interaction effects

Function setEffect has a keyword parameter to set the internal effect parameter, but the function includeInteraction does not have this keyword. The reason is that the internal effect parameters for the interaction are borrowed from the main effects; even if these are not included in the model. Setting the internal effect parameters for main effects, without including them in the model, can be done by

```
 \begin{split} & \text{setEffect}(\dots, \text{ parameter=}\dots, \text{ include=FALSE}) \\ & \text{as is done in the following example:} \\ & \text{myeff } <- \text{ setEffect}(\text{myeff}, \text{outActIntnX}, \\ & \text{name='mynet2'}, \\ & \text{interaction1='mynet1'}, \text{ interaction2='alc'}, \text{ parameter=1, include=FALSE}) \\ & \text{myeff } <- \text{ setEffect}(\text{myeff}, \text{divOutEgoIntn}, \\ & \text{name='mynet2'}, \\ & \text{interaction1='mynet1'}, \text{ parameter=1, include=FALSE}) \\ & (\text{myeff } <- \text{ includeInteraction}(\text{myeff}, \text{outActIntnX}, \text{divOutEgoIntn}, \\ & \text{name='mynet2'}, \\ & \text{interaction1=c('mynet1', 'mynet1'), interaction2=c('alc', '')))} \\ & \text{The main effects are defined by} \\ & \sum_{i} x_{ij} \sum_{h} w_{ih} v_{h} \end{aligned}
```

and

$$\sum_{j} x_{ij} \operatorname{divi}(1, w_{i+})$$

where  $\operatorname{divi}(a, b)$  is defined as a/b if  $b \neq 0$  and 0 if b = 0; so the interaction effect included in this way is

$$\sum_{j} x_{ij} \left( \sum_{h} w_{ih} v_{h} \right) \times \operatorname{divi}(1, w_{i+}) = \sum_{j} x_{ij} \frac{\sum_{h} w_{ih} v_{h}}{w_{i+}}$$

(where the ratio 0/0 is defined as 0), i.e., the ego effect of the average value of covariate V for those to whom actor i has a W-outgoing tie.

For a sienaFit object ans created from any effect object, the effect specification is given by

#### ans\$requestedEffects

while the effects object also including the main effect is given by

#### ans\$effects

The latter will show, for any included interaction effects, the internal effect parameters used for main effects even if they are not included themselves; this gives a possibility for checking the internal effect parameters used for the interactions.

## 5.12.2 The interactionType of interaction effects

All effects have a so-called interactionType, defined by the column interactionType in the effects data frame. This interaction type defines what is allowed for definition of interaction effects; the product of their change statistics should make sense as the change statistic of some effect. Further explanation is given in section 'Statistics for MoM' of Siena\_algorithms.pdf.

For network effects, the interaction type is "ego", "dyadic", or "" (blank); for behaviour effects, it is "OK" or "".

# 5.12.3 Interaction effects for network dynamics

The following kinds of user-defined interactions are possible for network dynamics of directed one-mode networks as well as bipartite networks.

- **a.** Ego effects of actor variables can interact with all effects.
- **b.** Dyadic effects can interact with each other.

For undirected networks, ego effects can only interact with ego effects and dyadic effects can only interact with dyadic effects. Whether an effect is an ego effect or a dyadic effect is defined by the column interactionType in the effects data frame. This column is shown in the list of effects that is displayed in a browser by using the function:

#### effectsDocumentation()

Thus, in directed networks, a two-way interaction must be between two dyadic effects or between one ego effect and another effect. A three-way interaction may be between three dyadic effects, two dyadic effects and an ego effect, or two ego effects and another effect. In undirected networks all three effects must be either ego or dyadic effect.

All effects used in interactions must be defined on the same network (in the role of dependent variable): that for which the 'unspecified interaction effects' is defined. And all must be of the same type (evaluation, endowment, or creation effects).

Examples of the use of includeInteraction are as follows.

Note the interaction1 keyword; this keyword is used also when defining these effects using includeEffects or setEffect. In this case, however, two effects are defined, and accordingly the interaction1 keyword has two components, combined by the c function. For effects such as recip that have no interaction1 keyword, the corresponding string is just the empty string, "". (Note that the name interaction1 does not itself refer to interactions in the sense of this section.)

Interactions between three effects are defined similarly, but now the interaction1 keyword must combine three components.

The number of network interaction effects that can be created is defined by the keyword nintn in function getEffects. If you need more interactions than permitted, just make a new effects object with the suitably increased value of nintn.

The list of effects in Chapter 12 contains a variety of interaction effects that cannot be created in this way; for example, those with short names transRecTrip, simRecipX, avSimEgoX, and covNetNet (there are many more).

For non-directed (symmetric) networks, what makes sense as standard interaction effects depends on the modelType as well as on the interactionType of the effects. These distinctions have not been fully implemented.

#### 5.12.4 Interaction effects for behavior dynamics

For behavior dynamics, interaction effects can be defined by the user, for each dependent behavior variable separately, as interactions of two or three actor variables, again using the function includeInteraction. These are interactions on the ego level, in line with the actor-oriented nature of the model.

There are some restrictions on what is permitted as interactions between behavior effects. Of course, they should refer to the same dependent behavior variable. What is permitted depends on the interactionType of the effects, which for behavior effects

can be "OK" or blank. A further explanation is given under the heading 'User-defined interaction effects' in Section 12.2. The interactionType of the effects is shown in the list of effects displayed in a browser by using the function:

```
effectsDocumentation()
```

The behavioral effects with non-"OK" (i.e., blank) interactionType include, in particular, all effects of which the name includes the word "similarity", or alternatively, the short name includes the string "sim".

The requirement for behavior interactions is that, of the interacting effects, all or all but one have the value "OK". Thus, for an interaction between two effects, one or both should be "OK"; for a three-effect interaction, two or all three should be "OK".

The number of behavior interaction effects that can be created is defined by the keyword behNintn in function getEffects. If you need more interactions than permitted, just make a new effects object with the suitably increased value of behNintn.

As an example, suppose that we have a data set with a dependent network variable friendship and a dependent behavior variable drinkingbeh (drinking behavior), and we are interested whether social influence, as represented by the 'average alter' effect, differs between actors depending on whether currently they drink little or much. Then the commands

define a model with the average alter effect (representing social influence) and an interaction between this and the quadratic shape effect. Recall that the latter can be regarded as the effect of drinking behavior on drinking behavior. Briefly, the interaction is between current drinking behavior and the average drinking behavior of friends. By consulting Section 12.2.1 on the mathematical definitions of the effects one can derive that this leads to the following objective function; where it is assumed that also the linear and quadratic shape effects are included in the model.

$$f_i^{\rm beh}(x,z) \,=\, \beta_1^{\rm beh} z_i \,+\, \beta_2^{\rm beh} z_i^2 \,+\, \beta_3^{\rm beh} z_i \, \frac{\sum_j x_{ij} \, z_j}{\sum_j x_{ij}} \,+\, \beta_4^{\rm beh} z_i^2 \, \frac{\sum_j x_{ij} \, z_j}{\sum_j x_{ij}} \;.$$

In addition, there are predefined interactions available between actor variables and influence, as described in Section 12.2.1.

# 5.13 Time heterogeneity in model parameters

When working with two or more periods, i.e., three or more waves, there is the question whether parameters are constant across the periods. This can be tested by the sienaTime-Test function, as explained in Section 8.6. To specify a model with time heterogeneous

<sup>&</sup>lt;sup>8</sup>The value is "OK" for the effects of which the formula as defined in Section 12.2.1 is given by  $z_i$  multiplied by something not dependent on  $z_i$ .

parameters, the function includeTimeDummy can be used, as follows. Consider the reformulation of the evaluation function into

$$f_{ij}^{(m)}(\mathbf{x}) = \sum_{k} \left( \beta_k + \delta_k^{(m)} h_k^{(m)} \right) s_{ik} \left( \mathbf{x} (i \leadsto j) \right)$$
 (8)

where m denotes the period (from wave m to wave m+1 in the panel data set) and  $\delta_k^{(m)}$  are parameters for the effects interacted with time dummies. You can include these in your model simply via the function

which would add three time dummy terms to each effect listed in the function.

We recommend that you start with simple models, and base the decision to include time heterogeneous parameters on your theoretical and empirical insight in the data (e.g., whether the different waves cover a period where the importance of some of the modeled 'mechanisms' may have changed) and the score type test that is implemented in the sienaTimeTest function, see Section 8.6.

See Lospinoso et al. (2011) for a technical presentation and examples of how the test works, and Lospinoso and Snijders (2019) for relations with testing goodness of fit (further treated in Section 5.15).

## 5.14 Limiting the maximum outdegree

It is possible to request that all networks simulated have a maximum outdegree less than or equal to some given value. This is meaningful only if the observed networks also do not have a larger outdegree than this number, for any actor at any wave.

This is carried out by specifying the maximum allowed value in the MaxDegree parameter of the sienaAlgorithmCreate function, which determines the settings of the algorithm.

MaxDegree is a named vector, which means that its elements have names. The length of this vector is equal to the number of dependent networks. Each element of this vector must have a name which is the name of the corresponding network. E.g., for one dependent network called mynet, one could use

MaxDegree = c(mynet = 10)

to restrict the maximum degree to 10. For two dependent networks called **friends** and **advisors**, one could use

MaxDegree = c(friends = 6, advisors = 4)

For a single network, the default value 0 is used to specify that the maximum is unbounded. For multiple networks, if for one network there is a bound for the maximum outdegree but for another network this should not be bounded, then the value 0 will not work, but one should use a bound which is at least n-1, where n is the number of actors in the network (or the largest number, if there are multiple groups).

If the MaxDegree parameter is used for data where all, or almost all, degrees are equal to this maximum value, then it is likely that the estimation algorithm will not converge. A fixed choice design for network data collection is not compatible with the free choice nature of the Stochastic Actor-Oriented Model. See Holland and Leinhardt (1973) for a discussion of fixed choice designs and Žnidaršič (2012) for references to more recent literature.

The MaxDegree option does not work for likelihood-based estimation; therefore, neither ML estimation using siena07 nor multilevel estimation using sienaBayes support this option.

## 5.15 Goodness of fit with auxiliary statistics: sienaGOF

The function sienaGOF in RSiena permits users to assess the fit of the model with respect to auxiliary statistics of networks, e.g. geodesic distributions, that are not explicitly fit by a particular effect, but are nonetheless important features of the network to represent by the probability model. This can be used to check, when one has followed the approach to model specification explained in Sections 5.2 to 5.8 – and explained also in Snijders et al. (2010b) –, whether the end result gives a good representation also of these other statistics.

The sienaGOF function, proposed and elaborated by Lospinoso (2012) and presented further in Lospinoso and Snijders (2019), operates basically by comparing the observed values, at the ends of the periods, with the simulated values for the ends of the periods. The differences are assessed by combining the auxiliary statistics using the Mahalanobis distance.

Various auxiliary statistics are available directly ('out of the box'), e.g., distributions of indegrees and outdegrees and of behavioral variables. These are listed in the help page for sienaGOF-auxiliary. Some other auxiliary statistics, requiring additional packages such as igraph and sna, are available by copying script from the examples in the same help page. (The reason for this construction is to reduce dependencies of RSiena on other packages.)

The examples in the help pages for sienaGOF and sienaGOF-auxiliary give ample help for how to use this function. Also see the script on the SIENA website.

## 5.15.1 Plots

The results of sienaGOF can be plotted which then produces violin plots (Hintze and Nelson, 1998), which present the distribution of the statistic as a combination of a box plot and a smooth approximation to the density (by a kernel density estimate), with the observed values superimposed. The violin plots tend to become squiggly when the probability distribution is concentrated on a few points (integers usually) and, as a consequence, the density plot tries to approximate a discrete distribution. For the associated plot function, options center and scale are available to equalize the centers and scales of the various statistics plotted. For distributions and cumulative distributions over sets of integers (e.g., of degrees or geodesic distances) it often is advisable to use the defaults center = FALSE, scale = FALSE, whereas for sets of statistics for which a common scale is less important, e.g., triad counts, a clearer picture may be obtained by plotting with center = TRUE, scale

#### **5.15.2** *p*-values

The p-values for sienaGOF compare, in the space of outcomes of the auxiliary statistic (which is a vector, with a dimension usually between 4 and 20), the position of the observed data to the cloud of points formed by the simulated data sets that correspond to the estimated model. This comparison is with respect to the 'distance' from the center of the cloud of points, where 'distance' is between quotation marks because it is the Mahalanobis distance, which takes into account the correlations and different variances of the components of the auxiliary statistic.

This means that very small values of p indicate poor fit. There is an important difference here between p=0 and p>0. Suppose that there are, e.g., 1,000 simulated data sets (this number is given by n3, the 'length of Phase 3'); then the smallest possible non-zero value of p is 0.001. If p=0, the observed data are further away from the center of the simulated data than any of the simulated data points – this could be very far away indeed. If p=0.001 or larger, then the observed data point must be at least somewhere close to or within the cloud of simulated points, because one or more of the simulated data points are further away from the center than the observed data. Concluding, if p=0, then with respect to the auxiliary statistic the fit is poor; it might be rather poor or extremely poor, and you do not know how extreme it is.

The customary value of p = 0.05 may be used as a threshold determining whether the fit is adequate, but this threshold is of even less importance here than it is in the case of regular hypothesis testing.

If you have a multi-group estimation (see Section 11.2), sienaGOF needs to be applied to each group separately, using the groupName keyword, usually groupName="Data1", groupName="Data2", etc. (see the help page for sienaGOF).

For combining sienaGOF results for multiple groups, the resulting p-values as calculated for each single group can be combined. The most suitable method here perhaps is the inverse normal method, also called Lipták's method; see Hedges and Olkin (1985, Section C.3). This methods transforms each p-value to a standard normal variate, adds these and divides by  $\sqrt{N}$  where N is the number of combined studies, and tests the result in the standard normal distribution. For the transformation to standard normal, you can use the R function qnorm (check whether you need qnorm(p) or qnorm(1-p)). For testing in the standard normal you can use pnorm.

Another good combination procedure is Fisher's method for combining independent p-values, described also in Hedges and Olkin (1985) and (briefly) in Snijders and Bosker (2012, Chapter 3). The basic idea is that, for groups k = 1, ..., K which have given the p-values  $p_k$ , the test statistic is  $-2\sum_k \ln(p_k)$  and it is tested in a chi-squared distribution with d.f. = 2K.

An illustration is in script

http://www.stats.ox.ac.uk/~snijders/siena/RscriptMultipleGroups.R.

## 5.15.3 Composition change, missing data, and structural values in sienaGOF

The method of joiners and leavers for representing composition change (Section 4.3.3) does not combine properly with the sienaGOF function.

Missing tie values and structurally determined tie values are treated in the estimation in such a way that they do not contribute directly to the target statistics. This behavior is mirrored in their treatment in sienaGOF. The aim is that such values do not contribute to any differences between observed and simulated values.

Tie variables that are missing at either the beginning or the end of the period are replaced by 0, both in the observed and in the simulated networks. For behavioral variables they are replaced by missings (NA).

If there are any differences between structural values at the beginning and at the end of a period, these are dealt with as follows. For tie variables that have a structural value at the start of the period, this value replaces the observed value at the end of the period (for the goodness of fit assessment only). For tie variables that have a structural value at the end of the period but a free value value at the start of the period, the reference value for the simulated values is lacking; therefore, the simulated values at the end of the period then are replaced by the structural value at the end of the period (again, for the goodness of fit assessment only).

# 6 Estimation

The model parameters are estimated under the specification given during the model specification part, using an iterative stochastic approximation algorithm. Four estimation procedures are implemented: the Method of Moments ('MoM'; Snijders, 2001; Snijders et al., 2007); the Generalized Method of Moments ('GMoM'; Amati et al., 2015, 2019); the Method of Maximum Likelihood ('ML'; Snijders et al., 2010a); and a Bayesian method (Koskinen, 2004; Koskinen and Snijders, 2007; Schweinberger and Snijders, 2007a). For non-constant rate functions, currently only MoM and GMoM estimation is available. The Method of Moments is the default; ML and Bayesian estimation methods require much more computing time. Given the greater efficiency but longer required computing time for the ML and Bayesian methods, these can be useful especially for smaller data sets and relatively complicated models (networks and behavior; creation or endowment effects).

In this chapter, the number of parameters is denoted by p. The algorithm is based on repeated (and repeated, and repeated...) simulation of the evolution process of the network. These repetitions are called 'runs' in the following. The MoM estimation algorithm is based on comparing the observed network (obtained from the data files) to the hypothetical networks generated in the simulations.

Note that the estimation algorithm is of a stochastic nature, so the results can vary! This is of course not what you would like. For well-fitting combinations of data set and model, the estimation results obtained in different trials will be very similar. It is good to repeat the estimation process at least once for the models that are to be reported in papers or presentations, to confirm that what you report is a stable result of the algorithm. If you wish to have identical results over different estimations, you can set the seed; see Section 6.2.

# 6.1 An overview of the estimation procedure

The estimation process is implemented in functions siena07 and sienacpp (alternatives for each other), using an algorithm specification defined in function sienaAlgorithmCreate. Estimation starts with initial values for the parameters, and the functions siena07 and sienacpp both return a so-called sienaFit object containing the estimates and their standard errors and a lot of further information. Since the estimation procedure is iterative (i.e., depending on the initial value) and stochastic, the results are not always completely satisfactory. We shall see below how the satisfactory convergence of the algorithm can be checked, and how to go on if this is not satisfactory.

Much of what follows is about the use of siena07 but applies equally to sienacpp. The difference between these two functions is that estimation by sienacpp stays entirely in the 'back end' C++ part of SIENA, contrasting with siena07 which carries out the simulations in C++ but the Robbins-Monro updates in the R 'front end' part; this yields greater computational efficiency for sienacpp. Since the simulations take the largest amount of processing time for medium-sized and large networks, the time difference is notable, in a proportional sense, mainly for data sets where simulations run very quickly (i.e., number of actors and distance between first and last simulations are small). Further, results from

sienacpp cannot used for sienaGOF. Note that sienacpp is available only in RSienaTest. The estimation algorithm is determined by a call of functions such as

```
algorithm1 <- sienaAlgorithmCreate(projname = "trypro")
results1 <- sienaO7(algorithm1, data = mydata, effects = myeff)</pre>
```

The function sienaAlgorithmCreate defines an algorithm specification object with options for the algorithm, and the function siena07 carries out the estimation. If you do not want to see the graphical interface with intermediate results, or if your computer has problems showing this, then add the option batch = TRUE, as in

If you wish to have detailed information at the console about the intermediate steps taken by the algorithm, then add the option verbose = TRUE, as in

Experience shows that parameters estimated by siena07 are not very large in magnitude, mostly they are less than 20 in absolute value. Therefore, when the algorithm at some moment leads to provisional values larger than, say, 50, this is a signal that the algorithm is diverging. This will lead to a stop of the algorithm; the default threshold value of 50 can be changed by the keyword thetaBound. E.g., one could use

Some effects associated with covariates will have values that scale inversely with the covariates. It was mentioned in Section 4.1.5 that actor covariates should be preferably have standard deviations between, say, 0.1 and 10. This will normally lead to parameter values less than 10. The same goes for dyadic covariates.

## 6.2 Parameters of the algorithm

Function sienaAlgorithmCreate defines the algorithm. The algorithm is described more in detail in Section 6.7. This function has a lot of parameters that you will never need to touch, and which have been put there for the purpose of investigating the best settings of the algorithm. The following parameters are worth knowing about.

- 1. projname: the 'project name', the root name of the file to which relevant intermediate and final information will be written. This file will be put in the working directory of your R session, and have the name ppp.txt, if projname = ppp. (In RSiena versions before 1.2-23, this file was called ppp.out).
- 2. n3: the number of iterations in the third phase of the estimation algorithm, determining the precision of the standard errors.

3. seed: the seed of the random number algorithm. This seed determines the starting value of the random number algorithm, and thereby leads to exactly reproducible results. Normally, if the estimation procedure has converged as indicated by the convergence criterion (see the next section) and parameter n3 is large enough, different random seeds will give results for converged models that are equivalent for practical purposes. (Well, this is a way to check whether indeed your convergence criterion is strict enough, and whether parameter n3 is large enough.)

Then there are some parameters that determine some aspects of the model, rather than the estimation algorithm. These are the following.

- 4. modelType: the type of model for non-directed networks, see Section 5.9.
- 5. behModelType: the type of model defining the behavior ministeps at the boundary (minimum and maximum) of the range, see Section 5.11.
- 6. MaxDegree: puts a maximum to the outdegree of all actors, see Section 5.14.
- 7. Offset: an offset is a constant added to the constant term in a generalized linear model, it is used for modelType = 3, see Section 5.9.

Some other parameters are sometimes important, in the case of convergence difficulties. Their role is described in the following sections.

- 8. useStdInits: whether the initial values in the effects object will be ignored and default values used instead (default: FALSE).
- 9. nsub: the number of subphases in the estimation algorithm (default: 4).
- 10. n2start: the number of iterations in the first subphase.
- 11. firstg: the initial value of the multiplier for the size in the stochastic approximation algorithm.

The default estimation algorithm is the Method of Moments. But it is also possible to use the method of Maximum Likelihood; further explanations are in Section 6.11. This is determined by the following parameters.

- 12. maxlike: whether to use Maximum Likelihood (ML) estimation (default: FALSE).
- 13. mult: the multiplication factor for the ML algorithm.

As described in Section 6.12.1, there is a choice between conditional and non-conditional estimation. Conditioning means that for one of the dependent variables, the simulated number of differences is always equal to the observed number of differences (note that the number of ministeps usually will be somewhat larger than the number of differences, because some ministeps may cancel earlier ministeps, and ministeps can have the outcome of 'no change'). The associated parameters are

14. cond: whether to use conditional estimation;

- 15. condvarno: the number of the conditioning variable;
- 16. condname: the name of the conditioning variable (an alternative to using condvarno).

Furthermore, there are some parameters with special purposes that sometimes can be used.

- 17. simOnly: to indicate that the interest is in simulation, not estimation; see Chapter 9.
- 18. lessMem: using lessMem = TRUE reduces storage in the sienaFit object produced by sienaO7 by leaving out some large arrays; but these are used by functions sienaTimeTest and sienaGOF, so running those functions will be impossible for sienaFit object obtained with lessMem = TRUE.

Finally, there is a parameter for which the default is always good; but you might try it out anyway.

19. dolby: the default is TRUE, which implies that the statistics are regressed on the augmented data score, which is known to have expected value 0. This is explained in Snijders (2013). This yields a considerable reduction of the variance of the statistics without affecting their expected values.

The regression coefficients are calculated in Phase 1. If prevAns = ans is used and the model has remained identical, the regression coefficients from Phase 3 of ans are used. These values are given in ans\$regrCoef; the corresponding correlations are in ans\$regrCor.

## 6.3 Results of the estimation

The estimation produces an output file in the current working directory, of which the name is defined by the projname option; in this example, the name is trypro.txt. To look at the information, you may either look at this file (which can be opened by any text editor), or produce results on the R console.

A brief summary of the results is given in the R console by typing the name of the sienaFit object. For example, if this name is results1,

#### results1

could give a summary such as

Estimates, standard errors and convergence t-ratios

	Estimate	e Standar Error		Convergence t-ratio
Rate parameters:				
0 Rate parame	eter 6.0803	(1.0220	)	
1. eval outdegree (	(density) -2.5270	(0.1589	)	0.0152
<ol><li>eval reciprocity</li></ol>	y 2.1021	( 0.3038	)	0.0039
<ol><li>eval transitive</li></ol>	triplets 0.5470	(0.1988	)	0.0214
4. eval 3-cycles	0.0805	( 0.3845	)	0.0369
<ol><li>5. eval smoke1 simi</li></ol>	ilarity 0.4400	(0.2560	)	-0.0427

Overall maximum convergence ratio: 0.1608

Requesting a longer summary by a command such as

```
summary(results1)
```

will produce more information, including, e.g., the covariance/correlation matrix of the estimators.

Convergence check

The column Convergence t-ratio shown above, also called t statistics for deviations from targets, is an indicator of convergence. If some of these values are higher in absolute value than 0.1, convergence is not adequate. The value Overall maximum convergence ratio is another, stricter, indicator of convergence. For adequate convergence, this value should be less than 0.25. The overall maximum convergence ratio for sienaFit object results1 is given by results1\$tconv.max.

In this example, convergence is good. If convergence is not adequate, the estimation must be repeated. Usually the best way to do this is by employing the argument prevAns in the call of siena07. Given that the earlier result was already called results1, this is done, e.g., by

The practical procedure therefore, in usual cases, is that one runs siena07, if necessary repeatedly with subsequent runs using the prevAns parameter as above, until the Overall maximum convergence ratio is less than 0.25 and all convergence t ratios are less than 0.1. (The threshold value of 0.25 is not hewn in stone, and small deviations are acceptable.) In some cases this aim is not easily reached; what to do then is treated in the following.

#### 6.3.1 Initial Values

The *initial values* can be given in three ways.

 The default: if useStdInits = FALSE and no prevAns parameter is given in the call of sienaO7, the initial values are taken from the sienaEffects object, in this example called myeff.

Requesting

myeff

will show the initial values. As long as no time dummies have been requested using sienaTimeFix, the initial values for the requested effects are in the vector

myeff\$initialValue[myeff\$include]

Changing these values is not often necessary, because the parameter prevAns, as explained in the next item, does this behind the scenes.

If one does wish to change the initial values contained in the effects object, this can be done using the function updateTheta, which copies the estimates from earlier results, contained in a sienaFit object, to the effects object. For a single effect the initial value can be changed by the setEffect function in which the initialValue then must be set.

2. If useStdInits = FALSE and the prevAns ('previous answer') parameter is used, such as in

the initial parameter estimates are taken from the results of what is given as the prevAns parameter. This must be a sienaFit object; in this example it is given as results0.

If the specification of the effects object used to obtain results0 was the same as myeff, then not only the initial values are copied, but also Phase 1 of the algorithm is skipped, because information for the sensitivity of the statistics with respect to the parameters is taken from the results of Phase 3 of results0.

If the specification of the effects object used to obtain results0 was not the same as myeff, then for those parameters that do match, the initial values are copied from results0 and Phase 1 is carried out as usual.

3. If useStdInits = TRUE is used in the call of sienaAlgorithmCreate, standard initial values are used.

These consist of some reasonable values for the rate parameters and the outdegree parameter, as well as for the linear shape parameter for behavioral dependent variables (if any); and 0 parameters for the rest.

The default is useStdInits = FALSE.

# 6.3.2 Convergence Check

When parameters have been estimated, first the convergence of the algorithm must be checked. This is done by looking at the t-ratios for convergence and the overall maximum convergence ratio. These are given in the output of the algorithm, presented above. This check considers the deviations between simulated values (in Phase 3, see below) of the statistics and their observed values (the latter are called the 'targets'). Ideally, these deviations should be 0. Because of the stochastic nature of the algorithm, when the process has properly converged the deviations are small but not exactly equal to 0. The program calculates the averages and standard deviations of the deviations and combines these in a t-ratio (in this case, average divided by standard deviation). The overall maximum

convergence ratio is the maximum value of the ratio

 $\frac{\text{average deviation}}{\text{standard deviation}}$ 

for any linear combination of the target values. A precise definition is given in Siena\_algorithms.pdf which can be downloaded from the SIENA website. The overall maximum convergence ratio for sienaFit object results1 is given by results1\$tconv.max.

Convergence is excellent when the overall maximum convergence ratio is less than 0.2, and for all the individual parameters the t-ratios for convergence all are less than 0.1 in absolute value; convergence is reasonable when the former is less than 0.30. For published results, it is suggested that estimates presented come from runs in which the overall maximum convergence ratio is less than 0.25. (These bounds are indications only, and are not meant as severe limitations.)

In the example above, the largest absolute value of the t-ratios for convergence is equal to 0.0427, and the overall maximum convergence ratio is 0.1608; both are quite good values.

If convergence is not adequate, the best way to continue is by making another estimation run, now carrying on from the last obtained result. This is done by using this result in the prevAns ('previous answer') parameter, while taking care that useStdInits = FALSE has been specified (but this is the default). An example is

In this case, this second estimation run produced good results, with a maximum absolute t-ratio for convergence equal to 0.0777. The output file gives more extensive results, viz., the averages and standard deviations of the deviations from targets and the resulting t-ratios:

End of stochastic approximation algorithm, phase 3.

Total of 1822 iterations.

Parameter estimates based on 822 iterations,
basic rate parameter as well as
convergence diagnostics, covariance and derivative matrices based on 1000 iterations.

Information for convergence diagnosis.

Averages, standard deviations, and t-ratios for deviations from targets:

- 1. 0.2460 16.1494 0.0152
- 2. 0.0560 14.3829 0.0039
- 3. 0.9520 44.5338 0.0214
- 4. 0.5380 14.5726 0.0369
- 5. -0.2080 4.8672 -0.0427

Good convergence is indicated by the t-ratios being close to zero. Overall maximum convergence ratio = 0.1608.

For example, for the fourth parameter (3-cycles), the average deviation from the target value was 0.5380, and the standard deviation across the 1000 simulations in Phase 3 was 14.5726. This yields a t-ratio of 0.5380/14.5726 = 0.0369. Large values of the averages and standard deviations are in themselves not at all a reason for concern; only the t-ratio is important.

### 6.3.3 Continued estimation to obtain convergence

Above, the prevAns parameter was mentioned which will lead to using the result from a previous estimation as the initial value for the next estimation. If convergence is difficult to obtain, one may use other settings of the estimation algorithm, given as parameters in the sienaAlgorithmCreate function, to try and improve convergence. The main parameters of sienaAlgorithmCreate that can be used for this purpose are the following. They are briefly explained in the help file for this function. For the technical background, see Siena\_algorithms.pdf which can be downloaded from the SIENA website.

#### • n2start

This is the minimum length of Phase 2.1, i.e., the first subphase of phase 2. The default value is  $2.52 \times (p+7)$ , where p is the number of estimated parameters, divided by nbrNodes, if multiple processes are used. The minimum lengths of the subsequent subphases<sup>9</sup> are  $(2.52)^{k-1} \times \text{n2start}$  for subphase k.

This implies the total duration of the algorithm will be roughly proportional to n2start. One may try using a value higher than the default.

#### • nsub

This is the number of subphases in Phase 2 of the algorithm. Usually, the default value of 4 is large enough. Below it is mentioned that sometimes it can be good to use nsub = 1 in conjunction with a large value of n2start to improve convergence after having found a reasonably good, but not totally satisfactory, estimate.

#### • firstg

This determines the step sizes in the estimation algorithm. If the algorithm is unstable, use a smaller value (but greater than 0). The default value is 0.2. Sometimes for difficult data-model combinations, the algorithm diverges very quickly, and this may be countered by smaller values of firstg, e.g., 0.01 or 0.05.

#### • diagonalize

This parameter may range from 0 to 1, and determines the extent to which the matrix of derivatives of expected values with respect to parameters is diagonalized. The value 1 (total diagonalization) gives greatest stability; smaller values give greater efficiency. The default for MoM is 0.2.

<sup>&</sup>lt;sup>9</sup>These values are meant to approximate  $(7+p) \times 2^{4(k+2)/3}$ . With the stepwise gain parameter  $a_N$  being halved for each new subphase, these values imply that  $a_N$  is asymptotically of order  $N^{-3/4}$ , which is good for convergence of the algorithm.

#### • doubleAveraging

This replaces the Robbins-Monro updating step by a double averaging step (Bather, 1989; Schwabe and Walk, 1996; Kushner and Yin, 2003) which can be more efficient. The default is doubleAveraging=0, which starts using this step from subphase 2.1.

If convergence is not very good even with repeated estimation with the prevAns option, sometimes it can be useful to try and use updateTheta to copy the results from the earlier estimation rather than prevAns; this will use the same starting values but not skip Phase 1 of the estimation algorithm, and sometimes this turns out to lead to faster convergence.

The following function will iterate the execution of siena07 until it has converged. It can be modified to suit your further purposes. The argument ans0 can be employed to use an earlier existing 'on track' estimation result, if available, as the initial value for the algorithm.

```
sienaO7ToConvergence <- function(alg, dat, eff, ansO = NULL, ...){
  numr <- 0
   ans <- siena07(alg, data = dat, effects = eff, prevAns = ans0, ...) # the first run
  repeat {
       save(ans, file = paste("ans",numr,".RData",sep = "")) # to be safe
       numr <- numr + 1
                                 # count number of repeated runs
       tm <- ans$tconv.max</pre>
                                 # convergence indicator
       cat(numr, tm,"\n")
                                 # report how far we are
       if (tm < 0.25) {break}
                                 # success
       if (tm > 10) {break}
                                 # divergence without much hope
                                 # of returning to good parameter values
       if (numr > 30) {break}
                                # now it has lasted too long
       ans <- siena07(alg, data = dat, effects = eff, prevAns = ans, ...)
       }
  if (tm > 0.25)
      cat("Warning: convergence inadequate.\n")
   }
   ans
}
```

Another approach that sometimes can be helpful to obtain convergence in difficult situations is to gradually build up the model, adding further effects while using the prevAns parameter to use previous estimates as starting values for the next, extended model. This may be more successful than estimating a complicated model right from the start.

When there are difficulties in obtaining convergence, note that one also should reconsider the model specification; good specification of the model often considerably improves the converge of the parameter estimation. When reading the words here because of a concern about convergence, please read on at least until and including Section 6.5.

## 6.4 Use of the algorithm parameters

The parameters mentioned in the preceding section can be used in the following way to try and achieve convergence if one does not obtain the desired low level of tconv.max by successive estimation using the prevAns parameter sequentially. The advice is to use (1.) or (2.), in both cases together with (3.). It is to be expected that (1.) will be computationally more efficient than (2.)

1. First do an estimation with the default parameters for the algorithm. If that has returned a somewhat reasonable provisional estimate (among other requirements, with a value of tconv.max that is not very high; and not with one or more parameter estimates being very high in absolute value), continue the estimation, using prevAns, with nsub=1; a high value for n3, e.g., 3000 or 5000; and a high value for n2start. What is 'a high value for n2start' depends on the data set - model combination, the number of parameters in the first place; and the number of processes used. The baseline value (see the preceding section 10) would be

 $(p+7) \times (2.52)^4/\text{nbrNodes}$ 

where p is the number of estimated parameters (excluding the rate parameters estimated by conditioning, if conditional estimation is used). For a 'high value', start with a value that is about twice as large, Note that computation time in Phase 2 will be proportional to n2start. Continue estimation with these algorithmic parameters using prevAns, further perhaps increasing n2start if necessary.

If estimation further was unstable, a possibility here is to use a value for firstg clearly less than the default 0.2; e.g., 0.01. But often the default works just fine here.

- 2. As an alternative to (1.): in estimations where prevAns is not<sup>11</sup> used, use more than 4 subphases: nsub=5 or, if that is not sufficient, nsub=6.

  Note that if prevAns is used, a larger value of nsub makes no sense; if anything, nsub should be decreased (and n2start increased).
- 3. Use for n3 a larger value than the default. E.g., n3 = 3000 or 5000; if it was necessary to further increase nsub or n2start, it may also be necessary to further increase n3.

All these choices clearly increase computing time; but less so, and with more likely success, than just carrying on very long with siena07ToConvergence using the default algorithmic parameters.

### 6.5 What to do if there are convergence problems

If there are persisting convergence problems even after repeated estimations using the prevAns parameter and trying out various settings for the algorithm as suggested in the preceding sections, this may have several reasons.

<sup>&</sup>lt;sup>10</sup>The value given here is the default number of simulation runs in the 4'th subphase.

<sup>&</sup>lt;sup>11</sup>Until December 2, 2019, unfortunately, the 'not' was erroneously omitted here.

- The data specification was incorrect (e.g., because the coding was not given properly).
- The starting values were poor. Try restarting from the standard initial values (a certain non-zero value for the density parameter, and zero values for the other parameters); or from values obtained as the estimates for a simpler model that gave no problems. The initial default parameter values can be obtained by choosing the algorithm option 'standard initial values'.
- The model does not fit well in the sense that even with well-chosen parameters it will not give a good representation of the data.

This can be the case, e.g., when there is a large heterogeneity between the actors which is not well represented by effects of covariates. The out-degrees and in-degrees are given in the begin of the RSiena output to be able to check whether there are outlying actors having very high in- or out-degrees, or a deviating dynamics in their degrees. Strong heterogeneity between the actors will have to be represented by suitable covariates; if these are not available, one may define one or a few dummy variables each representing an outlying actor, and give this dummy variable an ego effect in the case of deviant out-degrees, and an alter effect in the case of deviant in-degrees.

Sometimes transitivity can better be modeled by the GWESP effects (search for this term in the manual) than by transitive triplers. This may help with convergence.

For modeling large networks, it is important to represent meeting opportunities, e.g., by a suitable dyadic covariate, or by the sameX effect of a suitable categorical covariate.

Sometimes there are important differences between the actors in how many changes they make in their outgoing ties. This should then be reflected by including one or more rate effects. The experience is that the main rate effect to include is the rate effect of the outdegrees (outRate, see Section 12.1.5).

Another possibility is that there is time heterogeneity. Indications about this can be gathered also from the descriptives given in the start of the output file produced by print01Report: the number of changes upward and downward, in the network and also – if any – in the dependent behavioral variable. If these do not show a smooth or similar pattern across the observations, then it may be useful to include actor variables representing time trends. These could be smooth – e.g., linear – but they also could be dummy variables representing one or more observational periods; these must be included as an ego effect to represent time trends in the tendency to make ties (or to display higher values of the behavior in question). Further see Section 5.13 for how to discover and handle time heterogeneity.

• Too many weak effects are included. Use a smaller number of effects, delete non-significant ones, and increase complexity step by step. Retain parameter estimates from the last (simpler) model as the initial values for the new estimation procedure, provided for this model the algorithm converged without difficulties; here also

prevAns may be used.

Effects that are left out of the estimation can still be used in the model by specifying them with test=TRUE, fix=TRUE; this will not burden the estimation process, and give information (with a score-type test, see Section 8.2) about the significance of this excluded effect.

Usually this will be applied with initialValue=0, the default. But sometimes it may be done with a plausible non-zero value for initialValue.

- Two or more effects are included that are almost collinear in the sense that they can both explain the same observed structures. This will be seen in high absolute values of correlations between parameter estimates, presented in the summary of the results object and also in the output file. In this case it may be better to exclude one of these effects from the model.
- An effect is included that is large but of which the precise value is not well-determined (see above: section on fixing parameters). This will be seen in estimates and standard errors both being large and often in divergence of the algorithm. Fix this parameter to some large value. (Note: large here means, e.g., more than 5 or less than -5; depending on the effect, of course.)

Another trick that may be tried is the following. Sometimes one (or some) of the rate parameters are especially the causes of difficulties of convergence. Then one may fix this parameter at a good value, and estimate the rest of the parameters. Suppose that this is feasible, i.e., good convergence can be obtained provided that this rate parameter is fixed. Then by trial and error one may find a fixed value for this rate parameter for which the t-ratio for convergence for this parameter also is acceptable (less than 0.2, preferably less than 0.1). Since normally, rate parameters are nuisance parameters (i.e., not of focal interest), this can be an acceptable way out.

#### 6.6 Some important components of the sienaFit object

If a user would like to do further calculations, it can be useful to know about the following components of sienaFit objects. Suppose the object is called ans. Some of the components are the following. Further details are in the help file for siena07.

ans\$theta parameter estimates

(but not for the rate parameter used for conditioning; if time dummies were requested using sienaTimeFix,

these are also in theta)

ans\$covtheta covariance matrix of the estimates standard errors of the estimates

ans\$pp number of parameters

ans\$targets (observed statistics) for Method of Moments estimation

ans\$nrunMH number of Metropolis-Hastings step per wave

for Maximum Likelihood estimation

ans\$tconv t-ratios for convergence for each of the parameters

ans\$tmax maximum absolute value of these ratios for non-fixed parameters

ans\$tconv.max maximum t-ratio for convergence for

> any linear combination of the parameters, called the overall maximum convergence ratio

ans\$sf generated statistics in Phase 3 (targets subtracted)

ans\$msf covariance matrix of ans\$sf

ans\$dfra estimated derivative of expected statistics w.r.t. parameters,

ans\$ac autocorrelations of generated statistics in Phase 3

ans\$sims simulated values of dependent variables in Phase 3 of the algorithm

if returnDeps = TRUE in the call of siena07;

in case of Maximum Likelihood estimation (see Section 9.1), note that for observed tie variables the simulated values are equal to the observed;

for missing tie variables, the simulated values can be regarded

as model-based imputations

ans\$estMeans estimated expected values of the target statistics

> (if the Dolby option was chosen, this is not equal to the average of the simulations!)

ans\$requestedEffects

the effects object with only the requested effects

ans\$effects the effects object including also the main effects

for any requested interactions

ans\$x the algorithm object used

ans\$f everything needed for the calculations in C++;

> in particular, the data set is hidden in here, and can be reconstructed. Digging in will show as mat1 the network data, as mat2 the missings,

and as mat3 the structurally determined values. All these are stored as transposed edge lists.

Programmers can consult function initializeFRAN for the creation and hence contents of this object.

the RSiena/Test version ans\$version

ans\$startingDate date and time of the start of the estimation (since version 1.2-17).

Like for any R object, the internal structure of the sienaFit object can be requested by requesting

sink("ans.txt") str(ans) sink()

This writes the structure to the external file ans.txt, which may be better than printing it to the console, because it is a long story.

A limited representation of the structure of this object is obtained from

```
sink("ans.txt")
str(ans, 1)
sink()
```

To get some further understanding, one could investigate some of the components of this object as follows. Note that putting a statement between parentheses like in (A <- B) is just a way for constructing the object A and showing it at the same time.

```
# Compute the covariance matrix of the generated statistics
print(covsf <- cov(ans$sf))
# This is the same as ans$msf, provided there are no fixed parameters.
# The means and standard deviations of the generated statistics minus targets:
(v <- colMeans(ans$sf))
(s <- apply(ans$sf, 2, sd))
# This also allows to compute the convergence t-ratios
v / s
# To get the generated statistics without subtracting the targets,
# we have to add the targets.
# To do this, repeated transposition t can be used:
stats <- t(t(ans$sf) + ans$targets)
# or
stats <- ans$sf + rep(ans$targets, each = nrow(ans$sf))</pre>
```

The tconv components are used in the function siena07ToConvergence presented above.

### 6.7 Algorithm

The estimation algorithm is an implementation of a procedure of which the original version was proposed by Robbins and Monro (1951). The algorithm is described in Snijders (2001, 2005) and in Siena\_algorithms.pdf which can be downloaded from the SIENA website. It has three phases:

- 1. In Phase 1, the parameter vector is held constant at its initial value. This phase is for having a first rough estimate of the matrix of derivatives.
- 2. Phase 2 consists of several subphases. More subphases means a greater precision. The default number of subphases is 4.

Each subphase consists of a large number of runs, usually hundreds of them. Each run consists of a simulation of the network dynamics over all periods, using trial parameter values. From this simulation, the estimation statistics are calculated; the devations between the generated statistics and the observed values, which are the target values, are used to update the trial parameter values. For statistics of which the outcome is too high the corresponding parameter will be decreased, if it is too low the parameter will be increased. These changes in the parameter values are smaller in the later subphases.

If the option dolby is in force, the statistics are regressed on the score function of

the augmented data. This reduces their variance without affecting their expected value, see Snijders (2013).

The program searches for parameter values where these deviations average out to 0. This is reflected by what is called the 'quasi-autocorrelations' in the output screen. These are averages of products of successively generated deviations between generated and observed statistics. It is a good sign for the convergence of the process when the quasi-autocorrelations are negative (or positive but close to 0), because this means the generated values are jumping around the observed values. When estimating by the Method of Moments, it is usual for the quasi-autocorrelations to become close to 0. For estimation by Maximum Likelihood, they will usually eventually tend to fluctuate about some positive values determined by the multiplication factor (see Section 6.11). Large quasi-autocorrelations (larger than .5), when using the Method of Maximum Likelihood, suggest that either the estimation process is still far from its eventual limit (the final estimate), or the multiplication factor may be too small. But in this case, the autocorrelations given in the output file are more important information than those given on the screen.

3. In Phase 3, the parameter vector is held constant again, now at its final value. This phase is for estimating the covariance matrix and the matrix of derivatives used for the computation of standard errors.

The number of runs in Phase 3 is given by parameter n3, with the default value of 1000. For precise estimation of standard errors, higher values of n3 are desirable. This requires a lot of computing time, but when the number of Phase 3 runs is too low, the standard errors computed are rather unreliable.

The number of subphases in Phase 2, and the number of runs in Phase 3, are determined by parameters nsub and n3 in the call of sienaAlgorithmCreate.

During the estimation process, if the graphical user interface is used (the default batch = FALSE in the call of siena07), the user can break in and modify the estimation process in two ways:

- 1. it is possible to terminate the estimation;
- 2. in Phase 2, it is possible to terminate Phase 2 and continue with Phase 3.

### 6.8 Output

Output can be obtained in several ways.

- 1. On the R console.
  - When the sienaFit object produced by sienaO7 is called ans, requesting just ans or print(ans) produces output on the R console. The function summary(ans) produces more extensive output.
- 2. A table in latex or html format can be produced by the xtable.sienaFit method. For example,

```
xtable(ans, file = "ans1.htm", type = "html")
```

produces in the working directory a html file with the ans results in tabular form. The xtable package has many further options.

3. The function siena.table is available which serves a similar purpose, but not using xtable. This function writes the table to a file; the default file name of the table produced is the name of the sienaFit object. The choice between xtable.sienaFit and siena.table depends on the preference for the tables produced.

For importing the results of xtable or siena.table into MS-Word, the following steps can be used.

Request siena.table(ans) for some sienaFit object, called ans in this example. This will produce, under the default settings, the file ans.htm in the current working directory. Copy-paste this into the MS-Word file. In MS-Word then select this table, but only the lines of the header and the parameters, not any preceding blank line nor the footnote. In the MS-Word menu choose 'Insert — Convert text to table — Autofit to contents'. This will produce the table in your MS-Word file. You can then further modify the table; e.g., change the double minus sign — to the MS-Word minus sign — (available under 'insert — symbol') and replace the dots for the 't stat.' of the rate parameters by blanks.

4. The function siena07 writes an output file which is an ASCII ('text') file that can be read by any text editor. It is called *pname*.txt, where *pname* is the name specified in the call of sienaAlgorithmCreate.

This output file is divided into sections indicated by a line @1, subsections indicated by a line @2, subsubsections indicated by @3, etc. For getting the main structure of the output, it is convenient to have a look at the @1 marks first.

The primary information in the output of the estimation process consists of the following three parts.

#### 6.8.1 Convergence check

This was discussed in Section 6.3.2 above.

### 6.8.2 Parameter values and standard errors

The next crucial part of the output is the list of estimates and standard errors. Suppose that the following result was obtained on the R console.

Estimate Standard Convergence
Error t-ratio

Rate parameters:

```
Rate parameter
                             6.0742
                                     (1.0134
1. eval outdegree (density) -2.5341
                                      ( 0.1445
                                                     0.0571
2. eval reciprocity
                             2.1106
                                      (0.2625
                                                     0.0710
3. eval transitive triplets
                             0.5449
                                      ( 0.1781
                                                     0.0584
4. eval 3-cycles
                             0.0779
                                      ( 0.3425
                                                     0.0777
5. eval smoke1 similarity
                             0.4519
                                      (0.2497
                                                     0.0400
```

The rate parameter is the parameter called  $\rho_m^{\rm net}$  in Section 12.1.5 below (where m=1 because there is only one period). The value 6.0742 indicates that the estimated number of opportunities for change per actor (note that each actor corresponds to a row in the adjacency matrix) between the two observations is 6.07 (rounded in view of the standard error 1.01). Note that this refers to unobserved changes, and that some opportunities for change lead to the decision 'no change', and moreover some of these changes may cancel (make a new choice and then withdraw it again), so the average observed number of differences per actor will be smaller than this estimated number of unobserved changes.

The other five parameters are the weights in the evaluation function. The terms in the evaluation function in this model specification are the out-degree effect defined as  $s_{i1}$  in Section 12.1.1, the reciprocity effect  $s_{i2}$ , transitive triplets effect  $s_{i3}$ , three-cycles effect  $s_{i5}$ , sex similarity effect  $s_{i86}$ . Therefore the estimated evaluation function here is

$$-2.53 s_{i1}(x) + 2.11 s_{i2}(x) + 0.54 s_{i3}(x) + 0.08 s_{i5}(x) + 0.45 s_{i86}(x)$$

where again some rounding was applied in view of the standard errors. The parameter estimates can be combined with the standard errors to test the parameters. (Testing of parameters is discussed more extensively in Chapter 8.)

For the rate parameter, testing the hypothesis that it is 0 is meaningless because the fact that there are differences between the two observed networks implies that the rate of change must be positive. The weights in the evaluation function can be tested by t-statistics, defined as estimate divided by its standard error. (Do not confuse this t-test with the t-ratio for checking convergence; these are completely different although both are t ratios!) Here the t-values are, respectively, -2.5341/0.1445 = -17.54, 2.1106/0.2625 = 8.04, 0.5449/0.1781 = 3.06, 0.0779/0.3425 = 0.23, 0.4519/0.2497 = 1.81. Since the first three are larger than 2 in absolute value, they are significant at the 0.05 significance level. It follows that there is evidence that the actors have a 'preference' for reciprocal and transitive relations. For thee-cycles, the effect is not significant (t = 0.23), for smoking similarity it is significant at the 0.10 significance level. The value of the density parameter is not very important; it is important that this parameter is included to control for the density in the network, but as all other statistics are correlated with the density, the density is difficult to interpret by itself.

### 6.8.3 Collinearity check

In the output file, the covariance matrix of the estimates is presented. This can also be requested by summary(ans). For conditional estimation, the rate parameters of the dependent variables used for conditioning are not included in this matrix. In this case the covariance matrix is as follows.

Covariance matrix	of estimates	(correlations	below diagon	al):)
0.021	-0.018	-0.010	0.006	-0.008
-0.468	0.069	0.008	-0.034	-0.002
-0.395	0.180	0.032	-0.049	0.003
0.130	-0.378	-0.795	0.117	-0.001
-0.223	-0.037	0.074	-0.007	0.062

The diagonal values are the variances, i.e., the squares of the standard errors (e.g., for the reciprocity effect, 0.069 is the square of 0.2625). Below the diagonal are the correlations. E.g., the correlation between the estimated outdegree effect and the estimated reciprocity effect is -0.468. These correlations can be used to see whether there is an important degree of collinearity between the effects. Collinearity means that several different combinations of parameter values could represent the same data pattern, in this case, the same values of the network statistics. When one or more of the correlations are very close to -1.0 or +1.0, this is a sign of near collinearity. This will also lead to large standard errors of those parameters. It may then be advisable to omit one of the corresponding effects from the model, because it may be redundant given the other (strongly correlated) effect; but see below. It is possible that the standard error of the retained effect becomes much smaller by omitting the other effect, which can also mean a change of the t-test from non-significance to significance.

The suggestion of omitting effects that lead to high parameter correlations with other effect does not directly apply to effects that should be included for other reasons, such as the density effect for network dynamics and the linear and quadratic shape effects for behavior dynamics.

However, correlations between parameter estimates close to -1.0 or +1.0 should not be used too soon in themselves as reasons to exclude effects from a model. This is for two reasons. In the first place, network statistics often are highly correlated (for example, total number of ties and number of transitive triplets) and these correlations just are one of the properties of networks. Second, near collinearity is not a problem in itself, but the problem (if any) arises when standard errors are high, which may occur because the value of the parameters of highly correlated variables is very hard to estimate with any precision. The problem resides in the large standard errors, not in itself in the strong correlation between the parameter estimates. If for both parameters the ratio of parameter estimate to standard error, i.e., the t-ratio, is larger than 2 in absolute value, in spite of the high correlations between the parameter estimates, then the significance of the t-test is evidence anyway that both effects merit to be included in the model. In other words, in terms of the 'signal-to-noise ratio': the random noise is high but the signal is strong enough that it overcomes the noise.

As a rule of thumb for parameter correlations, usually for correlations of estimated structural network effects there is no reason for concern even when these correlations are as strong as .9.

In the example above, the strongest correlation was found between the parameter estimates for transitive triplets and three-cycles. This is not surprising, because both are triadic effects. In this case, the three-cycle effect was not significant, and can be dropped for that reason.

## 6.9 Other estimation procedures

RSiena can estimate models by four estimation methods: the (unconditional or conditional) Method of Moments ('MoM', the default; Snijders, 2001; Snijders et al., 2007); the Generalized Method of Moments ('GMoM', see Amati et al., 2015, 2019); the Maximum Likelihood method ('ML', see Snijders et al., 2010a), and Bayesian methods (see Koskinen, 2004; Koskinen and Snijders, 2007; Schweinberger and Snijders, 2007a; Koskinen and Snijders, 2023).

Relative to the MoM, the GMoM is theoretically more efficient and somewhat more time-consuming than the regular MoM since it requires a large number of simulations and iterations of the model estimation (see Section 6.10.1). In models for one network, the MoM and the GMoM yield similar estimates. However, the GMoM estimator of the parameters of the effects related to the new statistics is somewhat more efficient than the MoM estimator, especially for the co-evolution model. When the statistics are informative (i.e., when the number of simultaneous network and behavioral changes made by the same actor between any two observations is not negligible), and when the 'true model' has only evaluation effects for the GMoM statistics, the GMoM leads to more accurate estimates with smaller bias and standard errors than the regular MoM. This is documented in Amati et al. (2015, 2019).

Comparison among all the four methods indicates that, in models for one network, without co-evolution, in nice situations (data sets that are not too small, model specifications that do not request too much from the data, good fit between data and model specification), the four methods tend to agree and there seems to be no reason to use the more time-consuming ML or Bayesian methods. For co-evolution models and not-so-nice situations (very small network data sets, small network and behavior data sets in combination with complex models), however, GMoM, ML and Bayesian methods tend to produce more accurate results (i.e., smaller standard errors) than MoM. Statistical theory suggests that, if the model is well-specified, GMoM and ML are more efficient estimation methods than MoM in the sense of producing estimates with smaller standard errors. But in the 'nice situations' with only one dependent network variable the efficiency advantage of ML is very small. Bayesian estimation is based on a different statistical paradigm, and assumes and requires that the uncertainty about parameters is expressed itself in a probability distribution.

Estimation by the regular Method of Moments (MoM) and Generalized Method of Moments (GMoM) is implemented in the functions siena07 and sienacpp. ML estimation is done by the function siena07, using a set of options created by sienaAlgorithmCreate with maxlike = TRUE. Bayesian estimation is done by the function sienaBayes, but is as yet implemented only for multilevel network modeling (Section 11.4).

#### 6.10 Generalized Method of Moments estimation

# 6.10.1 Using the function siena07

Compared to the regular Method of Moments (MoM), the Generalized Method of Moments (GMoM) allows using more statistics than parameters, thereby exploiting more

information in the observed data. The additional information derives from the use of supplementary statistics, referred to as GMoM statistics. The GMoM statistics are defined only for evaluation effects. Thus, they are used under the assumption that effects operate in the same strength for the creation and the dissolution of ties, and the increasing and decreasing of the behavior. In other words, the creation and the endowment effects corresponding to the GMoM statistics are assumed to be zero (see Section 5.1.3).

Estimation by the GMoM with the function siena07 is obtained by specifying the GMoM statistics in the effects object using function includeGMoMStatistics, and setting option gmm=TRUE in the algorithm object created by the function sienaAlgorithmCreate.

Although the statistics are added to the effect object, one should keep in mind that the GMoM statistics are not effects in the stochastic model, i.e., they are not included as terms in the objective function. They are only used for parameter estimation to evaluate the moment conditions.

Several GMoM statistics have been implemented.

## GMoM statistics for network evolution

The statistics for the network evolution (Amati et al., 2015) complement the information provided by the regular statistics by considering two consecutive observations of the network. This formulation allows distinguishing between different antecedents of a same effect. The implemented statistics are named newrecip, persistrecip, realrecip, realtrans, agreetrans.

#### GMoM statistics for co-evolution of networks and behavior

The GMoM statistics for the co-evolution of networks and behaviors (Amati et al., 2019) supplement the information of the cross-lagged statistics, depending jointly on network and behavior, by considering the network and behavior at the same time point. Thus, the GMoM statistics account for the information on the simultaneous observation of network and behavioral changes. The names of the implemented GMoM statistics are avSim\_gmm, avAlt\_gmm, totAlt\_gmm, maxAlt\_gmm, minAlt\_gmm, egoX\_gmm, simX\_gmm.

For the GMoM, the additional statistics used for estimation are specified by the function includeGMoMStatistics. For example, the following lines

```
myeff <- includeGMoMStatistics(myeff, egoX_gmm, interaction1 = "mybeh")
myeff <- includeGMoMStatistics(myeff, simX_gmm, interaction1 = "mybeh")
specify the GMoM statistics for the covariate-related activity and similarity effects.</pre>
```

The distinction between model effects and GMoM statistics is recalled when printing the effect object in the R console. As illustrated by the example below, the output is divided into two parts. The first part lists the model effects. The second part reports the statistics used for the estimation: the regular statistics, included by default when an effect is specified in the probability model using includeEffects, and the GMoM statistics, specified with the function includeGMoMStatistics. The comparison between the number of effects and that of the statistics illustrates that, for the GMoM, the one-to-one correspondence between effects operating for the evolution of networks and/or behaviors, and statistics operating for parameter estimation is lost.

Effects and statistics for estimation by the Generalized Method of Moments

E:	ffects							
	name	effectName	include	fix	test	initialValue	parm	type
1	mynet	<pre>constant mynet rate (period 1)</pre>	TRUE	FALSE	FALSE	4.69604	0	rate
2	mynet	<pre>constant mynet rate (period 2)</pre>	TRUE	FALSE	FALSE	4.32885	0	rate
3	mynet	outdegree (density)	TRUE	FALSE	FALSE	-1.46770	0	eval
4	mynet	reciprocity	TRUE	FALSE	FALSE	0.00000	0	eval
5	${\tt mynet}$	transitive triplets	TRUE	FALSE	FALSE	0.00000	0	eval
6	mynet	3-cycles	TRUE	FALSE	FALSE	0.00000	0	eval
7	mynet	mybeh ego	TRUE	FALSE	FALSE	0.00000	0	eval
8	mynet	mybeh similarity	TRUE	FALSE	FALSE	0.00000	0	eval
9	${\tt mybeh}$	rate mybeh (period 1)	TRUE	FALSE	FALSE	0.70571	0	${\tt rate}$
10	${\tt mybeh}$	rate mybeh (period 2)	TRUE	FALSE	FALSE	0.84939	0	${\tt rate}$
11	${\tt mybeh}$	mybeh linear shape	TRUE	FALSE	FALSE	0.32237	0	eval
12	${\tt mybeh}$	mybeh quadratic shape	TRUE	FALSE	FALSE	0.00000	0	eval
13	${\tt mybeh}$	mybeh total similarity	TRUE	FALSE	FALSE	0.00000	0	eval
Re	•	and GMoM statistics						
		effectName	Statist	ic				
1		constant mynet rate (period 1)	-					
2	•	constant mynet rate (period 2)	Ū					
3	${\tt mynet}$	outdegree (density)	Regular					
4	${\tt mynet}$	reciprocity	Regular					
5	${\tt mynet}$	transitive triplets	Regular					
6	${\tt mynet}$	3-cycles	Regular					
7	${\tt mynet}$	mybeh ego	Regular					
8	${\tt mynet}$	mybeh similarity	Regular					
9	${\tt mybeh}$	rate mybeh (period 1)	Regular					
10	${\tt mybeh}$	rate mybeh (period 2)	Regular					
11	${\tt mybeh}$	mybeh linear shape	Regular					
12	${\tt mybeh}$	mybeh quadratic shape	Regular					
13	${\tt mybeh}$	mybeh total similarity	Regular					
14	${\tt mynet}$	mybeh ego	GMoM					
15	${\tt mynet}$	mybeh similarity	GMoM					

### GMoM statistics for co-evolution of multiple networks

Similarly to the co-evolution of networks and behavior as treated in (Amati et al., 2019), GMoM statistics have been implemented for the co-evolution of multiple networks, using contemporaneous statistics that supplement the cross-lagged statistics of the MoM. supplement the information of the cross-lagged statistics, depending jointly on network and behavior, by considering the network and behavior at the same time point. The implemented GMoM statistics are crprod\_gmm, to\_gmm, and from\_gmm, supplementing the effects with shortnames crprod, to, and from.

### Operation of GMoM estimation

The computation of the GMoM estimates is more time consuming than that of the regular MoM. The GMoM estimator is computed by minimizing the distance between the expected value of the statistics and their sample counterpart. The minimization demands

the computation of a matrix of weights describing the importance of the statistics and their sensitivity to the parameters. To estimate the matrix of the GMoM weights, a larger number of simulations in phase 1 and phase 3 and (at least) one iteration of the function siena07 is used.

The following estimation steps are recommended to reach the convergence of the algorithm quickly.

- 1. Run the function siena07 with one or two sub-phases (i.e., nsub = 1 or nsub = 2) in phase 2 and a large number of simulations in phase 3 (e.g., n3 = 5000) to get better initial estimates of the model parameters and a more stable estimation of the matrix of GMoM weights.
- 2. Re-estimate the model using the estimates produced in 1 by using four sub-phases in phase 2 (nsub = 4), the same number of simulations in phase 3 (e.g., n3 = 5000) and the argument prevAns = TRUE.

If convergence is not adequate, one runs siena07 repeatedly with prevAns = TRUE.

Difficulties in convergence might occur when the GMoM statistics are 'highly' correlated (> 0.9) with the regular statistics. Dropping the corresponding GMoM statistics would solve the problem. Intuitively, when the GMoM and the regular statistics are highly correlated, the GMoM statistics do not carry relevant information for the estimation of the parameters and can be discarded. Therefore, it is recommended to inspect the covariance matrix of the statistics to decide whether GMoM statistics should be used for the estimation.

We refer to the R script GMoMscript.R on the SIENA webpage for an illustration of the GMoM estimation, the steps described above and the inspection of the covariance matrix of the statistics.

Like the MoM, the GMoM allows conditional as well as unconditional estimation (see Section 6.12.1).

### 6.10.2 Using the function sienacpp

Estimation by the Generalized Method of Moments (GMoM) is also implemented in the function sienacpp, available in RSienaTest.

If the algorithm object created by sienaAlgorithmCreate uses maxlike = FALSE, the estimation function sienacpp will use the MoM or the GMoM; the latter will be used as soon as at least one of the effects were specified in the effects object with type = "gmm".

The specification of the effects object for GMoM estimation requires that in the effects object, apart from the basic rate effects, some of the effects were specified in includeEffects with type = "eval" (the default, which means that this does not need to be stated) and the others with type = "gmm". The first then are evaluation effects defining the model specification, the second are statistics used for estimation. The method requires that the number of statistics (type = "gmm") is equal to or larger than the number of evaluation effects (type = "eval"). For example, the commands

```
net <- sienaDependent(array(c(s501,s502,s503), dim = c(50,50,3)))
dataset <- sienaDataCreate(net)</pre>
eff <- getEffects(dataset)</pre>
eff <- includeEffects(eff, density)</pre>
eff <- includeEffects(eff, density, type = "gmm")</pre>
eff <- includeEffects(eff, recip)</pre>
eff <- includeEffects(eff, recip, realrecip, persistrecip, type = "gmm")
eff <- includeEffects(eff, transTrip)</pre>
eff <- includeEffects(eff, transTrip, agreetrans, realtrans, type = "gmm")</pre>
eff
will print the resulting effects object
  For estimation by the Generalized Method of Moments
  Effects
  effectName
                                 include fix
                                                 test initialValue parm type
1 constant net rate (period 1) TRUE
                                          FALSE FALSE
                                                          4.69604
                                                                     0
                                                                           rate
2 constant net rate (period 2) TRUE
                                          FALSE FALSE
                                                          4.32885
                                                                     0
                                                                           rate
3 outdegree (density)
                                 TRUE
                                          FALSE FALSE
                                                         -1.46770
                                                                     0
                                                                           eval
4 reciprocity
                                 TRUE
                                          FALSE FALSE
                                                          0.00000
                                                                     0
                                                                           eval
5 transitive triplets
                                 TRUE
                                          FALSE FALSE
                                                          0.00000
                                                                     0
                                                                           eval
 Statistics
  effectName
                        include type
1 outdegree (density) TRUE
                                gmm
2 reciprocity
                        TRUE
                                gmm
3 persistent recip.
                        TRUE
                                gmm
4 real recip.
                        TRUE
                                gmm
5 transitive triplets TRUE
                                gmm
6 real trans. trip.
                        TRUE
                                gmm
7 agree trans. trip.
                       TRUE
                                gmm
```

with three evaluation effects and seven statistics. Like the MoM, the GMoM allows conditional as well as unconditional estimation (see Section 6.12.1). In this example, for conditional estimation the number of statistics will be nine, including the two statistics for the rate parameters, viz., Hamming distances between subsequent waves.

## 6.11 Maximum Likelihood and Bayesian estimation

An important difference between estimation by the Method of Moments (MoM) and estimation by the Method of Maximum Likelihood (ML) is that the MoM tries to find parameters that will produce simulations, given the beginning of the waves, for which on average the target statistics are equal to their observed values; whereas ML tries to find parameters for which the probability to arrive in the observed data for the end of the waves, given the beginning of the waves, is maximal. The MoM simulations take account

only of the observations at the start of the wave; the ML simulations augment the data by connecting the observations at the start of each wave with the observation at the end of the wave. Such a connection may be called a *change path*. This is more complicated, and therefore more time-consuming. Technical accounts of the MoM are in Snijders (2001); Snijders et al. (2007), of the ML estimator in Snijders et al. (2010a). Both are presented in Snijders (2017).

ML estimation is done by the function siena07, using a set of options created by siena-AlgorithmCreate with maxlike = TRUE. Bayesian estimation is done by the function signa-Bayes. The following further information in this section is about ML estimation; Bayesian estimation is as yet implemented only for multilevel network modeling (see Section 11.4).

There are some restrictions to the application of ML estimation.

- 1. The method of joiners and leavers (Section 4.3.3) is not available.
- 2. The following data configurations are not allowed:
  - (a) tie variables in two consecutive waves changing from structural zero (code 10) to 1;
  - (b) tie variables in two consecutive waves changing from structural one (code 11)
  - (c) tie variables in three consecutive waves changing from structural zero (code 10) to NA to 1;
  - (d) tie variables in three consecutive waves changing from structural one (code 11) to NA to 0;
  - (e) and, for more than three consecutive waves, similar patterns with more NAs in between.
- 3. Using structural zeros to indicate intermittent absence of actors (i.e., structural zeros only at the beginning or only at the end of a period) is probably not a good idea. Above, some situations were mentioned that are logically impossible and therefore not allowed. In addition, the following is problematic:
  - tie variables that are 0 or 1 at the beginning of a wave and structurally zero (code 10) at the end of the wave, indicating that the actor has left the network. The reason is that this contains the information for RSiena that during this period, the tie variable has to change to the value 0; whereas probably you wish to convey the meaning that its value is unknown. At the end of a period, for a tie variable with the value 10, the contribution of this tie variable is not included in the computation of the log-likelihood, but through its dependence with other variables it still has consequences different from NA.

A solution for the problem of structural zeros at the end of a period may be to replace them by NA.

For data sets with multiple periods (i.e., three or more waves), the analysis using ML still is possible (as proposed by de la Haye et al., 2017) by arranging the data as a multi-group data set (Section 11.2). Unfortunately, this solution is not available for sienaBayes.

The ML estimation algorithm uses Metropolis-Hastings steps to simulate the score function, which is the expected value of the augmented-data score function; see Snijders et al. (2010a). For ML estimation, an important parameter for tuning the algorithm is the so-called multiplication factor, given in sienaAlgorithmCreate as the argument mult. This determines the number of Metropolis-Hastings steps taken for simulating each new change path. The number of steps (sometimes called 'sampling frequency' in the literature) is the multiplication factor multiplied by the sum over dependent variables of the distances between successive waves. When this is too low, the sequentially simulated change paths are too similar, which will lead to high autocorrelation in the generated statistics. This leads to poor performance of the algorithm. These autocorrelations are given in the output file. When some autocorrelations are more than 0.4, it is good to increase the multiplication factor. When the multiplication factor is unnecessarily high, on the other hand, computing time will be unnecessarily high. The advice is to aim at values for the autocorrelations between 0.1 and 0.4.

The number of Metropolis-Hastings steps (sampling frequency) is potentially different for each wave, and for the answer object ans these numbers are given by ans\$nrunMH.

A practical way to proceed is as follows. For initial tuning of the multiplication factor, use the model that is obtained as the default after creating the effects object, with very few effects included. The reason for using this model is the limited computation time and easy convergence. If the highest autocorrelation is more than 0.3, increase the multiplication factor (e.g., by making it twice as large; which will also lead to a twice as long computation time) and estimate the model again. If the highest autocorrelation is less than 0.1, then decrease the multiplication factor and estimate again. Tune the multiplication factor until the highest autocorrelation is between 0.1 and 0.3. Then start with estimating the models of interest. For other models the autocorrelations may change again, therefore it still can be important later on to adapt the multiplication factor to keep the highest autocorrelation less than 0.4.

Note that the multiplication factor can be given as one number, but also as a vector, with the number of elements equal to the number of basic rate parameters in the effects object; i.e., the number of periods times the number of waves. This permits setting a high value of the multiplication factor for only some period-wave combinations.

Some advice about how to set the value of the multiplication factor as a vector is in https://www.stats.ox.ac.uk/~snijders/siena/sienaBayes\_s.pdf; this is formulated there for sienaBayes, but applies equally well for ML estimation using siena07.

The Metropolis Hastings algorithm uses a proposal distribution composed of steps; see Snijders et al. (2010a) and Siena\_algorithms.pdf. Abbreviated names of these steps are 'InsDiag', 'CancDiag', 'Permute', 'InsPerm', 'DelPerm', 'InsMiss', 'DelMiss', 'InsMisdat', 'DelMisdat', and 'Move'. The 'Move' step was added in version 1.3.18, and is described in Greenan (2015b). The probabilities of the first 7 steps are determined by the parameter

prML in sienaAlgorithmCreate. The probability of 'Move' is 1 minus the sum of these 7 probabilities; 'InsMisdat' and 'DelMisdat' are offspring of 'InsPerm" and 'DelPerm' and don't need their own proposal probability. 'InsMiss' and 'DelMiss' are superfluous when there are no missing data, but this is taken care of automatically and does not need to be specified in prML. Currently, 'Move' steps cannot be taken for data sets with more than one dependent variable.

Another parameter of the algorithm that sometimes needs tuning (but less often than the multiplication factor) is the Initial value of the gain parameter. This determines the step sizes in the parameter updates in the iterative algorithm. It influences the stability and speed of moving of the algorithm. A too low value implies that it takes very long to attain a reasonable parameter estimate when starting from an initial parameter value that is far from the 'true' parameter estimate. A too high value implies that the algorithm will be unstable, and may be thrown off course into a region of unreasonable (e.g., hopelessly large) parameter values.

When using the Method of Moments (the default estimation procedure), it usually is unnecessary to change this. In the ML case, when the autocorrelations are smaller than 0.1 but the t statistics for deviations from targets are relatively small (less than, say, 0.3) but do not all become less than 0.1 in absolute value in repeated runs of the estimation algorithm, then it will be good to decrease the initial value of the gain parameter. Do this by dividing it by, e.g., a factor 2 or a factor 5, and then try again a few estimation runs.

### 6.12 Other remarks about the estimation algorithm

#### 6.12.1 Conditional and unconditional estimation

RSiena has two methods for MoM estimation and simulation: conditional and unconditional. They differ in the *stopping rule* for the simulations of the network evolution. In unconditional estimation, the simulations of the network evolution in each time period (and the co-evolution of the behavioral dimensions, if any are included) carry on until the predetermined time length (chosen as 1.0 for each time period between consecutive observation moments) has elapsed. This is explained in Snijders (2001, Section 4.2).

In conditional estimation, in each period the simulations run on until a stopping criterion is reached that is calculated from the observed data. Conditioning is possible for each of the dependent variables (network, or behavior), where 'conditional' means 'conditional on the observed number of changes on this dependent variable'. However, if there is more than one dependent variable (network and/or behavior), estimation can be conditional only on one of them.

Conditioning on the network variable means, for each period, running simulations until the number of entries that differ between the adjacency matrix of the initially observed network of this period and the simulated network is equal to the number of entries in the adjacency matrix that differ between the initially and the finally observed networks of this period.

Conditioning on a behavioral variable means, for each period, running simulations

until the sum of absolute score differences on the behavioral variable between the initially observed behavior of this period and the simulated behavior is equal to the sum of absolute score differences between the initially and the finally observed behavior of this period.

Conditional estimation is slightly more stable and efficient, because the corresponding rate parameters are not estimated by the Robbins Monro algorithm, so this method decreases the number of parameters estimated by this algorithm.

The choice between unconditional and the different types of conditional estimation is made in the sienaAlgorithmCreate function by setting the cond parameter. For data specifications with multiple dependent variables, at most one dependent variable can be used for conditioning. This choice is made dependent on the condvarno and condname parameters in this function.

If there are changes in network composition (see Section 4.3.3), only the unconditional estimation procedure is available.

If there are a lot of structurally determined values (see Section 4.3.1) then unconditional estimation is preferable.

#### 6.12.2 Fixing parameters

Sometimes an effect must be present in the model, but its precise numerical value is not well-determined. E.g., if the network at time  $t_2$  would contain only reciprocated choices, then the model should contain a large positive reciprocity effect but whether it has the value 3 or 5 or 10 does not make a difference. This will be reflected in the estimation process by a large estimated value and a large standard error, a derivative which is close to 0, and sometimes also by lack of convergence of the algorithm. (This type of problem also occurs in maximum likelihood estimation for logistic regression and certain other generalized linear models; see Geyer and Thompson (1992, section 1.6), Albert and Anderson (1984); Hauck and Donner (1977).) In such cases this effect should be fixed to some large value and not left free to be estimated. This can be specified by using the setEffect function with the fix = TRUE option.

#### 6.12.3 Automatic fixing of parameters

If the algorithm encounters computational problems, sometimes it tries to solve them automatically by fixing one (or more) of the parameters. This will be noticeable because a parameter is reported in the output as being fixed without your having requested this. This automatic fixing procedure is used, when in Phase 1 one of the generated statistics seems to be insensitive to changes in the corresponding parameter.

This is a sign that there is little information in the data about the precise value of this parameter, when considering the neighborhood of the initial parameter values. However, it is possible that the problem is not in the parameter that is being fixed, but is caused by an incorrect starting value of this parameter or one of the other parameters.

When the warning is given that the program automatically fixed one of the parameter, try to find out what is wrong.

In the first place, check that your data were entered correctly and the coding was given correctly, and then re-specify the model or restart the estimation with other (e.g., 0) parameter values. Sometimes starting from different parameter values (e.g., the default values implied by the algorithm option of 'standard initial values") will lead to a good result. Sometimes, however, it works better to delete this effect altogether from the model.

It is also possible that the parameter does need to be included in the model but its precise value is not well-determined. Then it is best to give the parameter a large (or strongly negative) value and indeed require it to be fixed (see Section 10.1).

### 6.12.4 Required changes from conditional to unconditional estimation

Even though conditional estimation is slightly more efficient than unconditional estimation, there is one kind of problem that sometimes occurs with conditional estimation and which is not encountered by unconditional estimation.

It is possible (but luckily rare) that the current parameter values are such that the conditional simulation does not succeed in ever attaining the condition required by its stopping rule (see Section 6.12.1). This will depend on the initial parameter values and the changes that occurred during the Robbins-Monro process. The solution is either to use different (perhaps standard) initial values or to go over to unconditional estimation.

# 6.13 Using multiple processes

1. If multiple processors are available, then using multiple processes can speed up the estimation in siena07, sienacpp, and sienaGOF. The help pages for these functions show how to do this. You can find out the number of processes possible by

```
library(parallel)
detectCores()
```

It is not advisable to utilize all available cores, because other processes also have to run. If you are not the only user of the machine, then there are obvious issues to be dealt with.

There are some examples in the help page for siena07.

2. For estimation by Method of Moments (MoM), in Phases 1 and 3 the simulations are performed in parallel. In Phase 2, multiple (viz., nbrNodes which is a parameter given in the call of siena07) simulations are done with the same parameters, and the resulting statistics are averaged for the updating step in the Robbins Monro algorithm. The gain parameter is increased and the number of iterations in Phase 2 reduced to take advantage of the increased accuracy of the update.

When using the MoM, decrease in computation time will be somewhat less than proportional to the number of processes used, if this number is less than (say) 10; larger number of processes will have diminishing proportional returns.

- 3. For estimation by Maximum Likelihood (ML) and by sienaBayes, parallelization goes by period; for multi-group data sets, period  $\times$  group. Therefore, for this type of estimation, the maximum meaningful number of parallel processes is (number of waves -1)  $\times$  (number of groups).
- 4. The parameters required to run all processes on one computer are fairly simple: in your call to siena07, set nbrNodes to the number of processes and useCluster and initC to TRUE. The Model Options screen also allows you to specify the number of processes, and will automatically set the other required parameters for you.
- 5. To use more than one machine is more complicated, but it can be done by using, in addition, the clusterString parameter. The computers need to be running incoming ssh.
- 6. For machines with exactly the same layout of R directories on each, simply set clusterString to a character vector of the names of the machines.
- 7. For other cases, e.g. using Macs alongside Linux, see the documentation for the package parallel.
- 8. RSiena uses sockets for inter-process communication.
- 9. On Windows, sub processes are always started using R scripts. On Linux and Mac there is an option available in R version 2.14.0 or later via the code interface to siena07 to ask for the sub processes to be formed by forking. See the help page for details.
- 10. Each process needs a copy of the data in memory. If there is insufficient memory available there will be no speed gain as too much time will be spent paging.
- 11. In each iteration the main process waits until all the other processes have finished. The overall speed is therefore that of the slowest process, and there should be enough processors to allow them all to run at speed.

## 7 Standard errors

The estimation of standard errors of the MoM estimates requires the estimation of derivatives, which indicate how sensitive the expected values of the statistics (see Section 6.7) are with respect to the parameters. The derivatives can be estimated by two methods:

- finite differences method with common random numbers,
- score function method.

The finite difference method is explained (briefly) in Snijders (2001), the score function method was developed in Schweinberger and Snijders (2007b) (where also the finite difference method is explained). The score function method is preferable, because it is unbiased and demands less computation time than finite differences, although it requires more iterations in Phase 3 of the estimation algorithm (see Section 6.7). It is recommended to use the score function method with at least 1000 iterations (default) in Phase 3. For published results, it is recommended to have at least 12 5000 iterations in Phase 3.

In some cases, with high correlations between parameters, the estimation of standard errors is unstable and even 5000 may be not enough. This can be checked this by running siena07 several times, with different random number seeds (or the default seed = NULL which also gives different seeds every run).

```
A more extensive way of checking this is presented in the scripts 
http://www.stats.ox.ac.uk/~snijders/siena/SE_checks.R 
and 
http://www.stats.ox.ac.uk/~snijders/siena/RscriptStandardErrors.R
```

To calculate the standard errors for a given set of estimates, when these estimates are good and do not need to be changed, algorithm settings can be used with nsub = 0 in sienaAlgorithmCreate, so phases 1 and 2 are skipped and the whole operation of sienaO7 takes considerably less time.

The method for estimating derivatives is set by the findiff parameter and the number of iterations in Phase 3 by the n3 parameter, both in function sienaAlgorithmCreate that creates the object with specifications for the algorithm.

### 7.1 Multicollinearity

Multicollinearity means that the matrix that is inverted to give the correlation matrix is ill-conditioned. Correlations between parameter estimates close to  $\pm 1$  are the most usual signs of this.

If the parameter estimates are perfectly collinear (standard errors of some parameters, or linear combinations of parameters, being infinitely large), standard errors are reported <sup>13</sup> as NA (the R term for "not available", missing). This can happen depending on the data-model combination (e.g., including the covariate-ego effect for a covariate with variance

<sup>&</sup>lt;sup>12</sup>Earlier recommendations in this manual were lower, '2000 to 4000'. This is adequate in many cases; the increase to 5000 has the purpose to be on the safe side.

 $<sup>^{13}</sup>$ Since version 1.1-285.

0; or including some effects that are collinear for any data set, such as the combination of outdegree, transitive triplets, outdegree activity, and balance effects — see Snijders, 2005), or on the combination of data, model and parameters (when a parameter value was given or was reached where the model is not sensitive to some parameter or combination of parameters). The remedy here usually is to drop some of the effects.

When siena07 finds near-perfect collinearity at the end of the estimation algorithm, it will give a warning message of the following kind.

```
*** Standard errors not reliable ***
The following is approximately a linear combination for which the data carries no information:
0.5 * beta[8] + -0.41 * beta[9] + 1 * beta[14]
It is advisable to drop one or more of these effects.
```

This is based on automatic detection of linear dependencies between the terms in the objective function (or linear predictor) (6). The coefficients are rounded. This warning message signals that, very probably, the combination of the effects mentioned causes the multicollinearity. This multicollinearity is signaled for the parameter values as obtained at the end of the algorithm.

In cases with strong but not complete multicollinearity, i.e., correlations between some parameter estimates (or some of their linear combinations) being close but not equal to -1 or +1, the estimated standard errors are less reliable. Estimates for these correlations are given under the heading Covariance matrix of estimates (correlations below diagonal) in the output file, and in the summary(...) of the estimation results (see Section 6.8.3). Strong collinearity may in practice lead to large differences between the estimated standard errors, and also to considerable differences between the parameter estimates, when comparing the results produced by different estimation runs. The remedy is to reduce the model to a more parsimonious one by excluding non-significant effects of which the parameter estimates are highly correlated with others.

Provisionally (until further experience has been collected), the following may be a reasonable guideline. High parameter correlations with the outdegree effect are not a reason for worry, but high parameter correlations with other effects are a reason for checking the stability of the estimated standard errors. The threshold for finding a parameter correlation 'too high' in this respect can be quite high, such as 0.95 (or 0.90). In cases of high parameter correlations, estimating the model twice (or more) and considering the stability of the standard errors will be a good way for seeing whether there are reasons for special caution. If the standard errors are stable, then parameter correlations above 0.90 still can be acceptable – in particular, when they are obtained for parameters that are significantly different from 0 and of which estimates as well as standard errors are stable across repeated runs of the estimation algorithm. Also see the preceding section about the stability of standard errors.

# 7.2 Precision of the finite differences method

The implementation of the finite differences method is not scale-invariant  $^{14}$ , with the result that the standard errors produced by this method are not very reliable if they are of the order of 0.02 or less.

<sup>&</sup>lt;sup>14</sup>The scales are determined by the variable z\$scale in function robmon. A better procedure would be to set the scale adaptively, but the finite differences method is hardly ever used any more, having been superseded by the score function method, and therefore this improvement has not been effectuated.

## 8 Tests

Estimated parameters in RSiena can be of three kinds: basic rate parameters, indicated by the letter  $\rho$  (rho), and specific for a dependent variable – period combination; other parameters for the rate function (which may be absent from the model), indicated by the letter  $\alpha$  (alpha); and parameters that are weights for the objective function, indicated by  $\beta$  (beta). The basic rate parameters are necessarily positive, so testing the null hypothesis that they are equal to 0 makes no sense. For a combined notation, we denote the combination of  $\alpha$  and  $\beta$  parameters by the letter  $\theta$  (theta); for non-conditional estimation,  $\theta$  will also include the basic rate parameters  $\rho$ .

For these parameters, two types of test are available in SIENA.

- 1. t-type tests of single parameters can be carried out by dividing the parameter estimate by its standard error. Under the null hypothesis that the parameter is 0, these tests have approximately a standard normal distribution. These may also be called Wald-type tests. Section 8.1 indicates how to construct multi-parameter tests from the same principle.
- 2. Score-type tests of single and multiple parameters are described in Section 8.2.

In addition, there are procedures for assessing goodness of fit as explained in Section 5.15.

## 8.1 Wald-type tests

Wald-type tests are based on the parameter estimates and their covariance matrix. Recall that the variances of the parameter estimates are on the diagonal of this covariance matrix, and the standard errors are the square roots of these diagonal elements.

In Section 6.6 we saw that, for a sienaFit object ans, the estimates are given in ans\$theta, the covariance matrix in ans\$covtheta, and the standard errors in ans\$se. For testing the null hypothesis that component k of the parameter vector is 0,

$$H_0: \theta_k = 0,$$

the t-test is based on

$$\frac{\hat{\theta}_k}{\text{s.e.}(\hat{\theta}_k)} = \text{ans\$theta[k] / ans\$se[k]} . \tag{9}$$

This can be easily calculated by hand from the RSiena results. It can also be obtained, e.g., from

Multipar.RSiena(ans, k)

where k is the sequence number of the effect. The function Multipar.RSiena can be used for testing multiple parameters simultaneously but, like in this example, can be used also for a single parameter.

In some cases, however, the t-statistic (9) does not have an approximate standard normal distribution under the null hypothesis, so that this test is not appropriate. This is the so-called Donner-Hauck phenomenon, named after Hauck and Donner (1977), who first drew attention to this phenomenon in the case of logistic regression. It is discussed also in Geyer and Thompson (1992, section 1.6) and Albert and Anderson (1984). For logistic regression this phenomenon occurs when there is complete, or almost complete, separation of the set of observed values for the vector of predictor variables such that this set consists of values in two half-spaces, where for one of these half-spaces the dependent variable always is 0, and for the other half-space always 1. The data then indicates that the parameter should be very large in absolute value, but not how large; mathematically, the estimated value may be infinite. The parameter as estimated by a practical algorithm then will be not infinite but large, and the standard error also will be large; but the point is that ratio (9) does not need to be large, and the Wald test may be non-significant while the data flatly contradicts a hypothetical parameter value of 0. In Section 6.12.2 we proposed that in such cases, it may be helpful to fix the parameter at some large value, without estimating it. Or, when it is being estimated and the overall maximum convergence ratio is small so convergence is judged as good, this estimate can be used, but in these cases, not its standard error. One possibility that then is available to test the significance is to make a second estimation run in which the parameter is fixed at the value 0, corresponding to the null hypothesis, and test this null value using the score-type test of Section 8.2. See the script RscriptSienaMultiple.R on the SIENA webpage for an example.

## 8.1.1 Multi-parameter Wald tests

Now suppose that we wish to test another null hypothesis, which can be represented as a linear constraint on  $\theta$ :

$$H_0: A\theta = 0,$$

where A is a  $r \times p$  matrix, the dimension of  $\theta$  being p. For example, if p = 5, for testing

$$H_0: \theta_2 = \theta_3$$

we would use the matrix

$$A = (0, 1, -1, 0, 0)$$
;

while for testing

$$H_0: \theta_2 = \theta_3 = 0$$

if p = 5, we would use the matrix

$$A = \left( \begin{array}{c} 0, 1, 0, 0, 0 \\ 0, 0, 1, 0, 0 \end{array} \right) \ .$$

Then the function Wald.RSiena can be used to produce the Wald-type test: the chi-squared value of the test statistic, the number of degrees of freedom, and the *p*-value.

As an example, for the klas12b data the following results were obtained, collected in the sienaFit object ans.

		Estimate		Standard Error		Convergence t-ratio
Rate p	parameters:					
0.1	Rate parameter period 1	8.7995	(	1.7061	)	
0.2	Rate parameter period 2	7.6486	(	1.2754	)	
0.3	Rate parameter period 3	8.2209	(	1.3090	)	
Other	parameters:					
1.	eval outdegree (density)	-1.9823	(	0.1374	)	-0.0200
2.	eval reciprocity	1.6659	(	0.2548	)	0.0153
3.	eval transitive triplets	0.3948	(	0.0504	)	-0.0225
4.	eval transitive recipr. triplets	-0.4540	(	0.0962	)	-0.0276
5.	eval primary	0.6062	(	0.1372	)	-0.0646
6.	eval sex alter	0.0732	(	0.1331	)	0.0069
7.	eval sex ego	0.3323	(	0.1347	)	-0.0526
8.	eval sex similarity	0.8828	(	0.1348	)	0.0525

Overall maximum convergence ratio: 0.1366

The output of summary(ans) also contains the covariance matrix of the estimates:

```
Covariance matrix of estimates (correlations below diagonal)
0.019 -0.023 -0.006  0.008 -0.002  0.001 -0.001 -0.004
-0.666  0.065  0.009 -0.020 -0.001 -0.001  0.002 -0.003
-0.817  0.664  0.003 -0.004  0.000  0.001  0.000  0.000
0.620 -0.795 -0.861  0.009 -0.001 -0.001  0.000  0.000
-0.083 -0.023  0.063 -0.104  0.019  0.000  0.000  0.002
0.045 -0.019  0.107 -0.097  0.020  0.018 -0.009 -0.001
-0.035  0.059 -0.073  0.012 -0.021 -0.481  0.018  0.005
-0.220 -0.099  0.054 -0.012  0.101 -0.049  0.255  0.018
```

You probably do not need this as a user, but this covariance matrix is used by RSiena to carry out the tests. To test the null hypothesis, e.g., that the three sex effects (ego, alter, similarity) are zero, the matrix A is constructed as follows, and the Wald test then is requested.

```
A <- matrix(0, 3, 8)
A[1, 6] <- 1
A[2, 7] <- 1
A[3, 8] <- 1
Wald.RSiena(A, ans)
The result is
chi-squared = 45.66, d.f. = 3; p < 0.001.
```

The value p < 0.001 expresses strong evidence that the network dynamics depends on sex.

The Wald test is frequently applied to test the null hypothesis that several parameters are 0. The extra work to define the matrix A above can be automated by using the following function Multipar.RSiena. The test of the preceding example is then produced by the command

```
Multipar.RSiena(ans, 6, 7, 8)
```

The output gives slightly more information:

#### Tested effects:

Multipar.RSiena(ans, 5)

```
friends: sex alter
friends: sex ego
friends: sex similarity
chi-squared = 45.66, d.f. = 3; p < 0.001.</pre>
```

If only one parameter is tested by this function, also the one-sided statistic is given. For example, the effect of knowing each other from primary school can be tested by

```
with the result
Tested effects:
  friends: primary
chi-squared = 19.52, d.f. = 1; one-sided Z = 4.42; p < 0.001.</pre>
```

The Z-value is just the square root of the chi-squared, but with the sign of the parameter attached.

#### 8.1.2 Tests of linear combinations

Sometimes there can be interest in a linear combination of parameters. For example, this can arise when an evaluation and an endowment function are included in the model. This situation was explained in Section 2.1.2 and further elaborated in Section 12.1.4. Equations (31) and (32) imply that, if there is no creation function, the split between creation and maintenance can be made as follows:

- creation of ties: evaluation parameter;
- maintenance of ties: evaluation parameter + endowment parameter.

In an example using the Glasgow data (160 students, with joiners and leavers), we obtained the following results (the LaTeX table was made by the function siena.table).

Effect	par.	(s.e.)
constant friendship rate (period 1)	13.801	(1.459)
constant friendship rate (period 2)	10.938	(1.192)
outdegree	-3.507	(0.223)
reciprocity: evaluation	4.392	(0.419)
reciprocity: endowment	-1.640	(0.703)
gwespFF (transitivity)	1.810	(0.077)
indegree-popularity	-0.053	(0.019)
outdegree-activity	0.144	(0.036)
reciprocated degree-activity	-0.346	(0.050)
sex alter	-0.137	(0.084)
sex ego	0.038	(0.095)
same sex	0.647	(0.076)

convergence t ratios all < 0.05.

Overall maximum convergence ratio 0.09.

The effect of reciprocation on creation of ties is 4.392, with s.e. = 0.419. This can be tested by the ratio t = 4.392/0.419 = 10.48, extremely strongly significant. Suppose the sienaFit object is called ans. The Wald test can also be obtained by the following. Requesting the dimensionality ans\$pp is helpful to confirm the numbering of the parameters. In this case, the dimension is 12, and the tested parameter, reciprocity evaluation, is the fourth. In terms of the formulae earlier in this section, the tested parameter is  $\theta_4$ .

```
\begin{array}{l} (p <- \, ans\$pp) \\ \text{mat} <- \, matrix(0,1,p) \\ \text{mat}[1,4] <- \, 1 \\ \text{Wald.RSiena(mat, ans)} \\ \text{with the result that } p=12, \, \text{and} \\ \\ \text{chi-squared = 110.03, d.f. = 1; one-sided Z = 10.49; } p < 0.001. \end{array}
```

The difference between the values 10.48 and 10.49 is due to rounding.

The estimated effect of reciprocation on maintenance of ties is  $\hat{\theta}_4 + \hat{\theta}_5 = 4.392 - 1.640 = 2.752$ . Here the Wald test is obtained by

```
(p <- ans$pp)
mat <- matrix(0,1,p)
mat[1,4] <- 1
mat[1,5] <- 1
Wald.RSiena(mat, ans)
with the result
chi-squared = 36.09, d.f. = 1; one-sided Z = 6.01; p < 0.001.</pre>
```

We can also get the standard error of the linear combination  $\hat{\theta}_4 + \hat{\theta}_5$  in the following way. If a is a vector with coefficients of the linear combination  $a'\theta$ , in which we are interested, then

```
sum(a*ans$theta)
is the corresponding estimate,
a %*% ans$covtheta %*% t(a)
is the variance, and
sqrt(a %*% ans$covtheta %*% t(a))
```

is the standard error. In this case, a is the matrix mat. Therefore the standard error is obtained as

```
sqrt(mat %*% ans$covtheta %*% t(mat)) with the result 0.458. Indeed 2.752/0.458 = 6.01.
```

In this example, a further question is whether the effect of reciprocation on creation of ties is equal to its effect on maintenance of ties. This is simply tested by the parameter  $\theta_5$ , which —as is implied by the above— is just the difference between the effects of reciprocation on maintenance and on creation of ties. In this case, the difference is -1.640, with t-ratio t = -1.640/0.703 = -2.33, p < 0.05. We may conclude that, in this model, the effect of reciprocation on creation of ties is significantly (two-sided p < 0.05) larger than on maintenance of ties; but the latter, on itself, still is significant.

## 8.2 Score-type tests

The Wald test is based on the estimate for the parameter, and thereby integrates estimation and testing. Sometimes, however, it can be helpful to separate these two types of statistical evaluation. This is the case notably when estimation is instable, e.g., when a model is considered with rather many parameters given the information available in the data set, or when the precise value of the estimate is not determined very well as happens under the Donner-Hauck phenomenon treated in the previous section. The score-type test gives the possibility of testing a parameter without estimating it.

This is done using the generalized Neyman-Rao score test that is implemented for the Method of Moments estimation method in SIENA as a part of function siena07, following the methods of Schweinberger (2012). For the ML estimation method, following the same steps produces the Rao (1947) efficient score test. Since the name of 'score test' is associated with likelihood-based analysis as in Rao (1947), the test of Schweinberger (2012) that is associated with the Method of Moments is called 'score-type test'.

When using the score-type test, some model is specified in which one or more parameters are restricted to some constant, in most cases 0 – these constant values define the null hypothesis being tested. This can be obtained in RSiena by appropriate choices in the effects dataframe. Parameters can be restricted by putting TRUE in the fix and test

columns, and the tested value in the initialValue column. The function setEffect is available to do this. For example, a score test for the evaluation effect of transitive ties in a network can be requested as follows.

```
myeff <- setEffect(myeff, transTies, fix = TRUE, test = TRUE, initialValue = 0.0)</pre>
```

The score-type test then proceeds by simply estimating the restricted model (not the unrestricted model, with unrestricted parameters) by the standard SIENA estimation algorithm. The result of the score-type test is presented in the summary of the estimation results (the sienaFit object obtained from siena07) and also in the output file. The t-ratios for convergence can be disregarded for the parameters that are fixed and estimated by the score test, as convergence is not an issue for these parameters.

Score tests can also be obtained by the function score. Test. This will provide score tests for any given subset of the parameters that were fixed and tested in the sienaFit object given to it. Note that while the summary of this object and the results in the output file give the score tests for the set of <u>all</u> fixed & tested parameters, and for <u>each</u> of these parameters separately, the function score. Test can give the combined test for any subvector.

It should be noted that using the prevAns option in siena07 overrides the initial values in the effects object, so that using siena07 for an effects object with fix = TRUE for some of the effects, jointly with the prevAns option, will lead to score-type tests of hypothesized values as given by the prevAns option. Since the presentation of the results includes the hypothesized value, there is no reason for doubt as to what has been done. However, mostly this is not what is desired, and therefore it usually will be preferable to proceed as follows. First update the initial values using updateTheta, then set the hypothesized value by setEffect, and then carry out the estimation and score-type test by siena07. The following is an example, assuming that an earlier reasonable estimate was found in sienaFit object myans0, and the user wishes to use this as starting values.

```
myeff <- updateTheta(myeff, myans0)
myeff <- setEffect(myeff, transTies, fix = TRUE, test = TRUE, initialValue = 0)
myans1 <- siena07(estimationSettings, data = mydata, effects = myeff)
summary(myans1)</pre>
```

The one-step estimates are given in the file pname.txt, mentioned in Section 6.8. These are 'quick and easy' estimates that are not totally reliable, but that can be used as approximations of the unrestricted estimates (that is, the estimates that would be obtained if the model were estimated once again, but without restricting the parameter by fix = TRUE). They are explained mathematically in Schweinberger (2012). Their computational advantage is based on the fact that they require simulations only for parameter vectors where this parameter is fixed (usually, to 0; but other values could be used, and this would be preferable if they are closer to the resulting estimate).

# 8.3 Example: one-sided tests, two-sided tests, and one-step estimates

Suppose that it is desired to test the goodness-of-fit of the model restricted by the null hypothesis that the reciprocity parameter is zero. The following output may be obtained in the file pname.txt, mentioned in Section 6.8:

```
@2
Generalised score test <c>
```

Testing the goodness-of-fit of the model restricted by

```
(1) eval: reciprocity = 0.0000

c = 3.9982 d.f. = 1 p-value = 0.0455
  one-sided (normal variate): 1.9996

-------

One-step estimates:

l: constant network rate (period 1) 6.3840
 l: constant network rate (period 2) 6.4112
 eval: outdegree (density) 0.9404
 eval: reciprocity 1.2567
```

To understand what test statistic <c> is about, consider the case where the network is observed at two time points, and let R be the number of reciprocated ties at the second time point. Then it can be shown that the test statistic is some function of

Expected R under the restricted model - observed R.

Thus, the test statistic has some appealing interpretation in terms of goodness-of-fit: when reciprocated ties do have added value for the firms—which means that the reciprocity parameter is not 0, other than the model assumes—then the deviation of the observed R from the R that is expected under the model will be large (large misfit), and so will be the value of the test statistic. Large values of the test statistic imply low p-values, which, in turn, suggests to abandon the model in favor of models incorporating reciprocity.

The null distribution of the test statistic c tends, as the number of observations increases, to the chi-square distribution, with degrees of freedom equal to the number of restricted parameters. The corresponding p-value is given in the output file.

In the present case, one parameter is restricted (reciprocity), hence there is one degree of freedom d.f. = 1. The value of the test statistic c = 3.9982 at one degree of freedom gives p = 0.0455. That is, it seems that reciprocity should be included into the model and estimated as the other parameters.

The one-sided test statistic, which can be regarded as normal variate, equals 1.9996 indicating that the value of the transitivity parameter is positive.

In this example, the one-step estimate of reciprocity is 1.2567, suggesting that this parameter is positive, which agrees with the one-sided test.

## 8.3.1 Multi-parameter tests

In the case where K > 1 model parameters are restricted, RSiena reports for the score test, in the summary and in the output file, the test statistic with K degrees of freedom. A low

p-value of the joint test would indicate that the goodness-of-fit of the model is intolerable. However, the joint test with K degrees of freedom gives no clue as to what parameters should be included into the model: the poor goodness-of-fit could be due to only one of the K restricted parameters, it could be due to two of the K restricted parameters, or due to all of them. Hence RSiena carries out, in addition to the joint test with K degrees of freedom, additional tests with one degree of freedom that test the single parameters one-by-one. The goodness-of-fit table is as follows:

```
Generalised score test <c>
Testing the goodness-of-fit of the model restricted by
                                                    = 0.0000
 (1)
      eval: covariate_ij (centered)
 (2)
                                                   = 0.0000
      eval: covariate_i alter
                                                   = 0.0000
 (3) eval: covariate_i similarity
-----
Joint test:
  c = 92.5111 d.f. = 3 p-value < 0.0001
(1) tested separately:
______
 - two-sided:
  c = 62.5964 d.f. = 1 p-value < 0.0001
 - one-sided (normal variate): 7.9118
(2) tested separately:
 - two-sided:
  c = 16.3001 d.f. = 1 p-value < 0.0001
 - one-sided (normal variate): 4.0373
(3) tested separately:
______
 - two-sided:
  c = 23.4879 d.f. = 1 p-value < 0.0001
 - one-sided (normal variate): 4.8464
One-step estimates:
1: constant network rate (period 1)
                                              7.4022
1: constant network rate (period 2)
                                              6.4681
eval: outdegree (density)
                                             -0.4439
eval: reciprocity
                                              1.1826
eval: transitive triplets
                                              0.1183
eval: covariate_ij (centered)
                                              0.4529
eval: covariate_i alter
                                              0.1632
eval: covariate_i similarity
                                              0.4147
```

In the example output, three parameters are restricted. The joint test has test statistic c, which has under the null hypothesis a chi-squared distribution with d.f. = 3. The p-value corresponding to the joint test indicates that the restricted model is not tenable. Looking at the separate tests, it seems that the misfit is due to all three parameters. Thus, it is sensible to improve the goodness-of-fit of the baseline model by including all of these parameters, and estimate them.

If a simultaneous score test is desired for a subset of the tested parameters with more than one element, the function score. Test can be used.

## 8.4 Alternative application: convergence problems

An alternative use of the score test statistic is as follows. When convergence of the estimation algorithm is doubtful, it is sensible to restrict the model to be estimated. Either 'problematic' or 'non-problematic' parameters can be kept constant at preliminary estimates (estimated parameters values). Though such strategies may be doubtful in at least some cases, it may be, in other cases, the only viable option besides simply abandoning 'problematic' models. The test statistic can be exploited as a guide in the process of restricting and estimating models, as small values of the test statistic indicate that the imposed restriction on the parameters is not problematic.

## 8.5 Testing differences between independent groups or periods

Sometimes it is interesting to test differences between parameters estimated for independent groups. For example, for work-related support networks analyzed in two different firms, one might wish to test whether the tendency to reciprocation of work-related support, as reflected by the reciprocity parameter, is equally strong in both firms. Such a comparison is meaningful especially if the total model is the same in both groups, as control for different other effects would compromise the basis of comparison of the parameters.

If the parameter estimates in the two networks are  $\hat{\beta}_a$  and  $\hat{\beta}_b$ , with standard errors  $s.e._a$  and  $s.e._b$ , respectively, then the difference can be tested with the test statistic

$$\frac{\hat{\beta}_a - \hat{\beta}_b}{\sqrt{(s.e_a)^2 + (s.e_b)^2}} \,\,\,(10)$$

which under the null hypothesis of equal parameters,  $\beta_a = \beta_b$ , has an approximating standard normal distribution.

The same method can be used for comparing estimates obtained from separate analyses for different periods from a data set with three or more waves. For example, if there are three waves and separate analyses were done for period 1 (wave 1 – wave 2) and period 2 (wave 2 – wave 3), then  $\hat{\beta}_a$  could be an estimate obtained for period 1 while  $\hat{\beta}_b$  would be the estimate for the same parameter for period 2. This is allowed because the analysis of period 2 conditions on wave 2.

## 8.6 Testing time heterogeneity in parameters

The model assumes that the parameter values  $\beta_k$  of the objective function are constant over the periods from 1 to M-1. This can be tested by the sienaTimeTest function, as explained in this section.

We initially assume that  $\beta$  does not vary over time, yielding a restricted model. Our data contains  $|\mathcal{M}|$  observations, and we estimate the restricted model the method of moments. We wish to test whether the restricted model is misspecified with respect to time heterogeneity. Formally, define a vector of time dummy terms  $\mathbf{h}$ :

$$h_k^{(m)} = \begin{cases} 1 & \{m : w_m \in \mathcal{W}, m \neq 1\} \\ 0 & \text{elsewhere} \end{cases} , \tag{11}$$

where k corresponds to an effect included in the model.<sup>15</sup> The explanation here is formulated for the network evaluation function, but the principle can be applied more generally. An *unrestricted model* which allows for time heterogeneity in all of the effects is considered as a modification of (15):

$$f_{ij}^{(m)}(\mathbf{x}) = \sum_{k} \left( \beta_k + \delta_k^{(m)} h_k^{(m)} \right) s_{ik} \left( \mathbf{x}(i \leadsto j) \right)$$
(12)

where  $\delta_k^{(m)}$  are parameters for interactions of the effects with time dummies. One way to formulate the testing problem of assessing time heterogeneity is the following:

$$H_0: \delta_k^{(m)} = 0 \text{ for all } k, m$$
  
 $H_1: \delta_k^{(m)} \neq 0 \text{ for some } k, m.$  (13)

This testing problem can be addressed by the score test in a way that no extra estimation is necessary. This method was elaborated and proposed by Lospinoso et al. (2011) and is implemented in RSiena. To apply the test to your dataset, run an estimation in the usual way, e.g. as follows (we specify nsub = 2, n3 = 100 just to have an example that runs very quickly):

and conduct the timetest through

## Conduct the score type test to assess whether heterogeneity is present.
tt2 <- sienaTimeTest(ans2)
plot(tt2, effects = 1:2)</pre>

<sup>&</sup>lt;sup>15</sup>The dummy  $\delta_k^{(1)}$  is always zero so that period  $w_1$  is (arbitrarily) considered the reference period.

When time heterogeneity is found, there are several options for how to continue. One option is to break apart the set of periods, and do the analysis separately for smaller subsets of waves. The extreme of this is to do a wave-by-wave analysis, i.e., an analysis for each pair of consecutive waves. Then there is no issue of heterogeneity, and the results will show in some cases that the heterogeneity affects only one or a few parameters. Depending on the non-constant pattern that is shown by some of the parameters, a new time-varying actor variable, or several, can be defined to capture this heterogeneity, and the analysis may be done for the set of all periods, for a model including interactions of this time varying covariate with the effects for which heterogeneity was found. This is the second option. The time-varying actor variable could be a linear function of time, a time dummy, or whatever. The summary of sienaTimeTest gives information that can be used to see which waves, and which effects, are mainly responsible for the time heterogeneity. A wave-by-wave analysis may give this information more clearly, but is more work.

One possibility for time-varying covariates is a time dummy variable. This is available in an automated way in the function includeTimeDummy. An example is

```
myeff <- includeTimeDummy(myeff, recip, balance, timeDummy = "2")
ans3 <- siena07(myalgorithm, data = mydata, effects = myeff, batch = TRUE)
after which you can test the new model again,
tt3 <- sienaTimeTest(ans3)
and so on.</pre>
```

# 9 Simulation

The simulation option simulates the network evolution for fixed parameter values. This is meaningful, e.g., for theoretical exploration of the model, for goodness of fit assessment, and for studying the sensitivity of the model to parameters. Simulations are produced by the siena07 function also used for parameter estimation, but by calling it in such a way that only Phase 3 is carried out (see section 6.7). This is done by requesting nsub = 0 in the model specification in function sienaAlgorithmCreate. By also requesting simOnly = TRUE the calculation of standard errors, which usually is not meaningful when simulating without estimating, is suppressed.

Mostly it is more meaningful to do this for non-conditional simulation (hence cond = FALSE), and a two-wave data set, so that the simulations are totally determined by the parameters and the first observation. The second wave then must be present in the data set only because RSiena requires it for estimation, and here we are using a function that is originally meant for estimation. Parameter values are obtained from the effects object, because of the option useStdInits = FALSE. If the name of the effects object is myeff, the current parameter values are obtained from requesting

### myeff

and different values can be specified by assigning the desired value to the vector

#### myeff\$initialValue[myeff\$include]

When artificial data sets are generated that have a close link to observed data, the restriction that simulations follow the monotonicity patterns that might be present in the data (see Section 4.2.4) can be undesirable. This restriction can be lifted by using allowOnly = FALSE in the call of sienaDependent (see the help file for this function). This parameter will then set any uponly and downonly flags to FALSE, precluding monotonicity constraints.

The statistics generated, which are the statistics corresponding to the effects in the model, can be accessed from the sienaFit object produced by siena07. Denoting the name of this object by sim\_ans, its component sim\_ans\$sf contains the generated deviations from targets. As discussed also in Section 6.6, the statistics can be recovered from the deviations and the targets as follows.

```
# To get the generated statistics without subtracting the targets,
# we have to add the targets to the deviations.
# To do this, repeated transposition t can be used:
stats <- t(t(sim_ans$sf) + sim_ans$targets)
# Calculate means and covariance matrix:
v <- apply(stats,2,mean)</pre>
```

```
covsf <- cov(stats)
# covsf is the same as sim_ans$msf</pre>
```

Of course, any other distributional properties of the generated statistics can also be obtained by the appropriate calculations and graphical representations in R.

## 9.1 Accessing the generated networks

If one is interested in the networks generated, not only in the statistics internally calculated, then the entire networks can be accessed. This is done by using the returnDeps option, as follows.

The returnDeps = TRUE option attaches a list sim\_ans\$sims containing all simulated networks as edge lists to the sim\_ans object. This uses rather a lot of memory. Since here the default n3 = 1000 was used, sim\_ans will be a list of 1000 elements; e.g., if there are three waves, the 568'th network generated for wave 2 is given by

```
sim_ans$sims[[568]][[1]][[1]][[1]]
```

The numbering is as follows: first the number of the simulation run (here, arbitrarily, 568); then the number of the group as defined in Section 11.2 (1 in the usual case of single-group data structures); then the number of the dependent variable (here 1, because it is supposed that there only is a dependent network); then the period number (i.e., wave number minus 1). This type of information can be found out by requesting

```
str(sim_ans$sims[[568]])
```

In the case of maximum likelihood estimation, if there is only one group and two waves, the list structure is simpler. Then the two superfluous list levels, group and wave/period, are omitted. The list then has three levels: dependent variable within group within simulation run.

The help page for siena07 contains an example for how to access the generated networks in the case of ML estimation. Note that this is meaningful only in view of the missing values of dependent variables for waves 2 and further, for which the simulations in Phase 3 can be regarded as a kind of model-based imputation.

Section 4.1.2 explains how such an edgelist can be transformed to an adjacency matrix:

```
# Determine number of actors (normally the user will know this)
n <- length(mydata$nodeSets[[1]])
# create empty adjacency matrix
adj <- matrix(0, n, n)
# Make shorter notation for edge list
edges <- sim_ans$sims[[856]][[1]][[1]][[1]]
# put edge values in desired places
adj[edges[, 1:2]] <- edges[, 3]</pre>
```

As an example, the following commands turn this list into a list of edgelists according to the format of the sna package (Butts, 2008), and then calculate the maximum k-core numbers in the networks. This assumes that a one-mode network is being analyzed.

```
# First define a function that extracts the desired component
# from the list element,
# gives the column names required for sna edgelists,
# and adds the attribute defining the number of nodes in the graph,
# as required by sna.
make.edgelist.sna <- function(x, n)</pre>
                      {
                          x \leftarrow x[[1]][[1]][[1]]
                          colnames(x) <- c("snd", "rec", "val")</pre>
                          attr(x, "n") <- n
# Note: if there is only one group and two waves,
# x[[1]][[1]][[1]] should be replaced by x[[1]]
# Apply this function to the list of simulated networks
simusnas <- lapply(sim_ans$sims, make.edgelist.sna, 50)</pre>
# Define a function that calculates the largest k-core number in the graph
library(sna)
max.kcores <- function(x)max(kcores(x))</pre>
# Apply this function and make a histogram
mkc <- sapply(simusnas, max.kcores)</pre>
hist(mkc)
```

Another possibility is to use the extractor functions, sparseMatrixExtraction, networkExtraction, or behaviorExtraction that are also used for sienaGOF.

# 9.2 Conditional and unconditional simulation

The distinction between conditional and unconditional simulation is the same for the simulation as for the estimation option of SIENA, described in Section 6.12.1. The choice between conditional and unconditional simulation is made in the sienaAlgorithmCreate function by setting the cond parameter, possibly also the condvarno and condname parameters

If the conditional option is chosen, then the simulations carry on until the desired distance is achieved on the dependent variable used for conditioning. For networks, the distance is the number of differences in the tie variables; for behavioral variables, the sum across actors of the absolute differences. This is determined as the distance between the consecutive networks (or, behaviors, if such a variable is used for conditioning) given in the call of sienaDataCreate. The rate parameter for this dependent variable then has no effect.

If the conditional simulation option was chosen (which is the default) and the simulations do not succeed in achieving the condition required by its stopping rule (see Section 6.12.1), then the simulation is terminated with an error message, saying *Unlikely to terminate this epoch*. In this case, you are advised to change to unconditional simulation.

### 9.3 Simulating with variable parameter values

The procedures described until now in this chapter conduct the simulations in Phase 3 with a constant parameter vector. If nsub = 0 this parameter value is given in the effects object, and returned for the resulting sienaFit object ans as ans\$theta. It is also possible to use a different parameter vector in every run of Phase 3. This is done by specifying a matrix of parameter values (runs in the rows, parameters in the columns) as the argument thetaValues for sienaO7. This requires specifying simOnly = TRUE. The parameter values actually used are then returned in ans\$thetaUsed.

If parallel processes are used (useCluster = TRUE) and the number of processes is nbrNodes > 1, the parameters used will be constant in each set of nbrNodes consecutively stored simulations. If the number of rows of thetaValues is not large enough given the values of n3 and nbrNodes, the last row of thetaValues will continue to be used. All this will be reflected by the value of ans\$thetaUsed.

# 10 Getting started

Note that there is a section 'Getting started' in the Minimal Introduction: Section 2. It may be best to go through that section first.

The best way to get started is to download the R scripts from the SIENA website and start reading and playing with them.

For carrying on and getting a first acquaintance with your own running of the model, the data set collected by Gerhard van de Bunt is useful; this data is discussed extensively in van de Bunt (1999); van de Bunt et al. (1999), and used as example also in Snijders (2001) and Snijders (2005). The data files are provided with the program and at the SIENA website. The digraph data files used are the two networks vrnd32t2.dat, vrnd32t4.dat. The networks are coded as 0 = unknown, 1 = best friend, 2 = friend, 3 = friendly relation, 4 = neutral, 5 = troubled relation, 6 = item non-response, 9 = actor non-response. Recode the network so that values 1, 2, and 3 are interpreted as ties for the first as well as the second network, and values 6 and 9 are missing data codes (NA).

The actor attributes are in the file vars.dat. Variables are, respectively, gender (1 = F, 2 = M), program, and smoking (1 = yes, 2 = no). See the Data sets tab at the SIENA website, and the references mentioned above for further information about this network and the actor attributes.

Create the various required objects, using functions sienaDataCreate, getEffects, and sienaAlgorithmCreate, as indicated in Chapters 4 and 6. At first, leave the whole model specification as it is by default (see Section 5): a constant rate function, the out-degree effect, and the reciprocity effect.

Then let the program estimate the parameters, using function siena07. You will see a screen with intermediate results: current parameter values, the differences ('deviation values') between simulated and observed statistics (these should average out to 0 if the current parameters are close to the correct estimated value), and the quasi-autocorrelations discussed in Section 6.

It is possible to intervene in the algorithm by clicking on the appropriate buttons: the algorithm may be restarted or terminated. In most cases this is not necessary.

A little bit of patience is needed to let the machine complete its three phases. When the algorithm has finished, look at the results in the output file or by the print or summary function of the resulting sienaFit object. Check that the overall maximum convergence ratio is small enough (ideally less than .25). If not, continue estimation with the prevAns option as discussed in Section 6.3.2. When satisfactory convergence has been obtained, make sense of the results: for example, is the reciprocity parameter significant?

As further steps, include some extra effects. First candidates are the transitive triplets effect or the gwesp effect, and any effects that may follow from your theories about the investigated network (see, e.g., Section 5); you can find their shortName, needed to specify them, in Chapter 12, where also the mathematical specifications are given. When these new effects have been added, follow the same steps: estimate, check convergence, if this is not yet satisfactory estimate again with the new initial values, and interpret the results when converged has been obtained.

To continue, non-significant effects may be excluded (but it is advised always to retain

the out-degree and the reciprocity effects) and other effects may be included, as suggested in Section 5.

### 10.1 Model choice

For the selection of an appropriate model for a given data set it is best to start with a simple model (including, e.g., 2 or 3 effects), delete non-significant effects, and add further effects in groups of 1 to 3 effects. Like in regression analysis, it is possible that an effect that is non-significant in a given model may become significant when other effects are added or deleted!

When you start working with a new data set, it is often helpful first to investigate the main endogenous network effects (reciprocity, transitivity, etc.) to get an impression of what the network dynamics looks like, and later add effects of covariates. The most important effects are discussed in Section 5; the effects are defined mathematically in Chapter 12.

Approaches to model specification are presented in Chapter 5 and in Snijders et al. (2010b).

When the distribution of the out-degrees is fitted poorly (which can be inspected using the sienaGOF function of Section 5.15), an improvement usually is possible either by including non-linear effects of the out-degrees in the evaluation function, or by other improvements of the model. This totally depends on the data set at hand.

# 11 Multilevel network analysis

For combining RSiena results of several independent networks, there are four options. ('Independent' networks here means that the sets of actors are disjoint, and it may be assumed that there are no direct influences from one network to another.) The first two options assume that the parameters of the actor-based models for the different networks are the same – except for the basic rate parameters and for those differences that are explicitly modeled by interactions with dummy variables indicating the different networks. All but the second option require that the number of observations is the same for the different networks. These methods can be applied for two or more networks.

In the following discussion, the terms 'networks' and 'groups' are used interchangeably. The four options are:

- 1. Combining the different networks in one large network, indicating by structural zeros that ties between the networks are not permitted. This is explained in Section 4.3.1. The special effort to be made here is the construction of the data files for the large (combined) network. This is superfluous; it is better to use option 2.
- 2. Combining different groups into one *multi-group* data set, and analyzing this by siena07. The 'groups' are the same as the 'different networks' mentioned here. This is explained in Section 11.2.
  - A difference between options 1 and 2 is that the use of structural zeros (option 1) will lead to a default specification where the rate parameters are equal across networks (this can be changed by making the rate dependent upon dummy actor variables that indicate the different networks) whereas the multi-group option yields rate parameters that are distinct across different networks.
  - In this option (like in option 1), the assumption is made that all parameters are the same for the various networks, except for the basic rate parameters; and except for explicitly specified interaction effects between variables depending on the group, and other effects. This is a very strong assumption, and undesirable unless options 3-4 are impossible.
- 3. Analyzing the different networks separately, without any assumption that parameters are the same but using the same model specification, and post-processing the output files by a meta-analysis using siena08. This is explained in Section 11.3.
- 4. Combining different groups into one *multi-group* data set as in option (2), but analyzing this by sienaBayes. This is explained in Section 11.4.

  Here the assumption for the parameters is that all basic rate parameters may differ arbitrarily between the groups; for the other parameters, some are identical and others vary randomly across groups according to a multivariate normal distribution. The distinction between 'some' and 'others' here is made by the parameter random in function setEffect.

The first and second options will yield nearly the same results, with the differences depending on the basic rate (and perhaps other) parameters that are allowed to differ between

the different networks, and of course also depending on the randomness of the estimation algorithm. The second option is more 'natural' given the design of RSiena and will normally run faster than the first. Therefore the second option seems preferable to the first.

The third option makes much less assumptions because parameters are not constrained at all across the different networks. This is preferable to the assumption of homogeneous parameters made in options 1 and 2. The fourth option is a middle ground between the first two and the third. The arguments usual in statistical modeling apply: as far as assumptions is concerned, options (3) and (4) are safer; but if the assumptions are satisfied (or if they are a good approximation), then options (1) and (2) have higher power and are simpler. Option (3) requires that each of the different network data sets is informative enough to lead to well-converged estimates; this will not always be the case for small data sets, and then option (4) may be preferable.

One difference between option 3 and the others is the centering of covariates; if this is used (which is the default!). For actor covariates, the multi-group data sets will center actor covariates by subtracting the grand mean; in the individual data sets analyzed in option 3, groupwise means are subtracted, This will affect the results. In some cases, for a multi-group analysis by option 2 or 4, it will be desirable not to center and/or to also use the groupwise means of covariates as an additional covariate.

When the data sets for the different networks are not too small individually, then a middle ground might be found in the following way. Start with option (3). This will show for which parameters there are important differences between the networks. Next follow option (2), with interactions between the group dummies and those parameters for which there were important between-network differences; or option (4), where the randomness of the effects is determined by these differences.

When the data sets for the different networks are quite small, then one might start by option (2), and use sienaTimeTest to test for which of the effects especially there is a large variation in parameter values across the groups; next one could follow approach (4), determining the randomness of the effects by the results about this variability.

In all cases, it is probably best to use an identical model specification for the various groups. A problem that may occur especially if the groups are small is that in some of the groups the change of the dependent variable (network or behavior) may be upward only or downward only, which by default then will be regarded by RSiena as a constraint for the simulations, as mentioned in Section 4.2.4. This leads to model differences that in most cases will be undesirable. Therefore it is advisable in the original construction of the datasets to use allowOnly = FALSE in the call of sienaDependent.

# 11.1 Function sienaGroupCreate

The function sienaGroupCreate operates on a list of siena data sets created by sienaData-Create, containing variables with the same names and of the same types. It creates a so-called sienaGroup object, which is a list of siena data objects with some additional information contained in attributes. This is used for options 2 and 4. For option 2 the numbers of waves may differ between the groups, for option 4 the groups all should have

the same number of waves.

For centered actor covariates, function sienaGroupCreate centers the original values again, now around the overall mean. This allows the use of centered actor covariates that are constant within groups. The egoX effect of a centered covariate, e.g., will be the sender effect of the original variable with grand-mean centering. For the altX effect the centering makes no difference.

For actor covariates, function sienaGroupCreate recalculates attributes range, range2, and simMean. Since version 1.3.4, the attributes range and range2 for each group are set at the range of this covariate in all groups. The simMean attribute of a group is set at the average of the similarity values for this group using the overall range. This implies that the parameters of the simX effect refer to the similarity transform (1) normalized with respect to the same range  $r_V$  for all groups; and the centering of this transform, which affects the parameter of the outdegree (density) effect, is group-specific.

## 11.2 Multi-group Siena analysis

The multi-group option 'glues' several Siena data sets (further referred to as groups) one after the other into one larger multi-group data set. These groups must have the same sets of variables of all kinds: that is, the list of dependent networks, dependent behavioral variables, actor covariates, and dyadic covariates must be the same for the various groups. Also their names must be the same. The number of actors and the number of waves can be different, however.

Gluing the groups together means that the observations (waves) of the groups are arranged after each other, so the total number of observations in the multi-group data set is the sum over all groups of the number of observations per group. In this way the groups then are combined into one Siena data set where the number of actors is the largest of the number of actors of the groups, and the number of observations is the sum of the observations of the groups. Constant covariates (whether monadic or dyadic) are transformed to changing covariates, because the groups now are regarded as multiple observations.

This is done by the function sienaGroupCreate which creates a so-called sienaGroup object, which is a list of siena objects with some additional information.

As an example, suppose that three Siena data sets with names sub1, sub2, and sub3 are combined. Suppose sub1 has 21 actors and 2 observations, sub2 has 35 actors and 4 observations, and sub3 has 24 actors with 5 observations. Then the combined multi-group group has 35 actors and 11 observations. The step from observation 2 to 3 switches from group sub1 to group sub2, while the step from observation 6 to 7 switches from group sub2 to sub3. These switching steps do not correspond to simulations of the actor-based model, because that would not be meaningful.

The different groups are considered to be unrelated except that they have the same model specification, the same variable names, and the same parameter values. It is important to check that this is a reasonable assumption. One aspect of this is by looking at the descriptives for change produced by print01Report, and checking that the tendencies in the dependent variable or variables, upward/stable/downward, are not too different

between the groups. The sienaTimeTest function can be used for formally testing this assumption. Moderate violations (p-values larger than 0.01) will probably be acceptable in the sense that the combined results still are a meaningful aggregate, strong violations are not acceptable and should be remedied by dropping some of the groups or by including an interaction term.

Given the potentially large number of periods that can be implied by the multi-group option, it probably is advisable, when using Method of Moments estimation, to use the conditional estimation option.

In multi-group groups, individual covariates are centered by subtracting the overall mean (across all groups), but dyadic covariates are centered by subtracting the withingroup means.

# 11.3 Meta-analysis of Siena results

The function siena08 is a meta-analysis method for SIENA. It combines estimates for a common model estimated for several data sets, that must have been obtained earlier. This function combines the estimates in a meta-analysis or multilevel analysis according to the methods of Snijders and Baerveldt (2003), and according to a Fisher-type combination of one-sided p-values.

The function siena08 takes as input the sienaFit objects produced by separate runs of siena08. These sienaFit objects must have exactly the same model specification and the same names of all variables; but it is allowed that there are differences with respect to parameters being fixed and perhaps tested. To get the same names of variables, the variables must be renamed in the call of sienaAlgorithmCreate; an example is in script RscriptMultipleGroups.R at the StOCNET website. If in some but not all groups a dependent variable has only upward or only downward changes, the automatic restriction to follow this pattern also in the simulations (see Section 4.2.4) must be lifted, because this would make the model specifications different. This must be done already in the original construction of the datasets that then later are combined by siena08, by using allowOnly = FALSE in the call of sienaDependent, as mentioned in Section 4.2.4.

If there are some parameters that cannot be estimated for some of the data sets (e.g., the effect of sex in a one-gender school; or because of near-multicollinearity), these parameters must still be included in the model for those data sets, but the parameters can be fixed to 0 (and perhaps tested by a score-type test).

Each parameter in the model is treated separately in the meta-analysis, without taking account of the dependencies between the parameters and their estimates. Denote the number of combined data sets by N. If we denote a given parameter (e.g., the coefficient of the reciprocity effect) by  $\theta$ , then the true parameter values for the N data sets are denoted  $\theta_1, \theta_2, \ldots, \theta_N$ , while their estimates are denoted  $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_N$ .

The package metafor can also be used for meta-analysis. This package is extensively documented in Viechtbauer (2010). In terms of Viechtbauer (2010), siena08 follows a random effects approach and presents the Hedges estimator which is the procedure of Snijders

and Baerveldt (2003), proposed by Cochran (1954); it also presents the maximum likelihood estimator. In Viechtbauer (2005), an extensive study is made comparing the various approaches, and it turns out that the comparison is not unequivocal. His recommendation, however, is to use the restricted maximum likelihood estimator. Since this is not implemented in siena08, this recommendation suggests that one should rather use metafor, with the option method = "REML".

Still another possibility is offered by the package mvmeta (Gasparrini et al., 2012). This was used in conjunction with RSiena by An (2015), who showed how to use this for incorporating group-level explanatory variables, using a fixed effects as well as a random effects approach, the latter with restricted maximum likelihood.

### 11.3.1 Meta-analysis directed at the mean and variance of the parameters

In this meta-analysis it is assumed that the data sets can be regarded as a sample from a population – i.e., a population of dynamic networks – and accordingly the true parameters  $\theta_j$  are a random sample from a population. If the number of data sets is small, e.g., less than 20, and especially if this number is less than 10, this assumption is not very attractive from a practical point of view, because the sample then would be quite small so the information obtained about the population is very limited.

The mean and variance in this population of parameters are denoted

$$\mu_{\theta} = \mathcal{E} \, \theta_j ,$$

$$\sigma_{\theta}^2 = \operatorname{var} \theta_j .$$

Each of these parameters must have been estimated in a run of siena07, yielding the estimate  $\hat{\theta}_j$ , which is the true parameter plus a statistical error  $E_j$ :

$$\hat{\theta}_j = \theta_j + E_j .$$

The standard error of this estimate is denoted by  $s_i$ .

For each of the parameters  $\theta$ , the function siena08 estimates the mean  $\mu_{\theta}$  and the variance  $\sigma_{\theta}^2$  of the distribution of  $\theta$ , and tests several hypotheses concerning these 'metaparameters':

- 1. Test  $H_0^{(0)}$ :  $\mu_\theta = \sigma_\theta^2 = 0$  (all  $\theta_j = 0$ ), i.e., effect  $\theta$  is nil altogether. This is done by means of a chi-squared test statistic  $T^2$  with N d.f.
- 2. Estimate  $\mu_{\theta}$ .
- 3. Test  $H_0^{(1)}$ :  $\mu_{\theta} = 0$ . This is done by means of a standard normal test statistic  $t_{\mu_{\theta}}$ , being the ratio of the estimate for  $\mu_{\theta}$  to its standard error.
- 4. Test  $H_0^{(2)}$ :  $\sigma_{\theta}^2 = 0$ , i.e.,  $\theta_j = \mu_{\theta}$  for all j. This is done by means of a chi-squared test statistic Q with N-1 d.f.

# 5. Estimate $\sigma_{\theta}^2$ .

Two approaches are followed and presented in the output. The first is an iterative weighted least squares method based on Cochran (1954) and Snijders and Baerveldt (2003). The second is a likelihood-based method under the assumption of normal distributions: the estimators are maximum likelihood estimators; the associated confidence intervals are based on profile likelihoods, and therefore will be asymmetric. The reported p-values for the population mean (hypothesis  $H_0^{(1)}$ ) are based on the t distribution with N-1 d.f. In all cases, it is possible that some of the data sets j are dropped for some of the parameters because the standard error  $s_j$  is too large (see below); in that case, the number N used here is the number of data sets actually used for the parameter under consideration.

For both of these two approaches, it is assumed that the true deviations  $\theta_j - \theta$  and the random errors  $E_j$  are uncorrelated. This is not always a plausible assumption; Fisher's combination, mentioned below, does not make this assumption. The plots of estimates versus standard errors, produced by using siena08 and following it up by plot.sienaMeta, can be used as information about the plausibility of this assumption.

For testing the hypotheses mentioned here, it is also assumed that, given the true parameter values  $\theta_j$ , the estimates  $\hat{\theta}_j$  are approximately normally distributed with mean  $\theta_j$  and variance  $s_j^2$ . This is often a reasonable assumption.

The likelihood-based methods also assume that the true values  $\theta_j$  are normally distributed in the population. If this is a reasonable approach, the likelihood-based methods are preferable. A disadvantage of the iterative weighted least squares method is that results are possible where the outcome of the test of  $H_0^{(2)}$  is significant at a usual level of significance, i.e.,  $\sigma_{\theta}^2$  is thought to be positive, whereas the estimate is  $\hat{\sigma}_{\theta}^2 = 0$ . This potential inconsistency is possible because the test and the estimator in this approach are not directly related (cf. Snijders and Baerveldt, 2003). The likelihood-based method does not suffer from this problem because the maximum likelihood estimate always is contained in the confidence interval based on the profile likelihood.

There may be reasons to distrust the estimates which are large with also a large standard error. (This is known as the Donner-Hauck phenomenon in logistic regression, discussed in Section 6.12.2.) Unfortunately, it is impossible to say in general what is to be regarded as a large standard error. A threshold of 4 or 5 for the standard error often is reasonable for most effects; if a tested parameter has a standard error larger than 4, then it is advisable to redo the analysis in a specification where this parameter only is fixed to 0 and a score test is carried out for this parameter. However, for some effects, in any case for the 'average similarity' effect for behavior dynamics, parameters and standard errors tend to be larger, and a larger threshold (e.g. 10) is appropriate. The same holds for effects of covariates with small variances (less than .1).

An alternative, probably better, for the estimation of standard errors is by using a non-parametric bootstrap confidence interval. For example, the adjusted percentile ( $BC_a$ ) method (Efron, 1987; Davison and Hinkley, 1997, Chapter 5) which is available in function boot.ci in R package boot.

### 11.3.2 Meta-analysis directed at testing the parameters

Another method for combining the various data sets, which does not make the assumption that the parameters are a sample from a population and also makes no assumptions of absence of correlation<sup>16</sup> between the true deviations  $\theta_j - \theta$  and the random errors  $E_j$ , is based on Fisher's method for combining independent p-values; the principle of this combination method of Fisher (1932) is described in Hedges and Olkin (1985) and (briefly) in Snijders and Bosker (2012, Chapter 3).

This principle here is applied in a double test:

1. for detecting if there are any networks with a positive parameter value, the null hypothesis tested is

 $H_0$ : For all networks, the value of this parameter is zero or less than zero; with the alternative hypothesis;

 $H_1$ : For at least one network, the value of this parameter is greater than zero;

2. for detecting if there are any networks with a negative parameter value, the null hypothesis tested is

 $H_0$ : For all networks, the value of this parameter is zero or greater than zero; with the alternative hypothesis

 $H_1$ : For at least one network, the value of this parameter is less than zero.

For each of these combined tests, the p-value is given. In the output these are denoted, respectively, as 'combination of right one-sided p-values' and 'combination of left one-sided p-values'.

It is advisable to use for each the significance level of  $\alpha/2$  (e.g., 0.025 if  $\alpha=0.05$ ) which yields an overall combined test at significance level  $\alpha$ . Note that four different overall results are possible. Indicating the right-sided and the left-sided p-values by  $p_r$  and  $p_l$ , respectively, these possible results are:

- (a)  $p_r > \alpha/2$ ,  $p_l > \alpha/2$ : No evidence for any nonzero parameter values;
- (b)  $p_r \leq \alpha/2$ ,  $p_l > \alpha/2$ : Evidence that some networks have a positive parameter value, no evidence for any negative parameter values;
- (c)  $p_r > \alpha/2$ ,  $p_l \le \alpha/2$ : Evidence that some networks have a negative parameter value, no evidence for any positive parameter values;
- (d)  $p_r \leq \alpha/2$ ,  $p_l \leq \alpha/2$ : Evidence that some networks have a negative parameter value, and some others have a positive parameter value.

<sup>&</sup>lt;sup>16</sup>This correlation is defined for the population of networks, and if the population does not exist then also the correlation is not defined.

If all networks have a zero true parameter value, i.e., under the combined null hypothesis that  $\theta_j = 0$  for all j, the probability of result (1) is less than or equal to  $\alpha$ ; this is the way in which this combined test respects the overall probability of an error of the first kind.

### 11.3.3 Contrast between the two kinds of meta-analysis

To understand the contrast between the method following the Cochran approach for inference about a population of networks, and the Fisher approach for combining independent tests, the following may be helpful. Inferring about a population always adds some uncertainty; this is more serious when the sample size (here: number of combined networks) is smaller. In the extreme case, consider the combination of N=2 networks, with estimates  $\hat{\theta}_1=1$ , standard error  $s_1=0.1$ , and  $\hat{\theta}_2=5, s_2=0.1$ . Then for both of the groups the t-statistic  $\hat{\theta}_j/s_j$  is very large, leading to the conclusion that parameters  $\theta_1$  and  $\theta_2$  are very likely to be positive. This will lead to a significant result for Fisher's combination of tests. On the other hand, the mean in the population of networks, given that there is available a sample of size as low as N=2, cannot be determined with any degree of precision, so the confidence interval for this mean  $\mu_{\theta}$  will be huge, and the result for testing the null hypothesis  $H_0^{(1)}$  will not be significant. However, the results for testing  $H_0^{(0)}$  and  $H_0^{(2)}$  will be significant.

# 11.4 Random coefficient multilevel Siena analysis

The function sienaBayes is for Bayesian estimation of one group or of multiple groups all having the same number of waves and the same model specification. It is available in package multiSiena (see the SIENA website). The methodology is explained in Koskinen and Snijders (2023). The parameters – excepting the basic rate parameters – can be either randomly varying between groups according to a multivariate normal distribution, or non-varying and constant across groups. (Note that the term 'non-varying' here does not mean that parameters are fixed, only that they do not vary between groups.) The difference is made by setting the keyword random in the function setEffect. The default is that only the out-degree (density) effect is randomly varying, but it is advisable to specify this for a larger set of effects. Specifying it for too many effects may, however, lead to unstable estimation.

The analysis is done by a Bayesian estimation method. This contrasts with the other functions in RSiena, which are based on the frequentist approach to statistical inference. There are many introductions to Bayesian inference and textbooks; see e.g., Kaplan and Depaoli (2013); Gill (2015); Donovan and Mickey (2019).

The operation of sienaBayes is explained in this section, and also in https://www.stats.ox.ac.uk/~snijders/sienaBayes\_s.pdf.

For the groupwise parameters normal distributions are assumed with conjugate priors. The prior distribution for the basic rate parameters is determined in a data-dependent way. For the non-varying parameters, the default is to assume a flat prior.

The procedure consists of three parts: initialization, warming, main phase.

1. In the initialization phase, initial parameter values and the proposal covariance matrix for Metropolis-Hastings ('MH') steps for groupwise parameters are obtained. This initialization has four parts.

First, Method of Moments estimation of a parameter vector assumed to be the same across the groups (in a multi-group estimation), with step size initgainGlobal. This can be replaced by a run of siena07 outside of sienaBayes, and giving the results of this run to sienaBayes as the prevAns parameter.

The second part, if initgainGroupwise is greater than 0, is one subphase of the Robbins-Monro algorithm for Method of Moments estimates for each group separately, with step size initgainGroupwise. Special precautions are taken here for dealing with group-level monadic variables, to prevent the issues that arise with such variables in normal use of siena07.

Third, nprewarm pre-warming steps are taken. These are MH steps where the proposal covariance matrices are used directly (i.e., without scaling) for random walk proposals. Prewarming is stopped when too few acceptances are observed.

The fourth part of the initialization scales the proposal covariance matrices, in the function improveMH, to achieve about 25 out of 100 acceptances of Bayes proposals after single MH steps.

- 2. After initialization and scaling of the proposal covariance matrices, a warming phase is done of nwarm Bayesian proposals each with a number of MH steps, followed again by the function improveMH.
- 3. Finally nmain repeats of  $\{\text{nrunMHBatches}\ \text{repeats}\ \text{of a number of MH steps sampling chains (the 'inner loop'), plus nSampVarying MH steps sampling the varying parameters <math>(\theta_j)$  plus nSampConst MH steps sampling the non-varying parameters  $(\eta)$  plus one Gibbs step sampling the global mean and covariance matrix of the varying parameters  $(\mu \text{ and } \Sigma)$  are performed. In the warming as well as the final phase, the number of MH steps in the inner loop is determined by parameter mult ('multiplication factor') in the call of sienaAlgorithmCreate that created the algorithm object.

The function sienaBayes is time-consuming. When starting to use it, it is advisable to start with low values of nmain to explore computing time. When the procedure seems to diverge, and for very small groups, it is advisable to use smaller values of the parameters initgainGlobal and initgainGroupwise; and perhaps reductionFactor.

# 11.4.1 Which data sets to use for sienaBayes

sienaBayes uses as data set a sienaGroup object with 2 or more groups. The number of waves should be the same for all groups.

The following data configurations are not allowed for groups included in sienaBayes estimation:

1. tie variables in two consecutive waves changing from structural zero (code 10) to 1;

- 2. tie variables in two consecutive waves changing from structural one (code 11) to 0;
- 3. tie variables in three consecutive waves changing from structural zero (code 10) to NA to 1;
- 4. tie variables in three consecutive waves changing from structural one (code 11) to NA to 0;
- 5. and, for more than three consecutive waves, similar patterns with more NAs in between.

To use data sets including such patterns for sienaBayes, you will have to make minimal changes to the data set so as to avoid these patterns. For example, replace sequences 10-1 by NA-1, and 10-NA-1 by NA-NA-1. An example script to make such replacements is at http://www.stats.ox.ac.uk/~snijders/siena/changeForbiddenChanges.R.

In addition to what is or is not allowed as data sets for sienaBayes, it is important to mention that using structural zeros to indicate intermittent absence of actors (i.e., structural zeros only at the beginning or only at the end of a period) is not a good idea for sienaBayes. Tie variables that are 0 or 1 at the beginning of a wave and structurally zero (code 10) at the end of the wave, have to assume the value 0 at the end of the period; and this is used as information for the estimation in sienaBayes of the parameters. Probably by such a structural zero you wish to convey the meaning that the value of this tie variable is unknown. If there are two waves of data, the simple solution is to replace all structural zeros for wave 2 by missing values (i.e., NA). There is no straightforward solution if there are three or more waves.

sienaBayes should be possible for groups as small as 5 actors. A restriction (maybe to be lifted later) is that the networks must not be empty at any wave; and consecutive waves of networks must not be identical (in other words, all Jaccard indices should be strictly less than 1).

Be prepared for long computation times. The reason is that likelihood-based computations are used (as distinct from the Method of Moments approach). If all individual groups have enough information for good estimation by the Method of Moments according to the intended model, the use of siena07 with the (default) Method of Moments, followed by a meta-analysis by means of siena08 or metafor, may be preferable.

It is advisable to first do a multi-group analysis of the same model, followed by a sienaTimeTest, to get an initial understanding of where problems might occur. You may then later use the result for the prevAns parameter of sienaBayes.

# 11.4.2 Model specification

Non-constant rate functions currently are not supported in sienaBayes.

The extra part of model specification for sienaBayes, compared to siena07, is that it is required to specify which parameters are randomly varying from group to group, and which are fixed across groups. The specification of fixed vs. randomly varying for the other parameters is done in the function setEffect, by its parameter random. To be able

to see which of the effects are specified in this way, please have a look at the help page for print.sienaEffects:

#### ?print.sienaEffects

and note the parameters includeRandoms and dropRates, which are helpful especially for random coefficient multilevel modeling. Requesting includeRandoms = TRUE will show the column indicating which effects are random; dropRates = TRUE omits the rows with rate effects, which can be a lot of superfluous information that perhaps you do not want to see.

The exception to the rate parameters being always randomly varying is that they may be fixed — in which case all of them need to be fixed, and input parameter priorRatesFromData needs to be set to -2.

There currently is little advice about which effects to specify as randomly varying. The basic issues are the following.

- interest: is variability of the effect across groups a primary part of the research question? (Usually not; such research questions about variability are of a quite secondary nature.)
- knowledge: is there prior knowledge about whether effects differ between groups? (Usually not. It is possible to first do a multi-group analysis of the same model, followed by a sienaTimeTest, and specify those effects as randomly varying for which the time heterogeneity is largest according to the groupwise results.)
- misspecification: if it would be erroneously assumed that the effect is fixed across groups, would this affect the parameter estimates of primary interest?

  (About this we have little general knowledge; it can be tried out by running estimations for different specifications of this part of the model.)
- convergence: specifying effects as random will influence convergence. Up to a point, randomly varying parameters will have better convergence properties than fixed parameters; but if their number is too large, given the number of groups, convergence can deteriorate again. This may then lead to outliers in the posterior means, which can be diagnosed by function plotPostMeansMDS.
- **amount of information:** which specification will use the information in the data most efficiently?

(Here finally we do have a provisional answer. Assuming that an effect is fixed across groups will give a smaller uncertainty —posterior standard deviation, interpreted as standard error— in the estimated parameter than assuming it varies randomly; cf. Section 11.4.8. This will be the more so as the number of groups is smaller. Therefore, for the coefficients for which there is no strong prior knowledge that they are variable across groups, and which are tested as a primary issue for answering the research question, from the point of view of statistical power it is advisable to specify that they are fixed.)

#### 11.4.3 How to enter your data in sienaBayes

See the example at the bottom of the sienaBayes help page for how a sienaGroup object can be created and used. If some but not all of the Siena data objects combined in the sienaGroup have periods where changes are only upward or only downward, it will be necessary to use allowOnly = FALSE in the call of sienaDependent; see the help page for sienaDependent. The scripts RscriptListGroups.R and RscriptMultipleGroups.R give further examples and explanation for creating sienaGroup objects.

If you have a large number of groups (more than 30), try first with a smaller number of groups (10-20). If you need or wish to make a selection of groups, select the one with few or no missing data and with Jaccard coefficients at least 0.30.

### 11.4.4 How to choose the parameter settings for sienaBayes

It is good to have an initial estimation as a multi-group estimation using the Method of Moments, i.e., using siena07, with an algorithm having nsub = 2 and a reduced value of firstg, e.g., 0.05 or less, and look at these results – they will give you a rough idea even if they are nowhere near convergence. If some of the estimated non-rate parameters are very large in absolute value, use a smaller value of firstg, such as 0.01 or 0.001. This result of siena07 can then be given as the prevAns parameter to sienaBayes (see the help page). Note that the parameter firstg corresponds to initgainGlobal of sienaBayes, if no prevAns is used.

It may be good to have an initial try run with nwarm = 5, nmain = 10, nrunMHBatches = 5, nImproveMH = 20 (for speed) and

silentstart = FALSE (for information about the initialization phase), and then print result. This will give information about the results of the initialization phase and about computing time. If some of the groups have some very high estimated rate parameters, you should either drop those groups or decrease the value of initgainGroupwise. The new value could be, e.g., 0.005 or 0.001 or 0.0. With the lower value of initgainGroupwise, dropping the groups concerned may be unnecessary, so don't drop groups too soon.

For normal use, nwarm = 500, nmain = 1000, nrunMHBatches = 20, nImproveMH = 100 may be reasonable. Computing time is roughly proportional to nmain × nrunMHBatches. We still have to develop guidelines about how to choose the number of iterations. If the tracelines show that the process is still quite unstable even in the later part of the runs, possibilities are to increase nrunMHBatches but also to increase the mult parameter in sienaAlgorithmCreate. Increasing these will for both of them lead to a proportional increase in computing time.

If multiple processors are available (which most computers have nowadays), you can make more speed by setting the nbrNodes parameter to a value larger than 1. Since parallelization goes by period × group, it is nice, but not necessary, to have a value for nbrNodes that is a divisor of (the number of periods multiplied by the number of groups); higher values are meaningless. Do not use such a high value for nbrNodes that your

computer gets too hot or overworked. For Windows machines this can be monitored by opening the Task Manager (you will find how to do this by right clicking on the bottom toolbar).

## 11.4.5 Prior distributions

The prior mean and prior variance need to be given for the vector of parameters that are randomly varying between the groups. These are the rate parameters and the parameters specified with

```
random = TRUE
```

in the call of setEffect. For an effects object called myeff, you can see which parameters are specified as randomly varying by

```
print(myeff, includeRandoms = TRUE)
```

This will also tell you the number of random effects, which are the dimensions for the parameters priorMu and priorSigma used to specify the prior distribution. The rate parameters can be dropped from the print result by requesting

```
print(myeff, includeRandoms = TRUE, dropRates = TRUE)
```

The list and order of only the randomly varying effects can be shown by requesting

myeff[(myeff\$randomEffects | (myeff\$basicRate & (myeff\$group==1) )) & myeff\$include, ]

The list and order of only the non-varying effects can be shown by requesting

```
myeff[myeff$include & (!myeff$basicRate) & (!myeff$randomEffects),]
```

Non-varying parameters

For the non-varying parameters ( $\eta$ , the greek letter 'eta'), by default an improper flat prior is used, and nothing needs to be specified by the user.

However, there is the possibility to specify normal priors with mean 0 and a given variance. This can be useful for effects of group-level variables. The prior variances of the non-varying parameters are given by the parameter priorSigEta. This needs to be a vector with length equal to the number of non-varying parameters. Values NA indicate that a flat prior should be used (which can be interpreted as an infinite variance).

#### Varying parameters

The population distribution of the randomly varying parameters is assumed to be multivariate normal  $\mathcal{N}(\mu, \Sigma)$ . The conjugate prior for the multivariate normal distribution is used to reflect the prior uncertainty in the parameters  $\mu$  and  $\Sigma$ ; cf. Gelman et al. (2014, Section 3.6) and O'Hagan and Forster (2004, Chapter 14). This conjugate prior is the inverse Wishart distribution for  $\Sigma$ ; and, conditional on  $\Sigma$ , for  $\mu$  a multivariate normal distribution, specified as follows:

$$\Sigma \sim \text{InvWishart}_{p_1}(\Lambda_0^{-1}, \nu_0)$$
, and conditionally on  $\Sigma$  (14a)

$$\mu \mid \Sigma \sim \mathcal{N}_{p_1}(\mu_0, \Sigma/\kappa_0)$$
. (14b)

Thus, the parameters of the prior are  $\nu_0$ ,  $\Lambda_0$ ,  $\mu_0$ , and  $\kappa_0$ . These are specified for sienaBayes in RSiena in the following way:

- $\nu_0 = \text{priorDf};$
- $\Lambda_0 = \text{priorDf} \times \text{priorSigma};$
- $\mu_0 = \text{priorMu};$
- $\kappa_0 = \text{priorKappa}$ .

This means the following for the prior parameters as specified for sienaBayes:

- priorDf can be regarded as the effective sample size that has led to the prior information. For mathematical reasons, this must be at least p + 2, where p is the number of varying parameters.
- priorSigma is your prior guess for the population covariance matrix of the varying parameters; in other words, this reflects the differences that may exist between the groups.
- priorMu is your prior guess for the population mean  $\mu$  of the varying parameters.
- priorSigma / priorKappa reflects your uncertainty about your guess for  $\mu$ .

It is quite a restriction that the covariance matrix expressing the prior uncertainty about  $\mu$  should be proportional to the population (i.e., 'true between-group') covariance matrix of the varying parameters. This is a mathematical consequence of the use of the conjugate prior. Often, the prior uncertainty about  $\mu$  will be greater than the assumed between-group differences, leading to a choice of **priorKappa** less than 1, e.g., 0.01; note that **priorKappa** is a ratio of variances, and the ratio of uncertainties will be better expressed by its square root.

More research is needed for advice about prior distributions. Especially for small numbers of groups, the priors may have a strong influence.

The prior mean of the varying parameters, priorMu, is a vector of length equal to the number of varying parameters. In the object produced by sienaBayes, let us call it ans, this length is stored as ans\$p1. For example, in a model with 4 waves, one dependent network variable, and varying parameters specified for outdegree, reciprocity, transitive ties, and similarity for some covariate V, the length would be 3+4=7 (3 for three periods + 4 for four varying parameters). The prior covariance matrix of the varying parameters, priorSigma, is a symmetric square matrix of dimension equal to the length of priorMu.

The treatment of these prior distributions should distinguish between the rate parameters and the parameters for the objective function.

## Rate parameters

If rate parameters are fixed (an exceptional case), they are not included in the prior.

In other cases, special attention must be given to the rate parameters. Sometimes, for small groups and complicated models, some of the rate parameters may be estimated in the multi-group option by very high numbers. This may be the case especially for groups with low Jaccard coefficients, or for groups that deviate strongly from the other groups. It may be advisable to take out the groups with extremely high rate parameters (e.g., larger than 50 or 80). To make it feasible to try and include some groups with high estimated rate parameters, a stronger prior distribution for these parameters may be employed.

By default, the prior for the basic rate parameters is data-dependent, with a mean and covariance matrix that are robust estimates based on the estimated rate parameters from the initialization phase. This mostly will be adequate.

To have explicit control of the prior for the basic rate parameters, the data dependence can be turned off by using priorRatesFromData = 0. Then the choice for priorMu and priorSigma is going to matter. Consider the rate parameters as estimated in the multigroup analysis of the same data set. Suppose the total number of groups is N. For each separate rate parameter (for a given wave and for a given dependent variable) there then are N estimates, some of which may be too high; denote the average and the variance of the subset of not-too-high values by m and  $s^2$ , respectively. These are specific for the given wave and the given dependent variable. These values for mean and variance should represent what one might find plausible values for this rate parameter.

The corresponding elements of priorMu and diagonal elements of priorSigma should be then set, respectively, to values close to these m and  $s^2$ .

As an example, suppose there are N=20 groups, 3 waves and one dependent variable (a network), and 5 varying parameters in addition to the rate parameters. Then  $\mathtt{p1}=2+5=7$ . Suppose that for the rate parameters the default data-dependent prior is used (priorRatesFromData = 2). Then in the parameters priorMu and priorSigma it still is necessary to include the prior rate parameters, to have the correct dimensionality of this vector and matrix, but their values do not matter. Suppose further that for the parameters of the objective function the prior mean for the density parameter is specified as -1, for the reciprocity parameter as +1, all other prior means as 0, and all prior between-group variances as 1. Further assume that the prior uncertainty about  $\mu$  is thought to be 10 times larger, on the variance scale, than the prior between-group differences. One then could use the following piece of code.

priorDf is the prior degrees of freedom for the covariance matrix. It can be interpreted as the sample size on which, hypothetically, the prior knowledge about the covariance matrix of the parameters is based. The influence of the prior on the results is larger when priorDf is larger.

Parameters of the objective function

For parameters of the objective function, it will be usually be possible to use some prior knowledge, together with neutrality with respect to the sign of tested parameters (in

order not to unduly bias results.) In most cases the outdegree parameter is expected to be negative and the reciprocity parameter positive. The researcher should consider earlier studies of similar network dynamics; reasonable values for the prior mean for the outdegree parameter might be -2 or -1, and for the reciprocity parameter +1.5 or +2. For homophily parameters on important attributes expressed by the  $\mathtt{simX}$  effect (which is standardized), as long as these are regarded as control effects, one might specify the prior mean conservatively as 0.3 or 0.5.

Continuing the example above, if the 5 varying non-rate parameters would start with an outdegree and a reciprocity parameter, the prior means could be modified, e.g., to

$$m7 \leftarrow c(5, 5, -2, 1.5, 0, 0, 0)$$

(if for the similarity parameters also the value of 0 is used...).

## 11.4.6 Operation of sienaBayes

In Section 11.4.4 it was already suggested to start with a run with very low values of the sample size settings for the MCMC procedure.

It is advisable to use multiple processes ('parallel computing') for sienaBayes; see Section 6.13 where it is explained that parallelization is different for sienaBayes than for MoM estimation by siena07.

When the procedure has made a good start but the MCMC sample seems too short, you can make a prolonged analysis using the prevBayes option, and then combine the earlier with the later results using glueBayes. This is illustrated in the example on the help page.

During operation of sienaBayes, partial results of the function are now and then stored as objects named z in files with the name PartialBayesResult.RData; see the help page. This is for the case that the computer or R stops inadvertently during the long computations. These are sienaBayesFit objects, and therefore can be used in the printand summary functions; they also can be used for the prevBayes option to continue estimation.

### 11.4.7 Assessing convergence

You can visually inspect convergence by looking at the tracelines of the various parameters. These can be plotted by the functions in BayesPlots.r (available from the Siena website). In many cases, the tracelines for the rate parameters already tell the story about convergence.

The file BayesPlots.r contains a variety of plotting functions that can be used to obtain trace plots and posterior density plots.

The parameters nwarm and nmain in the call of sienaBayes only imply that an extra improveMH step is made between the warming and the main iterations; there are no other differences between the warming and main iterations. It is possible that convergence has set in only later; depending on the case, the traceplots may give information about this. If you conclude that convergence has occurred later, then use this to define the nfirst parameter in the printand summary functions for sienaBayesFit object (see ?print.sienaBayesFit).

When the procedure seems to have diverged and this occurs right from the start, it is advisable to use smaller values of the parameters initgainGlobal and initgainGroupwise. If divergence sets in later and is most pronounced for the rate parameters, it may be advisable to use a smaller value of reductionFactor, e.g., 0.1. If generally the tracelines are irregular, it may be good to increase nrunMHBatches but another possibility is to increase the mult parameter set in sienaAlgorithmCreate.

Using other packages for convergence assessment

The advice of literature such as Gelman et al. (2014) is to use multiple sequences produced independently, preferably from overdispersed starting points, for assessing convergence. For example, one may use 3 to 5 such sequences. Function extract.sienaBayes can be used to extract from these sequences the draws from the posterior distributions of the parameters of interest. This function produces a three-dimensional array of iterations by chains by parameters, which then can be used, e.g., in function monitor of package rstan or with the help of package coda.

Currently there is no good way in sienaBayes to use overdispersed starting points. (The difficulty is to get overdispersion while still retaining a reasonable convergence for each sequence.) The best option currently is to use independent restarts of the whole algorithm; or to use one MoM estimation run as the basic starting point, and use that as prevAns parameter in several independent restarts, and perhaps continuations after this using prevBayes. Note that 'independence' is obtained by using different random number seeds in the sienaAlgorithm; and that this is implied by its default value NULL.

Function monitor gives information about the potential scale reduction  $\hat{R}$  of the posterior distribution if simulations were continued indefinitely, and the effective sample size  $n_{\rm eff}$  (i.e., the estimated equivalent sample size under independent sampling). Rules of thumb given in Gelman et al. (2014, p. 287) are that, for all parameters of interest,  $\hat{R} \leq 1.1$  and  $n_{\rm eff} \geq 5m$ , where m is the number of chains.

Note that what is given by monitor as the 'se\_mean' is the standard error of the mean of the posterior distribution as an estimator for the global mean  $\mu_i$ : i.e., this expresses the uncertainty due to the finite length of the MCMC chain, it is not a measure of spread of the posterior distribution itself.

## 11.4.8 Interpreting results of sienaBayes

The printand summary functions give posterior means, posterior standard deviations, 95% credibility intervals, and one-sided posterior p-values for testing whether the parameter is positive or negative. These are the Bayesian versions of estimates, standard errors, confidence intervals, and p-values. These functions can be used with a parameter nfirst, indicating the first iterations from which convergence is assumed. The default value is the number of warming iterations +1, but this may be inadequate. From the trace plots obtained by functions in script  $absolute{BayesPlots.r}$  is may be possible to make an improved guess for this iteration number, and use this for  $absolute{BayesPlots.r}$ 

The function siena.table can be used also for making html or LATEX tables of the results for the global parameters; here again, the parameter nfirst may be used.

The functions simpleBayesTest and multipleBayesTest are available for testing parameters; see the help page for these functions. To compute further properties of the sample of the posterior, the components ThinParameters, ThinPosteriorMu,

ThinPosteriorEta, and ThinPosteriorSigma of the sienaBayesFit object, as mentioned in the help page, may be useful.

The file BayesPlots.r, mentioned above, contains some plotting functions that are useful for making posterior density plots.

The function extract.posteriorMeans can be used to obtain posterior means and standard deviations of the groupwise estimated parameters. These are like the 'empirical Bayes' estimates known also from other statistical models. A multi-dimensional scaling ('MDS') plot of the posterior means can be obtained from the function

plotPostMeansMDS. This is useful especially for detecting outliers. If strong outliers are found, this may have two different reasons: truly deviating groups; or instability of the estimation of the posterior means, related perhaps to having too many randomly varying parameters. In the latter case, it may be quite arbitrary (in the sense of 'depending on the whim of the random numbers during the operation of sienaBayes') which groups are indicated in the plot as being the outliers.

It should be noted that due to the high correlations between parameters, the posterior means may be useful mainly for model specifications with a limited number of randomly varying parameters. If one is interested, for example, in assessing differences between groups with respect to reciprocation by comparing posterior means for the reciprocity parameter, it should be avoided to specify other parameters as randomly varying that are highly correlated with the reciprocity parameter.

It should be noted that the .txt files produced by sienaBayes are produced somewhere in the initial phase of the project and not meant to be informative for final results. They contain the results of the initial estimate by the multiple groups method and may be disregarded.

When comparing results for specifications that differ with respect to specifying the effects as fixed or varying across groups, it will be noted that posterior standard deviations for the means are larger when specifying the effects as randomly varying, as compared to specifying them as fixed. This is natural, and it is associated with a difference in interpretation. Specifying the effect as randomly varying implies that there also is an important step of generalization from the observed groups to the population of groups. The between-group variance then is a priori unknown and one is estimating a mean parameter from a sample of N groups; usually N is not very large and the uncertainty about the between-group differences will contribute considerably to the uncertainty of the population mean.

# 12 Mathematical definition of effects

The list of all effects available for any data set is obtained by the command

effectsDocumentation()

which produces a html file. For a given effects object, say with the name myeff, the command

effectsDocumentation(effects = myeff)

will give a file with all effects implemented for this effects object. See

?effectsDocumentation

for further options.

This chapter present the mathematical formulae for the definition of the effects. Further background to these formulae can be found for network dynamics in Snijders (2001, 2005); Snijders et al. (2010b); for network and behavior dynamics in the last reference and Steglich et al. (2010); and for co-evolution of multiple networks, including two-mode networks, in Snijders et al. (2013). The effects are grouped into effects for modelling network evolution and effects for modelling behavioral evolution (i.e., the dynamics of dependent actor variables). Within each group of effects, the effects are listed in the order in which they appear in SIENA. The short name of the effect (shortName), as it is specified in RSiena is specified in brackets.

For two-mode (bipartite) networks, only a subset of the effects is meaningful, since the first node set has only outgoing ties and the second only incoming; for example, the reciprocity effect is meaningless because there cannot be any reciprocal ties; the outdegree popularity effect is meaningless because it refers to incoming ties of actors with high out-degrees; and there are no direct similarity effects of actor covariates. However, distance-two effects such as simEgoInDist2, and the interaction of effects like altInDist2 with egoX can be used to represent homophily-like tendencies. There are some specific effects for two-mode networks, e.g., the four-cycle effect.

Some of the effects contain a number which is denoted in this section by p, and called in this manual an *internal effect parameter*. (These are totally different from the statistical parameters which are the weights of the effects in the objective function.) These are set or modified by the setEffect function, e.g.,

myeffects <- setEffect(myeffects, gwespFF, parameter = 69)</pre>

#### 12.1 Network evolution

The model of network evolution consists of the model of actors' decisions to establish new ties or dissolve existing ties (according to evaluation, creation, and endowment functions) and the model of the timing of these decisions (according to the rate function). The model, and the roles played by these three functions, were briefly explained in Section 5.1.

For some effects the creation and endowment functions are implemented not for estimation by the Method of Moments but only by the Maximum Likelihood or Bayesian method; this is indicated below by 'endowment effect only likelihood-based'.

(It may be noted that the network evaluation function was called objective function, and the creation and endowment functions were called gratification function, in Snijders (2001).)

## 12.1.1 Network evaluation function

The network evaluation function for actor i is defined as

$$f_i^{\text{net}}(x) = \sum_k \beta_k^{\text{net}} s_{ik}^{\text{net}}(x) \tag{15}$$

where  $\beta_k^{\text{net}}$  are parameters and  $s_{ik}^{\text{net}}(x)$  are effects as defined below. If the model also contains some elementary effects (see Section 5.1.1), the objective function is the sum of this and

$$f_i^{\text{el}}(x) = \sum_k \beta_k^{\text{el}} s_{ik}^{\text{el}}(x) , \qquad (16)$$

see Section 5.1.3. Elementary effects are of the type  $s_{ijk}^{el}(x) = x_{ij} s_{ijk}^{el0}(x)$ , where  $s_{ijk}^{el0}(x)$  does not depend on  $x_{ij}$ .

The potential effects in the network evaluation function are the following. Note that in all effects where a constants p occurs, this constant can be chosen and changed by the user; this is the internal effect parameter mentioned above, which can be modified by the function setEffect(..., parameter=..., ...). For non-directed networks, the same formulae are used, unless a different formula is given explicitly. Some of the effects are dropped for non-directed networks, because they are not meaningful; and some of the names differ in the non-directed case. Section 12.1.2 gives some effects that are specific to non-directed networks.

### Structural effects

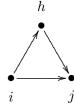
Structural effects are the effects depending on the network only. The following list also contains some elementary effects (see Section 5.1.1). The type of the elementary effects in RSiena still is eval, indicating that its parameter is applied both for creating new ties and for maintaining existing ties.

- 1. out-degree effect or density effect (density), defined by the out-degree  $s_{i1}^{\text{net}}(x) = x_{i+} = \sum_{j} x_{ij}$ , where  $x_{ij} = 1$  indicates presence of a tie from i to j while  $x_{ij} = 0$  indicates absence of this tie;
- 2. reciprocity effect (recip), defined by the number of reciprocated ties  $s_{i2}^{\text{net}}(x) = \sum_{i} x_{ij} x_{ji};$

3. transitive triplets effect (transTrip), defined by the number of transitive patterns in i's relations (ordered pairs of actors (j, h) to both of whom i is tied, while also j is tied to h),

combining the transitive triplets and transitive mediated triplets

both of whom 
$$i$$
 is tied, while also  $j$  is tied to  $h$ ),
$$s_{i3}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{ih} x_{hj};$$
(there was an error here until version 3.313, which amounted to

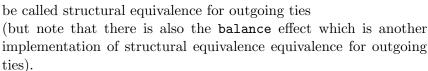


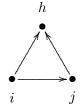
4. transitive triplets effect type 1 (transTrip1), may also be called transitive closure effect; the elementary effect corresponding to creating or maintaining the tie  $i \to j$ in the figure above; this is transitive closure in the strict sense of the term. The effect is

$$s_{i4}^{\text{el}}(x) = x_{ij} \sum_h x_{ih} x_{hj};$$

effects;)

5. transitive triplets effect type 2 (transTrip2), may also be called two-out-star closure effect; the elementary effect corresponding to creating or maintaining the tie  $i \to j$  in the figure here; this could be called structural equivalence for outgoing ties





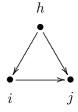
The effect is 
$$s_{i5}^{\text{el}}(x) = x_{ij} \sum_h x_{ih} x_{jh};$$

6. transitive mediated triplets effect (transMedTrip), defined by the number of transitive patterns in i's relations where i has the mediating position (ordered pairs of actors (j,h) for which h is tied to i and i to j, while also h is tied to j), which is different from the transitive triplets effect only for directed networks,

$$s_{i6}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{hi} x_{hj};$$

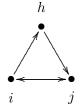
this cannot be used together with the transitive triplets effect in Method of Moments estimation, because of perfect collinearity of the fit statistics;

in the saying 'friends of my friends are my friends', the actor i here is the second 'friend';



7. transitive reciprocated triplets effect (transRecTrip), which can be regarded as an interaction between the transitive triplets effect and reciprocity, where the reciprocated tie is the tie  $i \leftrightarrow j$  that closes the two-path  $i \to h \to j$ ,

$$s_{i7}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{ji} x_{ih} x_{hj};$$



8. transitive reciprocated triplets effect (type 2) (transRecTrip2), another interaction between the transitive triplets effect and reciprocity, where the reciprocated tie is the tie  $h \leftrightarrow j$  in the closed the two-path  $\{i \to h \to j, i \to j\}$ ,

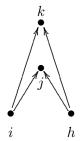
$$s_{i8}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{ih} x_{hj} x_{jh};$$

 $s_{i8}^{\rm net}(x) = \sum_{j,h} x_{ij} \, x_{ih} \, x_{hj} \, x_{jh}$ ; this represents the tendency to send ties simultaneously to pairs of actors who are mutually linked; but when outdegree-activity is also included in the model, it represents as well the tendency to send ties simultaneously to pairs of actors who are not linked to each other;

- 9. number of three-cycles (cycle3),  $s_{i9}^{\text{net}}(x) = \sum_{i,h} x_{ij} x_{jh} x_{hi};$
- 10. shared popularity sharedPop,

$$s_{i10}^{\text{net}}(x) = \frac{1}{4} \sum_{j,k,h;\text{all different}} x_{ij} x_{hj} x_{ik} x_{hk};$$

this is like a 4-cycle but in a special orientation, like the 2PU ('two-paths up') effect (for k=2) for directed ERGMs proposed in Robins et al. (2009). Therefore the statistic is called the 'number of 2-2PU' configurations;



11. for two-mode networks and for non-directed networks: the number of four-cycles with shortName (cycle4) for the two-mode and (cycle4ND) for the non-directed case,

$$s_{i11}^{\text{net}}(x) = \frac{1}{4} \sum_{j,k,h;\text{all different}} x_{ij} x_{ik} x_{hj} x_{hk};$$

for parameter p=2 the square root is taken;

$$s_{i11}^{\text{net}}(x) = \frac{1}{2} \sum_{j} x_{ij} \sqrt{\sum_{k,h;\text{all different}} x_{ik} x_{hj} x_{hk}};$$

note that this is like the sharedPop effect above,

but for the two-mode and non-directed cases

the directionality plays no role;

12. transitive ties effect (transTies) (earlier called (direct and indirect ties) effect), defined by the number of actors to whom i is directly as well as indirectly tied,  $s_{i12}^{\text{net}}(x) = \sum_{i} x_{ij} \max_{h} (x_{ih} x_{hj});$ 

- 13. betweenness count (between),  $s_{i13}^{\text{net}}(x) = \sum_{j,h} x_{hi} x_{ij} (1 x_{hj});$  note that this is one of the rare effects that depends on non-existence of a tie (here, the tie  $h \to j$ ); this makes it less attractive, use it only if you know what you are doing!
- 14. balance (balance), defined by the similarity between the outgoing ties of actor i and the outgoing ties of the other actors j to whom i is tied,

$$s_{i14}^{\text{net}}(x) = \sum_{j=1}^{n} x_{ij} \sum_{\substack{h=1\\h\neq i,j}}^{n} (b_0 - |x_{ih} - x_{jh}|),$$

where  $b_0$  is a constant included to reduce the correlation between this effect and the density effect, defined by

$$b_0 = \frac{1}{(M-1)n(n-1)(n-2)} \sum_{m=1}^{M-1} \sum_{\substack{i,j=1\\ h \neq i,j}}^{n} \sum_{\substack{h=1\\ h \neq i,j}}^{n} |x_{ih}(t_m) - x_{jh}(t_m)|.$$

This may also be regarded as structural equivalence with respect to outgoing ties. (In RSiena versions before 3.324, this was divided by n-2, which for larger networks tended to lead to quite large estimates and standard errors. Therefore in version 3.324, the division by n-2 — which had not always been there — was dropped.)

15. structural equivalence effect with respect to incoming ties (inStructEq), which is an analogue to the balance effect but now considering similarity with respect to incoming ties,

$$s_{i15}^{\text{net}}(x) = \sum x_{ij} d_{ij} \tag{17a}$$

with

$$d_{ij} = \sum_{\substack{h=1\\h\neq i,j}}^{n} \left( b_0 - |x_{hi} - x_{hj}| \right). \tag{17b}$$

This effect is not quite finalized yet, because provisionally  $b_0 = 0$  instead of a mean of the subtracted values like in the balance effect. Subtraction of the mean will lead to better convergence properties.

16. Jaccard similarity for outgoing ties effect (Jout), an elementary effect defined by the Jaccard similarity with respect to outgoing ties,

$$s_{i16}^{\text{net}}(x) = \sum_{j} x_{ij} J_{\text{out}}(i,j)$$
, where

$$J_{\text{out}}(i,j) \, = \, \frac{\sum_{h} x_{ih} \, x_{jh}}{x_{i+} + x_{j+} - \sum_{h} x_{ih} \, x_{jh}}$$

(where 0/0 is taken as 0).

Since the Jaccard measure has smaller variability than a lot of other effects, the parameter estimates of this will often be larger, with correspondingly larger standard errors, than many other parameter estimates. The same holds for the other Jaccard similarity effects.

17. Jaccard similarity for incoming ties effect (Jin), an elementary effect defined by the Jaccard similarity with respect to incoming ties,

$$s_{i17}^{\text{net}}(x) = \sum_{j} x_{ij} J_{\text{in}}(i,j)$$
, where

$$J_{\rm in}(i,j) = \frac{\sum_h x_{hi} x_{hj}}{x_{+i} + x_{+j} - \sum_h x_{hi} x_{hj}}$$

(where again 0/0 is taken as 0).

18. number of distances two effect (nbrDist2), defined by the number of actors to whom i is indirectly tied (through at least one intermediary, i.e., at sociometric distance 2),

$$s_{i18}^{\text{net}}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih} x_{hj}) > 0\};$$

endowment effect only likelihood-based because the Method of Moments estimators for endowment effects are based on the 'loss' associated with terminated ties, and this cannot be straightforwardly applied for the number of distances two effect.

This effect is difficult to interpret: actor i can increase this effect by making a new tie to an actor j at distance 2 but also by dropping a direct tie to an actor j to whom i is also connected via a two-path  $i \to h \to j$ .

When using this effect to represent a particular theoretical mechanism, it may be advisable to use it as separate creation and endowment effects (using type=creat and type=endow), to avoid this ambiguity. For representing network closure it may be preferable to use one of the many other effects representing it (transTrip, transTies, the various gwesp effects, the various Jaccard effects, etc.).

19. number of doubly achieved distances two effect (nbrDist2twice), defined by the number of actors to whom i is not directly tied, and tied through two paths via at least two intermediaries,

$$s_{i19}^{\text{net}}(x) = \#\{j \mid x_{ij} = 0, \sum_{h} (x_{ih} x_{hj}) \ge 2\};$$

evidently, this is even more difficult to interpret than (nbrDist2); both effects should be used if there are very specific and convincing reasons to include them in the model;

endowment effect only likelihood-based;

20. number of dense triads (denseTriads), defined as triads containing at least p ties,  $s_{i20}^{\text{net}}(x) = \sum_{j,h} x_{ij} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj} \} \ge p\}$ , where the 'indicator function'  $I\{A\}$  is 1 if the condition A is fulfilled and 0 otherwise,

and where p is either 5 or 6; (this effect is superfluous and undefined for symmetric networks);

21. five variations of the GWESP (geometrically weighted edgewise shared partners) effects: gwespFF, gwespBB, gwespFB, gwespBF, gwespRR, and for non-directed W networks the sixth version gwesp.

Note that there is a difference since version 1.1-251; see at the end of this item.

These are effects like those developed for exponential random graph models ('ERGMs') by Snijders et al. (2006), in the parametrisation of Hunter (2007). The five versions relate to the various directionalities of the ties, just like the proposals for directed networks in Robins et al. (2009).

The gwespFF effect is an alternative expression for transitivity. This concept here is specified in an actor-based way, by counting configurations in the local neighbourhood of a given actor, rather than in the tie-oriented way of the models in the ERGM family, for which the GWESP statistic was first developed. The actor-based gwespFF effect is defined, in direct analogy to the corresponding global statistic of Hunter (2007), by

$$GWESPFF(i,\alpha) = \sum_{k=1}^{n-2} e^{\alpha} \left\{ 1 - \left( 1 - e^{-\alpha} \right)^k \right\} EPFF_{ik} , \qquad (18a)$$

where EPFF<sub>ik</sub> (for 'edgewise partners') is the number of nodes j such that  $i \to j$  and there are exactly k other nodes h for which there is the two-path  $i \to h \to j$ .

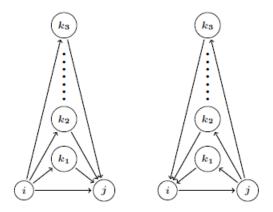


Figure 2: Geometrically weighted edgewise shared partners; left, transitive ('gwespFF'); right, cyclical ('gwespBB'); for F='forward', B='backward'.

An equivalent way of writing this is

GWESPFF
$$(i, \alpha) = \sum_{j=1}^{n} x_{ij} e^{\alpha} \left\{ 1 - \left( 1 - e^{-\alpha} \right)^{\sum_{h=1}^{n} x_{ih} x_{hj}} \right\} ,$$
 (18b)

where the convention is used that  $x_{jj} = 0$  for all j.

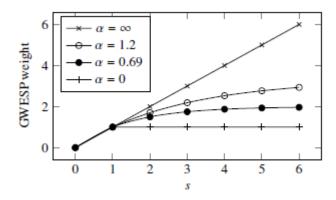


Figure 3: GWESP weight for a tie closing s two-paths for  $\alpha = \infty, 1.2, 0.69, 0.$ 

The parameter  $\alpha$  is a tuning parameter that may range from 0 to  $\infty$ . The internal effect parameter is defined as  $100 \times \alpha$ . For all  $\alpha$ , it holds that GWESP $(0,\alpha) = 0$ , GWESP $(1,\alpha) = 1$ , and GWESP $(k,\alpha)$  increases with k to a maximum slightly less than  $e^{\alpha}$ . For  $\alpha = 0$  the coefficients  $e^{\alpha} \{1 - (1 - e^{-\alpha})^k\}$  are equal to 1 for all  $k \geq 1$ , and for  $\alpha \to \infty$  they tend to k. Since we can write

$$\sum_{i,h} x_{ih} x_{hj} x_{ij} = \sum_{k=1}^{n-2} k \operatorname{EP}_{ik} ,$$

this implies that for  $\alpha \to \infty$  the regular number of transitive triplets is approached, while for smaller  $\alpha$  the extra contribution of a high number of intermediaries h is downweighted. An often used value is  $\alpha = \log(2) = 0.69$  (Snijders et al., 2006), corresponding to an internal effect parameter of 69,

### myeffects <- setEffect(myeffects, gwespFF, parameter=69)</pre>

but it is worthwhile to try out different values of  $\alpha$  to see which one gives the best fit.

Although the fit statistic of the GWESP effect is identical to that for transitive ties for  $\alpha=0$  and approximates the fit statistic for transitive triplets for large  $\alpha$ , the estimates are not the same because some other calculations are done differently. The issue is that the GWESP effects are not implemented as an evaluation effect, but as an elementary effect, where for the change statistic only changes of the tie  $i \to j$  in (18b) are considered, and not of the tie  $i \to h$ .

Thus, the GWESP effects are defined in RSiena as the elementary effects

$$s_{i21}^{\text{el}}(x) = x_{ij} e^{\alpha} \left\{ 1 - \left( 1 - e^{-\alpha} \right)^{\sum_{h=1}^{n} x_{ih} x_{hj}} \right\} . \tag{19}$$

It should be noted that although the GWESP statistic is not triadic but depends on higher-order configurations, still it is actor-oriented in the sense that only those configurations are considered that are part of the personal network, i.e., the set of actors immediately connected to the focal actor i.

The other types of GWESP effect are analogous, with different tie orientations. They are defined as follows:

gwespBB: uses EPBB<sub>ik</sub>, counting the number of nodes j with  $i \to j$  and there are exactly k other nodes h for which there is the two-path  $i \leftarrow h \leftarrow j$ ;

gwespFB: uses EPFB<sub>ik</sub>, counting the number of nodes j with  $i \to j$  and there are exactly k other nodes h for which there is the two-out-star  $i \to h \leftarrow j$ ;

gwespBF: uses EPBF<sub>ik</sub>, counting the number of nodes j with  $i \to j$  and there are exactly k other nodes h for which there is the two-in-star  $i \leftarrow h \to j$ ;

(the last two were incorrect in the manual before 23/06/2017)

gwespRR: uses EPRR<sub>ik</sub>, counting the number of nodes j with  $i \to j$  and there are exactly k other nodes h for which there are the reciprocal ties  $i \rightleftharpoons h \rightleftharpoons j$ .

For further interpretations and distinctions between the different directionalities in these effects, see Robins et al. (2009).

In version 1-1.251 the implementation was changed (thanks to Nynke Niezink), because earlier versions were not quite according to what is described above. The effect was until version 1-1.250 implemented as

$$c_1 + c_2 \times \text{GWESPFF}(i, \alpha')$$

for values  $c_1(\alpha)$  and  $c_2(\alpha)$  not dependent on x, and with positive parameters  $\alpha$ ,  $\alpha'$  related according to  $\exp(-\alpha) + \exp(-\alpha') = 1$ . Note that for the default value  $\alpha = \log(2)$  corresponding to the effect parameter 69 (see above),  $\alpha = \alpha'$ .

22. two variations of the GWDSP (geometrically weighted dyadwise shared partners) effects: gwdspFF and gwdspFB. These are defined, respectively, by

$$GWDSPFF(i,\alpha) = \sum_{h=1: h \neq i}^{n} e^{\alpha} \left\{ 1 - \left( 1 - e^{-\alpha} \right)^{\sum_{j=1}^{n} x_{ij} x_{jh}} \right\} , \qquad (20)$$

and

$$GWDSPFB(i,\alpha) = \sum_{h=1; h \neq i}^{n} e^{\alpha} \left\{ 1 - \left( 1 - e^{-\alpha} \right)^{\sum_{j=1}^{n} x_{ij} x_{hj}} \right\} . \tag{21}$$

It may be noted these are of a quite different nature than most other effects, with the outgoing tie variables from i tucked far away in the formula. These effects are somewhat similar with respect to what they mean for the networks as outPop and inPop, respectively.

Like the gwesp effects, these are similar to the effects developed for exponential random graph models ('ERGMs') by Snijders et al. (2006), in the parametrisation of Hunter (2007). They are motivated because they are natural companions to gwespFF and gwespFB, as the conditions for the transitive closure expressed by the latter two

effects.

These effects are implemented as regular effects, not as elementary effects.

23. number of (unilateral) peripheral relations to dense triads,

$$s_{i23}^{\text{net}}(x) = \sum_{j,h,k} x_{ij} (1 - x_{ji}) (1 - x_{hi}) (1 - x_{hi}) I\{(x_{jh} + x_{hj} + x_{jk} + x_{kj} + x_{hk} + x_{kh}) \ge p\},$$

where p is the same constant as in the *dense triads* effect;

for symmetric networks, the 'unilateral' condition is dropped, and the definition is  $s_{i23}^{\text{net}}(x) = \sum_{i,h,k} x_{ij} (1 - x_{hi}) (1 - x_{ki}) I\{(x_{jh} + x_{hj} + x_{jk} + x_{kj} + x_{hk} + x_{kh}) \ge p\};$ 

24. in-degree related popularity effect (inPop) (earlier called popularity or popularity of alter effect), defined by the sum of the in-degrees of the others to whom i is tied,

$$s_{i24}^{\text{net}}(x) = \sum_{j} x_{ij} x_{+j} = \sum_{j} x_{ij} \sum_{h} x_{hj} = \sum_{j} x_{ij} (\sum_{h \neq i} x_{hj} + 1)$$

 $s_{i24}^{\rm net}(x)=\sum_j x_{ij}\,x_{+j}=\sum_j x_{ij}\sum_h x_{hj}=\sum_j x_{ij}(\sum_{h\neq i} x_{hj}+1);$  in RSiena 3 until version 3.313, this effect was multiplied by a factor 1/n;

in RSiena this effect has had a bug until version 1.1-219:

in RSiena the target statistic for this effect was multiplied by a factor n until version 1.1-241;

25. in-degree related popularity effect (centered) (inPop.c), defined by the sum of the centered in-degrees of the others to whom i is tied,

$$s_{i25}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{+j} - d),$$

where d is the average in-degree across all waves;

26. in-degree related popularity (sqrt) effect (inPopSqrt) (earlier called popularity of alter (sqrt measure) effect), defined by the sum of the square roots of the in-degrees of the others to whom i is tied,

$$s_{i26}^{\text{net}}(x) = \sum_{i} x_{ij} \sqrt{x_{+j}} = \sum_{i} x_{ij} \sqrt{\sum_{h} x_{hj}}$$

 $s_{i26}^{\rm net}(x) = \sum_j x_{ij} \sqrt{x_{+j}} = \sum_j x_{ij} \sqrt{\sum_h x_{hj}};$  this often works better in practice than the raw popularity effect; also it is often reasonable to assume that differences between high in-degrees are relatively less important than the same differences between low in-degrees;

27. in-degree related popularity dyadic effect (inPop\_dya), defined as the elementary effect version of the half-centered squared in-degree of the other to whom i is tied,  $f_{i27}^{\text{net, el}}(x) = (x_{+j} - \bar{x}_{+.}),$ 

$$\bar{x}_{+.} = \frac{1}{MK} \sum_{m=1}^{M} \sum_{i,j} x_{ij}$$
,

where  $\bar{x}_{+.} = \frac{1}{MK} \sum_{m=1}^{M} \sum_{i,j} x_{ij}$ , with K the number of j (for one-mode networks this is n, for two-mode networks this is the number of nodes in the second mode);

for internal effect parameter p=2, it is defined by

$$f_{i27}^{\text{net, el}}(x) = \sqrt{x_{+i}} - \sqrt{\bar{x}_{+.}}$$
;

since this is an elementary effect, it can be used in interactions and in endowment and creation functions;

28. out-degree related popularity effect (outPop) (earlier called activity or activity of alter effect), defined by the sum of the out-degrees of the others to whom i is tied,

$$s_{i28}^{\rm net}(x) = \sum_j x_{ij} x_{j+} = \sum_j x_{ij} \sum_h x_{jh};$$
 until version 3.313, this effect was multiplied by a factor  $1/n$ ;

29. out-degree related popularity effect (centered) (outPop.c), defined by the sum of the centered out-degrees of the others to whom i is tied,

$$s_{i29}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{j+} - d),$$

where d is the average out-degree across all waves;

30. out-degree related popularity (sqrt) effect (outPopSqrt) (earlier called activity of alter (sqrt measure) effect), defined by the sum of the square roots of the out-degrees of the others to whom i is tied,

$$s_{i30}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{x_{j+}} = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{jh}};$$

 $s_{i30}^{\rm net}(x) = \sum_j x_{ij} \sqrt{x_{j+}} = \sum_j x_{ij} \sqrt{\sum_h x_{jh}};$  this often works better in practice than the raw activity effect for the same reasons as mentioned above for the sqrt measure of the popularity effect;

31. reciprocal degree-related popularity effect (reciPop) defined by the sum of the reciprocal degrees of the others to whom i is tied,

$$s_{i31}^{\text{net}}(x) = \sum_{i} x_{ij} x_{i}^{(r)},$$

 $s_{i31}^{\rm net}(x) = \sum_j x_{ij} x_j^{(r)},$  where the reciprocal degree is defined by

$$x_j^{(r)} = \sum_h x_{jh} x_{hj}.$$

32. reciprocal degree-related popularity (sqrt) effect (reciPopSqrt) defined by the sum of the square roots of the reciprocal degrees of the others to whom i is tied,

$$s_{i32}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \sqrt{x_{j}^{(r)}},$$

 $s_{i32}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{x_{j}^{(r)}},$  where the reciprocal degree is defined as above;

• for non-directed networks, the popularity and activity effects are taken together as 'degree effects', since in-degrees and out-degrees are the same in this case;

33. in-degree related activity effect, (inAct) defined as the cross-product of the actor's in- and out-degrees,

$$s_{i33}^{\text{net}}(x) = x_{i+} x_{+i};$$

endowment effect only likelihood-based;

34. in-degree related activity equipment (inAct\_ego), defined as the elementary effect version of the half-centered cross-product of the actor's in- and out-degrees,

$$f_{i34}^{\text{net, el}}(x) = (x_{+i} - \bar{x}_{+.}),$$

where 
$$\bar{x}_{+.} = \frac{1}{Mn} \sum_{m=1}^{M} \sum_{i,j} x_{ij}$$
; for internal effect parameter  $p=2$ , it is defined by  $f_{i34}^{\text{net, el}}(x) = \sqrt{x_{+i}} - \sqrt{\bar{x}_{+.}}$ ;

since this is an elementary effect, it can be used in interactions and in endowment and creation functions;

35. in-degree related activity effect (centered) (inAct.c), defined by the sum of the cross-product of the actor's out- and centered in-degrees,

$$s_{i35}^{\text{net}}(x) = x_{i+} (x_{+i} - d),$$

where d is the average in-degree across all waves;

- 36. in-degree related activity (sqrt) effect, (inActSqrt) defined by  $s_{i36}^{\text{net}}(x) = x_{i+} \sqrt{x_{+i}}$ ;
- 37. in-isolate Outdegree effect, (inIsDegree), the (additional) out-degree (or activity) effect for actors with in-degree zero, defined as the out-degree but only if the actor has in-degree zero,

$$s_{i37}^{\text{net}}(x,z) = I\{x_{+i} = 0\} \sum_{i} x_{ij} ;$$

38. out-degree related activity effect (outAct), defined as the squared out-degree of the

$$s_{i38}^{\mathrm{net}}(x) = x_{i+}^2;$$

endowment effect only likelihood-based;

39. out-degree related activity ego effect (outAct\_ego), defined as the elementary effect version of the half-centered squared out-degree of the actor,

$$f_{i39}^{\text{net, el}}(x) = (x_{i+} - \bar{x}_{.+}),$$

$$\bar{x}_{.+} = \frac{1}{Mn} \sum_{m=1}^{M} \sum_{i,j} x_{ij} ;$$

 $\bar{x}_{.+} = \frac{1}{Mn} \sum_{m=1}^{M} \sum_{i,j} x_{ij}$ ; for internal effect parameter p = 2, it is defined by

$$f_{i39}^{\text{net, el}}(x) = \sqrt{x_{i+}} - \sqrt{\bar{x}_{.+}}$$
;

since this is an elementary effect, it can be used in interactions and in endowment and creation functions;

40. out-degree related activity effect (centered) (outAct.c), defined by the out-degree times the centered out-degree of the actor,

$$s_{i40}^{\text{net}}(x) = x_{i+} (x_{i+} - d),$$

where d is the average out-degree across all waves;

41. out-degree related activity (sqrt) effect (outActSqrt) (earlier called out-degree ^(1.5)), defined by

$$s_{i41}^{\text{net}}(x) = x_{i+}^{1.5} = x_{i+} \sqrt{x_{i+}}$$

 $s_{i41}^{\text{net}}(x) = x_{i+}^{1.5} = x_{i+} \sqrt{x_{i+}}$  endowment effect only likelihood-based;

42. reciprocal degree-related activity effect (reciAct) defined by the degree of i multiplied by i's reciprocal degree,

$$s_{i42}^{\text{net}}(x) = x_{i+} x_i^{(r)};$$

where the reciprocal degree is defined as above;

for internal effect parameter p = 2, it is defined by

$$s_{i42}^{\text{net}}(x) = x_{i+} \sqrt{x_i^{(r)}}$$
;

43. reciprocal degree related activity ego effect (reciAct\_ego), defined as the elementary effect version of the degree of i multiplied by i's centered reciprocal degree,

$$f_{i43}^{\text{net, el}}(x) = (x_i^{(r)} - \bar{x}^{(r)}),$$

where

$$\bar{x}^{(r)} = \frac{1}{Mn} \sum_{m=1}^{M} \sum_{i,j} x_{ij} x_{ji} ;$$

 $\bar{x}^{(r)} = \frac{1}{Mn} \sum_{m=1}^{M} \sum_{i,j} x_{ij} x_{ji}$ ; for internal effect parameter p=2, it is defined by

$$f_{i43}^{\text{net, el}}(x) = \sqrt{x_i^{(r)}} - \sqrt{\bar{x}^{(r)}}$$
;

since this is an elementary effect, it can be used in interactions and in endowment and creation functions;

44. out-degree up to p (truncated out-degree) (outTrunc), where p is some constant (internal effect parameter, see above), there are two implementations here: outTrunc and outTrunc2, to allow the simultaneous use of this effect with 2 different internal effect parameters; the effect is defined by

$$s_{i44}^{\text{net}}(x) = \min(x_{i+}, p);$$

note that for p = 1 this represents –inversely– the tendency to be an isolate with respect to outgoing ties, i.e., have out-degree equal to 0:  $\min(x_{i+}, 1) = 0$  if  $x_{i+} = 0$ , and  $\min(x_{i+}, 1) = 1$  if  $x_{i+} \ge 1$ .

Since the representation is inverse, a result like a negative coefficient -1.2 for outTrunc (internal effect parameter p=1) is interpreted as a positive tendency +1.2 toward outdegrees equal to 0; the standard error is unchanged. In the case of p=1, an alternative and more directly comprehensible name is outdegree at least 1

Note that the outlso effect (see below) is equivalent to outlrunc for p = 1, but with the opposite sign.

Compare what is written below about the in-isolate effects: the isolation effects are difficult to interpret in words or pictures, because they are not based on subgraph counts. This holds for the outTrunc(p=1) effect but, in modified form, also for the outTrunc effect generally, The interpretation can best be understood from the change statistics. The outTrunc effect has a change statistic of +1 when actor i creates a link, if the previous outdegree of i is less than p. For p=1, this means a change statistic of +1 for the creation of the first outgoing tie of i, and of -1 for the termination of the last outgoing tie.

- 45. out-isolate (outIso), defined by  $s_{i45}^{\text{net}}(x) = I\{x_{i+} = 0\};$ 
  - this is equivalent to outTrunc for p = 1, but with a sign reversal for the parameter;
- 46. out-degree more than p where p is some constant (internal effect parameter, see above), there are two implementations here: outMore, outMore2, and outMore3, to allow the simultaneous use of this effect with three different internal effect parameters; they are defined by

$$s_{i46}^{\text{net}}(x) = \max(x_{i+} - p, 0);$$

this can be regarded as the mirror image of outTrunc;

47. square root out-degree, defined by

$$s_{i47}^{\text{net}}(x) = \sqrt{x_{i+}};$$

this is left out in later versions of SIENA;

- 48. squared (out-degree p), where p is some constant, defined by  $s_{i48}^{\text{net}}(x) = (x_{i+} p)^2$ , where p is chosen to diminish the collinearity between this and the density effect;
- 49.  $sum\ of\ (1/(out\text{-}degree + p)\ (outInv), where p is some constant, defined by <math>s_{i49}^{\text{net}}(x) = 1/(x_{i+} + p);$  endowment effect only likelihood-based;

this is left out in later versions of SIENA;

- 50.  $sum\ of\ (1/(out\text{-}degree+p)(out\text{-}degree+p+1))\ (outSqInv),\ where\ p\ is\ some\ constant,\ defined\ by <math display="block">s_{i50}^{\rm net}(x)=1/(x_{i+}+p)(x_{i+}+p+1);$  endowment effect only likelihood-based.
- 51. average out-degree (X: av. degree) (avDeg) defined by the out-degree of i multiplied by the average outdegree centered by the internal effect parameter p

$$s_{i51}^{\text{net}}(x) = \sum_{j} x_{ij} \left(\frac{1}{n} \sum_{h} x_{h+} - p\right) = x_{i+} \left(\frac{1}{n} \sum_{h} x_{h+} - p\right)$$
; this effect is a network-by-period level effect and therefore is useful only for multi-group data sets or for data sets with many periods; it may be beneficial for convergence to use a value of  $p$  that is close to the average degree (but constant, while this effect is for the dynamic average outdegree);

- $\bigcirc$  the following assortativity effects are, for p=1, various orientations of three-paths (plus a bit of two-paths):
- 52. out-out degree (1/p) assortativity (outOutAss), which represents the differential tendency for actors with high out-degrees to be tied to other actors who likewise have high out-degrees,

$$s_{i52}^{\rm net}(x) = \sum_j x_{ij} x_{i+}^{1/p} x_{j+}^{1/p};$$
 p can be 1 or 2 (the latter value is the default);

53. out-in degree (1/p) assortativity (outInAss), which represents the differential tendency for actors with high out-degrees to be tied to other actors who have high in-degrees,

$$s_{i53}^{\text{net}}(x) = \sum_{j} x_{ij} x_{i+}^{1/p} x_{+j}^{1/p};$$
  
  $p$  can be 1 or 2 (the latter value is the default);

54. in-out degree (1/p) assortativity (inOutAss), which represents the differential tendency for actors with high in-degrees to be tied to other actors who have high out-degrees,

$$s_{i54}^{\text{net}}(x) = \sum_{j} x_{ij} x_{+i}^{1/p} x_{j+}^{1/p};$$
  
  $p$  can be 1 or 2 (the latter value is the default);

55. in-in degree (1/p) assortativity (inInAss), which represents the differential tendency for actors with high in-degrees to be tied to other actors who likewise have high in-degrees,

$$s_{i55}^{\text{net}}(x) = \sum_{j} x_{ij} x_{+i}^{1/p} x_{+j}^{1/p};$$
  
p can be 1 or 2 (the latter value is the default);

56. network-isolate effect, (isolateNet), the effect of ego having in-degree as well as out-degree zero, i.e., being a total isolate,

$$s_{i56}^{\text{net}}(x,z) = I\{x_{+i} = x_{i+} = 0\}$$
;

57. anti isolates effect, (antiIso), the effect of wishing to connect to others who otherwise would be a total isolate, i.e., have no incoming or outgoing ties, and wishing not to sever connections to others who thereby would become a total isolate,

$$s_{i57}^{\text{net}}(x) = \sum_{i} I\{x_{+i} \ge 1, x_{j+} = 0\};$$

58. anti in-isolates effect, (antiInIso), the effect of wishing to connect to others who otherwise would have no incoming ties, and wishing not to sever connections to others who thereby would lose their last incoming connection:

$$s_{i58}^{\text{net}}(x) = \sum_{i} I\{x_{+i} \ge 1\};$$

the isolation effects are difficult to interpret in words or pictures, because they are not based on subgraph counts, and the in-isolation effects are especially difficult. The interpretation can best be understood from the change statistics. The anti inisolates effect has a change statistic of +1 when actor i creates a link to some other actor who previously had indegree 0; and (by implication) a change statistic of -1 when the actor was the only one to link to some other actor and now drops the link;

59. anti in-near-isolates effect = indegree at least 2 effect, (antiInIso2 = in2Plus), the effect of wishing to make a new connection to others who currently have an indegree equal to 1, and wishing not to sever connections to others who currently have an indegree equal to 2:

$$s_{i59}^{\text{net}}(x) = \sum_{j} I\{x_{+j} \ge 2\};$$

the anti in-near-isolates effect has a contribution of +1 when actor i creates a link to some actor who previously had indegree 1, and (by implication) -1 when the actor drops a link to some other actor who before this had indegree +2;

60. indegree at least 3 effect, (in3Plus), the effect of wishing to make a new connection to others who currently have an indegree equal to 2, and wishing not to sever connections to others who currently have an indegree equal to 3:

$$s_{i60}^{\text{net}}(x) = \sum_{j} I\{x_{+j} \ge 3\};$$

61. isolate popularity effect, (isolatePop), the effect of being tied to actors who further are isolates (the fact that such a tie does exist will give the other actor an in-degree of 1),

$$s_{i61}^{\text{net}}(x) = \sum_{i} x_{ij} I\{x_{+j} = 1, x_{j+} = 0\}$$
.

Note that perhaps this effect is of limited use, as other (third) actors might increase

the indegree of j to more than 1, and then the ex-isolate does not contribute any more to i's evaluation of the network; the three effects above ( 'anti isolates', 'anti in-isolates', and 'anti in-near-isolates') may be more useful instead.

Note that the network-isolate effect expresses the tendency for ego to be an isolate (not sending ties if ego has indegree 0), whereas the in-isolate and isolate popularity effect express the tendency for ego to connect to others who, without this connection, would have an indegree of 0, or be total isolates, respectively. Thus, in modeling the number of isolates, for the network-isolate effect the agency is in the isolate (see the i in the formula), whereas for the various anti isolates and the isolate popularity effects the agency is in others connecting (or not) to the isolate (see the j in the formulae).

62. only in RSienaTest: Simmelian outdegree effect, (simmelian), the effect of the Simmelian outdegree. The Simmelian transformation of the network is the network composed of all ties embedded in at least one complete triad:

$$x_{ij}^{\text{simm}} = \begin{cases} 1 & x_{ij} = x_{ji} = 1 \text{ and} \\ & \text{there is at least one } h \text{ for which } x_{hi} = x_{ih} = x_{hj} = x_{jh} = 1 \\ 0 & \text{else.} \end{cases}$$

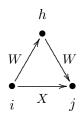
In other words, these ties must be reciprocal, and there must be at least one third actor to whom both have a mutual tie. The Simmelian outdegree of i is  $s_{i62}^{\rm net}(x) = \sum_j x_{ij}^{\rm simm}$ .

#### Dyadic covariate effects

The effects for a dyadic covariate  $w_{ij}$  are

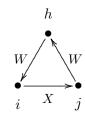
- 63. covariate (centered) main effect (X),  $s_{i63}^{\rm net}(x) = \sum_{j} x_{ij} (w_{ij} \bar{w})$  where  $\bar{w}$  is the mean value of  $w_{ij}$ ;
- 64. covariate (centered) × reciprocity (XRecip),  $s_{i64}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} (w_{ij} \bar{w}).$
- $\odot$  Various different ways can be modeled in which a triadic combination can be made between the dyadic covariate and the network. In the explanation, the dyadic covariate is regarded as a weighted network (which will be reduced to a non-weighted network if  $w_{ij}$  only assumes the values 0 and 1). By way of exception, the dyadic covariate is not centered in these effects (to make it better interpretable as a network). In the text and the pictures, an arrow with the letter W represents a tie according to the weighted network W.

65. WW => X closure of covariate (WWX),  $s_{i65}^{\rm net}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hj}$ ; this refers to the closure of W-W two-paths; each W-W two-path  $i \stackrel{W}{\to} h \stackrel{W}{\to} j$  is weighted by the product  $w_{ih}w_{hj}$  and the sum of these product weights measures the strength of the tendency toward closure of these W-W two-paths by a tie.

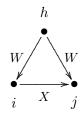


Since the dyadic covariates are represented by square arrays and not by edgelists, this and the following effects will be relatively time-consuming if the number of nodes is large.

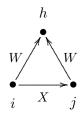
66. mixed cyclic WW => X closure, (X: cyclic closure of W) (cyWWX)  $s_{i66}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{jh} w_{hi};$  this refers to the cyclic closure of W-W two-paths (weighted if the dyadic covariate W does not only have 0 and 1 values); the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of product of weights of W-W two-paths  $j \xrightarrow{W_{jh}} h \xrightarrow{W_{hi}} i;$ 



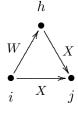
67. incoming shared WWX, (X: incoming shared W) (InWWX)  $s_{i67}^{\rm net}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj};$  this refers to shared incoming W ties contributing to the tie  $i \stackrel{X}{\to} j;$ 



68. incoming shared WWX, (X: incoming shared W) (OutWWX)  $s_{i68}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh};$  this refers to shared outgoing W ties contributing to the tie  $i \xrightarrow{X} j;$ 

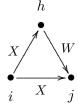


69. WX => X closure of covariate (WXX),  $s_{i69}^{\rm net}(x) = \sum_{j \neq h} x_{ij} \, w_{ih} \, x_{hj}$ ; this refers to the closure of mixed W-X two-paths; each W-Xtwo-path  $i \stackrel{W}{\rightarrow} h \rightarrow j$  is weighted by  $w_{ih}$  and the sum of these weights measures the strength of the tendency toward closure of these mixed W - X two paths by a tie; (this effect is implemented as an elementary effect; it may be noted that the corresponding evaluation effect is the (WXX) ef-



70. XW => X closure of covariate (XWX),  $s_{i70}^{\rm net}(x)=\sum_{j\neq h}x_{ij}\,x_{ih}\,w_{hj}$  ; this refers to the closure of mixed X-W two-paths; each X-Wtwo-path  $i \to h \stackrel{W}{\to} j$  is weighted by  $w_{hj}$  and the sum of these weights measures the strength of the tendency toward closure of these mixed X - W two paths by a tie.

fect for W + t(W), where t(W) is the transpose of W;



The covariate W here always is used non-centered.

There are two partial variants of this effect; they can be distinguished not by the Method of Moments, but only by Maximum Likelihood and Bayesian estimation.

71. XW = X closure-1 of covariate (XWX1),

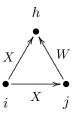
This is an elementary effect, not an evaluation effect, comprising of the 'XW = Xclosure of covariate' effect only the contribution of the the number of weighted X-Wtwo-paths  $i \xrightarrow{X} h \xrightarrow{W} j$ . In other words, only the  $i \to j$  tie in the figure above is the dependent variable here. The effect is defined as

$$s_{i71}^{\text{el}}(x) = x_{ij} \sum_{h;h\neq j} x_{ih} w_{hj};$$

 $s_{i71}^{\rm el}(x) = x_{ij} \sum_{h;h \neq j} x_{ih} w_{hj}$ ; The covariate W here always is used non-centered.

72. XW = X closure-2 of covariate (XWX2),

This is an elementary effect, not an evaluation effect, comprising of the 'XW = X closure of covariate' effect only the contribution of the number of weighted X - W two-in-stars  $i \stackrel{X}{\to} h$ ,  $j \stackrel{W}{\to} h$ . In other words, only the  $i \to j$  tie in the figure here to the right is the dependent variable. The effect is defined as



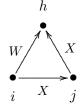
 $s_{i72}^{\rm el}(x)=x_{ij}\sum_{h;h\neq j}x_{ih}\,w_{jh}$  . The covariate W here always is used non-centered.

73. covariate-weighted twopaths (XXW),

$$s_{i73}^{\text{nec}}(x) = \sum_{j \neq h} x_{ij} \, x_{jh} \, w_{ih}$$

 $s_{i73}^{\rm net}(x)=\sum_{j\neq h}x_{ij}\,x_{jh}\,w_{ih}$  ; this refers to two\_paths weighted by the covariate: each X-Xtwo-path  $i \stackrel{X}{\to} j \stackrel{X}{\to} h$  is weighted by the value  $w_{ih}$  for the pair of nodes at the start and the end of the two-path.

The covariate W here always is used non-centered.



## Monadic covariate effects

For actor-dependent covariates  $v_i$  (recall that these are centered internally by SIENA, unless they are constructed with centered = FALSE), as well as for dependent behavior variables (for notational simplicity here also denoted  $v_i$ ; these variables also are centered), the following effects are available:

74. covariate-alter or covariate-related popularity (altX), defined by the sum of the covariate over all actors to whom i has a tie,

$$s_{i74}^{\text{net}}(x) = \sum_{j} x_{ij} v_j;$$

75. covariate squared - alter or squared covariate-related popularity (altSqX), defined by the sum of the squared covariate over all actors to whom i has a tie, (not included if the variable has range less than 2)

$$s_{i75}^{\text{net}}(x) = \sum_{j} x_{ij} v_{j}^{2};$$

76. right threshold covariate alter (altRThresholdX), defined by the number of actors to whom i has a tie and whose covariate value is greater than (to the right of) or equal to a threshold p,

$$s_{i76}^{\text{net}}(x) = \sum_{i} x_{ij} I\{v_j \ge p\};$$

for an actor covariate this (and the following effect) can also be obtained as altX for a transformed covariate, but this would be impossible for a behavioral dependent variable V;

note that behavioral dependent variables always are centered!

77. left threshold covariate alter (altLThresholdX), defined by the number of actors to whom i has a tie and whose covariate value is smaller than (to the left of) or equal to a threshold p,

$$s_{i77}^{\text{net}}(x) = \sum_{i} x_{ij} I\{v_j \le p\};$$

 $s_{i77}^{\rm net}(x) = \sum_j x_{ij} I\{v_j \leq p\};$  note that behavioral dependent variables always are centered; i.e., the overall mean is subtracted!

78. outdegree to actors with positive covariate more than p, outXMore, where p is some constant (internal effect parameter, see above), defined by

$$s_{i78}^{\text{net}}(x) = \max((\sum_{j} x_{ij} I\{v_j > 0\}) - p, 0);$$

this is meant primarily for non-centered dummy variables  $v_i$ ;

79. covariate-eqo or covariate-related activity (egoX), defined by i's out-degree weighted by his covariate value,

$$s_{i79}^{\text{net}}(x) = v_i x_{i+};$$

80. covariate squared - ego or squared covariate-related activity (egoSqX), defined by i's out-degree weighted by his covariate value squared,

$$s_{i80}^{\text{net}}(x) = v_i^2 x_{i+};$$

81. right threshold covariate ego (egoRThresholdX), defined by i's out-degree when i's covariate value is greater than (to the right of) or equal to a threshold p,

$$s_{i81}^{\text{net}}(x) = I\{v_i \ge p\} x_{i+};$$

for an actor covariate this (and the following effect) can also be obtained by transforming the covariate, but this would be impossible for a behavioral dependent variable V;

note that behavioral dependent variables always are centered!

82. left threshold covariate ego (egoLThresholdX), defined by i's out-degree when i's covariate value is less than (to the right of) or equal to a threshold p,

$$s_{i \otimes 2}^{\text{net}}(x) = I\{v_i \leq p\} x_{i+};$$

note that behavioral dependent variables always are centered!

83. absolute difference outdegree – covariate (degAbsDiffX), defined by the absolute difference between i's out-degree and his covariate value,

$$s_{i83}^{\text{net}}(x) = |x_{i+} - \lfloor v_i \rfloor|;$$

here  $\lfloor v_i \rfloor$  is  $v_i$  truncated to the largest integer not larger than  $v_i$ ; working with a non-centered covariate may be advisable for this effect;

84. positive difference outdegree – covariate (degPosDiffX), defined by the positive part of the difference between i's out-degree and her covariate value,

$$s_{i84}^{\text{net}}(x) = \max\{x_{i+} - \lfloor v_i \rfloor, 0\};$$

here  $\lfloor v_i \rfloor$  is  $v_i$  truncated to the largest integer not larger than  $v_i$ ; working with a non-centered covariate may be advisable for this effect;

85. negative difference outdegree – covariate (degNegDiffX), defined by the negative part of the difference between i's out-degree and her covariate value,

$$s_{i,85}^{\text{net}}(x) = -\min\{x_{i+} - |v_i|, 0\} = \max\{|v_i| - x_{i+}, 0\};$$

here  $\lfloor v_i \rfloor$  is  $v_i$  truncated to the largest integer not larger than  $v_i$ ; working with a non-centered covariate may be advisable for this effect;

86. covariate-related similarity (simX), defined by the sum of centered similarity scores  $\sin_{ij}^v$  between i and the other actors j to whom he is tied,

$$s_{i86}^{\text{net}}(x) = \sum_{i} x_{ij} (\sin_{ij}^{v} - \widehat{\sin^{v}}),$$

where  $\widehat{\sin^v}$  is the mean of all similarity scores, which are defined as  $\sup_{ij}^v = \frac{\Delta - |v_i - v_j|}{\Delta}$  with  $\Delta = \max_{ij} |v_i - v_j|$  being the observed range of the covariate v (this mean is given in the output file just before the 'initial data description'; it is also given, e..g., for data set mydata and constant covariate mycov, by attr(mydata\$cCovars\$mycov, "simMean"));

the definition of  $sim_{ij}^{v}$  is discussed in section 4.2, see (1);

87. covariate-difference or covariate-related difference (diffX), defined by the alterminus-ego difference of the covariate over all actors to whom i has a tie,

$$s_{i87}^{\text{net}}(x) = \sum_{j} x_{ij} (v_j - v_i);$$

88. absolute covariate-difference or covariate-related absolute difference (absDiffX), defined by the absolute value of the alter-ego difference of the covariate over all actors

to whom 
$$i$$
 has a tie,  

$$s_{i88}^{\text{net}}(x) = \sum_{i} x_{ij} | v_i - v_i |;$$

89. covariate-squared-difference or covariate-related squared difference (diffSqX), defined by the squared alter-minus-ego difference of the covariate over all actors to whom i has a tie,

$$s_{i89}^{\text{net}}(x) = \sum_{j} x_{ij} (v_j - v_i)^2;$$

90. covariate-ego  $\times$  difference or covariate-related ego-difference interaction (egoDiffX), defined by ego's value times the alter-minus-ego difference of the covariate over all actors to whom i has a tie,

$$s_{i90}^{\text{net}}(x) = \sum_{j} x_{ij} v_i (v_j - v_i);$$

91.  $covariate-ego \times alter$  (egoXaltX), defined by the product of i's covariate and the sum of those of his alters,

$$s_{i91}^{\text{net}}(x) = v_i \sum_j x_{ij} v_j;$$

92.  $covariate-ego \times alter \times reciprocity$  (egoXaltXRecip), defined by the product of i's covariate and the sum of those of his reciprocated alters,

$$s_{i92}^{\text{net}}(x) = v_i \sum_j x_{ij} x_{ji} v_j;$$

93. covariate-related similarity  $\times$  reciprocity (simRecipX), defined by the sum of centered similarity scores for all reciprocal dyads in which i is situated,

$$s_{i93}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{v} - \sin^{v});$$

94. same covariate, which can also be called covariate-related identity (sameX), defined by the number of ties of i to all other actors j who have exactly the same value on the covariate,

$$s_{i94}^{\rm net}(x) = \sum_j x_{ij} I\{v_i = v_j\},$$

where the indicator function  $I\{v_i = v_j\}$  is 1 if the condition  $\{v_i = v_j\}$  is satisfied, and 0 if it is not;

for equality between different variables for ego and alter, there is the **sameXV** effect, mentioned below;

- 95. same covariate  $\times$  reciprocity (sameXRecip) defined by the number of reciprocated ties between i and all other actors j who have exactly the same value on the covariate,  $s_{i95}^{\rm net}(x) = \sum_j x_{ij} x_{ji} I\{v_i = v_j\};$
- 96. different covariate (unequalX), defined by the number of ties of i to all other actors j who have a different value on the covariate,

$$s_{i96}^{\text{net}}(x) = \sum_{i} x_{ij} I\{v_i \neq v_j\},$$

where the indicator function  $I\{v_i \neq v_j\}$  is 1 if the condition  $\{v_i \neq v_j\}$  is satisfied, and 0 if it is not;

this is the converse of the sameX effect, and can be useful in interactions;

97. in-degree popularity weighted by sender's V (inPopX) defined by the sum of values of V for actors h for which there are ties  $i \to j \leftarrow h$ ,

i.e., an in-two-star; the value  $\sum_h x_{hj} v_h$  can be regarded as a V-weighted version of the indegree of j:

$$s_{i97}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{hj} v_h$$

or for 
$$p=2$$

$$s_{i97}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{hj} v_h};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered;

further note that if V is a behavioral variable, it will be used in the centered version, so that p must be 1; if weighting by a non-centered behavioral variable is desired, a new effect is necessary;

# 98. in-degree activity weighted by sender's V (inActX)

defined by the sum of values of V for actors h for which there are ties  $h \to i \to j$ , i.e., a two-path with i in the middle; the value  $\sum_h x_{hi} v_h$  can be regarded as a Vweighted version of the indegree of i:

$$s_{i98}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{hi} v_h$$
  
or for  $p = 2$ 

or for 
$$p=2$$

$$s_{i98}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{hi} v_{h}};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered;

# 99. out-degree popularity weighted by receiver's V (outPopX)

defined by the sum of values of V for actors h for which there are ties  $i \to j \to h$ , i.e., at out-distance 2; the value  $\sum_h x_{jh} v_h$  can be regarded as a V-weighted version of the outdegree of j:

$$s_{i99}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{jh} v_{h}$$
 or for  $p = 2$ 

or for 
$$p=2$$

$$s_{i99}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{jh} v_{h}};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered;

## 100. out-degree activity weighted by receiver's V (outActX)

defined by the sum of values of V for actors h for which there are ties  $h \leftarrow i \rightarrow j$ , i.e., an out-two-star; the value  $\sum_h x_{ih} v_h$  can be regarded as a V-weighted version of the outdegree of i:

$$s_{i100}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{ih} v_{h}$$
 or for  $p=2$ 

or for 
$$p=2$$

$$s_{i100}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{ih} v_h};$$

note that to use this effect for p = 2 the variable V must be nonnegative, which implies that it must be non-centered.

101. indegree popularity from same covariate (sameXInPop) defined by the number of incoming ties received by those to whom i is tied and sent by others who have the same covariate value as i,

$$s_{i101}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{hj} I\{v_i = v_h\};$$

for internal effect parameter 
$$p=2$$
, it is defined by  $s_{i101}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{hj}} I\{v_i = v_h\};$ 

102. indegree popularity from different covariate (diffXInPop) defined by the number of incoming ties received by those to whom i is tied and sent by others who have a different covariate value than i,

$$s_{i102}^{\rm net}(x) = \sum_j x_{ij} \sum_h x_{hj} I\{v_i \neq v_h\};$$
 for internal effect parameter  $p=2$ , it is defined by 
$$s_{i102}^{\rm net}(x) = \sum_j x_{ij} \sqrt{\sum_h x_{hj}} I\{v_i \neq v_h\};$$

103.  $outdegree\ activity\ to\ same\ covariate$  (sameXOutAct) defined by the squared number of ties to those others who have the same covariate value as i,

$$s_{i103}^{\text{net}}(x) = \left(\sum_{j} x_{ij} I\{v_i = v_j\}\right)^2;$$
 for internal effect parameter  $p = 2$ , it is defined by  $s_{i103}^{\text{net}}(x) = \sum_{j} x_{ij} I\{v_i = v_j\} \sqrt{\sum_{h} x_{ih} I\{v_i = v_h\}};$ 

104.  $outdegree\ activity\ to\ different\ covariate\ (diffXOutAct)\ defined\ by\ the\ squared\ number of\ ties\ to\ those\ others\ who\ have\ a\ different\ covariate\ value\ than\ i,$ 

$$\begin{array}{l} s_{i104}^{\rm net}(x) = \left(\sum_j x_{ij} \, I\{v_i \neq v_j\}\right)^2; \\ \text{for internal effect parameter } p = 2, \text{ it is defined by} \\ s_{i104}^{\rm net}(x) = \sum_j x_{ij} \, I\{v_i \neq v_j\} \sqrt{\sum_h x_{ih} \, I\{v_i \neq v_h\}}; \end{array}$$

105. outdegree activity different-same covariate (crossXOutAct) defined by the combination of ties to others who have a different covariate value than i and others who have the same covariate value as i. The internal effect parameter p has three values, depending on where the square root is taken. The effect is defined by

$$s_{i105}^{\text{net}}(x) = \sum_{j} x_{ij} I\{v_i = v_j\} \sum_{h} x_{ih} I\{v_i \neq v_h\},$$
  $(p=1)$ 

$$s_{i105}^{\text{net}}(x) = \sum_{j} x_{ij} I\{v_i = v_j\} \sqrt{\sum_{h} x_{ih} I\{v_i \neq v_h\}},$$
  $(p=2)$ 

$$s_{i105}^{\text{net}}(x) = \sum_{j} x_{ij} I\{v_i \neq v_j\} \sqrt{\sum_{h} x_{ih} I\{v_i = v_h\}} ;$$
  $(p=3)$ 

106. outdegree activity to homogeneous covariate (homXOutAct) defined by the sum of outgoing ties weighted by the number of outgoing ties to those with the same covariate value (as alter),

$$s_{i106}^{\text{net}}(x) = \sum_{i} x_{ij} \sum_{h} x_{ih} I\{v_h = v_j\};$$

107. outdegree activity to homogeneous covariate (homXOutAct2), almost the same as homXOutAct, but much more efficiently programmed, and only for covariates with integer values from 0 to 20; defined by the sum of outgoing ties weighted by the number of outgoing ties to those with the same covariate value (as alter), but it should be positive:

$$s_{i107}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{ih} I\{v_h = v_j > 0\};$$

for internal effect parameter p=2 it is defined by

$$s_{i107}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} x_{ih} I\{v_h = v_j > 0\}};$$

note that ties to alters with  $v_i = 0$  do not contribute anything;

108. outdegree activity weighted by alter's covariate (altXOutAct) defined by the squared sum of ties weighted by alter's covariate values,

$$s_{i108}^{\text{net}}(x) = \left(\sum_{j} x_{ij} v_j\right)^2;$$

 $s_{i108}^{\text{net}}(x) = \left(\sum_{j} x_{ij} v_{j}\right)^{2}$ ; since the covariate here is used as a weight, this probably makes sense especially for non-centered covariates;

109. reciprocal degree activity to same covariate (sameXReciAct) defined by the outdegree of i, multiplied by the number of i's reciprocated ties to others who have the same covariate value as i,

$$s_{i109}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} x_{ih} x_{hi} I\{v_i = v_h\};$$

110. reciprocal degree activity to different covariate (diffXReciAct) defined by the outdegree of i, multiplied by the number of i's reciprocated ties to others who have a different covariate value than i,

$$s_{i110}^{\text{net}}(x) = \sum_{i} x_{ij} \sum_{h} x_{ih} x_{hi} I\{v_i \neq v_h\};$$

111. only in RSienaTest: Simmelian alter X effect, (simmelianAltX), the effect of Simmelian ties weighted by alter's value of X. The Simmelian transformation of the network is the network composed of all ties embedded in at least one complete triad:

$$x_{ij}^{\text{simm}} = \begin{cases} 1 & x_{ij} = x_{ji} = 1 \text{ and} \\ & \text{there is at least one } h \text{ for which } x_{hi} = x_{ih} = x_{hj} = x_{jh} = 1 \\ 0 & \text{else.} \end{cases}$$

In other words, these ties must be reciprocal, and there must be at least one third actor to whom both have a mutual tie. The Simmelian alter V effect is  $s_{i111}^{\text{net}}(x) = \sum_{j} x_{ij}^{\text{simm}} v_j$ .

112. covariate × transitive triplets (transTripX), defined by the sum of transitive triplets  $i \to h \to j \leftarrow i$  weighted by the covariate value for the intermediate actor h,  $s_{i112}^{\text{net}}(x) = \sum_{j} v_h \, x_{ij} \, x_{ih} \, x_{hj};$ 

this is meaningful especially if covariate V is non-centered; else some weights would be negative, and the term 'weight' would not be appropriate. Note that weighting by the covariate values for i and/or j can be obtained by 'user-defined' interactions of transTrip with egoX and/or altX.

- 113. same covariate × transitive triplets (sameXTransTrip), defined by the number of transitive triplets  $i \to h \to j \leftarrow i$  that have the same covariate value for i and j,  $s_{i113}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ih} x_{hj} I\{v_i = v_j\};$
- 114. different covariate × transitive triplets (diffXTransTrip), defined by the number of transitive triplets  $i \to h \to j \leftarrow i$  that have different covariate values for i and j,  $s_{i114}^{\text{net}}(x) = \sum_{i} x_{ij} x_{ih} x_{hj} I\{v_i \neq v_j\};$

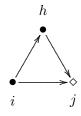
115. covariate-related similarity  $\times$  transitive triplets (simXTransTrip), defined by the sum of transitive triplets  $i \to h \to j \leftarrow i$  weighted by the centered similarity scores between i and j,

$$s_{i115}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ih} x_{hj} (\sin_{ij}^{v} - \widehat{\sin^{v}});$$

116. homogeneous covariate  $\times$  transitive triplets (homXTransTrip), defined by the number of transitive triplets  $i \to h \to j \leftarrow i$  that have the same covariate value for i, j, and h.

$$s_{i116}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ih} x_{hj} I\{v_i = v_j = v_h\};$$

117. transitive triplets jumping to different V (jumpXTransTrip),  $s_{i117}^{\rm net}(x) = \sum_{j \neq h} x_{ij} \, x_{ih} \, x_{hj} \, I\{v_i = v_h \neq v_j\};$  this refers to transitive closure, restricted to 'jump outside of V-groups' in the sense that the focal actor and the mediating actor have the same value of V, but the target actor has a different value;



118. same covariate  $\times$  transitive reciprocated triplets (sameXTransRecTrip), defined by the number of transitive reciprocated triplets  $i \to h \to j \leftrightarrow i$  that have the same covariate value for i and j,

$$s_{i118}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} x_{ih} x_{hj} I\{v_i = v_j\};$$

119. homogeneous covariate  $\times$  transitive reciprocated triplets (homXTransRecTrip), defined by the number of transitive reciprocated triplets  $i \to h \to j \leftrightarrow i$  for which the covariate values for i, j, h all are the same,

$$s_{i119}^{\text{net}}(x) = \sum_{i} x_{ij} x_{ji} x_{ih} x_{hj} I\{v_i = v_j = v_h\};$$

120. different covariate  $\times$  transitive reciprocated triplets (diffXTransRecTrip), defined by the number of transitive reciprocated triplets  $i \to h \to j \leftrightarrow i$  that have different covariate values for i and j,

$$s_{i120}^{\rm net}(x) = \sum_j x_{ij} \, x_{ji} \, x_{ih} \, x_{hj} \, I\{v_i \neq v_j\};$$

121. ego > alter for covariate (higher), defined by the number of ties where i's covariate is larger than alter's, while equality counts for half,

$$s_{i121}^{\text{net}}(x) = \sum_j x_{ij} \operatorname{dsign}(v_i - v_j),$$
  
where  $\operatorname{dsign}(d) = 0$  for  $d < 0, 0.5$  for  $d = 0$ , and 1 for  $d > 0$ .

122. covariate of indirect ties (IndTies), defined by the sum of the covariate over the actors to whom i is tied indirectly (at a geodesic distance of 2),

$$s_{i122}^{\text{net}}(x) = \sum_{j} (1 - x_{ij}) \left( \max_{h} x_{ih} x_{hj} \right) v_j;$$

123. number of four-cycles from same covariate (sameXCycle4), defined by the number of four-cycles sent by actors with the same covariate value,

$$s_{i123}^{\text{net}}(x) = \frac{1}{4} \sum_{j,k,h;\text{all different}} x_{ij} x_{ik} x_{hj} x_{hk} I\{v_i = v_h\};$$
 for parameter  $p = 2$  the square root is taken.

124. group average (avGroupEgoX), defined by ego's value multiplied by the average of the covariate for this period,

$$s_{i124}^{\text{beh}}(x,z) = \sum_{i} x_{ij} \, \overline{v};$$

here  $\overline{v}$  is the mean of the values  $v_{ih}$  for all actors h in the actor set: for changing covariates, the mean of the wave at the start of the period; for behavioral dependent variables, the current mean. This effect is not meaningful for constant covariates in single-group data. It is useful especially for multi-group data sets, where the average value varies between groups, and where there are no constant covariates in the multi-group object. Note that always, the average  $\overline{v}$  is centered by the global mean. For multi-group data, global centering means centering over all groups.

The following group of effects uses an auxiliary variable  $\check{v}_j$  which can be called 'alters' v-average'. It is described as the average value of  $v_h$  for those h to whom j is tied, and defined mathematically by

$$\tilde{v}_{j} = \begin{cases}
\frac{\sum_{h} x_{jh} v_{h}}{x_{j+}} & \text{if } x_{j+} > 0 \\
\bar{v} & \text{if } x_{j+} = 0,
\end{cases}$$
(22)

where  $\bar{v}$  is the overall observed mean of V (for changing actor covariates and behavioral variables, the mean over all waves).

(It may be noted that, for centered V, this constructed variable  $\check{v}_j$  will not itself have exactly a zero mean generally.)

Note that this value is being updated during the simulations. Network changes will change  $\check{v}_j$ ; if  $v_j$  is a dependent behavior variable, then behaviour changes will also change  $\check{v}_j$ .

In the following list, there is no ego effect, because the ego effect of  $v_j$  would be the same as the alter effect of  $v_j$ .

124. covariate - alter at distance 2 (altDist2). This effect is associated with an effect parameter which can have values 1 or 2. For parameter 1, it is defined as the sum of alters' covariate-average over all actors to whom i has a tie,

$$s_{i124}^{\text{net}}(x) = \sum_{i} x_{ij} \breve{v}_{j}$$
 (parameter 1)

For parameter 2, it is defined similarly, but for an alters' covariate-average excluding ego:

$$s_{i124}^{\text{net}}(x) = \sum_{j} x_{ij} \check{v}_{j}^{(-i)}$$
 (parameter 2)

where

To compute the contribution for this effect, note that

$$\sum_{j} x_{ij} \check{v}_{j}^{(-i)} = \sum_{j} x_{ij} \frac{x_{j+} \check{v}_{j} - x_{ji} v_{i}}{x_{j+} - x_{ji}}$$

This shows that, given that  $\check{v}_j$  is being updated for all j, the contribution for this effect for parameter 2 can be computed as

$$\frac{x_{j+}\breve{v}_j - x_{ji}v_i}{x_{j+} - x_{ji}}$$

(where 0/0 is interpreted as 0).

125. total covariate - alter at distance 2 (totDist2). This is like the previous effect, but using the total instead of the average value of alter's covariate. For parameter 1, it is defined as

$$s_{i125}^{\text{net}}(x) = \sum_{i} x_{ij} x_{j+} \breve{v}_j \qquad \text{(parameter 1)}$$

and for parameter 2, as

$$s_{i125}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{j+} - x_{ji}) \check{v}_{j}^{(-i)}$$
 (parameter 2)

where  $\check{v}_j$  and  $\check{v}_j^{(-i)}$  are as above.

126. ego-(alter-distance-2) covariate - similarity (simEgoDist2), defined as the sum of centered similarity between i and alters' covariate-average, for all actors j to whom i has a tie,

$$s_{i126}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \text{sim}(\hat{v})_{ij} - \widehat{\text{sim}^v} \right) ,$$

where the similarity scores  $sim(\grave{v})_{ij}$  are defined as

$$\sin(\grave{v})_{ij} = \frac{\Delta - |v_i - \breve{v}_j^{(-i)}|}{\Delta} ,$$

where  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate  $v, \check{v}_j^{(-i)}$  is as above in effect (altDist2) and  $\widehat{\text{sim}}^v$  is the mean of all similarity scores as used also for the simX effect; this centering is applied since version 1.1-285. For a constant covariate mycov, this mean is given by attr(mydata\$cCovars\$mycov, "simMean").

127. covariate - similarity at distance 2 (simDist2), defined as the sum of centered similarity values for alters' covariate-average between i and all actors j to whom i has a tie,

$$s_{i127}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \text{sim}(\check{v})_{ij} - \widehat{\text{sim}^{v}} \right) ,$$

where the similarity scores  $sim(\check{v})_{ij}$  are defined as

$$sim(\breve{v})_{ij} = \frac{\Delta - |\breve{v}_i - \breve{v}_j|}{\Delta} ,$$

while  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate v and  $\widetilde{\text{sim}^v}$  is the mean of all similarity scores as used also for the  $\widetilde{\text{sim}X}$  effect; this centering is applied since version 1.1-285. For a constant covariate  $\widetilde{\text{mycov}}$ , this mean is given by  $\widetilde{\text{attr}(\text{mydata}ccovars}, \$ "simMean").

Note that for both ego (i) and alter (j) their alters' covariate-average is used, so that this effect is about a comparison between the out-neighbourhoods of i and j.

128. ego-(alter-distance-2) same covariate (sameEgoDist2), defined as the sum, for all actors j to whom i has a tie, of the proportion of values  $v_h$  equal to  $v_i$  in their outneighbourhoods (i.e., for all actors h, other than i, with ties from j; configurations  $i \to j \to h$ ):

$$s_{i128}^{\rm net}(x) = \sum_{j;\, j \neq i,\, x_{j+} > x_{ij}} \frac{x_{ij}}{x_{j+} - x_{ij}} \, \sum_{h \neq i} x_{jh} \, I\{v_h = v_i\} \ ,$$

where 0/0 is defined as 0; for internal effect parameter p = 0 it is defined as the number of out-ties j who have any outgoing tie to an actor h with  $v_h = v_i$ :

$$s_{i128}^{\text{net}}(x) = \sum_{i} x_{ij} I\left\{\left(\sum_{h \neq i} x_{jh} I\{v_h = v_i\}\right) > 0\right\}.$$

129. covariate - in - alter at distance 2 (altInDist2). This is defined as the sum of alters' values for the average of the covariate values of all their incoming ties, except for the possibly incoming tie from ego:

$$s_{i129}^{\text{net}}(x) = \sum_{i} x_{ij} \check{v}_{j}^{(-i)}$$

where

$$\check{v}_{j}^{(-i)} = \begin{cases}
\frac{\sum_{h \neq i} x_{hj} v_{h}}{x_{+j} - x_{ij}} & \text{if } x_{+j} - x_{ij} > 0 \\
\bar{v} & \text{if } x_{+j} - x_{ij} = 0,
\end{cases}$$
(24)

where  $\bar{v}$  is (again) the global mean of V.

130. total covariate - in - alter at distance 2 (totInDist2). This is defined as the sum of alters' values for the total of the covariate values of all their incoming ties, except for the possibly incoming tie from ego:

$$s_{i130}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{+j} - x_{ij}) \check{v}_{j}^{(-i)} = \sum_{j} x_{ij} \sum_{h \neq i} x_{hj} v_{h}$$
.

For a non-centered binary variable V with values  $v_j = 0$  or 1, this is a V-restricted version of indegree popularity.

131. ego-(in-alter-distance-2) covariate - similarity (simEgoInDist2), defined as the sum of centered similarity between i and alters' covariate-in-average, for all actors j to whom i has a tie,

$$s_{i131}^{\text{net}}(x) = \sum_{j} x_{ij} \left( \operatorname{sim}(\acute{v})_{ij} - \widehat{\operatorname{sim}^{v}} \right) ,$$

where the similarity scores  $sim(\acute{v})_{ij}$  are defined as

$$sim(\acute{v})_{ij} = \frac{\Delta - |v_i - \check{v}_j^{(-i)}|}{\Delta} ,$$

while  $\check{v}_j^{(-i)}$  is as in the definition of altInDist2,  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate v, and  $\widehat{\sin}^v$  is as above (see simDist2).

Note that for ego (i) the own value is used and for alter (j) the alters' covariate-average (these are alter's in-alters!), so that this effect is about a comparison between i and the in-neighbourhoods of the j's.

132. ego-(in-alter-distance-2) same covariate (sameEgoInDist2), defined as the sum, for all actors j to whom i has a tie, of the proportion of values  $v_h$  equal to  $v_i$  in their inneighbourhoods (i.e., for all actors h, other than i, who send ties to j; configurations  $i \to j \leftarrow h$ ):

$$s_{i132}^{\text{net}}(x) = \sum_{j: j \neq i, x_{+j} > x_{ij}} \frac{x_{ij}}{x_{+j} - x_{ij}} \sum_{h \neq i} x_{hj} I\{v_h = v_i\} ,$$

where 0/0 is defined as 0; for internal effect parameter p = 0 it is defined as the number of out-ties j who have any incoming tie from an actor h with  $v_h = v_i$ :

$$s_{i132}^{\text{net}}(x) = \sum_{j} x_{ij} I\left\{\left(\sum_{h \neq i} x_{hj} I\{v_h = v_i\}\right) > 0\right\}.$$

133. ego-(in-alter-distance-2) same covariate (sameEgoInDist2), defined as the sum, for all actors j to whom i has a tie, of the proportion of values  $v_h$  equal to  $v_i$  in their inneighbourhoods (i.e., for all actors h, other than i, who send ties to j; configurations

$$i \rightarrow j \leftarrow h$$
):

$$s_{i133}^{\text{net}}(x) = \sum_{j; j \neq i, x_{+j} > x_{ij}} \frac{x_{ij}}{x_{+j} - x_{ij}} \sum_{h \neq i} x_{hj} I\{v_h = v_i\} ,$$

where 0/0 is defined as 0; for internal effect parameter p = 0 it is defined as the number of out-ties j who have any incoming tie from an actor h with  $v_h = v_i$ :

$$s_{i133}^{\rm net}(x) = \sum_j x_{ij} \, I\Big\{ \Big( \sum_{h \neq i} x_{hj} \, I\{v_h = v_i\} \Big) > 0 \Big\} \ .$$

## Monadic double covariate effects

For pairs of actor-dependent covariates V and U with values  $v_i$  and  $u_j$ , the following effects are available. These are useful, e.g., for bipartite (two-mode) networks. Then V must be a covariate for the first mode, and U for the second mode.

If it is desired to use them for the same variables (V = U), it is necessary to define a data set with two identical variables (i.e., with all variables equal) bearing the same name.

134.  $same\ covariate1$ -covariate2, which can also be called covariate-related  $identity\ (sameXV)$ , defined by the number of ties of i to all other actors j for which the value of i on covariate V is the same as the value of j on covariate U, and larger than 0,

$$s_{i134}^{\text{net}}(x) = \sum_{i} x_{ij} I\{v_i = u_j \ge 1\},$$

where the indicator function  $I\{v_i = u_j\}$  is 1 if the condition  $\{v_i = u_j \ge 1\}$  is satisfied, and 0 if it is not.

This effect is implemented only for U and V that are non-centered categorical actor variables with integer values in the range from 0 to 20.

The exclusion of values 0 is for the treatment of missing values: these can be given imputationValues equal to 0, see the help page for coCovar and varCovar.

135. indegree popularity from same covariate1 to same covariate2, (sameXVInPop) and (sameXVInPop2). This is an elementary effect that is implemented only for non-centered categorical actor variables with values within the range of numbers from 0 to 20.

It represents the frequency, or relative frequency, of ties from actors in the same category of covariate V to nodes in the same category of covariate U.

It is defined, for the tie from i to j, by the number of incoming ties sent by actors h who have the same non-zero covariate value  $v_h = v_i$  as i and received by those k who have the same non-zero covariate value  $u_k = u_j$  as j:

$$s_{i135}^{\text{el}}(x) = x_{ij} \sum_{h,k} x_{hk} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}$$
,  $(p=1)$ 

where the indicator function  $I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}$  is 1 if the condition  $\{v_h = v_i \ge 1, u_k = u_j \ge 1\}$  is satisfied, and 0 if it is not.

The exclusion of values 0 is for the treatment of missing values: these can be given imputationValues equal to 0, see the help page for coCovar and varCovar.

The effect parameter p can take the values 1, 2, 3 and 4. The formula given above is for p = 1.

For p=3, the effect is defined by the <u>proportion</u> of ties from same-V actors to same-V nodes, as given by the formula

$$s_{i135}^{\text{el}}(x) = x_{ij} \frac{\sum_{h,k} x_{hk} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}}{\sum_{h,k} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}} .$$
 (p = 3)

For p=2 and p=4, the square root is taken:

$$s_{i135}^{\text{el}}(x) = x_{ij} \sqrt{\sum_{h,k} x_{hk} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}}$$
,  $(p=2)$ 

and

$$s_{i135}^{\text{el}}(x) = x_{ij} \sqrt{\frac{\sum_{h,k} x_{hk} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}}{\sum_{h,k} I\{v_h = v_i \ge 1, u_k = u_j \ge 1\}}} .$$
  $(p = 4)$ 

The default is p = 3.

This effect is perhaps rather time-consuming because it iterates over all  $x_{hk}$ . For this reason, sameXVInPop and sameXVInPop2 are different implementations of the same effect. The former iterates over all ties, the second over all nodes h, and then over their outgoing ties. You can choose whichever is more efficient for your data set.

Differences between sameXInPop, sameXVInPop, sameXCycle4:

All work with an actor variable V, and sameXVInPop works in addition with a nodal variable U.

All three express (for positive parameters) that actors with the same categories of V make related choices of other nodes.

For sameXInPop, they tend to choose the same nodes. For sameXVInPop, they tend to choose nodes of the same U category. For sameXCycle4, if they tend to choose some particular node in common, they will also tend to choose some other nodes in common.

Note effect homXOutAct2, which is an ego version of sameXVInPop, expressing that ego has differential propensities to ties with alters of different categories; the role of U in sameXVInPop is taken by V in the definition of homXOutAct2.

### 12.1.2 Special effects for non-directed networks

For non-directed (symmetric) networks (see Section 5.9 and Snijders and Pickup, 2016), in principle the same effects are used as above; you may search in this manual for 'non-directed' to see the cases where the definition of an effect for non-directed networks deviates from the definition for directed networks. Here we present a few effects that are defined specifically for non-directed networks. The reason is that non-directed data will not allow to differentiate between senders and receivers of ties, and instead of working hypothetically with either ego or alter effects, it may be preferable to utilize effects where ego and alter are assumed to have equal contributions.

1. degree activity plus popularity effect (degPlus). This is defined as the sum of the in=outdegree popularity and in=outdegree activity effects; equivalently, this effect operates as the degree popularity combined with the degree activity effect under the assumption that the parameters for both are the same:

$$s_{i1}^{\text{net}}(x) = \sum_{j} x_{ij}(x_{j+} + x_{i+})$$
.

For internal effect parameter equal to 2, the square roots are taken:

$$s_{i1}^{\text{net}}(x) = \sum_{i} x_{ij} (\sqrt{x_{j+}} + \sqrt{x_{i+}}) .$$

2. A centered version of degPlus:

degree activity plus popularity effect (centered) (degPlus.c); This is defined as the sum of the centered in=outdegree popularity and in=outdegree activity effects:

$$s_{i2}^{\text{net}}(x) = \sum_{j} x_{ij}(x_{j+} + x_{i+} - 2d) ,$$

where d is the average degree across all waves.

3. covariate effect (egoPlusAltX). This is defined as the sum of the covariate-ego and covariate-alter effects; equivalently, this effect operates as the covariate-ego combined with the covariate-alter effect under the assumption that the parameters for both are the same:

$$s_{i3}^{\text{net}}(x) = \sum_{j} x_{ij}(v_i + v_j)$$
.

4. covariate-squared effect (egoPlusAltSqX). This is defined as the sum of the covariate-squared ego and covariate-squared alter effects; equivalently, this effect operates as the covariate-squared ego combined with the covariate-squared alter effect under the assumption that the parameters for both are the same:

$$s_{i4}^{\text{net}}(x) = \sum_{i} x_{ij} (v_i^2 + v_j^2) ;$$

5. transitive triads effect (transTriads), defined by the number of transitive patterns in i's relations, which for non-directed networks can be written as  $s_{i5}^{\text{net}}(x) = \sum_{j < h} x_{ij} x_{ih} x_{hj}$ ;

For non-directed networks there is only one degree assortativity effect, the degree assortativity effect which is the outdegree-indegree version<sup>17</sup> still with short name outInAss. There is also only one GWESP effect, called simply gwesp.

 $<sup>^{-17}</sup>$ Of the four versions this has the correct change statistic, because the change in  $x_{ij}$  is associated with a change in both the outdegree of i and the indegree of j.

## 12.1.3 Multiple network effects

An introduction to the analysis of multiple (multivariate) networks, with a discussion of the basic effects, is given in Snijders et al. (2013).

See Section 5.7 for the possibility that two networks have a deterministic constraint by being mutually exclusive, or by one being implied by the other, or (rarely used, if ever) by the union of them being the complete graph.

If there are multiple dependent networks, the definition of cross-network effects is such that always, one network has the role of the dependent variable, while the other network, or networks, have the role of explanatory variable(s). In the following list the network in the role of dependent variable is denoted by the tie variables  $x_{ij}$ , while the tie variables  $w_{ij}$  denote the network that is the explanatory variable.

Various of these effects are applicable only if the networks X and W satisfy certain conditions of conformability; for example, the first effect of W on X is meaningful only if W and X have the same dimensions, i.e., either both are one-mode networks, or both are two-mode networks with the same actor set for the second mode; as another example, the second effect, of incoming W on X, is applicable only if W and X are one-mode networks. These conditions are hopefully clear from logical considerations, and drawing a little diagram of the involved nodes and arrows will be helpful in cases of doubt.

In the RSiena output for groups with multiple networks, the dependent network in each given effect is indicated by the first part of the effect name. In the list below, a more or less normally formulated name is given first, then the name used in RSiena between parentheses, using X as the name for the dependent network and W as the name for the explanatory network, then between parentheses in typewriter font the shortName as used by RSiena. Since this is a co-evolution model, RSiena will include also the effects where the roles of X and W are reversed.

The first three effects are dyadic. The first can be regarded as a main effect; the reciprocity and mutuality effects will require rather big data sets to be empirically distinguished from each other.

- 1. Effect of W on X(X: W) (crprod),  $s_{i1}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij}$ ;  $i \xrightarrow{W} j$  leads to  $i \xrightarrow{X} j$ ; this is also called the dyadic entrainment effect; if one of the contraints of Section 5.7 is in use, this effect should not be included because its role is taken over by the constraint;
- 2. Effect of incoming W on X (X: reciprocity with W) (crprodRecip),  $s_{i2}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ji}$ ; this can be regarded as generalized exchange:  $j \stackrel{W}{\to} i$  leads to  $i \stackrel{X}{\to} j$ ;
- 3. Effect of mutual ties in W on X(X): mutuality with W) (crprodMutual),  $s_{i3}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij} w_{ji}$ ;  $j \stackrel{W}{\leftrightarrow} i \text{ leads to } i \stackrel{X}{\rightarrow} j.$

The following are degree-related effects, where nodal degrees in the W network have effects on popularity or activity in the X network. They use an internal effect parameter p, which mostly will be 1 or 2.

To decrease correlation with other effects, the W-degrees are centered by subtracting the value  $\bar{w}$ , which is the average degree of W across all observations.

4. Effect of in-degree in W on X-popularity (X: indegree<sup>1/p</sup> W popularity) (inPopIntn) defined by the sum of the W-in-degrees of the others to whom i is tied, for parameter p=2 the square roots of the W-in-degrees:

$$s_{i4}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{+j} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i4}^{\text{net}}(x) = \sum_{j} x_{ij} (\sqrt{w_{+j}} - \sqrt{\bar{w}});$ 

5. Effect of in-degree in W on X-activity (X: indegree<sup>1/p</sup> W activity) (inActIntn) defined by the W-in-degrees of i (for p=2 the square roots) times i's X-out-degrees:

$$s_{i5}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{+i} - \bar{w}) = x_{i+} (w_{+i} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i5}^{\text{net}}(x) = \sum_{j} x_{ij} (\sqrt{w_{+i}} - \sqrt{\bar{w}}) = x_{i+} (\sqrt{w_{+i}} - \sqrt{\bar{w}});$ 

6. Effect of out-degree in W on X-popularity (X: outdegree<sup>1/p</sup> W popularity) (outPopIntn) defined by the sum of the W-out-degrees of the others to whom i is tied, for parameter p=2 the square roots of the W-out-degrees:

$$s_{i6}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{j+} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i6}^{\text{net}}(x) = \sum_{j} x_{ij} (\sqrt{w_{j+}} - \sqrt{\bar{w}});$ 

7. Effect of out-degree in W on X-activity (X: outdegree<sup>1/p</sup> W activity) (outActIntn) defined by the W-out-degree of i (for p = 2 its square root) times i's X-out-degree:

$$s_{i7}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{i+} - \bar{w}) = x_{i+} (w_{i+} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i7}^{\text{net}}(x) = \sum_{j} x_{ij} (\sqrt{w_{i+}} - \sqrt{\bar{w}}) = x_{i+} (\sqrt{w_{i+}} - \sqrt{\bar{w}});$ 

The following four effects are meant mainly for interactions. Recall from Section 5.12 that interactions are defined by the product of the change statistics. In many cases, the change statistics are defined by omitting  $\sum_j x_{ij}$  from the formula for the effect. In Section 5.12.1 we saw an example in which the ego effect of the average value of covariate V for those to whom actor i has a W-outgoing tie can be defined as the interaction between outActIntnX, the effect of out-degree in W, weighted by receiver's V, on X-activity, with divOutEgoIntn. For a given ego, outActIntnX gives the sum of the V-values of ego's W-ties, and divOutEgoIntn divides this by the number of ego's W-ties; this yields the average of the V-values of ego's W-ties.

For such interaction effects, the internal effect parameters should match; see Section 5.12.1.

8. 1 divided by ego's out-degree in W (X: 1/outdeg.  $^{1/p}$  W ego) (divOutEgoIntn) defined by i's X-out-degree divided by the W-out-degree of i (for p=2 its square root), or 0 if the W-out-degree is 0:

$$s_{i8}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, w_{i+}) \text{ or}$$
  
for  $p = 2$   $s_{i8}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, \sqrt{w_{i+}});$   
where  $\operatorname{divi}(a, b)$  is defined as  $a/b$  if  $b \neq 0$  and 0 if  $b = 0$ ;

9. 1 divided by ego's in-degree in W (X: 1/indeg.  $^{1/p}$  W ego) (divInEgoIntn) defined by i's X-out-degree divided by the W-in-degree of i (for p=2 its square root), or 0 if the W-indegree is 0:

$$s_{i9}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, w_{+i}) \text{ or}$$
  
for  $p = 2$   $s_{i9}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, \sqrt{w_{+i}});$   
where  $\operatorname{divi}(a, b)$  is defined as above;

10. 1 divided by alter's out-degree in W (X: 1/outdeg.  $^{1/p}$  W alter) (divOutAltIntn) defined by 1 divided by the sum of the W-out-degrees of those to whom i has an outgoing X-tie (for p=2 its square root), or 0 if the W-out-degree is 0:

$$s_{i10}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, w_{j+}) \text{ or}$$
  
for  $p = 2$   $s_{i10}^{\text{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, \sqrt{w_{j+}});$   
where  $\operatorname{divi}(a, b)$  is defined as above;

11. 1 divided by alter's in-degree in W (X: 1/indeg.  $^{1/p}$  W alter) (divInAltIntn) defined by 1 divided by the sum of the W-out-degrees of those from whom i has an incoming X-tie (for p=2 its square root), or 0 if the W-indegree is 0:

$$s_{i11}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, w_{+j})$$
 or for  $p = 2$   $s_{i11}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \operatorname{divi}(1, \sqrt{w_{+j}});$  where  $\operatorname{divi}(a, b)$  is defined as above;

12. Effect of average out-degree in W on X (X: av. degree W) (avDegIntn) defined by the X-out-degree of i multiplied by the average W-out-degree, centered by the internal effect parameter p

$$s_{i12}^{\text{net}}(x) = \sum_{j} x_{ij} \left(\frac{1}{n} \sum_{h} w_{h+} - p\right) = x_{i+} \left(\frac{1}{n} \sum_{h} w_{h+} - p\right)$$
; this effect is a network-by-period level effect and therefore is useful only for multi-group data sets or for data sets with many periods; it may be beneficial for convergence to use a value of  $p$  that is close to the average  $W$ -degree (but constant, while this effect is for the dynamic average outdegree):

13. Interaction  $W \times in$ -degree W activity (X: W x indegree  $^{1/p}$  W activity) (crprodInActIntn) defined by the entrainment of W by X weighted by W-in-degrees of i (for p=2 the square roots):

$$s_{i13}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij} (w_{+i} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i13}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij} (\sqrt{w_{+i}} - \sqrt{\bar{w}});$ 

14. Effect of absolute difference of both W out-degrees  $(X: outdegree^{1/p} W abs. diff.)$  (absOutDiffIntn)

defined by the sum of the absolute difference of the W-out-degrees of alter and ego for all alters to whom i is tied; for parameter p=2 the square roots of the W-out-degrees are taken:

$$s_{i14}^{\text{net}}(x) = \sum_{j} x_{ij} |w_{i+} - w_{j+}| \text{ or}$$
  
for  $p = 2$   $s_{i14}^{\text{net}}(x) = \sum_{j} x_{ij} |\sqrt{w_{i+}} - \sqrt{w_{j+}}|;$ 

this can be regarded as minus the similarity of the W-out-degrees (but without the division by the range);

15. Effect of both in-degrees in W on X-popularity (X: both indegrees  $^{1/p}$  W) (both) defined by the sum of the W-in-degrees of the others to whom i is tied multiplied by the centered W-in-degree of i, for parameter p=2 the square roots of the W-in-degrees:

$$s_{i15}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{+i} - \bar{w}) (w_{+j} - \bar{w}) \text{ or}$$
  
for  $p = 2$   $s_{i15}^{\text{net}}(x) = \sum_{j} x_{ij} (\sqrt{w_{+i}} - \sqrt{\bar{w}}) (\sqrt{w_{+j}} - \sqrt{\bar{w}});$ 

this can be regarded as an interaction between the effect of W-in-degree on Xpopularity and the effect of W-in-degree on X-activity;

16.  $Duplex\ XW\ out\text{-}degree\ activity\ effect\ (X: duplex\ W\ outdegree^{1/p}\ activity)$  (doubleOutAct)

which is like the out-degree activity effect, but now for the degrees in the joined X and W network; for parameter p=2 the square root of the out-degrees is taken:

$$s_{i16}^{\text{net}}(x) = \left(\sum_{j} x_{ij} w_{ij}\right)^2 \text{ or}$$

for p=2  $s_{i16}^{\rm net}(x)=\sum_j x_{ij}\,w_{ij}\sqrt{\sum_h x_{ih}\,w_{ih}}$ ;

17.  $Duplex\ XW\ in-degree\ popularity\ effect\ (X: duplex\ W\ indegree^{1/p}\ popularity)$  (doubleInPop)

which is like the in-degree popularity effect, but now for the degrees in the joined X and W network; for parameter p=2 the square root of the out-degrees is taken:

$$s_{i17}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij} \sum_{h} x_{hj} w_{hj} \text{ or}$$
  
for  $p = 2$   $s_{i17}^{\text{net}}(x) = \sum_{j} x_{ij} w_{ij} \sqrt{\sum_{h} x_{hj} w_{hj}}$ .

The betweenness effect is another positional effect: a positional characteristic in the W network affects the ties in the X network, but now the position is the betweenness count, defined as the number of pairs of nodes that are not directly connected:  $j \stackrel{W}{\to} h$ , but that are connected through  $i : j \stackrel{W}{\to} i \stackrel{X}{\to} h$ . Again there is an internal effect parameter p, usually 1 or 2.

18. Effect of W-betweenness on X-popularity (X: betweenness  $^{1/p}$  W popularity) (betweenPop) defined by the sum of the W-betweenness counts of the others to whom i is tied:

$$s_{i18}^{\text{net}}(x) = \sum_{j} x_{ij} \left( \sum_{h,k;h \neq k} w_{hj} w_{jk} (1 - w_{hk}) \right)^{1/p};$$

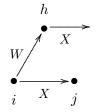
19. mixed two path activity effect (X: mixed two path  $^{1/p}$  W activity) (outOutActIntn) defined by the outdegree of i multiplied by sum of X-outdegrees of those to whom i has a W-tie: for p=1, this is

$$s_{i19}^{\text{net}}(x) = x_{i+} \sum_{h} w_{ih} (x_{h+} - \bar{x})$$

and for p = 2, there is a square root transformation:

$$s_{i19}^{\text{net}}(x) = x_{i+} \sum_{h} w_{ih} \left( \sqrt{x_{h+}} - \sqrt{\bar{x}} \right)$$

where  $\bar{x}$  is the average outdegree over all waves.



20. mixed distance-2 out-in activity effect (X: nbr out-in dist.2 W activity) (nDist2ActIntn) defined by the outdegree of i multiplied by the number of actors at out-in distance 2 in the W network:

for p = 1, this is, for one-mode networks:

$$s_{i20}^{\text{net}}(x) = x_{i+} \sum_{h} \{ \max_{k} (w_{ik} w_{hk}) - w_{ih} \}$$

and for two-mode (bipartite) networks

$$s_{i20}^{\text{net}}(x) = x_{i+} \sum_{h} \left\{ \max_{k} \left( w_{ik} w_{hk} \right) \right\} ;$$

and for p = 2, there is a square root transformation:

$$s_{i20}^{\text{net}}(x) = x_{i+} \sqrt{\sum_{h} \left\{ \max_{k} \left( w_{ik} w_{hk} \right) - w_{ih} \right\}}$$

and

$$s_{i20}^{\text{net}}(x) = x_{i+} \sqrt{\sum_{h} \left\{ \max_{k} \left( w_{ik} w_{hk} \right) \right\}}$$

respectively.

Further there are a number of mixed triadic effects.

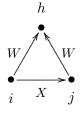
21. agreement along W leading to X, (X: from W agreement) (from)

$$s_{i21}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh};$$

this refers to agreement of actors with respect to their W-choices (structural equivalence with respect to outgoing W-choices); the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of joint W choices of others,  $i \stackrel{W}{\to} h \stackrel{W}{\leftarrow} j$ .

For internal effect parameter  $p \geq 2$ , the effect is

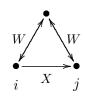
$$s_{i21}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h \neq i,j} w_{ih} w_{jh}}$$
.



22. agreement along mutual W-ties leading to X, (X: from W mu)tual agreement) (fromMutual)

$$s_{i22}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hi} w_{jh} w_{hj};$$

this refers to agreement of actors with respect to their mutual W-choices (structural equivalence with respect to mutual Wchoices); the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of joint mutual W choices of others,  $i \stackrel{W}{\leftrightarrow} h \stackrel{W}{\leftrightarrow} j$ .

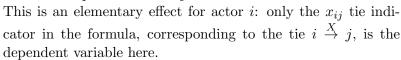


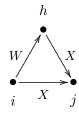
For internal effect parameter  $p \geq 2$ , the effect is

$$s_{i22}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h \neq i,j} w_{ih} w_{hi} w_{jh} w_{hj}}$$
.

23. W leading to agreement along X, (X: W to agreement) (to)  $s_{i23}^{\text{net}}(x) = \sum_{i \neq h} x_{ij} w_{ih} x_{hj};$ 

this refers to the closure of mixed W-X two-paths; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of mixed W-X two-paths  $i \stackrel{W}{\rightarrow} h \stackrel{X}{\rightarrow} j$ .





The interpretation is that actors have the tendency to make the same outgoing X-choices as those to whom they have a W-tie.

For internal effect parameter 
$$p=2$$
, the effect is

$$s_{i23}^{
m net}(x) = \sum_j x_{ij} \sqrt{\sum_{h 
eq i,j} w_{ih} \, x_{hj}}$$
. For internal effect parameter  $p=3$ , the effect is

$$s_{i23}^{
m net}(x) = \sum_{j} x_{ij} \, I\{\sum_{h \neq i,j} w_{ih} \, x_{hj} \ge 1\}$$
 .

24. W leading to any agreement along X, (X: W to any agreement) (toAny)

$$s_{i24}^{\text{net}}(x) = \sum_{h} w_{ih} \max_{j} \{x_{ij} x_{hj}\};$$

this is a truncated version of the to effect, somewhat similar to the transTies effect: the effect is the number of  $i \xrightarrow{W} h$  ties accompanied by at least one X - X out-two-

The interpretation is that with others to whom actors have a W-tie, they have the tendency to make at least one outgoing X-choice in common.

25. W (backwards) leading to agreement along X, (X: W) (backwards) to agreement) (toBack) (earlier called (MixedInWX))

$$s_{i25}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} x_{hj};$$

this refers to the closure of mixed incoming W and X ties; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of mixed W - X out-two-stars  $h \stackrel{W}{\rightarrow} i, h \stackrel{X}{\rightarrow} j$ .

This is an elementary effect for actor i: only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

For internal effect parameter p = 2, the effect is

$$s_{i25}^{
m net}(x) = \sum_j x_{ij} \sqrt{\sum_{h 
eq i,j} w_{hi} \, x_{hj}}$$
. For internal effect parameter  $p=3$ , the effect is

$$s_{i25}^{\rm net}(x) = \sum_{j} x_{ij} I\{\sum_{h \neq i,j} w_{hi} x_{hj} \ge 1\}$$
.

26. mutual W leading to agreement along X, (X: mutual W to)agreement) (toRecip)

$$s_{i26}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{ih} x_{hj};$$

this refers to the closure of mixed mutual W and X ties; the contribution of the tie  $i \stackrel{X}{\rightarrow} j$  is proportional to the number of mixed W-X out-two-stars  $h \overset{W}{\leftrightarrow} i, h \overset{X}{\rightarrow} j$ .

This is an elementary effect for actor i: only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

For internal effect parameter p = 2, the effect is

$$s_{i26}^{
m net}(x) = \sum_j x_{ij} \sqrt{\sum_{h 
eq i,j} w_{hi} \, w_{ih} \, x_{hj}}$$
. For internal effect parameter  $p=3$ , the effect is

$$s_{i26}^{\rm net}(x) = \sum_{j} x_{ij} I\{\sum_{h \neq i,j} w_{hi} w_{ih} x_{hj} \ge 1\}$$
.

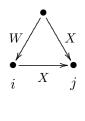
27. XWX closure of W, (cl.XWX)

$$s_{i27}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} x_{ih} w_{hj};$$

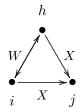
this refers to the closure of mixed X-W two-paths; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of mixed X-W two-paths  $i \stackrel{X}{\rightarrow} h \stackrel{W}{\rightarrow} j$  plus the number of mixed X-Wtwo-in-stars  $i \stackrel{X}{\to} h$ ,  $j \stackrel{W}{\to} h$ .

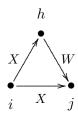
The interpretation is the closure of  $X \to W$  paths: if there is a tie  $h \stackrel{W}{\rightarrow} j$ , then ties  $i \stackrel{X}{\rightarrow} j$  and  $i \stackrel{X}{\rightarrow} h$  will tend to entrain each

(The reported target statistic is multiplied by 2.)



h





There are two partial variants of this effect; they can be distinguished not by the Method of Moments, but only by Maximum Likelihood and Bayesian estimation.

28. XWX closure-1 of W, (cl.XWX1)

This is an elementary effect, not an evaluation effect, comprising of the 'XWX closure of W' effect only the contribution of the the number of mixed X-W two-paths

So the dependent variable here is only the tie variable  $i \to j$  in the figure above. The effect is defined as

$$s_{i28}^{\text{el}}(x) = x_{ij} \sum_{h;h \neq j} x_{ih} w_{hj};$$

29. XWX closure-2 of W, (cl.XWX2)

This is an elementary effect, not an evaluation effect, comprising of the 'XWX closure of W' effect only the contribution of the number of mixed X - W two-in-stars  $i \stackrel{X}{\to} h$ ,  $j \stackrel{W}{\to} h$ . In other words, only the  $i \rightarrow j$  tie in the figure here is the dependent variable. The effect is defined as

$$X \bigvee_{i}^{W} W$$

 $s_{i29}^{el}(x) = x_{ij} \sum_{h:h \neq j} x_{ih} w_{jh}$ .

30. shared mixed incoming X and W, (X: mixed incoming withW) (mixedInXW)

$$s_{i30}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} x_{hi} w_{hj};$$

this refers to the closure of mixed incoming X and W ties; the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of mixed X - W out-two-stars  $h \stackrel{\tilde{X}}{\to} i, h \stackrel{W}{\to} j$ .

For internal effect parameter p = 2, the effect is

$$s_{i30}^{\rm net}(x) = \sum_j x_{ij} \sqrt{\sum_{h \neq i,j} x_{hi} w_{hj}}$$
. For internal effect parameter  $p=3$ , the effect is

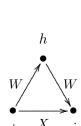
$$s_{i30}^{\text{net}}(x) = \sum_{i} x_{ij} I\{\sum_{h \neq i, j} x_{hi} w_{hj} \ge 1\}$$

31.  $mixed WW => X \ closure, (X: closure of W)$  (closure)

$$s_{i31}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hj};$$

this refers to the closure of W-W two-paths; the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of W - W two-paths  $i \xrightarrow{W} h \xrightarrow{W} j$ .

The interpretation is that actors have the tendency to make and maintain X-ties to those to whom they have an indirect (distance 2) W-tie: 'W-ties of W-ties tend to become X-ties';

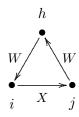


32.  $mixed\ cyclic\ WW => X\ closure,\ (X:\ cyclic\ closure\ of\ W)$  (cyClosure)

$$s_{i32}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{jh} w_{hi};$$

this refers to the cyclic closure of W-W two-paths; the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of W-W two-paths  $j \xrightarrow{W} h \xrightarrow{W} i$ .

The interpretation is that actors have the tendency to make and maintain X-ties to those from whom they receive an indirect (distance 2) W-tie: 'W-ties of W-ties tend to become reciprocated by X-ties'.



33. closure of shared incoming WW => X, (X: shared incoming W) (sharedIn)

$$s_{i33}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj};$$

this refers to shared incoming W ties contributing to the tie  $i \stackrel{X}{\rightarrow} j$ .



The following two Jaccard similarity effects also are triadic, but not expressed as sums over triads.

34. Jaccard similarity with respect to outgoing W ties effect (JoutMix), the Jaccard similarity with respect to outgoing W-ties,  $s_{i34}^{\text{net}}(x) = \sum_{j} x_{ij} J_{W,\text{out}}(i,j)$ , where

$$J_{W, \text{ out}}(i, j) = \frac{\sum_{h} w_{ih} w_{jh}}{w_{i+} + w_{j+} - \sum_{h} w_{ih} w_{jh}}$$

(where 0/0 is taken as 0).

35. Jaccard similarity with respect to incoming W ties effect (JinMix), defined by the Jaccard similarity with respect to incoming W-ties,  $s_{i35}^{\text{net}}(x) = \sum_{j} x_{ij} J_{W,\text{in}}(i,j)$ , where

$$J_{W, \text{in}}(i, j) = \frac{\sum_{h} w_{hi} w_{hj}}{w_{+i} + w_{+j} - \sum_{h} w_{hi} w_{hj}}$$

(where again 0/0 is taken as 0).

36. Five variations of mixed GWESP (geometrically weighted edgewise shared partners) effects: gwespFFMix, gwespBBMix, gwespFBMix, gwespBFMix, gwespBFMix, and for non-directed and two-mode networks the sixth version gwespMix. See above for the GWESP effects for one-mode networks for their further background. These are geometrically downweighted analogues: gwespFFMix of closure,

gwespBBMix of cyClosure, gwespFBMix of from, gwespBFMix of sharedIn, and gwespRRMix of fromMutual. The gwespFFMix effect is defined by

GWESPFF
$$(i, \alpha) = \sum_{j=1}^{n} x_{ij} e^{\alpha} \left\{ 1 - (1 - e^{-\alpha})^{\sum_{h=1}^{n} w_{ih} w_{hj}} \right\},$$

where the convention is used that  $w_{jj} = 0$  for all j. See the figures above in the presentation of the gwespFF effect.

The parameter  $\alpha$  is a tuning parameter that may range from 0 to  $\infty$ . The internal effect parameter is defined as  $100 \times \alpha$ . An often used value is  $\alpha = \log(2) = 0.69$  (Snijders et al., 2006), corresponding to an internal effect parameter of 69, but it is worthwhile to try out different values of  $\alpha$  to see which one gives the best fit. For large  $\alpha$  the gwespffmix effect will approximate the closure effect, and similarly for the other correspondences mentioned above.

The other types of mixed GWESP effect are analogous, with different tie orientations. They are defined as follows:

gwespBBMix:  $\sum_{h=1}^{n} w_{ih} w_{hj}$  is replaced by  $\sum_{h=1}^{n} w_{hi} w_{jh}$ ;

gwespFBMix:  $\sum_{h=1}^{n} w_{ih} w_{hj}$  is replaced by  $\sum_{h=1}^{n} w_{ih} w_{jh}$ ;

gwespBFMix:  $\sum_{h=1}^{n} w_{ih} w_{hj}$  is replaced by  $\sum_{h=1}^{n} w_{hi} w_{hj}$ ;

gwespRRMix:  $\sum_{h=1}^{n} w_{ih} w_{hj}$  is replaced by  $\sum_{h=1}^{n} w_{ih} w_{hi} w_{jh} w_{hj}$ ;

gwespMix for two-mode  $W: \sum_{h=1}^{n} w_{ih} w_{hj}$  is replaced by  $\sum_{h=1}^{m} w_{ih} w_{jh}$ .

Then there are effects using mixed configurations on four nodes (cf. sharedPop).

37. mixed twopath average effect (X: mixed twopath<sup>1/p</sup> W average) (outOutAvIntn), defined by the outdegree of i multiplied by the average of the X-outdegrees of those to whom i has a W-tie:

for p = 1, this is

$$s_{i37}^{\text{net}}(x) = x_{i+} \frac{\sum_{h} w_{ih} (x_{h+} - \bar{x})}{\sum_{h} w_{ih}}$$

and for p = 2, there is a square root transformation:

$$s_{i37}^{\text{net}}(x) = x_{i+} \frac{\sum_{h} w_{ih} (\sqrt{x_{h+}} - \sqrt{\bar{x}})}{\sum_{h} w_{ih}}$$

where  $\bar{x}$  is the average outdegree over all waves.

38. W twopath activity effect  $(X: W \text{ twopath}^{1/p} \text{ activity})$ (dist2OutInActIntn) defined by the outdegree of i multiplied by sum of W-indegrees of those k to whom i has a W-tie  $i \to k$ : for p = 1, this is

$$s_{i38}^{\text{net}}(x) = x_{i+} \sum_{k} w_{ik} (w_{+k} - \bar{w})$$

and for p = 2, there is a square root transformation:

$$s_{i38}^{\text{net}}(x) = x_{i+} \sum_{k} w_{ik} \left( \sqrt{w_{+k}} - \sqrt{\bar{w}} \right)$$

where  $\bar{w}$  is the average W-outdegree over all waves.

39. shared W outdegree activity effect (X: shared W outdegree  $^{1/p}$ activity) (outOutDist2ActIntn) defined by the outdegree of i multiplied by sum of X-outdegrees of those h to whom i has a distance-2 W-tie  $i \to k \leftarrow h$  (counting multiplicities).

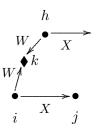
This is an elementary effect. For p = 1, it is defined by

$$s_{ij39}^{\text{el}}(x) = x_{ij} \sum_{h} (x_{h+} - \bar{x}) \{ \sum_{k} w_{ik} w_{hk} \}$$

and for p = 2, there is a square root transformation:

$$s_{ij39}^{\rm el}(x) = x_{ij} \sum_{h} (x_{h+} - \bar{x}) \sum_{h} \sqrt{\sum_{k} w_{ik} w_{hk}}$$

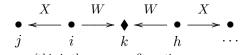
where  $\bar{x}$  is the average outdegree over all waves.



40. average two-mode W alter by total (X: av. 2M alter W by tot) (avAlt.2M.tot), defined by the outdegree of i multiplied by the average of the X-outdegrees of those h to whom i has a distance-2 W-tie  $i \to k \leftarrow h$  (counting multiplicities). This is an elementary effect. For p = 1, it is defined by

for p=1, this is

$$s_{ij40}^{\text{el}}(x) = \frac{x_{ij} \sum_{h} (x_{h+} - \bar{x}) \{ \sum_{k} w_{ik} w_{hk} \}}{\sum_{k,h} w_{ik} w_{hk}}$$



and for p = 2, there is a square root transformation:

$$s_{ij40}^{\text{el}}(x) = \frac{x_{ij} \sum_{h} (x_{h+} - \bar{x}) \sum_{h} \sqrt{\sum_{k} w_{ik} w_{hk}}}{\sqrt{\sum_{k,h} w_{ik} w_{hk}}}$$

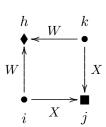
where  $\bar{x}$  is the average outdegree over all waves, and 0/0 is defined as 0.

41. shared W leading to agreement along X, (X: shared W to agreement) (sharedTo).

This is an elementary effect. It is defined by 
$$s_{ij41}^{\text{el}}(x) = x_{ij} \sum_{h,k:k\neq i} x_{kj} (w_{ih} w_{kh} - c)$$
;

this refers to the closure of mixed W-X three-paths; the contribution of the tie  $i \stackrel{X}{\rightarrow} j$  is proportional to the number of mixed W-X three-paths  $i \stackrel{W}{\rightarrow} h \stackrel{W}{\leftarrow} k \stackrel{X}{\rightarrow} j$ .

This can be regarded also as a mixed 4-cycle effect.



The effect parameter p can take the values 1, 2, 3 and 4. The value c, a constant for centering, is the average of the observed values  $w_{ih} w_{kh}$ :

$$c = \begin{cases} (1/Mn(n-1)) \sum_{m,h} (w_{+h}^2(t_m) - w_{+h}(t_m)) & (p \ge 3) \\ 0 & (p < 3) \end{cases}.$$

Note that since this is the evaluation function for actor i with respect to network X, only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

The interpretation is that actors have the tendency to make the same outgoing X-choices as those (k) with whom they share many outgoing W-ties.

For p = 2 and p = 4, the square root is taken:

$$s_{i41}^{\text{net}}(x) = \sum_{j,k;k \neq i} x_{ij} x_{kj} \left( \sqrt{\sum_h w_{ih} w_{kh}} - \sqrt{c} \right).$$

This can be regarded as a higher-order effect related to the inPopOutW effect (see below) as well as indegree-popularity. The differences between the centered and non-centered versions amount to a multiple of the indegree-popularity (without  $\checkmark$ ) inPop effect. Therefore, when the sharedTo effect is used, it may be advisable also to include the inPop, outActIntn, and inPopOutW effects, so as to avoid misinter-pretations.

42.  $average\ two-mode\ W\ alter\ by\ tie,\ (X:\ av.\ 2M\ alter\ W\ by\ tie)\ (avAlt.2M.tie).$ 

This is an elementary effect. It is defined by

$$s_{ij42}^{\rm el}(x) = x_{ij} \, \frac{\sum_{h,k;k \neq i} x_{kj} \, (w_{ih} \, w_{kh})}{\sum_{h,k;k \neq i} (w_{ih} \, w_{kh})} \, ;$$

this refers to the average closure of mixed W-X three-paths, and is an alternative for sharedTo. This can be also regarded as a mixed 4-cycle effect.

For internal effect parameter p = 2, it is defined by

$$s_{ij42}^{\text{el}}(x) = x_{ij} \frac{\sum_{k;k \neq i} x_{kj} \sqrt{\sum_{h} (w_{ih} w_{kh})}}{\sum_{k:k \neq i} \sqrt{\sum_{h} (w_{ih} w_{kh})}}.$$

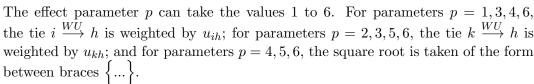
43. shared W weighted by U leading to agreement along X,  $(X: \text{shared } W \text{ wghtd by } U \Rightarrow \text{agrmnt along } X)$  (sharedToU).

This is an elementary effect. It is defined by

$$\begin{array}{l} s_{i43}^{\mathrm{el}}(x) = x_{ij} \, \sum_{k;k \neq i} x_{kj} \, \Big\{ \, \sum_h \left( w_{ih} \, w_{kh}[u_{ih}] \, [u_{kh}] \right) \Big\} \,; \\ \text{this refers to the closure of mixed } W - X \text{ three-paths,} \\ \text{where the } W \text{ ties are weighted by the dyadic covariate} \\ U; \text{ the contribution of the tie } i \stackrel{X}{\to} j \text{ is proportional to} \\ \text{the weighted sum of mixed } W - X \text{ three-paths } i \stackrel{WU}{\to} h \stackrel{X}{\longleftrightarrow} j. \end{array}$$

Usually, it will be good to work with a non-centered dyadic covariate.

This can be regarded also as a weighted mixed 4-cycle effect.



This means that for p = 3, 6, both ties are weighted, and for p = 1, 2, 4, 5, only one of them.

Note that since this is the evaluation function for actor i with respect to network X, only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

The interpretation is that actors have the tendency to make the same outgoing X-choices as those (k) with whom they share many outgoing W-ties, weighted by U. There is no centering as in sharedTo.

average two-mode W alter by tie, (X: av. 2M alter W by tie) (avAlt.2M.tie).

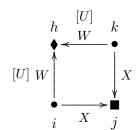
44. average two-mode W alter weighted by U, by tie, (X: av. 2M alter W weight <math>U by tie) (avAltU.2M.tie).

This is an elementary effect. It is defined for internal effect parameter p = 1, 2, 3 by

$$s_{ij44}^{\text{el}}(x) = x_{ij} \frac{\sum_{k;k \neq i} x_{kj} \sum_{h} (w_{ih} w_{kh}[u_{ih}] [u_{kh}])}{\sum_{k;k \neq i} \sum_{h} (w_{ih} w_{kh}[u_{ih}] [u_{kh}])};$$

this refers to the average closure of mixed W-X three-paths, where the W ties are weighted by the dyadic covariate U; it is an alternative to (sharedToU). This can be also regarded as a weighted mixed 4-cycle effect.

The weighting depends on the internal effect parameter p just like it is for (sharedToU). This can be also regarded as a weighted mixed 4-cycle effect.



For internal effect parameter p = 4, 5, 6, it is defined by

$$s_{ij44}^{\rm el}(x) = x_{ij} \frac{\sum_{k;k \neq i} x_{kj} \sqrt{\sum_{h} \left(w_{ih} \, w_{kh}[u_{ih}] \, [u_{kh}]\right)}}{\sum_{k;k \neq i} \sqrt{\sum_{h} \left(w_{ih} \, w_{kh}[u_{ih}] \, [u_{kh}]\right)}} \, .$$

45. indegree-popularity weighted by W-outdegree, (X: ind. pop. by outd. W) (inPopOutW)

$$s_{i45}^{\text{net}}(x) = \sum_{i,k:k \neq i} x_{ij} x_{kj} (w_{k+} - c);$$

 $s_{i45}^{\rm net}(x)=\sum_{j,k;k\neq i}x_{ij}\,x_{kj}\left(w_{k+}-c\right);$  this refers to mixed X-X-W three-paths; the contri-

bution of the tie  $i \stackrel{X}{\rightarrow} j$  is proportional to the number of mixed X - W three-paths  $j \stackrel{X}{\leftarrow} k \stackrel{W}{\rightarrow} h$ .

This can be regarded also as a mixed 3-path effect.

The effect parameter p can take the values 1, 2, 3 and 4. For p = 1, 2, there is no centering: c = 0. For p = 3, 4, the value c, a constant for centering, is the average of the W-outdegrees over all waves.

For p=2 and p=4, the square root is taken:

$$s_{i45}^{\mathrm{net}}(x) = \sum_{j,k;k \neq i} x_{ij} x_{kj} \left( \sqrt{w_{k+}} - \sqrt{c} \right).$$

Note that since this is the evaluation function for actor i with respect to network X, only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

The interpretation is that actors have the tendency to make more outgoing X-choices to those (j) who have many outgoing W-ties.

This can be regarded as a higher-order effect related to indegree-popularity. The differences between the centered and non-centered versions amount to a multiple of the indegree-popularity (without  $\sqrt{\phantom{a}}$ ) inPop effect. Therefore, when the inPopOutW effect is used, it is advisable also to include the inPop effect, so as to avoid misinterpretations.

# Monadic covariate effects

The mixed degree-related effects can be weighted with monadic covariates V, as implemented by the following four effects. (For two-mode networks, these effects are meaningful only in special cases; however, this is not checked when creating the effects object, so errors can lead here to meaningless results or to crashes.) Note that, unlike the unweighted effects inPopIntn, inActIntn, outPopIntn, outActIntn, the W-degrees here are not centered (well, they are also not really degrees).

Weights at the other side of the W-tie can be obtained by interactions of inPopIntn, inActIntn, outPopIntn, outActIntn with the ego or alter effects of V. Different types of weighting are available through the distance-two effects altDist2W and totDist2W, see

For these effects the other network W is given as interaction1, and the covariate Vas interaction2.

These weighted effects are defined by <u>sums</u> over all outgoing or incoming W-ties. To obtain averages, they can be interacted by the appropriate one of divOutEgoIntn, divInEgoIntn, divOutAltIntn, or divInAltIntn. The internal effect parameters should match; see Section 5.12.1.

46. Effect of in-degree in W, weighted by sender's V, on X-popularity (X: indegree  $^{1/p}$ W weighted by V popularity) (inPopIntnX)

defined by the sum of values of V for actors h for which there are ties  $i \stackrel{X}{\to} j \stackrel{W}{\leftarrow} h$ , i.e., at mixed X-W out-in distance 2; the value  $\sum_h w_{hj} v_h$  can be regarded as a V-weighted version of the W-indegree of j:

$$s_{i46}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \sum_{h} w_{hj} v_{h}$$
 or for  $p=2$ 

$$s_{i46}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} w_{hj} v_h};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered:

47. Effect of in-degree in W, weighted by sender's V, on X-activity (X: indegree  $^{1/p}$  W weighted by V activity) (inActIntnX)

defined by the sum of values of V for actors h for which there are ties  $h \xrightarrow{W} i \xrightarrow{X} j$ , i.e., a mixed W-X two-path with i in the middle; the value  $\sum_h w_{hi} v_h$  can be regarded as a V-weighted version of the W-indegree of i:

$$s_{i47}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \sum_{h} w_{hi} v_{h}$$
 or for  $p = 2$ 

$$s_{i47}^{\text{net}}(x) = \sum_j x_{ij} \sqrt{\sum_h w_{hi} v_h};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered;

48. Effect of out-degree in W, weighted by receiver's V, on X-popularity (X: outdegree  $^{1/p}$ W weighted by V popularity) (outPopIntnX)

defined by the sum of values of V for actors h for which there are ties  $i \xrightarrow{X} j \xrightarrow{W} h$ , i.e., at mixed X-W outgoing distance 2; the value  $\sum_h w_{jh} v_h$  can be regarded as a V-weighted version of the W-outdegree of j:

$$s_{i48}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} w_{jh} v_{h}$$
 or for  $p = 2$ 

$$s_{i48}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} w_{jh} v_{h}};$$

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered:

49. Effect of out-degree in W, weighted by receiver's V, on X-activity (X: outdegree  $^{1/p}$ W weighted by V activity) (outActIntnX)

defined by the sum of values of V for actors h for which there are ties  $h \stackrel{W}{\leftarrow} i \stackrel{X}{\rightarrow} j$ , i.e., a mixed W-X two-star; the value  $\sum_h w_{ih} v_h$  can be regarded as a V-weighted version of the W-outdegree of i:

$$s_{i49}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} w_{ih} v_{h}$$

or for 
$$p = 2$$
  
 $s_{i49}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} w_{ih} v_{h}};$ 

note that to use this effect for p=2 the variable V must be nonnegative, which implies that it must be non-centered.

Similarly, there are some mixed degree effects restricting the summation to third actors h with the same covariate value as i. Here again, the other network W is given as interaction1, and the covariate V as interaction2.

50. Effect of in-degree in W, restricted to same V, on X-popularity (X: indegree  $^{1/p}$  W from same V - popularity) (sameXinPopIntn)

defined by the number of incoming W ties to alter from other actors h having the same value of V as ego:

$$\begin{array}{l} s_{i50}^{\rm net}(x) = \sum_{j} x_{ij} \sum_{h} w_{hj} I\{v_i = v_h\} \\ \text{or for } p = 2 \\ s_{i50}^{\rm net}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} w_{hj} I\{v_i = v_h\}}; \end{array}$$

51. Effect of in-degree in W, restricted to same V, on X-activity (X: indegree  $^{1/p}$  W from same V - activity) (sameXinActIntn)

defined by the number of incoming W ties to ego from other actors h having the same value of V as ego:

$$\begin{array}{l} s_{i51}^{\rm net}(x) = \sum_{j} x_{ij} \, \sum_{h} w_{hi} \, I\{v_i = v_h\} \\ \text{or for } p = 2 \\ s_{i51}^{\rm net}(x) = \sum_{j} x_{ij} \, \sqrt{\sum_{h} w_{hi} \, I\{v_i = v_h\}}; \end{array}$$

52. Effect of out-degree in W, restricted to same V, on X-popularity (X: outdegree  $^{1/p}$  W to same V - popularity) (sameXoutPopIntn)

defined by the number of outgoing W ties from alter to other actors h having the same value of V as ego:

$$s_{i52}^{\text{net}}(x) = \sum_{j} x_{ij} \sum_{h} w_{jh} I\{v_i = v_h\}$$
 or for  $p = 2$  
$$s_{i52}^{\text{net}}(x) = \sum_{j} x_{ij} \sqrt{\sum_{h} w_{jh} I\{v_i = v_h\}};$$

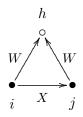
53. Effect of out-degree in W, restricted to same V, on X-activity (X: outdegree<sup>1/p</sup> W to same V - activity) (sameXoutActIntn)

defined by the number of outgoing W ties from ego to other actors h having the same value of V as ego:

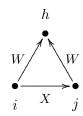
$$\begin{array}{l} s_{i53}^{\rm net}(x) = \sum_j x_{ij} \, \sum_h w_{ih} \, I\{v_i = v_h\} \\ {\rm or \,\, for \,\,} p = 2 \\ s_{i53}^{\rm net}(x) = \sum_j x_{ij} \, \sqrt{\sum_h w_{ih} \, I\{v_i = v_h\}}. \end{array}$$

Then there are some mixed triadic effects restricted to triples with the same or different values on a monadic covariate V.

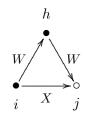
54. agreement along W leading to X, for same V,  $(X: \text{from } W \text{ agr.} \times \text{same } V) \text{ (covNetNet)},$   $s_{i54}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh} I\{v_i = v_j\};$  (specified with interaction1 = W, interaction2 = V) this refers to agreement of actors with respect to their W-choices (structural equivalence with respect to outgoing W-choices), but only for actors sharing the same value of a covariate V; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of joint W choices of others,  $i \stackrel{W}{\to} h \stackrel{W}{\leftarrow} j$ , counting only those for which  $v_i = v_j$ .



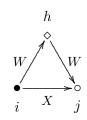
55. agreement along W leading to X, for homogeneous V,  $(X: \text{from } W \text{ agr.} \times \text{hom. } V)$  (homCovNetNet),  $s_{i55}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh} I\{v_i = v_j = v_h\};$  (specified with interaction1 = W, interaction2 = V) this refers to agreement of actors with respect to their W-choices (structural equivalence with respect to outgoing W-choices), but only for actors and choices sharing the same value of a covariate V; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the number of joint W choices of others,  $i \stackrel{W}{\to} h \stackrel{W}{\leftarrow} j$ , counting only those for which  $v_i = v_j = v_h$ .



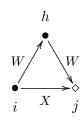
56.  $mixed\ WW \Rightarrow X\ closure,\ same-V\ path\ (X:\ mixed\ WW\ closure\ same\ V),\ (sameWWClosure)$  (specified with interaction1 = W, interaction2 = V)  $s_{i56}^{\rm net}(x) = \sum_{j\neq h} x_{ij}\ w_{ih}\ w_{hj}\ I\{v_i=v_h\};$  this refers to the closure by X of W-W two-paths, restricted to two-paths  $i\stackrel{W}{\to} h\stackrel{W}{\to} j$  in which the focal actor and the mediating actor have the same value of V.



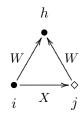
57. mixed  $WW \Rightarrow X$  closure, different-V path (X: mixed WW closure diff. <math>V), (diffWWClosure) (specified with interaction 1 = W, interaction 2 = V)  $s_{i57}^{\rm net}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hj} I\{v_i \neq v_h\};$  this refers to the closure by X of W-W two-paths, restricted to two-paths  $i \stackrel{W}{\to} h \stackrel{W}{\to} j$  in which the focal actor and the mediating actor have a different value of V.



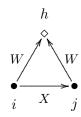
58.  $mixed\ WW \Rightarrow X\ closure,\ same-V\ path\ jumping\ to\ different\ V\ (X:\ W\ closure\ jumping\ V),\ (jumpWWClosure)\ (specified\ with\ interaction1=W,\ interaction2=V)\ s_{i58}^{\rm net}(x)=\sum_{j\neq h}x_{ij}\ w_{ih}\ w_{hj}\ I\{v_i=v_h\neq v_j\};$  this refers to the closure of W-W paths, restricted to 'jump outside of V-groups' in the sense that the focal actor and the mediating actor have the same value of V, but the target actor has a different value.



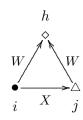
59. agreement along W, same-V, leading to X for different V (X: from W agreement jumping V), (jumpFrom) (specified with interaction1 = W, interaction2 = V)  $s_{i59}^{\rm net}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh} I\{v_i = v_h \neq v_j\};$  this refers to agreement about W ties, restricted to 'jump outside of V-groups' in the sense that the focal actor and the agreed actor have the same value of V, but the target actor has a different value.



60. agreement along W, same-V about contrasting V, leading to X (X: from W agreement contrasting V), (contrastCovNetNet) (specified with interaction1 = W, interaction2 = V)  $s_{i60}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh} I\{v_i = v_j \neq v_h\};$  this refers to agreement 'outside of V-groups' about W ties, in the sense that the focal actor and the target actor have the same value of V, but the agreed actor has a different value;

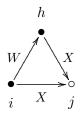


61. agreement along W, all different V, leading to X (X: from W agreement all diff. V), (allDifCovNetNet) (specified with interaction1 = W, interaction2 = V)  $s_{i61}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{jh} I\{v_i, v_j, v_h \text{ all different}\};$  this refers to agreement about W ties between actors who have all three a different value for V;

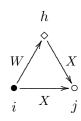


62. mixed WX ⇒ X closure, same-V path (X: mixed WX closure same V), (sameWXClosure) (specified with interaction1 = W, interaction2 = V) s<sub>i62</sub><sup>net</sup>(x) = ∑<sub>j≠h</sub> x<sub>ij</sub> w<sub>ih</sub> x<sub>hj</sub> I{v<sub>i</sub> = v<sub>h</sub>}; this refers to the closure of W - X two-paths, restricted to two-paths i <sup>W</sup>→ h <sup>X</sup>→ j in which the focal actor and the mediating actor have the same value of V.
A variant of this effect with weighting by a dyadic covariate is

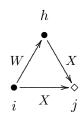
the toU effect.



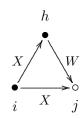
63.  $mixed\ WX \Rightarrow X\ closure,\ different\ V\ path\ (X:\ mixed\ WX\ closure\ diff.\ V),\ (diffWXClosure)$  (specified with interaction 1=W, interaction 2=V)  $s_{i63}^{\rm net}(x) = \sum_{j\neq h} x_{ij}\ w_{ih}\ x_{hj}\ I\{v_i\neq v_h\};$  this refers to the closure of W-X two-paths, restricted to two-paths  $i\stackrel{W}{\to} h\stackrel{X}{\to} j$  in which the focal actor and the mediating actor have a different value of V.



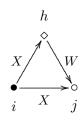
64.  $mixed\ WX \Rightarrow X\ closure,\ same-V\ path\ jumping\ to\ different\ V\ (X:\ mixed\ WX\ closure\ jumping\ V),\ (jumpWXClosure)\ (specified\ with\ interaction1=W,\ interaction2=V)\ s_{i64}^{\rm net}(x) = \sum_{j\neq h} x_{ij}\ w_{ih}\ x_{hj}\ I\{v_i=v_h\neq v_j\};$  this refers to the closure of W-X paths, restricted to 'jump outside of V-groups' in the sense that the focal actor and the mediating actor have the same value of V, but the target actor has a different value.



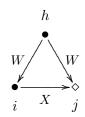
65.  $mixed\ XW \Rightarrow X\ closure,\ same-V\ path\ (X:\ mixed\ XW\ closure\ same\ V),\ (sameXWClosure)$  (specified with interaction1 = W, interaction2 = V)  $s_{i65}^{\rm net}(x) = \sum_{j \neq h} x_{ij}\ x_{ih}\ w_{hj}\ I\{v_i = v_h\};$  this refers to the closure of X-W two-paths, restricted to two-paths  $i \stackrel{X}{\to} h \stackrel{W}{\to} j$  in which the focal actor and the mediating actor have the same value of V.



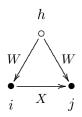
66.  $mixed\ XW \Rightarrow X\ closure,\ different-V\ path\ (X:\ mixed\ XW\ closure\ diff.\ V),\ (diffXWClosure)$  (specified with interaction1 = W, interaction2 = V)  $s_{i66}^{\rm net}(x) = \sum_{j\neq h} x_{ij}\ x_{ih}\ w_{hj}\ I\{v_i\neq v_h\};$  this refers to the closure of X-W two-paths, restricted to two-paths  $i\stackrel{X}{\to} h\stackrel{W}{\to} j$  in which the focal actor and the mediating actor have a different value of V.



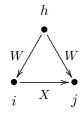
67. closure of shared incoming WW => X same-V, leading to X for different V, (X: shared incoming W jumping V) (jumpSharedIn)  $s_{i67}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj} I\{v_i = v_h \neq v_j\};$  (specified with interaction1 = W, interaction2 = V) this refers to shared incoming W ties contributing to the tie  $i \xrightarrow{X} j$ , restricted to 'jump outside of V-groups' in the sense that the focal actor and the count of shared incoming actors is for those having the same value of V, but the target actor has a different value.



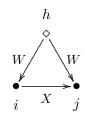
68. shared incoming W leading to X, for same V,  $(X: \operatorname{shared incoming } W \operatorname{same } V)$  ( $\operatorname{covNetNetIn}$ ),  $s_{i68}^{\operatorname{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj} I\{v_i = v_j\};$  (specified with interaction1 = W, interaction2 = V) this refers to shared incoming W-choices (structural equivalence with respect to incoming W-choices), but only for actors sharing the same value of a covariate V; the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of joint W choices of others,  $i \xleftarrow{W} h \xrightarrow{W} j$ , counting only those for which  $v_i = v_j$ .



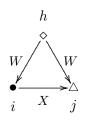
69. shared incoming W leading to X, for homogeneous V, (X: shared incoming W hom. V) (homCovNetNetIn),  $s_{i69}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj} I\{v_i = v_j = v_h\};$  (specified with interaction1 = W, interaction2 = V) this refers to shared incoming W-choices (structural equivalence with respect to incoming W-choices), but only for actors and choices sharing the same value of a covariate V; the contribution of the tie  $i \xrightarrow{X} j$  is proportional to the number of joint incoming W choices of others,  $i \xleftarrow{W} h \xrightarrow{W} j$ , counting only those for which  $v_i = v_j = v_h$ .



70. shared incoming W, same-V about contrasting V, leading to X (X: shared incoming W contrasting V), (contrastCovNetNetIn) (specified with interaction1 = W, interaction2 = V)  $s_{i70}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj} I\{v_i = v_j \neq v_h\};$  this refers to shared incoming W-choices (structural equivalence with respect to incoming W-choices), restricted in the sense that the focal actor and the target actor have the same value of V, but the actors making the shared incoming choices have a different value;



71. shared incoming W, all different V, leading to X (X: shared incoming W all different V), (allDifCovNetNetIn) (specified with interaction1 = W, interaction2 = V)  $s_{i71}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{hi} w_{hj} I\{v_i, v_j, v_h \text{ all different}\};$  this refers to shared incoming W-choices (structural equivalence with respect to incoming W-choices), restricted in the sense that the focal actor, the target actor, and the actors making the shared incoming choices all have a different value of V;

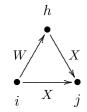


72. mixed  $WX \Rightarrow X$  closure, homogeneous on V

(X: mixed W closure homog. V), (homWXClosure) (specified with interaction1 = 
$$W$$
, interaction2 =  $V$ )

$$s_{i72}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} x_{hj} I\{v_i = v_j = v_h\};$$

this refers to the closure of W-X paths, restricted to triples that are homogeneous with respect to V.



73. W, weighted by U, leading to agreement along X,

(X: W weighted by U to agreement) (toU)

$$s_{i73}^{\text{net}}(x) = \sum_{i \neq h} x_{ij} w_{ih} u_{ih} x_{hj};$$

this refers to the closure of mixed W-X two-paths, weighted by the dyadic covariate U; the contribution of the tie  $i \stackrel{X}{\to} j$  is proportional to the sum of  $u_{ih}$  for all mixed W-X two-paths  $i \stackrel{W}{\to} h \stackrel{X}{\to} j$ .

This is an elementary effect for actor i: only the  $x_{ij}$  tie indicator in the formula, corresponding to the tie  $i \stackrel{X}{\to} j$ , is the dependent variable here.

The interpretation is that actors have the tendency to make the same outgoing X-choices as those to whom they have a W-tie, weighted by U.

Here W is interaction1 and U is interaction2.

This will be suitable mainly for non-centered covariates U, for which the term 'weight' is appropriate.

There are several effects similar to the 'alter' and 'similarity' effects described above, now depending on the auxiliary variable  $\check{v}_i$ , 'alters' v-average'. Here the 'alter' is defined, however, by the other network W. Thus, W-alters' v-average  $\check{v}_i^W$  is defined by

$$\tilde{v}_{i}^{W} = \begin{cases}
\frac{\sum_{j} w_{ij} v_{j}}{w_{i+}} & \text{if } w_{i+} > 0 \\
\bar{v} & \text{if } w_{i+} = 0,
\end{cases}$$
(25)

where  $\bar{v}$  is the overall mean of variable V.

74. covariate - alter at W-distance 2 (altDist2W)

This effect is associated with an effect parameter which can have values 1 or 2. For parameter 1, it is defined as the sum of W-alters' covariate-average over all actors to whom i has a tie,

$$s_{i74}^{\text{net}}(x) = \sum_{j} x_{ij} \breve{v}_{j}^{W}$$
 (parameter 1).

For parameter 2, it is defined similarly, but for an alters' covariate-average excluding ego:

$$s_{i74}^{\text{net}}(x) = \sum_{i} x_{ij} \check{v}_{j}^{W(-i)}$$
 (parameter 2)

where

75. total covariate - alter at W-distance 2 (totDist2W)

This effect is like the previous effect, but now defined defined as the sum of W-alters' covariate-total over all actors to whom i has a tie. For parameter 1 it is

$$s_{i75}^{\text{net}}(x) = \sum_{i} x_{ij} x_{j+} \breve{v}_{j}^{W}$$
 (parameter 1).

For parameter 2, it is defined similarly, but for an alters' covariate-total excluding ego:

$$s_{i75}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{j+} - x_{ji}) \, \breve{v}_{j}^{W(-i)}$$
 (parameter 2)

where  $\breve{v}_j$  and  $\breve{v}_i^{W(-i)}$  are as above.

76. ego-(alter-W-distance-2) covariate - similarity (simEgoDist2W), defined as the sum of centered similarity between i and alters' covariate-average, for all actors j to whom i has a tie,

$$s_{i76}^{\text{net}}(x) = \sum_{j} x_{ij} \left( \sin(\hat{v}^W)_{ij} - \widehat{\sin^v} \right) ,$$

where the similarity scores  $sim(\hat{v}^W)_{ij}$  are defined as

$$sim(\tilde{v}^W)_{ij} = \frac{\Delta - |v_i^W - (\tilde{v}_j^W)^{(-i)}|}{\Delta} ,$$

while  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate v,  $(\check{v}_j^W)^{(-i)}$  is as above in effect (altDist2W),

and  $\sin^v$  is the mean of all similarity scores as used also for the simX effect; this centering is applied since version 1.1-285. For a constant covariate mycov, this mean is given by attr(mydata\$cCovars\$mycov, "simMean").

77. covariate - similarity at W-distance 2 (simDist2W), defined as the sum of centered similarity values for alters' covariate-average between i and all actors j to whom i has a tie,

$$s_{i77}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \text{sim}(\breve{v}^W)_{ij} - \widehat{\text{sim}^v} \right) ,$$

where the similarity scores  $sim(\check{v}^W)_{ij}$  are defined as

$$\sin(\breve{v}^W)_{ij} = \frac{\Delta - |\breve{v}_i^W - \breve{v}_j^W|}{\Delta} ,$$

while  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate v, and  $\widehat{\sin}^v$  is as above.

## 78. covariate - in-alter at W-distance 2 (altInDist2W)

This effect is associated with an effect parameter which can have values 1 or 2. For parameter 1, it is defined as the sum of W-in-alters' covariate-average over all actors to whom i has a tie,

$$s_{i78}^{\text{net}}(x) = \sum_{i} x_{ij} \, \check{v}_{j}^{W} \qquad \text{(parameter 1)},$$

where

$$\tilde{v}_{j}^{W} = \begin{cases}
\frac{\sum_{h \neq i} w_{hj} v_{h}}{x_{+j} - x_{ij}} & \text{if } x_{+j} > 0 \\
\bar{v} & \text{if } x_{+j} = 0,
\end{cases}$$
(27)

where  $\bar{v}$  is (again) the global mean of V.

For parameter 2, it is defined similarly, but for an alters' covariate-average excluding ego:

$$s_{i78}^{\text{net}}(x) = \sum_{i} x_{ij} \, \check{v}_{j}^{W(-i)} \qquad \text{(parameter 2)}$$

where

$$\tilde{v}_{j}^{W(-i)} = \begin{cases}
\frac{\sum_{h \neq j} w_{hj} v_{h}}{w_{j+} - w_{ji}} & \text{if } w_{+j} - w_{ij} > 0 \\
\bar{v} & \text{if } w_{+j} - w_{ij} = 0.
\end{cases}$$
(28)

# 79. total covariate - in-alter at W-distance 2 (totInDist2W)

This effect is like the previous effect, but now defined defined as the sum of W-in-alters' covariate-total over all actors to whom i has a tie. For parameter 1 it is

$$s_{i79}^{\text{net}}(x) = \sum_{j} x_{ij} x_{j+} \check{v}_{j}^{W}$$
 (parameter 1).

For parameter 2, it is defined similarly, but for an alters' covariate-total excluding ego:

$$s_{i79}^{\text{net}}(x) = \sum_{j} x_{ij} (x_{j+} - x_{ji}) \, \check{v}_{j}^{W(-i)}$$
 (parameter 2)

where  $\check{v}_j$  and  $\check{v}_j^{W(-i)}$  are as above.

80. ego-(in-alter-W-distance-2) covariate - similarity (simEgoInDist2W), defined as the sum of centered similarity between i and alters' covariate-in-average, for all actors j to whom i has a tie,

$$s_{i80}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \sin(\hat{v}^W)_{ij} - \widehat{\sin^v} \right) ,$$

where the similarity scores  $\operatorname{sim}(\acute{v}^W)_{ij}$  are defined as

$$\sin(\acute{v})_{ij} = \frac{\Delta - |v_i - \check{v}_j^{(-i)}|}{\Delta} ,$$

while  $\check{v}_{j}^{(-i)}$  is as in the definition of altInDist2W,  $\Delta = \max_{ij} |v_i - v_j|$  is the observed range of the *original* covariate v, and  $\widehat{\sin}^v$  is as above.

The following is an effect for three networks: the dependent network is X, the two explanatory networks are W and Z.

81. from W agreement weighted by Z indegrees (from.w.ind), defined as the sum of Z-indegrees of all actors who have incoming W-ties from i and from those actors j to whom i has a tie,

$$s_{i81}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \sum_{h} w_{ih} \, w_{jh} \, z_{+h} \right) .$$

For internal effect parameter p = 2, the effect is

$$s_{i81}^{\text{net}}(x) = \sum_{j} x_{ij} \left( \sum_{h} w_{ih} \, w_{jh} \, \sqrt{z_{+h}} \right) .$$

For internal effect parameter p = -1, the effect is

$$s_{i81}^{\text{net}}(x) = \sum_{i} x_{ij} \left( \sum_{h} \frac{w_{ih} w_{jh}}{(z_{+h} + 1)} \right).$$

W is given as interaction1, Z as interaction2.

This effect also is available if all or some of X, W, and Z are the same. If X and W are the same, this effect is defined as an elementary effect (see Section 5.1.1).

#### 12.1.4 Network creation and endowment functions

The *network creation function* is one way of modeling effects which operate in different strengths for the creation and the dissolution of relations. The network creation function is zero for dissolution of ties, and is given by

$$c_i^{\text{net}}(x) = \sum_k \zeta_k^{\text{net}} s_{ik}^{\text{net}}(x) \tag{29}$$

for creation of ties. In this formula, the  $\zeta_k^{\rm net}$  are the parameters for the creation function. The potential effects  $s_{ik}^{\rm net}(x)$  in this function, and their formulae, are the same as in the evaluation function; except that not all are available, as indicated in the preceding subsection. For further explication, consult Snijders (2001, 2005); (here, the 'gratification function' is used rather than the creation function), Snijders et al. (2007), and Steglich et al. (2010) (here only the endowment function is treated and not the creation function, but they are similar in an opposite way).

The *network endowment function* is another way of modeling effects which operate in different strengths for the creation and the dissolution of relations. The network endowment function is zero for creation of ties, and is given by

$$e^{\text{net}}(x) = \sum_{k} \gamma_k^{\text{net}} s_{ik}^{\text{net}}(x)$$
 (30)

for dissolution of ties. In this formula, the  $\gamma_k^{\rm net}$  are the parameters for the endowment function. The potential effects  $s_{ik}^{\rm net}(x)$  in this function, and their formulae, are the same as in the evaluation function; except that not all are available, as indicated in the preceding subsection. For further explication, consult Snijders (2001, 2005); (here, the 'gratification function' is used rather than the endowment function), Snijders et al. (2007), and Steglich et al. (2010).

A better term than endowment is perhaps maintenance.

These functions are combined in the following way. For the creation of ties, the objective function used is

$$f_i^{\text{net}}(x) + c_i^{\text{net}}(x) , \qquad (31)$$

in other words, the parameters for the evaluation and creation effects are added. For the dissolution of ties, on the other hand, the objective function is

$$f_i^{\text{net}}(x) + e_i^{\text{net}}(x)$$
, (32)

in other words, the parameters for the evaluation and endowment effects are added. Therefore, a model with a parameter with some value  $\beta_k$  for a given evaluation effect, and for which there are no separate creation and endowment effects, has exactly the same consequences as a model for which this evaluation effect is excluded, and that includes a creation as well as an endowment effect, both with the same parameter value  $\zeta_k = \beta_k$  and  $\gamma_k = \beta_k$ .

Of the three types of effect — evaluation, creation, and endowment —, one therefore should use one or two, not all three, because this leads to collinearity.

#### 12.1.5 Network rate function

To define rate effects by the function include Effects or set Effect, note that in the call of these functions the specification type = "rate" should be included.

The network rate function  $\lambda^{\rm net}$  (lambda) is defined for Model Type 1 (which is the default Model Type) as a product

$$\lambda_i^{\rm net}(\rho,\alpha,x,m) = \lambda_{i1}^{\rm net} \lambda_{i2}^{\rm net} \lambda_{i3}^{\rm net}$$

of factors depending, respectively, on period m, actor covariates, and actor position (see Snijders, 2001, p. 383). The corresponding factors in the rate function are the following:

1. The dependence on the period (Rate) can be represented by a simple factor

$$\lambda_{i1}^{\mathrm{net}} = \rho_m^{\mathrm{net}}$$

for m = 1, ..., M - 1.

This basic rate parameter is always period-dependent (i.e., it may and will be different for different values of m); this is in contrast to all other parameters.

2. The effect of actor covariates (RateX) with values  $v_{hi}$  can be represented by the factor

$$\lambda_{i2}^{\text{net}} = \exp(\sum_{h} \alpha_h \, v_{hi}) \,.$$

If  $v_h$  is a behavior variable, the non-centered value is used.

3. The dependence on the position of the actor can be modeled as a function of the actor's out-degree (outRate), in-degree (inRate), and number of reciprocated relations (recipRate), the 'reciprocated degrees'. Define these by

$$x_{i+} = \sum_{j} x_{ij}, \qquad x_{+i} = \sum_{j} x_{ji}, \qquad x_{i(r)} = \sum_{j} x_{ij}x_{ji}$$

(recalling that  $x_{ii} = 0$  for all i).

The contribution of the out-degrees to  $\lambda_{i3}^{\rm net}$  is a factor

$$\exp(\alpha_h x_{i+})$$
,

if the associated parameter is denoted  $\alpha_h$  for some h, and similarly for the contributions of the in-degrees and the reciprocated degrees.

Nonlinear dependence of the exponent on the degrees can also be specified:

• inverse outdegree effect (outRateInv) Denoting again the corresponding parameter by  $\alpha_h$  (but always for different index numbers h), this effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\alpha_h/(x_{i+}+1)\right)$$
.

• logarithmic outdegree effect (outRateLog) This effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\ln\left(\alpha_h(x_{i+}+1)\right)\right) = (x_{i+}+1)^{\alpha_h}.$$

• inverse indegree effect (inRateInv) This effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\alpha_h/(x_{+i}+1)\right)$$
.

• logarithmic indegree effect (inRateLog) This effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\ln\left(\alpha_h(x_{+i}+1)\right)\right) \,=\, \left(x_{+i}+1\right)^{\alpha_h}\,.$$

• inverse reciprocal degree effect (recipRateInv) This effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\alpha_h/(x_{i(r)}+1)\right)$$
.

• logarithmic reciprocal degree effect (recipRateLog) This effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

$$\exp\left(\ln\left(\alpha_h(x_{i(r)}+1)\right)\right) = \left(x_{i(r)}+1\right)^{\alpha_h}.$$

The exponential link function and logarithmic transformation collaborate to produce direct proportionality to (outdegree + 1) or (indegree + 1), respectively, in case the parameter is  $\alpha_h = 1$ .

For these effects, the addition of 1 to the degrees avoids problems (division by 0, logarithm of 0) that otherwise would occur when  $x_{i+} = 0$  or  $x_{+i} = 0$ .

For making the rates dependent on the degrees, the logarithmic effects often give the best fit.

These effects work properly only for non-conditional estimation (set cond = FALSE in sienaAlgorithmCreate).

When rate effects are included, sometimes it can be helpful to specify a smaller value for the initial gain parameter in sienaAlgorithmCreate; e.g., firstg = 0.01 or even firstg = 0.001. When the estimation algorithm diverges and any rate effects are in the model, this option should be considered. With a smaller firstg parameter convergence will be slower, but also there is a smaller probability that the parameter will diverge accidentally.

# 12.2 Behavioral evolution

The model of the dynamics of a dependent actor variable consists of a model of actors' decisions (according to evaluation, creation, and endowment functions) and a model of the timing of these decisions (according to a rate function), just like the model for the network dynamics. The decisions now do not concern the creation or dissolution of network ties, but whether an actor increases or decreases his score on the dependent actor variable by one, or keeps it as it is.

### 12.2.1 Behavioral evaluation function

The behavior evaluation function for actor i is defined as

$$f_i^{\text{beh}}(x,z) = \sum_k \beta_k^{\text{beh}} s_{ik}^{\text{beh}}(x,z)$$
(33)

where  $\beta_k^{\text{beh}}$  are parameters and  $s_{ik}^{\text{beh}}(x,z)$  are effects as defined below. The behavioral dependent variable is denoted by z and the dependent network variable by x. Here the dependent variable is transformed to have an overall average value of 0; in other words, z denotes the original input variable minus the overall mean<sup>18</sup>, which is given in the output file produced by print01Report under the heading Reading dependent actor variables.

First there are effects that have to do only with the behavioral variable itself.

- 1. behavioral shape effect (linear),  $s_{i1}^{\text{beh}}(x,z) = z_i$ , where  $z_i$  denotes the value of the dependent behavior variable of actor i;
- 2. quadratic shape effect, or effect of the behavior upon itself (quad), where the attractiveness of further steps up the behavior 'ladder' depends on where the actor is on the ladder:

$$s_{i2}^{
m beh}(x,z)=z_i^2$$
; note that here  $z_i$  is the centered dependent behavioral variable;

3. threshold shape effect (threshold), (threshold2), (threshold3), (threshold4), where the attractiveness of the behavior depends on whether it is larger than the internal effect parameter p:

$$s_{i3}^{\text{beh}}(x,z) = I\{z_i \ge p\}$$
;

note that by exception  $z_i$  here is the <u>uncentered</u> dependent behavioral variable! Multiple versions of this effect are available to allow the use of more than one threshold.

4. p-near similarity effect (simAllNear),  $s_{i4}(z) = \sum_{j\neq i} \left( \max\{p - |z_j - z_i|, 0\} \right) = \sum_{j\neq i} I\{|z_j - z_i| \leq p\} \left( p - |z_j - z_i| \right)$  where  $p \geq 1$  is an internal effect parameter; note that this is an influence effect from all actors in the network, which does not take a network into account;

<sup>&</sup>lt;sup>18</sup>More precisely: this is the mean of the means per wave, and the means per wave are means of the non-missing observations for that wave.

- 5. p-far similarity effect (simAllFar),  $s_{i5}(z) = \sum_{j \neq i} \min\{p |z_j z_i|, 0\} = \sum_{j \neq i} I\{|z_j z_i| \geq p\} (p |z_j z_i|)$  where  $p \geq 1$  is an internal effect parameter; note that this also is an influence effect from all actors in the network, which does not take a network into account;
- 6. group average (avGroup), defined by ego's value multiplied by the average of all values  $z_h$  for this period,

$$s_{i6}^{\text{beh}}(x,z) = z_i (\overline{z} - c_p);$$

here  $\overline{z}$  is the mean of the values  $z_h$  for all actors h in the group, and  $c_p$  is a centering constant depending on the internal effect parameter p. Denote the overall mean used for centering the observed Z values by  $m_Z$  (see the footnote on p. 193). If  $p \leq 0.5$ , centering is no different than for the  $z_i$  values generally, so  $c_p = 0$ ; if p > 0.5, centering is by  $c_p = p - m_Z$ . For the original non-centered values of Z, this means that the mean is centered around the value p, the effect parameter itself. Note that this can be any real value, not necessarily integer.

This effect is useful especially for multi-group data sets, where the average value varies between groups. For multi-group data sets, centering by the groupwise mean may be less desirable, and it will be better to center by a value p close to the median of Z. (Note that the centering in a multi-group data set will affect the linear shape parameter.)

7. constant effect (constant),

```
an elementary effect for behavior, indicating that the behavior stays constant: s_{i7}^{\text{beh}}(x, z^0, z) = I\{z_i = z_i^0\}.
```

The advice for this effect is not to use it, unless you have very specific reasons. For estimation, this effect is totally collinear with the rate effect; therefore its parameter cannot be estimated. It is only of theoretical importance, and can be used perhaps in interaction with other effects.

Next there is a list of effects that have to do with the influence of the network on the behavior. To specify such effects in RSiena using, e.g., function includeEffects, it is necessary 19 to specify the dependent behavior variable in the keyword name as well as the network in the keyword interaction1. For example,

The list of these effects is the following.

8. average similarity effect (avSim), defined by the average of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is tied,

$$s_{i8}^{\text{beh}}(x, z) = x_{i+}^{-1} \sum_{j} x_{ij} (\sin_{ij}^{z} - \widehat{\sin}^{z});$$
  
(and 0 if  $x_{i+} = 0$ );

the definition of  $sim_{ij}^z$  is discussed in section 4.2, see (1);

<sup>&</sup>lt;sup>19</sup>If this behavior variable is the only dependent variable, then this is not necessary. But this seldom happens.

9. total similarity effect (totSim), defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between i and the other actors j to whom he is tied,

$$s_{i9}^{\text{beh}}(x,z) = \sum_{j} x_{ij} (\operatorname{sim}_{ij}^{z} - \widehat{\operatorname{sim}}^{z});$$

it may be helpful for estimating this effect also to include the outdeg effect, with which it is correlated;

10. average incoming similarity effect (avInSim), defined by the average of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j who send a tie to i,

$$s_{i10}^{\mathrm{beh}}(x,z) = x_{+i}^{-1} \sum_{j} x_{ji} (\sin_{ij}^{z} - \widehat{\sin}^{z});$$
 (and 0 if  $x_{+i} = 0$ );

11. total incoming similarity effect (totInSim), defined by the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j who send a tie to i,

$$s_{i11}^{\text{beh}}(x,z) = \sum_{j} x_{ji} (\sin_{ij}^z - \widehat{\sin}^z);$$

12. indegree effect (indeg),  $s_{i12}^{\text{beh}}(x,z) = z_i \sum_i x_{ii};$ 

13. outdegree effect (outdeg),  $s_{i13}^{\text{beh}}(x,z) = z_i \sum_i x_{ij};$ 

14. reciprocated degree effect (recipDeg),  $s_{i14}^{\text{beh}}(x,z) = z_i \sum_j x_{ij} x_{ji};$ 

15. in-isolate effect (isolate), the differential attractiveness of the behavior for inisolates,

$$s_{i15}^{\text{beh}}(x,z) = z_i I\{x_{+i} = 0\},\,$$

where again  $I\{A\}$  denotes the indicator function of the condition A; (before version 1.2-16 this was called the *isolate effect*;)

16. out-isolate effect (outIsolate), the differential attractiveness of the behavior for out-isolates,

$$s_{i16}^{\text{beh}}(x,z) = z_i I\{x_{i+} = 0\}$$

 $s_{i16}^{\mathrm{beh}}(x,z)=z_i\,I\{x_{i+}=0\},$  where  $I\{A\}$  still denotes the indicator function of the condition A;

17. average similarity × reciprocity effect (avSimRecip), defined by the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is reciprocally

$$s_{i17}^{\text{beh}}(x,z) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} (\sin^{z}_{ij} - \widehat{\sin^{z}});$$
 (and 0 if  $x_{i(r)} = 0$ );

18. total similarity × reciprocity effect (totSimRecip), defined by the sum of centered similarity scores  $sim_{ij}^z$  between i and the other actors j to whom he is reciprocally

$$s_{i18}^{\text{beh}}(x,z) = \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{z} - \widehat{\sin}^{z});$$

19. average similarity × popularity alter effect (avSimPopAlt), defined by the average of centered similarity scores  $\sin_{ij}^{z}$  between i and the other actors j to whom he is tied, multiplied by their indegrees,

$$s_{i19}^{\text{beh}}(x,z) = x_{i+}^{-1} \sum_{j} x_{ij} x_{+j} (\sin^{z}_{ij} - \widehat{\sin^{z}});$$
  
(and 0 if  $x_{i+} = 0$ );

20. average incoming similarity × popularity alter effect (avInSimPopAlt), defined by the average of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j who send a tie to i, multiplied by their indegrees,

$$s_{i20}^{\text{beh}}(x,z) = x_{+i}^{-1} \sum_{j} x_{ji} x_{+j} (\sin^{z}_{ij} - \widehat{\sin^{z}});$$
 (and 0 if  $x_{+i} = 0$ );

21. popularity alter effect (popAlt), defined by the average in-degrees of the other actors j to whom i is tied,

$$\begin{array}{l} s_{i21}^{\rm beh}(x,z) = z_i \, x_{i+}^{-1} \, \sum_j x_{ij} x_{+j}; \\ ({\rm and} \ 0 \ {\rm if} \ x_{i+} = 0) \ ; \end{array}$$

this effect may be useful, e.g., as a control effect for the average similarity × popularity alter effect;

22. total similarity × popularity alter effect (totSimPopAlt), defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between i and the other actors j to whom he is tied, multiplied by their indegrees,

$$s_{i22}^{\text{beh}}(x,z) = \sum_{j} x_{ij} x_{+j} (\operatorname{sim}_{ij}^z - \widehat{\operatorname{sim}}^z);$$

23. total similarity × popularity alter effect (totInSimPopAlt), defined by the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j who send a tie to

$$i$$
, multiplied by their indegrees,  $s_{i23}^{\text{beh}}(x,z) = \sum_{j} x_{ji} x_{+j} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ 

24. average similarity  $\times$  reciprocity  $\times$  popularity alter effect (avSimRecPop), defined by the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is reciprocally tied, multiplied by their indegrees,  $s_{i24}^{\text{beh}}(x,z) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} x_{+j} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ 

$$s_{i24}^{\text{beh}}(x,z) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} x_{+j} (\sin^{z}_{ij} - \widehat{\sin^{z}});$$
  
(and 0 if  $x_{i(r)} = 0$ );

25.  $total\ similarity imes reciprocity imes popularity\ alter\ effect\ (totSimRecPop),\ defined\ by$ the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is reciprocally tied, multiplied by their indegrees,

$$s_{i25}^{\text{beh}}(x,z) = \sum_{j} x_{ij} x_{ji} x_{+j} (\sin^{z}_{ij} - \widehat{\sin^{z}});$$

26. average attraction toward higher effect (avAttHigher), defined by the average of non-centered similarity scores  $\sin_{ij}^z$  between i and the actors j to whom he is tied,

where the similarity is replaced by 1 for those who have a lower behavior than i,

$$s_{i26}^{\text{beh}}(x,z) = x_{i+}^{-1} \sum_{j} x_{ij} \left( I\{z_j > z_i\} \sin_{ij}^z + I\{z_j \le z_i\} \right)$$
$$= x_{i+}^{-1} \sum_{j} x_{ij} \left( 1 - \frac{\max(z_j - z_i, 0)}{r_Z} \right);$$

(and 0 if  $x_{i+} = 0$ );

the definition of  $\sin_{ij}^z$  is discussed in section 4.2, see (1);  $r_Z$  is the range of Z;

27. average attraction toward lower effect (avAttLower), defined by the average of non-centered similarity scores  $\sin_{ij}^z$  between i and the actors j to whom he is tied, where the similarity is replaced by 1 for those who have a higher behavior than i,

$$s_{i27}^{\text{beh}}(x,z) = x_{i+}^{-1} \sum_{j} x_{ij} \left( I\{z_j < z_i\} \sin_{ij}^z + I\{z_j \ge z_i\} \right)$$
$$= x_{i+}^{-1} \sum_{j} x_{ij} \left( 1 - \frac{\max(z_i - z_j, 0)}{r_Z} \right);$$

(and 0 if  $x_{i+} = 0$ );

28. total attraction toward higher effect (totAttHigher), defined by the sum of non-centered similarity scores  $\sin_{ij}^z$  between i and the actors j to whom he is tied, where the similarity is replaced by 1 for those who have a lower behavior than i,

$$s_{i28}^{\text{beh}}(x, z) = \sum_{j} x_{ij} \left( I\{z_j > z_i\} \sin_{ij}^z + I\{z_j \le z_i\} \right)$$
$$= \sum_{i} x_{ij} \left( 1 - \frac{\max(z_j - z_i, 0)}{r_Z} \right);$$

29. total attraction toward lower effect (totAttLower), defined by the sum of non-centered similarity scores  $\sin_{ij}^z$  between i and the actors j to whom he is tied, where the similarity is replaced by 1 for those who have a higher behavior than i,

$$s_{i29}^{\text{beh}}(x,z) = \sum_{j} x_{ij} \left( I\{z_j < z_i\} \sin_{ij}^{z} + I\{z_j \ge z_i\} \right)$$
$$= \sum_{i} x_{ij} \left( 1 - \frac{\max(z_i - z_j, 0)}{r_Z} \right);$$

30. average alter effect (avAlt), defined by i's behavior multiplied by the average behavior of his alters (a kind of ego-alter behavior covariance),

$$s_{i30}^{\rm beh}(x,z)=z_i\big(\sum_j x_{ij}\,z_j\big)/\big(\sum_j x_{ij}\big)$$
 (and the mean behavior, i.e. 0, if the ratio is 0/0) ;

31. total alter effect (totalt), defined by i's behavior multiplied by the sum of behavior of his alters,

$$s_{i31}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} z_j \right);$$

32. average in-alter effect (avInAlt), defined by i's behavior multiplied by the average behavior of his in-alters (a kind of ego-alter behavior covariance),

$$s_{i32}^{\rm beh}(x,z)=z_i\big(\sum_j x_{ji}\,z_j\big)/\big(\sum_j x_{ji}\big)$$
 (and the mean behavior, i.e. 0, if the ratio is 0/0) ;

33. total alter effect (totInAlt), defined by i's behavior multiplied by the sum of behavior of his in-alters,

$$s_{i33}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ji} z_j \right);$$

34. average reciprocated alter effect (avRecAlt), defined by i's behavior multiplied by the average behavior of his reciprocated alters,

$$s_{i34}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} x_{ji} z_j \right) / \left( \sum_j x_{ij} x_{ji} \right)$$
 (and 0 if the ratio is 0/0);

35. total reciprocated alter effect (totRecAlt), defined by i's behavior multiplied by the sum of behavior of his reciprocated alters,

$$s_{i35}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} x_{ji} z_j \right);$$

36. average alter × popularity effect (avAltPop), defined by the behavior multiplied by the average behavior of his alters, multiplied by their indegrees,

$$s_{i36}^{\text{beh}}(x,z) = z_i x_{i+}^{-1} \left( \sum_j x_{ij} x_{+j} z_j \right);$$
  
(and 0 if  $x_{i+} = 0$ );

37.  $total\ alter imes popularity\ effect\ (totAltPop)$ , defined by the behavior multiplied by the sum of the behavior of his alters, multiplied by their indegrees,

$$s_{i37}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} x_{+j} z_j \right);$$

38. only in RSienaTest: average Simmelian alter effect, (avSimmelianAlt), the effect of the average behavior for the Simmelian alters. The Simmelian transformation of the network is the network composed of all ties embedded in at least one complete triad:

$$x_{ij}^{\text{simm}} = \begin{cases} 1 & x_{ij} = x_{ji} = 1 \text{ and} \\ & \text{there is at least one } h \text{ for which } x_{hi} = x_{ih} = x_{hj} = x_{jh} = 1 \\ 0 & \text{else.} \end{cases}$$

In other words, these ties must be reciprocal, and there must be at least one third actor to whom both have a mutual tie. The average Simmelian alter effect is

$$s_{i38}^{\mathrm{beh}}(x,z) = z_i \left(\sum_j x_{ij}^{\mathrm{simm}} z_j\right) / \left(\sum_j x_{ij}^{\mathrm{simm}}\right)$$
 (and the mean behavior, i.e. 0, if the ratio is 0/0);

- 39. only in RSienaTest: total Simmelian alter effect, (totSimmelianAlt), the effect of the sum of behavior for the Simmelian alters, defined by  $s_{i39}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij}^{\text{simm}} z_j\right);$
- 40. average alter effect at distance 2 (avAltDist2), defined by i's behavior multiplied by the average of the alter-averages  $\check{z}_j^{(-i)}$  of his alters, excluding the contribution from a tie  $j \to i$  (if any), defined by

$$\tilde{z}_{j}^{(-i)} = \begin{cases}
\frac{\sum_{h \neq i} x_{jh} z_{h}}{x_{j+} - x_{ji}} & \text{if } x_{j+} - x_{ji} > 0 \\
0 & \text{if } x_{j+} - x_{ji} = 0.
\end{cases}$$
(34)

(also see (23)),  $s^{\rm beh}_{i40}(x,z) = z_i \big( \sum_j x_{ij} \, \check{z}_j^{(-i)} \big) / \big( \sum_j x_{ij} \big)$  (and the mean behavior, i.e. 0, if the ratio is 0/0) ;

41. total alter effect at distance 2 (totAltDist2), defined by i's behavior multiplied by the total of the alter-totals of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i41}^{\text{beh}}(x,z) = z_i \sum_j x_{ij} \sum_{h \neq i} x_{jh} z_h = z_i \sum_j x_{ij} (x_{j+} - x_{ji}) \, \breve{z}_j^{(-i)};$$

42. average total alter effect at distance 2 (avTAltDist2), defined by i's behavior multiplied by the average of the alter-totals of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i42}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij} (x_{j+} - x_{ji}) z_j^{(-i)}\right) / \left(\sum_j x_{ij}\right) = z_i \left(\sum_j x_{ij} \sum_{h \neq i} x_{jh} z_h\right) / \left(\sum_j x_{ij}\right)$$
 (and the mean behavior, i.e. 0, if the ratio is  $0/0$ );

43. total average alter effect at distance 2 (totAltDist2), defined by i's behavior multiplied by the total of the alter-averages  $z_j^{(-i)}$  of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i43}^{\text{beh}}(x,z) = z_i (\sum_i x_{ij} \, \tilde{z}_i^{(-i)});$$

44. average incoming alter effect at distance 2 (avInAltDist2), defined by i's behavior multiplied by the average of the incoming alter averages  $\check{z}_j^{(-i)}$  of his alters, excluding the contribution from a tie  $i \to j$  (if any), defined by

$$\tilde{z}_{j}^{(-i)} = \begin{cases}
\frac{\sum_{h \neq i} x_{hj} z_{h}}{x_{+j} - x_{ij}} & \text{if } x_{+j} - x_{ij} > 0 \\
0 & \text{if } x_{+j} - x_{ij} = 0.
\end{cases}$$
(35)

(cf. (34)), 
$$s_{i44}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} \, \check{z}_j^{(-i)} \right) / \left( \sum_j x_{ij} \right)$$
 (and the mean behavior, i.e. 0, if the ratio is 0/0);

45. total incoming alter effect at distance 2 (totInAltDist2), defined by i's behavior multiplied by the total of the incoming alter-totals of his alters, excluding the contribution from a tie  $i \rightarrow j$  (if any),

$$s_{i45}^{\text{beh}}(x,z) = z_i \sum_j x_{ij} \sum_{h \neq i} x_{hj} z_h = z_i \sum_j x_{ij} (x_{+j} - x_{ij}) \check{z}_j^{(-i)};$$

46. average total incoming alter effect at distance 2 (avTInAltDist2), defined by i's behavior multiplied by the average of the incoming alter-totals of his alters, excluding the contribution from a tie  $i \rightarrow j$  (if any),

$$s_{i46}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} (x_{+j} - x_{ij}) \check{z}_j^{(-i)} \right) / \left( \sum_j x_{ij} \right)$$
 (and the mean behavior, i.e. 0, if the ratio is  $0/0$ ):

47. total average incoming alter effect at distance 2 (totAInAltDist2), defined by i's behavior multiplied by the total of the incoming alter-averages  $z_j^{(-i)}$  of his alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i47}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} \check{z}_j^{(-i)} \right);$$

48. average in-similarity effect at distance 2 (avInSimDist2), defined by i's average similarity to the incoming alter averages  $\check{z}_j^{(-i)}$  of his out-alters, excluding the contribution from a tie  $i \to j$  (if any),

substitute that it to 
$$t = f(x)$$
 (it easy),  $s_{i48}^{\text{beh}}(x,z) = \left(\sum_{j} x_{ij} I\{x_{+j} - x_{ij} > 0\} \left( \text{sim}^z(z_i, \check{z}_j^{(-i)}) - \widehat{\text{sim}}^z \right) \right) / \sum_{j} x_{ij}$ , where the similarity scores  $\text{sim}^z$  are defined as usual; this is the behavior/influence effect parallel to  $\text{simEgoInDist2}$ ;

49. total in-similarity effect at distance 2 (totInSimDist2), defined by the sum of i's similarity to the incoming alter averages  $\check{z}_j^{(-i)}$  of his out-alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i49}^{\text{beh}}(x,z) = \sum_{j} x_{ij} I\{x_{+j} - x_{ij} > 0\} \left( \sin^{z}(z_{i}, \tilde{z}_{j}^{(-i)}) - \widehat{\sin^{z}} \right),$$
 where again the similarity scores  $\sin^{z}$  are defined as usual;

50. average in-similarity effect at distance 2 (avInSimDist2), defined by i's average similarity to the incoming alter averages  $\check{z}_{j}^{(-i)}$  of his out-alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i50}^{\mathrm{beh}}(x,z) = \left(\sum_{j} x_{ij} I\{x_{+j} - x_{ij} > 0\} \left( \mathrm{sim}^z(z_i, \check{z}_j^{(-i)}) - \widehat{\mathrm{sim}}^z \right) \right) / \sum_{j} x_{ij},$$
 where the similarity scores  $\mathrm{sim}^z$  are defined as usual; this is the behavior/influence effect parallel to  $\mathrm{simEgoInDist2}$ ;

51. total in-similarity effect at distance 2 (totInSimDist2), defined by the sum of i's similarity to the incoming alter averages  $\check{z}_j^{(-i)}$  of his out-alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i51}^{\rm beh}(x,z) = \sum_j x_{ij} I\{x_{+j} - x_{ij} > 0\} \left( \sin^z(z_i, \tilde{z}_j^{(-i)}) - \widehat{\sin^z} \right),$$
 where again the similarity scores  $\sin^z$  are defined as usual;

52. maximum alter effect (maxAlt), defined by i's behavior multiplied by the maximum behavior of his alters,

```
s_{i52}^{\text{beh}}(x,z) = z_i (\max_j x_{ij} z_j) (and the mean behavior, i.e. 0, if \sum_i x_{ij} = 0);
```

53. minimum alter effect (minAlt), defined by i's behavior multiplied by the minimum behavior of his alters,

```
s_{i53}^{\mathrm{beh}}(x,z)=z_i \big( \min_j x_{ij} \, z_j \big) (and the mean behavior, i.e. 0, if \sum_j x_{ij}=0);
```

54. dense triads effect (behDenseTriads), defined by i's behavior multiplied by the number of dense triads in which actor i is located,

$$s_{i54}^{\text{beh}}(x,z) = z_i \sum_{j,h} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj} \ge p\},$$
 where the internal effect parameter  $p$  is either 5 or 6;

- 55. similarity in dense triads effect (simDenseTriads), defined by the total sum of i's behavior similarity to others in the dense triads in which actor i is located,  $s_{i55}^{\text{beh}}(x,z) = \sum_{j,h} (\sin(v_i,v_j) + \sin(v_i,v_h)) I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj} \ge p\}$ , where p is either 5 or 6;
- 56. peripheral effect, defined by i's behavior multiplied by the number of dense triads to which actor i stands in a unilateral-peripheral relation,

$$\begin{split} s^{\text{beh}}_{i56}(x,z) &= \\ z_i \sum_{j,h,k} x_{ij} (1-x_{ji}) (1-x_{hi}) (1-x_{ki}) I\{x_{ij}+x_{ji}+x_{ih}+x_{hi}+x_{jh}+x_{hj} ) \geq p\} \,, \end{split}$$
 where  $p$  is the same constant as in the  $dense\ triads$  effect; for symmetric networks, the unilateral condition is dropped, and the effect is  $s^{\text{beh}}_{i56}(x,z) = z_i \sum_{j,h,k} x_{ij} (1-x_{hi}) (1-x_{ki}) I\{x_{ij}+x_{ji}+x_{ih}+x_{hi}+x_{jh}+x_{hj} ) \geq p\} \,; \end{split}$ 

57. average similarity  $\times$  popularity ego effect (avSimPopEgo), defined by the sum of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is tied, multiplied by ego's indegree,

$$s_{i57}^{\text{beh}}(x, z) = x_{+i} x_{i+}^{-1} \sum_{j} x_{ij} (\sin_{ij}^{z} - \widehat{\sin}^{z});$$
 (and 0 if  $x_{i+} = 0$ );

because of collinearity, under the Method of Moments this cannot be estimated together with the average similarity  $\times$  popularity alter effect;

### Effects of multiple networks

If there are more than one dependent network variables, denoted  $X_1$  and  $X_2$ , they can operate jointly on the behavioural dependent variable using the following effects.  $X_1$  is given as interaction1,  $X_2$  as interaction2.

For the combined degree-effects, mentioned first, F means 'Forward', B means 'Backward', and R means 'Reciprocal'.

$$s_{i58}^{\text{beh}}(x,z) = z_i \sum_j x_{1ij} x_{2ij};$$

if the internal effect parameter p is at least 2, the outdegree for  $X_1$  is subtracted (perhaps this leads to confusion...):

$$s_{i58}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{1ij} x_{2ij} - \sum_j x_{1ij} \right);$$

note that in this case, the factor between (...) is non-positive;

# 59. double indegree effect (BBDeg),

$$s_{i59}^{\text{beh}}(x,z) = z_i \sum_j x_{1ji} x_{2ji};$$

if the internal effect parameter p is at least 2, the indegree for  $X_1$  is subtracted:

$$s_{i59}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{1ji} x_{2ji} - \sum_j x_{1ji} \right);$$

note that also in this case, the factor between (...) is non-positive;

60. combined out-indegree effect (FBDeg),

$$s_{i60}^{\text{beh}}(x,z) = z_i \sum_j x_{1ij} x_{2ji};$$

if the internal effect parameter p is at least 2, the outdegree for  $X_1$  is subtracted:

$$s_{i60}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{1ij} x_{2ji} - \sum_j x_{1ij} \right);$$

note that also in this case, the factor between (...) is non-positive;

 $61.\ combined\ out\text{-}reciprocated\ degree\ effect\ (\texttt{FRDeg}),$ 

$$s_{i61}^{\text{beh}}(x,z) = z_i \sum_j x_{1ij} x_{2ij} x_{2ji};$$

if the internal effect parameter p is at least 2, the outdegree for  $X_1$  is subtracted:

$$s_{i61}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{1ij} x_{2ij} x_{2ji} - \sum_j x_{1ij} \right);$$

note that also in this case, the factor between (...) is non-positive;

62. combined in-reciprocated degree effect (BRDeg),

$$s_{i62}^{\text{beh}}(x,z) = z_i \sum_{j=1}^{n} x_{1ji} x_{2ij} x_{2ji};$$

if the internal effect parameter p is at least 2, the indegree for  $X_1$  is subtracted:

$$s_{i62}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{1ji} x_{2ij} x_{2ji} - \sum_j x_{1ji} \right);$$

note that also in this case, the factor between (...) is non-positive.

# Monadic covariate effects

For each actor-dependent covariate  $v_j$  (recall that by default these are centered internally by SIENA) as well as for each of the other dependent behavior variables (for notational simplicity here also denoted  $v_j$ ), there are the following effects.

63. main covariate effect (effFrom),

$$s_{i63}^{\text{beh}}(x,z) = z_i v_i;$$

here too, the other dependent behavioral variables are centered so that they have overall mean 0;

64. alter's covariate average effect on behavior z (avXAlt), formerly called (AltsAvAlt), defined as the product of i's behavior  $z_i$  and i's alters' covariate-average  $\check{v}_i$  as defined in (22),

 $s_{i64}^{\text{beh}}(x,z) = z_i \, \breve{v}_i.$ 

This is similar to the 'average alter' effect; for  $v_i = z_i$  it would reduce to the latter effect.

65. alter's covariate total effect on behavior z (totXAlt), defined as the product of i's behavior  $z_i$  and i's alters' covariate sum  $x_{i+} \check{v}_i$  as defined in (22),

 $s_{i65}^{\text{beh}}(x,z) = z_i x_{i+} \breve{v}_i.$ 

This is similar to the 'total alter' effect; for  $v_i = z_i$  it would reduce to the latter effect.

66. maximum alter's covariate effect (maxXAlt), defined by i's behavior multiplied by the maximum covariate value of his alters,

```
\begin{array}{l} s_{i66}^{\rm beh}(x,z) = z_i \bigl( \max_j x_{ij} \, v_j \bigr) \\ ({\rm and \ the \ mean \ covariate \ if \ } \sum_j x_{ij} = 0) \ ; \end{array}
```

67. minimum alter's covariate effect (minXAlt), defined by i's behavior multiplied by the minimum covariate value of his alters,

$$s_{i67}^{\text{beh}}(x,z) = z_i (\min_j x_{ij} v_j)$$
  
(and the mean covariate if  $\sum_j x_{ij} = 0$ );

68. absolute contrast outdegree – covariate (degAbsContrX), defined as the product of i's behavior  $z_i$  and the absolute difference between i's out-degree and his covariate value,

$$s_{i68}^{\text{beh}}(x,z) = z_i | x_{i+} - v_i |;$$

working with a non-centered covariate may be advisable for this effect;

69. positive contrast outdegree – covariate (degPosContrX), defined as the product of i's behavior  $z_i$  and the positive part of the difference between i's out-degree and her covariate value,

$$s_{i69}^{\text{net}}(x) = z_i \max\{x_{i+} - v_i, 0\};$$

working with a non-centered covariate may be advisable for this effect;

70. negative contrast outdegree – covariate (degNegContrX), defined as the product of i's behavior  $z_i$  and the negative part of the difference between i's out-degree and her covariate value,

$$s_{i70}^{\text{net}}(x) = -z_i \min\{x_{i+} - v_i, 0\} = z_i \max\{v_i - x_{i+}, 0\};$$
 working with a non-centered covariate may be advisable for this effect;

71. alter's covariate total effect on behavior z (totXAlt), defined as the product of i's behavior  $z_i$  and i's alters' covariate sum  $x_{i+} \check{v}_i$  as defined in (41),

$$s_{i71}^{\text{beh}}(x,z) = z_i x_{i+} \breve{v}_i.$$

This is similar to the 'total alter' effect; for  $v_i = z_i$  it would reduce to the latter effect.

72. in-alter's covariate average effect on behavior z (avXInAlt), defined as the product of i's behavior  $z_i$  and i's in-alters' covariate-average  $\check{v}_i$  as defined in

$$\check{v}_i = \begin{cases}
\frac{\sum_j x_{ji} v_j}{x_{+i}} & \text{if } x_{+i} > 0 \\
0 & \text{if } x_{+i} = 0,
\end{cases}$$
(36)

the effect being

$$s_{i72}^{\text{beh}}(x,z) = z_i \, \check{v}_i.$$

This is similar to the 'average in-alter' effect; for  $v_i = z_i$  it would reduce to the latter effect.

73. in-alter's covariate total effect on behavior z (totXInAlt), defined as the product of i's behavior  $z_i$  and i's alters' covariate sum  $x_{i+} \check{v}_i$  as defined in (36).

$$s_{i73}^{\text{beh}}(x,z) = z_i x_{i+} \check{v}_i.$$

This is similar to the 'total in-alter' effect; for  $v_i = z_i$  it would reduce to the latter effect.

74. alter's distance-two covariate average effect on behavior z (avXAltDist2), defined by i's behavior multiplied by the average of the alter-averages  $v_j^{(-i)}$  of his alters, excluding the contribution from a tie  $j \to i$  (if any), defined by

$$\tilde{v}_{j}^{(-i)} = \begin{cases}
\frac{\sum_{h \neq i} x_{jh} v_{h}}{x_{j+} - x_{ji}} & \text{if } x_{j+} - x_{ji} > 0 \\
0 & \text{if } x_{j+} - x_{ji} = 0.
\end{cases}$$
(37)

(also see (23) and (34)),

$$s_{i74}^{\mathrm{beh}}(x,z) = z_i \left(\sum_j x_{ij} \, \breve{v}_j^{(-i)}\right) / \left(\sum_j x_{ij}\right)$$

(and the mean behavior, i.e. 0, if the ratio is 0/0);

this is similar to avAltDist2, but now for covariate V instead of the behavior Z;

75. alter's distance-two covariate total effect on behavior z (totXAltDist2), defined by i's behavior multiplied by the total of the alter-totals on V of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i75}^{\text{beh}}(x,z) = z_i \sum_j x_{ij} \sum_{h \neq i} x_{jh} v_h = z_i \sum_j x_{ij} (x_{j+} - x_{ji}) \check{v}_j^{(-i)};$$

this is similar to totAltDist2, but now for covariate V instead of the behavior Z;

76. average total covariate alter effect at distance 2 (avTXAltDist2), defined by i's behavior multiplied by the average of the alter-totals on V of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i76}^{\rm beh}(x,z) = z_i \left(\sum_j x_{ij} \left(x_{j+} - x_{ji}\right) \breve{v}_j^{\left(-i\right)}\right) / \left(\sum_j x_{ij}\right)$$

(and the mean behavior, i.e. 0, if the ratio is 0/0); this is similar to avTAltDist2, but now for covariate V instead of the behavior Z;

77. total average covariate alter effect at distance 2 (totAXAltDist2), defined by i's behavior multiplied by the total of the alter-averages on V of his alters, excluding the contribution from a tie  $j \to i$  (if any),

$$s_{i77}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} \, \breve{v}_j^{(-i)} \right);$$

this is similar to totAAltDist2, but now for covariate V instead of the behavior Z;

78. alter's distance-two incoming covariate average effect on behavior z (avXInAltDist2), defined by i's behavior multiplied by the average of the incoming alter-averages  $\check{v}_{j}^{(-i)}$  of his alters, excluding the contribution from a tie  $j \to i$  (if any), defined by

$$\check{v}_{j}^{(-i)} = \begin{cases}
\frac{\sum_{h \neq i} x_{hj} v_{h}}{x_{+j} - x_{ij}} & \text{if } x_{+j} - x_{ij} > 0 \\
0 & \text{if } x_{+j} - x_{ij} = 0.
\end{cases}$$
(38)

(also see (35)),

$$s_{i78}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij} \, \check{v}_j^{(-i)}\right) / \left(\sum_j x_{ij}\right)$$

(and the mean behavior, i.e. 0, if the ratio is 0/0);

this is similar to avInAltDist2, but now for covariate V instead of the behavior Z;

79. alter's distance-two incoming covariate total effect on behavior z (totXInAltDist2), defined by i's behavior multiplied by the total of the incoming alter-totals on V of his alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i79}^{\text{beh}}(x,z) = z_i \sum_j x_{ij} \sum_{h \neq i} x_{hj} v_h = z_i \sum_j x_{ij} (x_{+j} - x_{ij}) \check{v}_j^{(-i)};$$

this is similar to totInAltDist2, but now for covariate V instead of the behavior Z;

80. average total incoming covariate alter effect at distance 2 (avTXInAltDist2), defined by i's behavior multiplied by the average of the incoming alter-totals on V of his alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i80}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} (x_{+j} - x_{ij}) \, \check{v}_j^{(-i)} \right) / \left( \sum_j x_{ij} \right)$$

(and the mean behavior, i.e. 0, if the ratio is 0/0);

this is similar to avTInAltDist2, but now for covariate V instead of the behavior Z;

81. total average incoming covariate alter effect at distance 2 (totAXInAltDist2), defined by i's behavior multiplied by the total of the incoming alter-averages on V of his alters, excluding the contribution from a tie  $i \to j$  (if any),

$$s_{i81}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} \, \check{v}_j^{(-i)} \right);$$

this is similar to totAInAltDist2, but now for covariate V instead of the behavior Z;

There are also a number of interaction effects between actor covariates (which includes other dependent behavior variables; note that these are used in their <u>centered</u> versions) and influence effects. These have to be specified using **interaction1** for the covariate and **interaction2** for the network, e.g.,

Between parentheses: the functionality of the 'ego'-variant of these effects is duplicated by interaction effects created, for example, as follows:

Furthermore, note that when the same dependent variable is used (i.e., name and interaction1 are the same), the dependent variable is used as a weight with its value before the ministep, and the potential change is not taken into account; this departs from the concept of evaluation effect, and is a case of an elementary effect.

For the 'alter'-variant, this way of construction will not work because the effect statistic cannot be decomposed into a product of two ego-level statistics. The available effects of this type are the following.

- 82. interaction of ego (tie sender) variable with average similarity, (avSimEgoX)  $s_{i82}^{\text{beh}}(x,z) = (v_i/x_{i+}) \sum_j x_{ij} (\sin_{ij}^z \widehat{\sin}^z);$  (and the mean behavior, i.e. 0, if  $x_{i+} = 0$ );
- 83. interaction of ego (tie sender) variable with total similarity, (totSimEgoX)  $s_{i83}^{\text{beh}}(x,z) = v_i \sum_i x_{ij} (\sin_{ij}^z \widehat{\sin}^z);$
- 84. interaction of ego (tie sender) variable with average alter, (avaltegox)  $s_{i84}^{\rm beh}(x,z) = v_i \, z_i \left( \sum_j x_{ij} \, z_j \right) / \left( \sum_j x_{ij} \right)$  (and the mean behavior, i.e. 0, if the ratio is 0/0);
- 85. interaction of ego (tie sender) variable with total alter, (totAltEgoX)  $s_{i85}^{\text{beh}}(x,z) = v_i z_i \left( \sum_j x_{ij} z_j \right)$ ;
- 86. interaction of alter (tie receiver) variable with average similarity, (avSimAltX)  $s_{i86}^{\text{beh}}(x,z) = (1/x_{i+}) \sum_{j} x_{ij} v_{j} (\sin^{z}_{ij} \widehat{\sin^{z}});$  (and the mean behavior, i.e. 0, if  $x_{i+} = 0$ );
- 87. interaction of alter (tie receiver) variable with total similarity, (totSimAltX)  $s_{i87}^{\text{beh}}(x,z) = \sum_{j} x_{ij} v_{j} (\sin \frac{z}{ij} \widehat{\sin}^{z});$
- 88. interaction of alter (tie receiver) variable with average alter, (avAltAltX)  $s_{i88}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij} v_j z_j\right) / \left(\sum_j x_{ij}\right)$  (and the mean behavior, i.e. 0, if the ratio is 0/0);

89. interaction of alter (tie receiver) variable with total alter, (totAltAltX)  $s_{i89}^{\text{beh}}(x,z) = z_i \left( \sum_j x_{ij} v_j z_j \right)$ ;

# Dyadic covariate effects

For each dyadic covariate W there are the following effects. Recall that the default is to center dyadic covariates; however, it often will be preferable here to use non-centered covariates. It is then important how to define the zero point of the covariate!

90. alter's average of dyadic covariate effect on behavior z (avWAlt), defined as the product of i's behavior  $z_i$  and i's alters' dyadic covariate-average  $\check{w}_i$  as defined by

$$\check{w}_{i} = \begin{cases}
\frac{\sum_{h} x_{ih} w_{ih}}{x_{i+}} & \text{if } x_{i+} > 0 \\
0 & \text{if } x_{i+} = 0.
\end{cases}$$
(39)

leading to the effect  $s_{i90}^{\text{beh}}(x,z) = z_i \, \check{w}_i$ .

91. in-alter's average of dyadic covariate effect on behavior z (avWInAlt), defined as the product of i's behavior  $z_i$  and i's in-alters' dyadic covariate-average  $\check{w}_h$  as defined by

leading to the effect  $s_{ig1}^{beh}(x, z) = z_i \, \check{w}_h$ .

92. in-alter's average of dyadic covariate effect on behavior z (avWInAlt), defined as the product of i's behavior  $z_i$  and i's in-alters' dyadic covariate-average  $\check{w}_h$  as defined by

$$\widetilde{w}_h = \begin{cases}
\frac{\sum_h x_{hi} w_{ih}}{x_{+i}} & \text{if } x_{+i} > 0 \\
0 & \text{if } x_{+i} = 0.
\end{cases}$$
(41)

leading to the effect  $s_{i92}^{\text{beh}}(x,z) = z_i \, \check{w}_h$ .

93. alter's total of dyadic covariate W effect on behavior z (totWAlt), defined as the product of i's behavior  $z_i$  and the sum of i's alters' values of the dyadic covariate,

$$s_{i93}^{\text{beh}}(x,z) = z_i \sum_h x_{ih} w_{ih}.$$

94. in-alter's total of dyadic covariate W effect on behavior z (totWInAlt), defined as the product of i's behavior  $z_i$  and the sum of i's in-alters' values of the dyadic covariate,

$$s_{i94}^{\text{beh}}(x,z) = z_i \sum_h x_{hi} w_{ih}.$$

95. average alter weighted by dyadic covariate W (avAltW), a weighted version of avAlt; depending on the parameter value (1 or 2):

```
parameter = 1 : s_{i95}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij} w_{ij} z_j\right) / \left(\sum_j x_{ij}\right)
parameter = 2 : s_{i95}^{\text{beh}}(x,z) = z_i \left(\sum_j x_{ij} w_{ij} z_j\right) / \left(\sum_j w_{ij} x_{ij}\right); (and the mean behavior, i.e. 0, if the ratio is 0/0);
```

96. average in-alter weighted by dyadic covariate W (avInAltW), a weighted version of avInAlt;

```
depending on the parameter value (1 or 2): parameter = 1 : s_{i96}^{\rm beh}(x,z) = z_i \left(\sum_j x_{ji} \, w_{ij} \, z_j\right) / \left(\sum_j x_{ji}\right) parameter = 2 : s_{i96}^{\rm beh}(x,z) = z_i \left(\sum_j x_{ji} \, w_{ij} \, z_j\right) / \left(\sum_j w_{ij} \, x_{ji}\right); (and the mean behavior, i.e. 0, if the ratio is 0/0);
```

- 97. total alter weighted by dyadic covariate W (totAltW), a weighted version of totAlt;  $s_{i97}^{\rm beh}(x,z)=z_i\sum_j x_{ij}\,w_{ij}\,z_j;$
- 98. total in-alter weighted by dyadic covariate W (totInAltW), a weighted version of totInAlt;

$$s_{i98}^{\text{beh}}(x,z) = z_i \sum_j x_{ji} w_{ij} z_j;$$

99. average similarity weighted by dyadic covariate W (avSimW), a weighted version of avSim;

100. total similarity weighted by dyadic covariate W (totSimW), a weighted version of totSim;

$$s_{i100}^{\text{beh}}(x,z) = \sum_{j} x_{ij} w_{ij} \left( \sin_{ij}^{z} - \widehat{\sin}^{z} \right).$$

## User-defined interaction effects

The user-defined interaction effects of Section 5.12.4 are defined as follows. Suppose we consider two effects  $s_{ia}^{\text{beh}}(x,z)$  and  $s_{ib}^{\text{beh}}(x,z)$ . Then their interaction effect is defined by

$$s_{i[a*b]}^{\text{beh}}(x,z) = \frac{1}{z_i} s_{ia}^{\text{beh}}(x,z) s_{ib}^{\text{beh}}(x,z) .$$
 (42)

For three effects  $s_{ia}^{\text{beh}}(x,z)$ ,  $s_{ib}^{\text{beh}}(x,z)$  and  $s_{ic}^{\text{beh}}(x,z)$ , the interaction effect is defined by

$$s_{i[a*b*c]}^{\text{beh}}(x,z) = \frac{1}{z_i^2} s_{ia}^{\text{beh}}(x,z) s_{ib}^{\text{beh}}(x,z) s_{ic}^{\text{beh}}(x,z) .$$
(43)

The division by  $z_i$  or  $z_i^2$ , respectively, is necessary to offset the fact that all behavior effects of interactionType = OK contain a factor  $z_i$ , and further do not depend on  $z_i$ . For example, the interaction between two main effects (effFrom) of actor covariates  $v_{1i}$  and  $v_{2i}$ , is the same as the main effect of the product variable  $v_{1i} \times v_{2i}$ , with the proviso that the user-defined interaction does not center the product variable; the user-defined interaction then is defined by

$$s_{i[v_1*v_2]}^{\text{beh}}(x,z) = \frac{1}{z_i} (z_i v_{1i}) (z_i v_{2i}) = z_i v_{1i} v_{2i} , \qquad (44)$$

where both component variables  $v_{1i}$  and  $v_{2i}$  are internally centered, but the product variable will generally not have mean 0.

### 12.2.2 Behavioral creation function

Also the behavioral model knows the distinction between evaluation, creation, and endowment effects. The formulae of the effects that can be included in the behavioral creation function  $c^{\text{beh}}$  are the same as those given for the behavioral evaluation function. However, they enter calculation of the creation function only when the actor considers increasing his behavioral score by one unit (downward steps), not when downward steps (or no change) are considered. For more details, consult Snijders et al. (2007) and Steglich et al. (2010) and replace 'going down' by 'going up'.

The statistics reported as *inc. beh.* (increase in behavior) are the sums of the changes in actor-dependent values for only those actors who increased in behavior. More precisely, it is

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} I\{z_i(t_{m+1}) > z_i(t_m)\} \left(s_{ik}^{\text{beh}}(x(t_{m+1})) - s_{ik}^{\text{beh}}(x(t_m))\right), \tag{45}$$

where M is the number of observations,  $x(t_m)$  is the observed situation at observation m, and the indicator function  $I\{A\}$  is 0 if event A is true and 0 if it is untrue.

## 12.2.3 Behavioral endowment function

Also the behavioral model knows the distinction between evaluation and endowment effects. The formulae of the effects that can be included in the behavioral endowment function  $e^{\text{beh}}$  are the same as those given for the behavioral evaluation function. However, they enter calculation of the endowment function only when the actor considers decreasing his behavioral score by one unit (downward steps), not when upward steps (or no change) are considered. For more details, consult Snijders et al. (2007) and Steglich et al. (2010).

The statistics reported as *dec. beh.* (decrease in behavior) are the sums of the changes in actor-dependent values for only those actors who decreased in behavior. More precisely, it is

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} I\{z_i(t_{m+1}) < z_i(t_m)\} \left(s_{ik}^{\text{beh}}(x(t_m)) - s_{ik}^{\text{beh}}(x(t_{m+1}))\right), \tag{46}$$

where M is the number of observations,  $x(t_m)$  is the observed situation at observation m, and the indicator function  $I\{A\}$  is 0 if event A is true and 0 if it is untrue.

## 12.2.4 Behavioral rate function

The behavioral rate function  $\lambda^{\text{beh}}$  consists of a constant term per period,

$$\lambda_i^{\mathrm{beh}} = \rho_m^{\mathrm{beh}}$$

for m = 1, ..., M - 1, which can be called the basic rate; multiplied potentially by the following further effects.

1. The dependence on the position of the actor can be modeled as a function of the actor's out-degree (outRate), in-degree (inRate), and number of reciprocated relations (recipRate), the 'reciprocated degrees'. These can be defined by

$$x_{i+} = \sum_{j} x_{ij}, \qquad x_{+i} = \sum_{j} x_{ji}, \qquad x_{i(r)} = \sum_{j} x_{ij} x_{ji}$$

(recalling that  $x_{ii} = 0$  for all i).

The contribution of the out-degrees to the rate function is a factor

$$\exp(\alpha_h x_{i+})$$
,

if the associated parameter is denoted  $\alpha_h$  for some h, and similarly for the contributions of the in-degrees and the reciprocated degrees.

2. The dependence on actor covariates or behavioral variables (RateX) with values  $v_{hi}$  multiplies the rate by a factor

$$\exp(\alpha_h v_{hi})$$
.

If  $v_h$  is a behavioral variable, the non-centered value is used.

• For the analysis of diffusion of innovations, which is applicable if the behavior variable is a non-decreasing variable with values 0 and 1, there are various contagion effects that render a model that would reduce to a proportional hazards model if the network were constant; see Greenan (2015a). This holds if they are part of the rate function, but not if they are included in the evaluation function. This also holds for effects

depending on actor covariates. For all these effects, the rate function is multiplied by

$$\exp\left(\alpha_h a_{ih}(x)\right)$$
,

if the associated parameter is denoted  $\alpha_h$  for some h, and the effect is  $a_{ih}(x)$ .

The dependence on the internal effect parameter p is similar for all these diffusion effects, and is described after the susceptAvCovar effect.

For models including these effects, it is recommended to use an algorithm that leads to smaller step sizes, i.e., with a value for firstg lower than the default of 0.2.

3. average exposure effect (avExposure), defined as the proportion of i's alters who have adopted the innovation,

$$a_{i3}(y) = \frac{\sum_{j=1}^{n} z_j x_{ij}}{\sum_{j=1}^{n} x_{ij}};$$

4. total exposure effect (totExposure), defined as the number of i's alters who have adopted the innovation,

$$a_{i4}(y) = \sum_{j=1}^{n} z_j x_{ij};$$

5. infection by indegree effect (infectIn), defined as the sum of indegrees of i's alters who are also adopters of the innovation,

$$a_{i5}(y) = \sum_{j=1}^{n} z_j x_{ij} x_{+j};$$

for non-directed networks this is called the *infection by degree effect* (infectDeg);

6. infection by outdegree effect (infectOut), defined as the sum of outdegrees of i's alters who are also adopters of the innovation,

$$a_{i6}(y) = \sum_{j=1}^{n} z_j x_{ij} x_{j+};$$

7. *infection by covariate effect* (infectCovar), defined as the sum of covariate values of *i*'s alters who are also adopters of the innovation,

$$a_{i7}(y) = \sum_{j=1}^{n} z_j x_{ij} v_j;$$

8. susceptibility to average exposure by indegree effect (susceptAvIn), defined as the interaction between i's indegree and i's average exposure,

$$a_{i8}(y) = x_{+i} \frac{\sum_{j=1}^{n} z_j x_{ij}}{\sum_{j=1}^{n} x_{ij}};$$

for non-directed networks the equivalent effect is totExposure;

9. susceptibility to average exposure by covariate effect (susceptAvCovar), defined as the interaction between i's covariate value and i's average exposure,

$$a_{i9}(y) = v_i \frac{\sum_{j=1}^n z_j x_{ij}}{\sum_{j=1}^n x_{ij}}.$$

For all these diffusion effects there is an additional dependence on the internal effect parameter p; the formulae given above are the default, valid for p = 0.

The parameter p must be integer-valued. If p > 0, the contribution is changed to  $a_{i9}(y) = 0$  if less than p alters have adopted the innovation, i.e., if

$$\sum_{j=1}^n z_j x_{ij} < p.$$

E.g., if p = 3, then there is influence on actor i only if this actor has at least three alters who have adopted the innovation.

For the effects avExposure, totExposure, susceptAvIn, and susceptAvCovar, if p < 0, the same rule is applied with the minimum |p|, but if there are more than |p| alters who have adopted the innovation, only the value |p| is used. This means that for p < 0,  $\sum_{j=1}^{n} z_j x_{ij}$  in the formula for  $a_{ik}(y)$  is replaced by

$$\begin{cases} 0 & \text{if } \sum_{j=1}^{n} z_{j} x_{ij} < |p| \\ |p| & \text{if } \sum_{j=1}^{n} z_{j} x_{ij} \ge |p| \end{cases}.$$

# 12.3 Effects for estimation by Generalized Method of Moments

A variety of effects can be specified with type = "gmm", as shown in effectsDocumentation. These are not effects for the definition of the probability model, but for the estimation by the Generalized Method of Moments. This is documented in Section 6.10.

# 13 Parameter interpretation

The main 'driving force' of the actor-oriented model is the evaluation function (extended with the creation and/or endowment function, if these are part of the model specification). For the network, this is given in formula (15) as

$$f_i^{\text{net}}(x) = \sum_k \beta_k^{\text{net}} s_{ik}^{\text{net}}(x) .$$

The evaluation function can be regarded as the 'attractiveness" of the network (or behavior, respectively) for a given actor. For getting a feeling of what are small and large values, is is helpful to note that the evaluation functions are used to compare how attractive various different tie changes are, and for this purpose random disturbances are added to the values of the evaluation function with standard deviations equal<sup>20</sup> to 1.28.

An alternative interpretation is that when actor i is making a 'ministep', i.e., a single change in his outgoing ties (where no change also is an option), and  $x_a$  and  $x_b$  are two possible results of this ministep, then  $f_i^{\text{net}}(x_b) - f_i^{\text{net}}(x_a)$  is the log odds ratio for choosing between these two alternatives – so that the ratio of the probability of  $x_b$  and  $x_a$  as next states is

$$\exp\left(f_i^{\text{net}}(x_b) - f_i^{\text{net}}(x_a)\right)$$
.

Note that, when the current state is x, the possibilities for  $x_a$  and  $x_b$  are x itself (no change), or x with one extra outgoing tie from i, or x with one fewer outgoing tie from i. Explanations about log odds ratios can be found in texts about logistic regression and loglinear models. For dependent behavior variables, the interpretation is similar, keeping in mind that permitted changes in the behavior variable are -1, 0, and +1 (as far as these changes do not lead beyond the permitted range).

If one wishes an extremely short description of what the parameters of the objective function stand for, one could write 'non-standardized contributions to log-probabilities' as a shorthand for 'contributions to log-probabilities of increasing the dependent variable by 1 unit when the effect is increased by 1 unit'. Here the dependent variable is either the tie variable  $X_{ij}$  or the behavior variable  $Z_i$ ; the effect is  $s_{ik}(x)$ .

# 13.1 Networks

The evaluation function is a weighted sum of 'effects'  $s_{ik}^{\text{net}}(x)$ . Their formulae can be found in Section 12.1.1. These formulae, however, are defined as a function of the whole network x, and in most cases the contribution of a single tie variable  $x_{ij}$  is just a simple component of this formula. The contribution to  $s_{ik}^{\text{net}}(x)$  of adding the tie  $i \to h$  minus the contribution of adding the tie  $i \to j$  is the log probability ratio comparing the probabilities of i sending a new tie to i versus sending the tie to i, if all other effects  $s_{ik}^{\text{net}}(x)$  yield the same values for these two hypothetical new configurations.

<sup>&</sup>lt;sup>20</sup>More exactly, the value is  $\sqrt{\pi^2/6}$ , the standard deviation of the Gumbel distribution; see Snijders (2001).

For example, suppose that actors j and h, actual or potential relation partners of actor i, have exactly the same network position and the same values on all variables included in the model, except that for some actor variable V for which only the popularity (alter) effect is included in the model, actor h is one unit higher than actor j:  $v_h = v_j + 1$ . It can be seen in Section 12.1.1 that the popularity (alter) effect is defined as

$$s_{ik}^{\text{net}}(x) = \sum_{j} x_{ij} v_j .$$

The contribution to this formula made by a single tie variable, i.e., the difference made by filling in  $x_{ij} = 1$  or  $x_{ij} = 0$  in this formula, is just  $v_j$ . Let us denote the weight of the V-alter effect by  $\beta_k$ . Then, the difference between extending a tie to h or to j that follows from the V-alter effect is  $\beta_k \times (v_h - v_j) = \beta_k \times 1 = \beta_k$ .

Thus, in this situation,  $\beta_k$  is the log probability ratio of the probability that h is chosen compared to the probability that j is chosen. E.g., if i currently has a tie neither to j nor to h, and supposing that  $\beta_k = 0.3$ , the probability for i to extend a new tie to h is  $e^{0.3} = 1.35$  times as high as the probability for i to extend a new tie to j.

### 13.2 Behavior

The evaluation function for behavior is given by

$$f_i^{\text{beh}}(x,z) = \sum_k \beta_k^{\text{beh}} s_{ik}^{\text{beh}}(x,z) ,$$

see (33). In many cases<sup>21</sup> the effect has the form of a product

$$s_{ik}^{\text{beh}}(x,z) = z_i \, s_{ik}^0(x,z) ,$$
 (47)

where  $s_{ik}^0(x,z)$  is not dependent on  $z_i$  (although it might depend on  $z_j$  for other actors j), and therefore would not be affected by the outcome of a behavior ministep of actor i. Examples are the main effect of an actor attribute, but also the average alter effect. For such effects, when a ministep in behavior occurs, the contribution on the probability distribution of the change is as follows: a change of  $z_i$  by -1 will decrease the evaluation function by  $\beta_k^{\text{beh}} s_{ik}^0(x,z)$ , and a change by +1 will increase it by the same amount. (Note that this amount does not depend on the value of  $z_i$  because of the mentioned condition.) Therefore, the log probability-ratio of an increase in behavior compared to staying constant that can be attributed to a difference of +1 in the value of the predictor function  $s_{ik}^0(x,z)$ , is equal to  $\beta_k^{\text{beh}}$ . The probability ratio is  $\exp(\beta_k^{\text{beh}})$ .

For example, later in this section results are presented where, for an analysis of drinking behavior, the estimated parameter for average alter is 1.1414. This means that when comparing two individuals who are equal in all respects except that the friends of the first on average are 1 higher on the drinking scale than those of the second individual, the odds of increasing drinking behavior compared to no change (in the event of a ministep with

<sup>&</sup>lt;sup>21</sup>For effects satisfying this condition, the interactionType is defined as "OK".

respect to drinking behavior) are  $\exp(1.1414) = 3.1$  times higher for the first individual than for the second.

The average similarity effect is defined by

$$s_{i9}^{\text{beh}}(x,z) = x_{i+}^{-1} \sum_{j} x_{ij} \left( \sin_{ij}^{z} - \widehat{\sin}^{z} \right)$$

(see Section 12.2.1), where  $\sin_{ij}^z$  is given in (1) in Section 4.2. (If the ratio is 0/0, the effect is defined as 0.) Subtracting  $\widehat{\sin}^z$  is done just for the purpose of centering. Since the underlying measure  $\sin_{ij}^z$  ranges from 0 (if  $z_i$  and  $z_j$  are the two different extreme values of Z) to 1 (if  $z_i = z_j$ ), this effect has a range of 1 independently of the scaling of Z. Therefore the average similarity effect, and also the total similarity effect, differ from the total and average alter effects by the fact that the latter are dependent on the scaling of Z, whereas the former are not directly dependent on this scaling. This means that parameters for average similarity effects can be regarded as being on the same scale for different variables, but parameters for average alter are not.

The interpretations of total and average similarity are more laborious to explain than the interpretation of the average alter effect. This is because total and average similarity are not of the form (47). To explain the log-probabilities or probability ratios due to these effects, it has to be understood how a change in the behavior  $z_i$  will affect the values of these effects. Examining their formulae leads to the following.

For a given actor i, the out-degree (number of friends) is denoted  $x_{i+}$ . Let the number of friends whose values  $z_j$  are less than, equal to, or greater than the value  $z_i$  of i, be denoted by a, b, and c. Denote the range (maximum minus minimum value) of the behavior by r. Then, in the event of a ministep with respect to behavior, the contributions of the total similarity effect to the log-probabilities of changes -1, 0, or +1, are given by  $\beta_k^{\text{beh}}(a-b-c)/r$ , 0, and  $\beta_k^{\text{beh}}(c-a-b)/r$ , respectively. The contributions for the average similarity effect are  $\beta_k^{\text{beh}}(a-b-c)/(rx_{i+})$ , 0, and  $\beta_k^{\text{beh}}(c-a-b)/(rx_{i+})$ . This shows that the influence of the friends in the similarity effects depends only on whether they have larger or smaller values than the focal actor, not on how much larger the values are. It also shows that for the similarity effects the dispersion of the friends' values matters and not only their average, whereas for the average alter effect only the average matters.

To have a compact formulation, without all this detail, one could say the following. We use the example on one of the following pages, where an average similarity effect on drinking is reported of  $\hat{\beta}_k^{\text{beh}} = 3.9689$ , where drinking has a range of r = 5 - 1 = 4. For an individual all of whose friends drink more than this individual does, the contribution of friends' influence to the probability ratio of an increase in drinking as compared to no change is a factor  $\exp(3.9689/4) = 2.7$ . (In this formulation, the condition 'in the event of a ministep with respect to drinking behavior" is left implicit.)

In the same situation, if hypothetically a total average similarity effect were found of 0.82, then one could say that having *one* additional friend who drinks more than oneself increases the probability ratio of an increase in drinking as compared to no change by a factor  $\exp(0.82/4) = 1.23$ . In general, parameters for the total similarity effect will

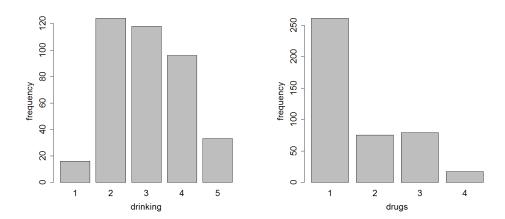


Figure 4: Frequencies of drinking and drug use over 3 waves combined.

tend to be smaller than those for the average similarity effect, because the former refer to comparisons about a single friend, and the latter to comparisons about all friends.

# 13.3 Ego – alter selection tables

When some variable V occurs in several effects in the model, then its effects can best be understood by considering all these effects simultaneously. This section treats this issue for the case that V is an actor variable with more than two values (for dichotomous variables the situation is simpler).

As an example, consider the Glasgow data set from the Teenage Friends and Lifestyle Study of West et al. (Michell and West, 1996; Pearson and West, 2003; Steglich et al., 2010). We refer to any of these papers for a further description of the data. The data is used for the 129 students who were present at all three measurement times; this data set is available from the SIENA website. We treat friendship as the dependent network, drinking as the dependent behaviour variable, and cannabis use ('drugs') as an actor covariate. Drinking has categories 1–5, coded as 1 (none), 2 (once or twice a year), 3 (once a month), 4 (once a week), and 5 (more than once a week); cannabis use is coded 1 (none), 2 (tried once), 3 (occasional) and 4 (regular). The frequencies (over the three waves combined) of these substance use variables are given in Figure 4.

For both actor variables, in the network part we use the four-parameter specification of Snijders and Lomi (2019), which is a combination of the ego minus alter squared, alter squared, ego, and alter effects.

The formulae in Section 12.1.1 imply that the components in the network evaluation function corresponding to the effects of variable V are

$$\beta_{\text{sq. diff}} \sum_{j} x_{ij} (v_j - v_i)^2 + \beta_{\text{ego}} (v_i - \bar{v}) x_{i+} + \beta_{\text{alter}} \sum_{j} x_{ij} (v_j - \bar{v}) + \beta_{\text{sq. alter}} \sum_{j} x_{ij} (v_j - \bar{v})^2.$$
(48)

The contribution to this formula of the single tie variable  $x_{ij}$  — i.e., the difference between the values of (48) for  $x_{ij} = 1$  and  $x_{ij} = 0$  — is equal to

$$\beta_{\text{sq. diff}}(v_j - v_i)^2 + \beta_{\text{ego}}(v_i - \bar{v}) + \beta_{\text{alter}}(v_j - \bar{v}) + \beta_{\text{sq. alter}}(v_j - \bar{v})^2. \tag{49}$$

It should be noted that by default all variables are internally centered by SIENA. The mean values used for the centering are given near the beginning of the file produced by print01Report and are given by the "mean" attributes of the covariates, as explained in Section 4.2.2. This is made explicit in the formulae by the subtraction of the mean  $\bar{v}$ . From this equation (49) a table, or a plot, can be made that gives these contributions for some values of  $v_i$  and  $v_j$ .

From the output of print01Report for these data it can be seen that the alcohol use variable assumes values from 1 to 5, with overall mean<sup>22</sup> equal to  $\bar{v} = 2.99$ . Drug use is a changing actor variable, with range 1–4, and mean  $\bar{v} = 1.5$ .

Suppose that we fit a model of network-behavior co-evolution to this data set with for the network evolution the effects of outdegree, reciprocity, geometrically weighted edgewise shared partners ('gwesp', in the gwespFF version, representing transitivity), indegree popularity, outdegree activity, outdegree popularity, and reciprocal degree activity; ego, alter, and same effect of sex; for drinking and drug use the four effects in (48); and for the behavior (i.e., alcohol drinking) dynamics the effects linear shape, quadratic shape (effect of drinking on itself), and average alter.

The results obtained are as follows.

<sup>&</sup>lt;sup>22</sup>The mean of the three means per wave.

Effect	par.	(s.e.)
Network Dynamics		
constant friendship rate (period 1)	12.643	(1.595)
constant friendship rate (period 2)	10.201	(1.024)
outdegree (density)	-2.723	(0.304)
reciprocity	3.387	(0.244)
gwesp (transitivity)	1.847	(0.088)
indegree – popularity	-0.030	(0.025)
outdegree – popularity	-0.107	(0.048)
outdegree – activity	0.075	(0.037)
reciprocal degree – activity	-0.287	(0.051)
sex alter	-0.120	(0.096)
sex ego	0.054	(0.108)
same sex	0.664	(0.089)
drinking alter	0.024	(0.082)
drinking squared alter	-0.113	(0.091)
drinking ego	0.104	(0.088)
drinking (alter-ego) squared	-0.117	(0.056)
drugs alter	-0.137	(0.091)
drugs squared alter	0.164	(0.067)
drugs ego	-0.123	(0.068)
drugs (alter-ego) squared	-0.051	(0.025)
$Behaviour\ Dynamics$		
rate drinking (period 1)	1.598	(0.302)
rate drinking (period 2)	2.431	(0.508)
drinking linear shape	0.452	(0.138)
drinking quadratic shape	-0.609	(0.181)
drinking average alter	1.281	(0.488)

convergence t ratios all < 0.04.

Overall maximum convergence ratio 0.15.

We interpret here the parameter estimates for the effects of drinking behavior and drug use without being concerned with the significance, or lack thereof. For the drinking behavior, formula (49) yields

$$-0.117 (v_j - v_i)^2 + 0.104 (v_i - \bar{v}) + 0.024 (v_j - \bar{v}) - 0.113 (v_j - \bar{v})^2 .$$

These values can be calculated for  $v_i$  and  $v_j$  ranging from 1 to 5, with  $\bar{v}=2.99$ . This is called the *selection table*. On the SIENA website the script SelectionTables.r is available, which contains several functions that can be helpful for doing this.

Given that this script is available in the working directory, the data set is called G129\_Data, the answer (sienaFit) object is called ans, the network "friendship", and the actor variable "drinking", the following script can be used.

source("SelectionTables.r") # needs to be done only once in the session
sm.drink <- selectionMatrix(ans, G129\_Data, "friendship", "drinking", 1:5)
# It can be displayed
sm.drink</pre>

This will produce the following table.

$v_i \setminus v_j$	1	2	3	4	5
1	-0.70	-0.46	-0.68	-1.35	-2.49
2	-0.72	-0.24	-0.22	-0.66	-1.57
3	-0.96	-0.25	0.00	-0.21	-0.88
4	-1.45	-0.50	-0.01	0.01	-0.42
5	-2.17	-0.98	-0.26	0.00	-0.20

Table 3: Selection table for friendship with respect to drinking.

It is instructive to make a plot of this table. This can be done, e.g., using package ggplot2 (Wickham, 2009). The following script will produce a png plot in the current working directory. Of course all elements of this script can be changed to accommodate the wishes given the purpose of the figure.

```
library(ggplot2)
vname <- "drinking"</pre>
name <- "friendship"</pre>
levls <- 1:5
vselect <- selectionTable(ans, G129_Data, name, vname, levls)</pre>
sp <- ggplot(vselect, aes(valter, select, group = ego, colour = ego))</pre>
png(filename=paste("selectionTable_",vname,".png",sep = ""),
    width = 1000, height = 800)
sp + geom_point() + geom_smooth(size = 1.2, span = 3) +
        scale_colour_hue() +
        scale_x_continuous(breaks = levls) +
        theme(legend.key = element_blank())+
        labs(x = paste(vname),
                 y = paste("selection function"),
                title = paste("Effect", vname, "on", name),
                 colour = paste(vname)) +
        theme_grey(base_size = 26, base_family = "") +
        theme(legend.key.width = unit(1, "cm")) +
        theme(plot.title = element_text(hjust = 0.5))
```

### graphics.off()

In this case, what was produced can be seen in Figure 5.



Figure 5: Plot of selection table for friendship with respect to drinking.

From the table it can be concluded that there is a significant tendency toward homophily (negative effect of ego minus alter squared). We see that the maximum of the curves increases with ego's value  $v_i$ , but for low  $v_i$  the maximum is larger than  $v_i$ , and for high  $v_i$  it is less than  $v_i$ : homophily is combined with a kind of 'regression to the mean'.

For drug use, formula (49) yields

$$-0.051 (v_j - v_i)^2 + -0.123 (v_i - \bar{v}) - 0.137 (v_j - \bar{v}) + 0.164 (v_j - \bar{v})^2 ,$$

which leads to the following table.

$v_i \setminus$	$v_j$	1	2	3	4
1		0.16	-0.02	0.03	0.30
2		-0.02	-0.09	0.06	0.43
3		-0.30	-0.27	-0.01	0.47
4		-0.68	-0.54	-0.19	0.39

Table 4: Selection table for friendship with respect to drug use.

The corresponding figure is in Figure 6.

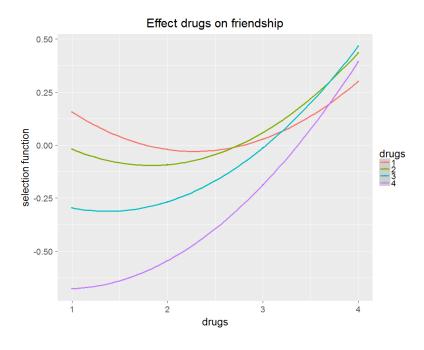


Figure 6: Plot of selection table for friendship with respect to drug use.

Here we see a totally different pattern. Since the sum of the alter squared and the difference squared effects is positive, the parabolas have a minimum rather than a maximum. Pay attention to the vertical scale: in Figure 5 the difference between the highest and lowest values is about 2.5, in Figure 6 it is about 1.2, half as much. This signifies that overall, drug use has smaller effects on friendship than smoking. For ego's who do not use cannabis  $(v_i = 1)$ , drug use of alters hardly matters; egos who use cannabis regularly  $(v_i = 4)$  have a somewhat lower attraction to alters with lower cannabis use. For this actor variable, the specification with the effects of ego, alter, and the product interaction ego × alter might fit just as well, or perhaps better.

### 13.4 Ego – alter influence tables

In quite a similar way as in the preceding section, from the parameter estimates as presented in the output tables, combined with the formulae for the effects, we can construct tables indicating how likely are changes to various different values of the behavior, depending on the behavior of the actor's friends. The functions used to define the effects can be found in Section 12.2.1, and it must not be forgotten that all variables are internally centered in RSiena, and the subtracted means are reported in the initial output produced by print01Report, but for the covariates they also can be obtained by requesting the 'mean' attributes of the covariates, see Section 4.2.2.

### 13.4.1 Average alter

In the model from the preceding section, the estimated coefficients in the behavior evaluation function are as follows.

Effect	par.	(s.e.)
Behaviour Dynamics		
rate drinking (period 1)	1.598	(0.302)
rate drinking (period 2)	2.431	(0.508)
drinking linear shape	0.452	(0.138)
drinking quadratic shape	-0.609	(0.181)
drinking average alter	1.281	(0.488)

The dependent behavior variable now is indicated Z. (In the preceding section the letter V was used, but this referred to any actor variable predicting network dynamics, irrespective of whether it was also a dependent behavior variable.)

The formulae in Section 12.2.1 show that the evaluation function for this model specification is

$$f_i^{\text{beh}} = \beta_{\text{linear}} (z_i - \bar{z}) + \beta_{\text{quad}} (z_i - \bar{z})^2 + \beta_{\text{avAlt}} (z_i - \bar{z}) (\bar{z}_{(i)} - \bar{z}) ,$$
 (50)

where  $\bar{z}_{(i)}$  is the average Z value of i's friends,

$$\bar{z}_{(i)} = \frac{1}{x_{i+}} \sum_{j} x_{ij} z_j ,$$

and 0 if this formula would be 0/0.

The *Influence Table* presents a matrix with in the rows the average of Z for i's friends, in the columns the possible values of  $z_i$ , and in the cells the evaluation function (50). For the table above, the evaluation function is

$$\hat{f}_i^{\text{beh}} = 0.452 (z_i - \bar{z}) - 0.609 (z_i - \bar{z})^2 + 1.281 (z_i - \bar{z})(\bar{z}_{(i)} - \bar{z}) .$$

At the SIENA website there is a script InfluenceTables.r which can be used to produce the table and the corresponding plot.

Given that this script is available in the working directory, the data set is called G129\_Data, the answer (sienaFit) object is called ans, the network 'friendship', and the actor variable 'drinking', the following script can be used.

The influence table is as follows.

$ar{z}_{(i)}$	\	$z_i$	1	2	3	4	5
1			1.76	1.48	-0.02	-2.74	-6.67
2			-0.79	0.21	-0.01	-1.44	-4.10
3			-3.34	-1.06	0.00	-0.15	-1.53
4			-5.89	-2.33	0.02	1.14	1.05
5			-8.44	-3.59	0.03	2.43	3.62

The interpretation<sup>23</sup> is that each row corresponds to a given average behavior of the focal actor's friends; comparing the different values in the row shows the relative 'attractiveness' of the different potential values of ego's own behavior. The maximum in each row is assumed at the diagonal. This means that for each value for the average friends' behavior  $\bar{z}_{(i)}$ , the focal actor 'prefers' to have the same behavior as all these friends. The differences in the bottom rows are larger than in the top rows, indicating that in the case where the friends who do not drink at all, the 'preference' (or social pressure) toward imitating their behavior is less strong than in the case where all the friends drink a lot.

A figure of this table can be produced, e.g., by the following script.

<sup>&</sup>lt;sup>23</sup>The interpretation is given here in terms of attraction and preferences; this is for the sake of having a straightforward and easy way of expressing the interpretation, which strictly should be only about probabilities.

```
scale_colour_hue() +
scale_x_continuous(breaks = levls) +
theme(legend.key = element_blank())+
labs(x = paste(name),
        y = paste("evaluation function"),
        title = paste("Influence effect",zname,"on",name),
        colour = paste(name,"\nalter")) +
theme_grey(base_size = 26, base_family = "") +
theme(legend.key.width = unit(1, "cm")) +
theme(plot.title = element_text(hjust = 0.5))
graphics.off()
```

The corresponding figure is in Figure 7.

# Influence effect friendship on drinking drinking alter 1 2 3 4 5 drinking

Figure 7: Plot of influence table for friendship with respect to drinking.

The five plotted curves intersect at the same point. This is indeed implied by formula (50), which is 0 if  $z_i = \bar{z} = 2.99$ .

Another way to look at the behavior evaluation function is to consider mathematically the location of its maximum. This function here can be written also as

$$f_i^{\text{beh}} = (0.452 + 1.281(\bar{z}_{(i)} - \bar{z}))(z_i - \bar{z}) - 0.609(z_i - \bar{z})^2$$
.

Differentiating with respect to  $z_i$  shows that this function is maximal for

$$z_i = \bar{z} + \frac{0.452 + 1.281(\bar{z}_{(i)} - \bar{z})}{2 \times 0.609} = 0.22 + 1.05\,\bar{z}_{(i)}$$

just a little bit larger than  $\bar{z}_{(i)}$ . Indeed in the table and in the plot we see that, if  $\bar{z}_{(i)}$  has integer values 1, 2, 3, 4, or 5, the highest values are obtained exactly for  $z_i = \bar{z}_{(i)}$ .

### 13.4.2 Average similarity

The formulae in Section 12.2.1 show that when average similarity is used as the specification of social influence, the evaluation function is given by

$$f_i^{\text{beh}}(z_i) = \beta_{\text{trend}}(z_i - \bar{z}) + \beta_{\text{drink}}(z_i - \bar{z})^2 + \beta_{\text{av. sim}} \frac{1}{x_{i+}} \sum_j x_{ij} \left( \operatorname{sim}(z_i, z_j) - \widehat{\operatorname{sim}}^z \right)$$
(51)

where the similarity score is

$$sim(z_i, z_j) = 1 - \frac{|z_i - z_j|}{\Delta}$$

in which  $\Delta$  is the range (maximum minus minimum) of Z, while  $\widehat{\sin}^z$  is the mean of the similarity scores. The latter is just a constant that does not have any influence on the change probabilities of the behavior. Therefore we use the equivalent form

$$f_i^{\text{beh}}(z_i) = \beta_{\text{trend}}(z_i - \bar{z}) + \beta_{\text{drink}}(z_i - \bar{z})^2 + \beta_{\text{av. sim}} \frac{1}{x_{i+}} \sum_j x_{ij} \left( \frac{|\bar{z} - z_j| - |z_i - z_j|}{\Delta} \right),$$

$$(52)$$

which has the advantage that it is zero for  $z_i = \bar{z}$ , and therefore comparable across i.

With this specification for the effect of drinking (and a modified specification of the effect of drug use), the estimation results are as follows.

Effect	par.	(s.e.)
Network Dynamics		
constant friendship rate (period 1)	12.769	(1.331)
constant friendship rate (period 2)	10.262	(0.987)
outdegree (density)	-2.708	(0.320)
reciprocity	3.376	(0.247)
gwesp (transitivity)	1.837	(0.084)
indegree – popularity	-0.028	(0.027)
outdegree – popularity	-0.108	(0.053)
outdegree – activity	0.075	(0.037)
reciprocal degree – activity	-0.285	(0.052)
sex alter	-0.123	(0.092)
sex ego	0.051	(0.107)
same sex	0.661	(0.087)
drinking alter	0.037	(0.075)
drinking squared alter	-0.108	(0.095)
drinking ego	0.097	(0.091)
drinking squared ego	0.010	(0.098)
drinking (ego – alter) squared	-0.108	(0.052)
drugs alter	-0.020	(0.054)
drugs ego	-0.183	(0.066)
drugs ego $\times$ drugs alter	0.126	(0.052)
Behaviour Dynamics		
rate drinking (period 1)	1.696	(0.326)
rate drinking (period 2)	2.659	(0.565)
drinking linear shape	0.461	(0.136)
drinking quadratic shape	-0.003	(0.088)
drinking average similarity	6.190	(1.810)

convergence t ratios all < 0.08.

Overall maximum convergence ratio 0.17.

Equation (52) is less simple than equation (50), because (50) is a quadratic function of  $z_i$ , with coefficients depending on the Z values of i's friends as a function of their average, whereas (52) depends on the entire distribution of the Z values of i's friends.

Suppose that, in model (52), the similarity coefficient  $\beta_{\text{av. sim}}$  is positive, and compare two focal actors,  $i_1$  all of whose friends have  $z_j = 3$  and  $i_2$  who has four friends, two of whom with  $z_j = 2$  and the other two with  $z_j = 4$ . Both actors are then drawn toward the preferred value of 3; but the difference between drinking behavior 3 on one hand and 2 and 4 on the other hand will be larger for  $i_1$  than for  $i_2$ . In model (50), on the other hand, since the average is the same, both actors would be drawn equally strongly toward the average value of 3.

Since the objective function for model (52) depends on all behavior values of the actor's friends, not just on their average, here we present a table only for the special case of actors all whose friends have the same behavior  $z_j$ . For the parameters given above, the behavior evaluation function then reads

$$f_i^{\text{beh}}(z_i) = 0.461 (z_i - \bar{z}) - 0.003 (z_i - \bar{z})^2 + 6.190 \left( \frac{|\bar{z} - z_j| - |z_i - z_j|}{\Delta} \right).$$

The same script used above for average alter, InfluenceTables.r, will for this model specification produce the following table and plot.

$z_{j}$	\	$z_i$	1	2	3	4	5
1			2.15	1.07	-0.01	-1.10	-2.19
2			-0.94	1.07	-0.01	-1.10	-2.19
3			-4.01	-1.99	0.02	-1.07	-2.17
4			-4.01	-1.99	0.02	2.02	0.93
5			-4.01	-1.99	0.02	2.02	4.02

The interpretation of this table is that each row corresponds to a given common behavior of the focal actor's friends; comparing the different values in the row shows the relative 'attractiveness' of the different potential values of ego's own behavior. The maximum in each row is assumed at the diagonal. This means that for each value for the common friends' behavior  $z_j$ , the focal actor prefers to have the same behavior as these friends. The differences in the bottom rows are larger than in the top rows, indicating that in the case where the friends who do not drink at all, the preference (or social pressure) toward imitating their behavior is less strong than in the case where all the friends drink a lot.

The values far away from the maximum contrast in this case less strongly than in the case of the model with the average alter effect, which is a consequence of the use of the absolute rather than squared deviation. However, neither of these models fits clearly better than the other. The fact that the t-ratio for testing the social influence effect is larger for average similarity (6.190/1.810 vs. 1.281/0.488) suggest that this specification has a slightly better fit.

These tables present only the contribution of some of the terms of the objective function, and the behavior dynamics will of course be compounded if the objective function contains more effects.

Figure 8 gives the same information as the table.

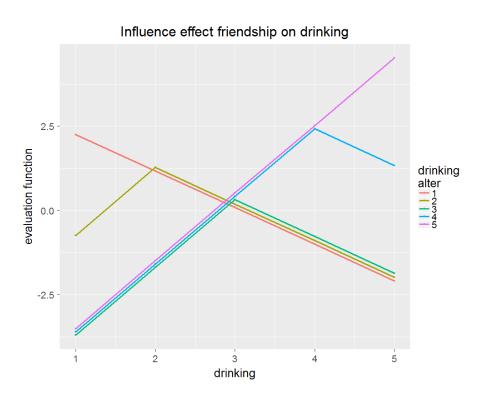


Figure 8: Plot of influence table for friendship with respect to drinking, for specification by average similarity.

Small vertical displacements were added to make the curves visually distinguishable.

### 13.5 Effect sizes and measures of fit

For linear regression models we are used to the coefficient of determination, usually denoted  $R^2$ , defined as the proportion of variance that is explained by the model. Such a measure is not easily available for most other statistical models; linear regression is special in allowing such a well-interpretable measure of 'fit'. The following measures that have something of the same purpose are available in RSiena through function sienaRI.

### 13.5.1 Relative importance of effects

The measure for relative importance of effects in Stochastic Actor-Oriented Models, proposed by Indlekofer and Brandes (2013), together with a measure for the (non-relative) importance of effects, can be calculated by sienaRI.

Ministeps in the Stochastic Actor-Oriented Model are random events with c possible outcomes; for one-mode networks c = n, for two-mode networks c is the number of second-mode nodes +1 (for 'no change'), for behavior c = 3 (changes by -1, 0, or +1); there may be constraints diminishing the number of choices. Let us denote the probabilities with which actor i in a ministep may change one of the outgoing ties by  $p_i(\beta, x) = (p_{i1}(\beta, x), \ldots, p_{ic}(\beta, x))$ . The Indlekofer-Brandes measures are numerical answers to the question, 'How strongly do these probabilities depend on each of the effects?'. In their paper, measures are proposed for expressing this per actor per wave; these measures can be relative (so that the total importance for each actor always is 1), or raw. The relative importances are calculated by sienaRl as RIActors, the raw importances as IActors (see the help page). The relative importances can be averaged over actors to give expectedRI, which is printed by the print method for sienaRl.

### 13.5.2 Entropy / relative certainty

The uncertainty in a random categorical variable can be measured by the *entropy* (Shannon, 1948). For a probability distribution with probabilities  $p = (p_1, \ldots, p_c)$  for an outcome with categories  $1, \ldots, c$ , it is defined as

$$H(p) = -\sum_{k=1}^{c} p_k^2 \log(p_k) . {53}$$

This is 0 if the variable is deterministic, i.e., one of the categories has probability 1; the maximum value is  $^2 \log(c)$ , attained if all categories have the same probability 1/c (maximum uncertainty).

Using a minus sign, the degree of certainty (as opposed to uncertainty) in the outcome of the ministep with probabilities  $p_i(\beta, x) = (p_{i1}(\beta, x), \dots, p_{in}(\beta, x))$  can be expressed by

$$R_H(i,\beta,x) = 1 - \frac{H(p_i(\beta,x))}{2\log(n)}$$
 (54)

For models with a constant rate function, this can be averaged over actors to obtain a

measure for a ministep taken by a random actor:

$$R_H(\beta, x) = \frac{1}{n} \sum_{i} R_H(i, \beta, x) . \tag{55}$$

This is a measure between 0 and 1. It was proposed in Snijders (2004), where further discussion may be found.

The values of the degree of certainty will generally be low, because which tie to create or to drop normally will have a very large degree of 'chance variability'. They are calculated by sienaRl as RHActors (see the help page). The mean over actors is printed by the print method for sienaRl.

Note that the values given by sienaRI refer to each of the waves, including the first wave; for this purpose, x in (55) is taken as the observed network (and behaviour...) at each wave.

### 13.5.3 Standard deviation of change statistics

The change statistics  $\delta_{ijk}(x)$  for an effect  $s_{ik}(x)$  are the changes in the value of this effect, for a given actor i, a given current state of the network x, and a given alter j, when the tie variable  $x_{ij}$  would be toggled (and analogous for behavioral dependent variables). These are the basic 'variables' in the conditional multinomial logistic regression model that is at the heart of the Stochastic Actor-Oriented Model. Their within-actor standard deviations  $\sigma_k(x)$  can be used to turn the parameters  $\beta_k$  into comparable quantities,

$$\sigma_k(x)\,\beta_k$$
 (56)

These express the parameters  $\beta_k$ , for different k and a common x, on a common scale.

These standard deviations are computed by sienaRI as sigma and printed by the print method for sienaRI if printSigma = TRUE. The change statistics themselves are potentially a lot of numbers ( $n^2 \times M \times K$ , where M is the number of waves and K the number of effects in the model), and are only retained in the object produced by sienaRI if getChangeStats = TRUE; see the help page for sienaRI.

### 14 Error messages

This chapter contains some error messages with their explanations. Currently it is not very extensive; new error messages will be added as answerable questions about them arise. For understanding errors, general advice is to read in this manual, read the help pages for functions, and look at the News page of the Siena website.

For diagnosing errors in R in general, a useful function is

### traceback()

This is to be called after an error was generated. It will give information, usually very helpful, about where exactly the error was caused. For example, it will give you the function generating the error message, with recursive information.

Note that it is not difficult to find the source of error messages in the code. The easiest way is searching for the text of the error message by some search machine such as google. You will probably find it points to GitHub or another repository of the code. There you can see what led to the error message, if you can understand some R. An alternative, if you know the function generating the error message (which you can find out by traceback(), see above) is to search directly in the code for this function. If the function is called, e.g., siena07, you can write the code to a file called listing.txt by the commands

```
sink("listing.txt")
siena07
sink()
```

Note that you should give the function name without parentheses. When the function is not an exported function, e.g., robmon, you can request it via

```
sink("listing_robmon.txt")
RSiena:::robmon
sink()
```

### 14.1 After updating

Updates of RSiena are not always completely backward compatible. When updating the version, you may have to create effects objects and algorithm objects again. If there is incomplete backward compatibility with respect to useability of scripts, this will be mentioned at the *News page* of the SIENA website.

Creating a new effect object given that you have an earlier effect object old.effects for an older RSiena version can become easier using the function updateSpecification, with commands such as the following.

```
eff0 <- getEffects(data_object)
new.effects <- updateSpecification(eff0, old.effects)</pre>
```

See the help file for updateSpecification; the RSiena version used to create old.effects does not matter.

### 14.2 During estimation

 $\Diamond$  Unlikely to terminate this epoch: more than 1000000 steps.

This can happen in function siena07; in conditional estimation (cond = FALSE in sienaAlgorithmCreate), when the rate parameter provisionally has hit a value such that the desired number of changes will probably never be reached; or in non-conditional estimation when the number of ministeps has become too large before arriving at the time for the next wave. See Section 6.12.1.

### Potential solutions.

- 1. Check whether your model specification is reasonable; for example, there might be doubts about the specification of the rate function.
- 2. Use non-conditional estimation (Section 6.12.1) if you were not already doing so.
- 3. If the model includes the outRate effect, consider replacing it by the outRateLog effect; but note that this works properly only for non-conditional estimation (cond = FALSE in sienaAlgorithmCreate).
- 4. If you are working with a data set with more than two waves, there might be unmodeled heterogeneity between the periods. Try modeling the periods separately.
- 5. Set the parameter firstg in sienaAlgorithmCreate to a lower value than the default 0.2. This will make the algorithm move more slowly, hopefully avoiding the problematic region in the parameter space.
  - The advice would be to set firstg to 0.02 and expect the necessity to do a second estimation using the prevAns parameter in siena07. If the problem still occurs for firstg = 0.02, use a smaller value (but less than 0.001 probably makes no sense). firstg determines the step sizes in the stochastic approximation algorithm; it is mentioned in some places earlier in this manual. Especially for models with additional rate effects the default value of 0.2 might be too large. firstg is the initial value of parameter  $a_N$  mentioned on p. 393 of Snijders (2001).
- 6. Use multiple processes (useCluster = TRUE, nbrNodes = ...).
- 7. In combination with trying out a lower value for firstg, you might try setting a higher value for doubleAveraging in sienaAlgorithmCreate. Double averaging is a different definition of the Robbins-Monro update step; see Section 6.3.3. You could try setting doubleAveraging to 1 or 2, which uses single averaging in the first 1 or 2 subphases; or you could set it to 4, avoiding double averaging altogether (if you have 4 subphases).

```
\Diamond Error in solve.default(z$dfra): system is computationally singular: reciprocal condition number = 2.34809e-35 (or some other very small number – note that 'e-35' means 10 to the power -35.)
```

or

### $\Diamond$ Error for inversion of d11

This can happen at the end of estimation in function siena07, when the covariance matrix is singular. It means that some effects in the model are linearly related, or are always 0.

Solutions.

Check your data (look at the description of the variables given as the output of print01Report).

Check your model specification.

Find out which effect might be especially correlated with some other effects, and treat it by a score-type test without estimating (i.e., specify it with fix = TRUE, test = TRUE). (Also see the discussion of the Hauck-Donner phenomenon in Section 6.12.2; this issue might be relevant here.)

### $\Diamond$ Error in initializeFRAN(z, x, initC = FALSE, ...) :

Unexpected effect name: someShortName

(where some Short Name is a short name you supplied in the model specification).

Probably your effects object was made by an earlier or newer version of RSiena than the version you are using now.

Solution.

First check that the short name was correctly spelled.

If this is all right, then recreate the effects object.

You can check that an effect is available in your version of RSiena by requesting

### effectsDocumentation(myeff)

(where you should substitute the name of your effects object for myeff).

### $\Diamond$ Endowment effect not supported

(for a specified endowment or creation effect).

This endowment or creation effect was not implemented.

Solution.

First check that the short name was correctly spelled and is available in your version of RSiena, by requesting

### effectsDocumentation(myeff)

(where of course you should substitute the name of your effects object for myeff). If this is all right, the recreate the effects object.

### ♦ total probability non-positive

The calculation of probabilities for the next ministep failed, probably because no change was allowed for this actor in this period, given the current state of the model.

This may have to do with structurally determined values (see Section 4.3.1) and/or with joiners and leavers (see Section 4.3.3); or with a restriction that a variable (tie variable, behavioral variable) is monotonic (Section 4.2.4), i.e., can only increase or only decrease.

Perhaps the most likely possibility is that you are working with a multiple group data set (perhaps using sienaBayes) for which some groups are only-increasing or only-decreasing, which then is taken by RSiena to apply also to the simulations carried out for this group, as mentioned in Section 4.2.4.

Solution.

Try to find out why it is possible in your data set that some actor has no options at all to choose from.

If you are working with a multi-group data set, use allowOnly = FALSE in the call of sienaDependent, as recommended in Section 4.2.4; also see the help file for sienaDependent.

Note that if you wish to forbid some actors from making any changes in some dependent variable, you can use a rate function dependent on a dummy actor covariate (effect rateX), with parameter fixed to a large negative value such as -100, which is sufficiently close to infinity (recall the exponentation in the calculation of the rate function) to effectively prevent this actor from taking any ministeps.

```
\Diamond Error in matrix(0, nrow = sum(y[x, ] != 0), ncol = 3) : invalid "nrow" value (too large or NA)
```

You are using a dyadic covariate containing only missing values.

Solution.

Correct the definition of this dyadic covariate.

### 14.3 As the result of a score-type test (including time test)

 $\Diamond$  Error in solve.default(v9): Lapack routine dgesv: system is exactly singular Error in if (cvalue < 0) cvalue < -0: missing value where TRUE/FALSE needed.

This can happen as the result of a score test requested in function siena07; or the score test requested in function sienaTimeTest. In the first case it indicates that there are linear dependencies in the list of effects (estimated and fixed) that are used for siena07. If this error message occurs for sienaTimeTest, it indicates linear dependencies in the list of effects estimated in the siena07 run analyzed by this sienaTimeTest, together with the interactions with time dummies tested by sienaTimeTest. See Sections 5.13 and 8.2.

Solutions.

For siena07: drop some of the requested score-type test, retaining only a set of tested effects between which there are no linear dependencies.

For sienaTimeTest: exclude some of the requested time heterogeneity tests by the excludeEffects parameter.

### 14.4 In sienaGOF

 $\diamondsuit \ Error \ in \ if \ (attr(obsData[[groupName]]\$depvars[[varName]], \ "sparse")) \ \{ \ : \ argument \ is \ of \ length \ zero$ 

This can happen directly when calling sienaGOF. It indicates that you used a wrong name for groupName or varName. See the help file for sienaGOF.

Solutions.

Use a correct groupName and varName.

 $\Diamond$  Error in if (isbipartite) { : argument is of length zero

This can happen directly when calling sienaGOF. It indicates that you used a wrong name for groupName or varName. See the help file for sienaGOF.

Solutions.

Use a correct groupName and varName.

### 15 For programmers: Get the source code

To do something with the source code, first you must get access to it. In the first place, it is good to know that for any R function that can be called, the source code is listed by writing the function name. Thus, e.g., if RSiena is loaded, the command

### sienaAlgorithmCreate

will list the code for the function with this name.

To get insight into a package, and certainly to modify or personalize it, it is necessary, however, to get the source code of the whole package. This is available from GitHub at https://github.com/stocnet/rsiena/.

A lot of programmers' documentation is also in the file Siena\_algorithms.pdf which can be downloaded from the SIENA website.

### 16 For programmers: Other tools you need

- Windows 1. Download and install the appropriate (version number depending on which R you are using) Rtools.exe from http://www.murdoch-sutherland.com/Rtools/. (I think this is not the right place any more should be downloaded from CRAN.)
  - 2. Make sure you check the box to amend your path during installation.
  - 3. Beware: if you later install other programs containing utilities such as tar (delphi is one offender), you may need to uninstall and reinstall Rtools, as you need Rtools at the start of your path.
  - 4. Add the path-to-the file R.exe to your path. Right-click on My Computer icon, select Properties/Advanced/Environment variables...

    Restart your computer to put the new path into effect.

Mac 1. Make sure the Xcode tools are installed.

2. Add the path-to-the-file R.exe to your path.

linux Add the path-to-the-file R.exe to your path.

# 17 For programmers: Building, installing and checking the package

In a command prompt or terminal window, navigate to the directory immediately above the siena source tree. Here we assume the source tree is in a directory called RSiena. (You may have minor difficulties if it is not the same as the name of the package you are trying to build or install.)

For Windows computers, the following 'type' instructions are Dos commands. A convenient way to apply them are by including them in a batch file (extension name .bat)

followed by a name with pause so that the Dos window – that will contain the error messages if there are any – will still be there when all is over.

Install Installing will recreate the binary and install in your normal R library path. Type
R CMD INSTALL RSiena

Build Building will create a tar ball. Type

R CMD build RSiena

This may give warning messages about the line endings if you run it on Windows. Do not worry, unless you have created any new source files, when it might remind you to set the property of eol-style on them when you add them to the repository.

**Check** Checking is a process designed to ensure that packages are likely to work correctly when installed. Type

```
R CMD check RSiena_1.4.14.tar.gz (where the version number is adjusted to match the tar ball name.)
```

**zip file** To make a zip file that can be used in Windows for 'installing from a local zip file', and therefore is easy for distribution to others, type

```
R CMD INSTALL --build RSiena_1.4.14.tar.gz (where again the version number is adjusted to match the tar ball name.)
```

If you make a change you need to INSTALL the new version in order to test it, and before you commit any changes to a repository you should check your new version. Make sure you get *no* warnings or errors from the check.

You can also INSTALL from a tar ball, and check a source tree.

If you have permission problems on Linux or Mac, you may need to do the first install from within R, so that the necessary personal library directories will be created. Use install.packages(tarballname, repos = NULL)

after creating a tar ball. (Or possibly just try to install some other package within R which will create the directories for you.)

You can unpack a tar ball by using

tar xf tar-ball-name

### 18 For programmers: Understanding and adding an effect

If you wish to check the definition of an effect, you can locate it in the source code and study it. You may also add effects to your personalized version of RSiena. If you think the effect could be useful for others, too, it will be appreciated if you propose it for inclusion through one of the discussion lists or directly to the maintainer of the package. This section gives the outline of the procedure for adding an effect, and then presents an elaborate example.

If you only wish to understand an effect without creating a new one, then you may follow the appropriate steps of this section. The main things then are to go to the Effectfactory.cpp file, find the name of the effect you are interested in and from there the function that implements it and read the code of this function.

The explanations here are not yet given for generic effects, which allow a more streamlined construction of effects. Looking at the code, starting with the file EffectFactory.cpp, may be helpful for understanding this construction of generic effects. For example, compare the construction of sameXTransTrip to that of sameXInPop.

1. Work out the definition of the effect and the contribution or change statistic. For network effects, the change statistic is

$$\Delta_{kij}(x) = s_{ki}(x^{+ij}) - s_{ki}(x^{-ij}) \tag{57}$$

where  $x^{+ij}$  is the network with the tie  $i \to j$  and  $x^{-ij}$  is the network without this tie.

- 2. The list of all defined effects can be obtained from effectsDocumentation, which produces a file effects.html or effects.pdf. All effects are also listed, with their definitions, in the manual (Section 12).
  - Determine an existing effect that is most similar to this effect (or perhaps more than one). In the file effects.html or effects.pdf the effects are grouped by effectGroup.
- 3. Open the file allEffects.csv located in "RSiena\data". The default program for opening a .csv file usually is Excel, but other editors may be more helpful for opening this particular file; e.g., NotePad or NotePad++. It must <u>not</u> be saved as a Excel file!
  - You will see that the first column is called effectGroup. These groups define combinations of dependent variable, effect type, and covariates (if any) (e.g. non-SymmetricObjective, bipartiteSymmetricObjective). Identify the effectGroup where this effect belongs. Determine which is which by considering some examples in this file or in the result of effectsDocumentation().
    - For covariate-related effects for two-mode networks, see extra remarks in Section 18.3.
  - Insert a new row in this group. Copy a row that corresponds best to your new effect and modify effectName, functionName, shortName, and more if this seems necessary. Perhaps your new effect is suitable in more than one group; then a new row can be made for all these groups, differing only in the name of the effect group; e.g., check that there are three versions of inPop, for directed, non-directed, and bipartite (two-mode) networks. Assume our new effect has shortName newEf.

In some cases, the new function will have extra parameters, as you can see from other examples; this is mostly the case, if one function is being used to define more than one effect.

For how to deal with internal effect parameters, look up a function defining an effect that has such a parameter.

It is advisable to use the hash sign (#) in the effect and function names. See subsection 18.1.

- Build the package and install it. Check from R that the new effect (which has only been created nominally) appears now in the effects object in RSiena. If this is not the case, there may be further changes necessary in the file effects.r; also see Section 18.3.
- 4. Open the folder "RSiena\src\model\effects". In an editor open the files AllEffects.h and EffectFactory.cpp.

These are C++ files; using a C++ editor is convenient but not necessary. Note however that you must save the files as ASCII (raw text) files without changing their names. Let us use the name *NewEffect* as the function name to be used (replace this by whatever is appropriate).

- In the file AllEffects.h you need to add the line #include "NewEffect.h" where it is alphabetically appropriate.
- In the file EffectFactory.cpp, at the appropriate place, add the lines

```
else if (effectName == "newEf")
{
  pEffect = new NewEffect(pEffectInfo);
}
```

- Now you will need to create two files (namely header and source files for C++) that should be called NewEffect.h and NewEffect.cpp. If there is a similar effect to the one you want to add it is usually easier to use it as a template. We recommend opening any effect file to see how the syntax works, but creating a new effect will be hard without knowing at least a bit of C++.
- Add the name NewEffect.cpp to the file sources.list. This is a long file without any hard returns. Separation of filenames is by blanks. It does not matter where you put it .
- 5. Once you are done editing you should build the package again and install it (from the command prompt) and then go to R to see if it is available to you.

It is a good idea to check the target statistics computed for a simple two-wave data set such as s50. Examples are in the script https://github.com/stocnet/rsiena/blob/main/checkEffects.R.

As examples, start with simple effects. For example, a network effect depending on a nonlinear transformation of outdegree, or a behavior effect depending nonlinearly on the behavior and nothing else. After having obtained experience with such a simple effect, continue with the effect that you are interested in.

If you want to use the effect in interactions, consult the section in Siena\_algorithms.pdf about interaction effects. If you wish to construct a dyadic or ego effect, note that you should define a tieStatistic rather than an egoStatistic.

Note that if your new effect could usefully be used as part of a multiple network effect you should use the generic effect approach and not the following.

## 18.1 The use of the hash (#) sign in names of effects and estimation statistics

If an effect contains an internal effect parameter, it is advisable to report the value of the internal effect parameter in the names of the effect and the estimation statistics reported when printing the effects object and the estimation results. These names are the columns effectName and functionName in the effects object, and their origins are the corresponding columns in file allEffects.csv. The value of the internal effect parameter can be included by the use of the hash sign (#). In function setEffect, occurrences of "#" in columns effectName and functionName are replaced by the value of the internal effect parameter (given explicitly by the user, or else the default value).

Thus, the hash sign (#) is a placeholder for the value of the internal effect parameter. When you look in the file allEffects.csv, you will see many occurrences of this symbol.

A difficulty occurs when the replacement has been done (so there is no more "#"), but a new call of setEffect is made with a new value of the internal effect parameter. To make the correct replacement then, the following rule is followed for defining the names in allEffects.csv:

• The symbol # should occur in the effectName and functionName in allEffects.csv in only the following substrings:

```
- "1/#"
- "(#)"
- "#-"
- "=_#"
- "+_#"
```

where " $_{\sqcup}$ " denotes one blank space (used here for the sake of clarity, not for use in allEffects.csv).

• No confusion should be possible, i.e., no substrings in allEffects.csv of these kinds are permitted where # is any number.

### 18.2 Example: adding the truncated out-degree effect

As an example, we show how the truncated out-degree effect (short name outTrunc) was added. It is defined by

$$s_i^{\text{net}}(x) = \min(x_{i+}, p) \tag{58}$$

where p is an internal effect parameter.

1. The change statistic (57) is

$$\Delta_{ij}(x) = \begin{cases} 1 & \text{if } \{x_{i+} < p, x_{ij} = 0\} \text{ or } \{x_{i+} \le p, x_{ij} = 1\} \\ 0 & \text{else.} \end{cases}$$
 (59)

Note that for this effect the case for going up  $(x_{ij} = 0)$  must be distinguished from the case for going down  $(x_{ij} = 1)$ .

2. In the file allEffects.csv the name of the effect group nonSymmetricObjective seems to cover the type of effect we are considering, and also contains other effects such as out-degree activity which are very similar to this effect.

The row for the out-degree activity (sqrt) effect was copied and inserted below this row. The 'effectName' was changed to outdegree-trunc(#), the 'functionName' to Sum of outdegrees trunc(#), and the 'shortname" to outTrunc. The hash sign (#) in these names will be replaced by the value of the internal effect parameter in the written output. The parm column, which defines the default value of the internal effect parameter, was set to 5.

The package was built. Loading it in R and creating an RSiena data set showed that indeed the effect was there.

3. The name TruncatedOutdegreeEffect was chosen for the new function. In the file AllEffects.h the line

```
#include 'TruncatedOutdegreeEffect.h'
```

was included at the appropriate alphabetic place.

In the file EffectFactory.cpp, after the piece referring to effectName == "outActSqrt", the lines

```
else if (effectName == "outTrunc")
{
    pEffect = new TruncatedOutdegreeEffect(pEffectInfo);
}
```

were inserted. This refers the program, when it encounters short name outTrunc, to the function TruncatedOutdegreeEffect. The next step was to construct this function.

4. To choose a template for TruncatedOutdegreeEffect, we could make various different choices; here it is important to have a look at the various effects defined in Chapter 12 that depend only on the outdegree. Consider the effects Outdegree activity - sqrt (short name outActSqrt) and sum of (1/(out-degree + p) (short name outInv) as possible examples. A look in EffectFactory.cpp shows that these are implemented using the functions OutdegreeActivitySqrtEffect and InverseOutdegreeEffect, respectively. Therefore look at the files OutdegreeActivitySqrtEffect.cpp and InverseOutdegreeEffect.cpp where these functions are defined.

The former defines the effect through a 'calculateContribution' function, which defines the tie flip contribution (the function called  $\Delta_{ij}(x)$  above) and tieStatistic, which is the function  $r_{ij}(x)$  when the effect can be defined as

$$s_i^{\text{net}}(x) = \sum_j x_{ij} \, r_{ij}(x) \ . \tag{60}$$

The latter defines the effect through a calculateContribution function and an egoStatistic function, which is the effect as defined in (58). It should be noted that generally effects can be defined either by the one or the other combination.

Since our new effect cannot be expressed in a straightforward way by an equation of the type (60), we chose to use the files InverseOutdegreeEffect.h and

InverseOutdegreeEffect.cpp as templates. This has a second advantage: the outInv effect has an effect parameter, which we also need to represent the parameter p in (58).

As a first step, the files InverseOutdegreeEffect.h and InverseOutdegreeEffect.cpp were saved under the new names TruncatedOutdegreeEffect.h and TruncatedOutdegreeEffect.cpp.

For the header file TruncatedOutdegreeEffect.h, all strings 'inverseoutdegreeeffect' were changed into 'truncatedoutdegreeeffect' while retaining the original use of upper and lower case. The explanation also was adapted. The header file now implies that for the function TruncatedOutdegreeEffect functions are needed of the types calculateContribution, endowmentStatistic and egoStatistic.

This was implemented in the file TruncatedOutdegreeEffect.cpp, which just was created by renaming InverseOutdegreeEffect.cpp. First all strings 'outdegreeactivitysqrteffect' were changed into 'truncatedoutdegreeeffect', again retaining the original use of upper and lower case.

To understand the C++ syntax, keep into account the object-oriented nature of C++. The keyword this is a pointer referring to the object in which the current function is defined, and the arrow -> indicates a further pointer; thus, the variable this->pNetwork()->outDegree(this->ego())

refers to the outdegree of ego (ego being denoted in our mathematical formulae by i) in the current network – in other words,  $x_{i+}$ .

The variable this->lc refers to the internal effect parameter, denoted in our formulae by p. The return statement defines the function value that is returned when the function is called.

Armed with this knowledge, we specified the change statistic, implementing (59), as follows.

```
double TruncatedOutdegreeEffect::calculateContribution(int alter) const
{
   double change = 0;
   // Current out-degree
```

```
int d = this->pNetwork()->outDegree(this->ego());
    if (this->outTieExists(alter))
        // After a tie withdrawal, the new out-degree would be d-1, and
        // the new effect value would have decreased by 1 if d <= this->lc
        if (d <= this->lc)
             change = 1;
        }
    }
    else
    {
        // When introducing a new tie, the new out-degree would be d+1, and
        // the new effect value would have increased by 1 if d < this->lc
        if (d < this->lc)
             change = 1;
    }
    return change;
}
The effect statistic, implementing (59), was specified as follows.
double TruncatedOutdegreeEffect::egoStatistic(int ego,
    const Network * pNetwork)
{
    // Current out-degree
    int d = this->pNetwork()->outDegree(this->ego());
    if (d <= this->lc)
    {
         return d;
    }
    else
    {
         return this->lc;
    }
}
```

- 5. To the file src\sources.list, the item model/effects/TruncatedOutdegreeEffect.cpp was added.
- 6. Having done this, the package was built and installed again. (To be honest, there first were some errors; but the error messages from the compiler are quite clear and easily led to solving the errors.)

Upon starting R and loading RSiena, indeed the new effect was available. For an easy check, the following commands were used.

```
mynet     <- sienaDependent(array(c(s501, s502), dim = c(50, 50, 2)))
mydata     <- sienaDataCreate(mynet)
myalgorithm <- sienaAlgorithmCreate(projname = "s50_12")
myeff     <- getEffects(mydata)
myeff     <- setEffect(myeff, outTrunc, parameter = 3)
ans          <- siena07(myalgorithm, data = mydata, effects = myeff)
summary(ans)</pre>
```

The parameter was set at 3, because the maxima of the observed out-degrees in the two data sets  ${\tt s501}$  as well as  ${\tt 502}$  were 5, so the 'outdegree-trunc(#)' effect would be highly collinear with the outdegree effect if the default parameter of 5 were used. This led to good convergence. To check the calculation of the statistics, it was noted that the output file mentioned the target values

Observed values of target statistics are

```
1. Number of ties 116.0000
2. Number of reciprocated ties 70.0000
3. Sum of outdegrees trunc(3) 105.0000
```

The value of the target statistic for the new effect should be

$$\sum_{i} s_{i}^{\text{net}}(x(t_{2})) = \sum_{i} \min(x_{i+}(t_{2}), 3) .$$

This can be directly calculated in R by requesting

```
sum(pmin(rowSums(s502),3))
```

which indeed returns the value 105, confirming that the calculation of the ego statistic seems correct.

7. To complete the extension of the package by this effect, it also was added to the set of effects for symmetric and bipartite networks. This was done by inserting, at appropriate places in the file allEffects.csv, the same line but now with effectGroup changed to bipartiteObjective and symmetricObjective, respectively.

### 18.3 Notes on effectGroups and two-mode networks

For two-mode networks the difference between the two node sets implies some peculiarities. Recall that effectGroups are used in the file allEffects.csv and in function effects.r.

In the function getEffects in file effects.r, some additional measures are taken for effects in effectGroup covarBipartiteEff. This implies that for adding such effects, it will be necessary to see whether this function (i.e., getEffects) also must be modified; this will have to be done in function covarBipartiteEff. (Note: it would be preferable perhaps to have separate effect classes for covariates on the first and on the second mode, as done for the effect groups in the following paragraphs.)

A different approach was taken for effectGroups covarABehaviorBipartiteObjective and covarBBehaviorBipartiteObjective: the former is for covariates on the first node set, the second for covariates on the second node set.

For effects defined for two dependent networks and one actor covariate, the following effectGroups are defined:

- covarNetNetObjective is for effects where the second network is one-mode;
- covarABNetNetObjective is without restriction on the modes;
- covarANetNetObjective is for effects where, if the second network is two-mode, the covariate is defined for the first node set;
- covarBNetNetObjective is for effects where, if the second network is two-mode, the covariate is defined for the second node set.

For effects defined for a two-mode dependent network, another dependent network (in the role of explanatory; whether one- or two-mode), and one actor covariate, the following effectGroup is defined:

• covarABipNetObjective for covariates defined for the first node set.

Further some of the complex effectGroups are the following:

- nonSymmetricNonSymmetricObjective is for combination of two directed networks, one in the role of dependent network and the other in the role of explanatory network;
- nonSymmetricSymmetricObjective is for the combination of a directed network in the role of dependent network and a non-directed network in the role of explanatory network;
- nonSymmetricSObjective is for the combination of a directed network in the role of dependent network and a non-directed network in the role of explanatory network, where also the primary setting can have the role of explanatory network (and is treated with cross-lagged statistics as usual for co-evolution models);
- tripleNetworkObjective is for combinations of three networks where for the two in the role of explanatory networks,
- bipartiteNonSymmetricObjective is for combination of a two-mode dependent network and a directed network in the role of explanatory network;
- bipartiteSymmetricObjective is for combination of a two-mode dependent network and a non-directed network in the role of explanatory network;
- bipartiteBipartiteObjective is for combination of two two-mode networks, one in the role of dependent network and the other in the role of explanatory network;

- dyadANetNetObjective is for combinations of two networks and a dyadic covariate, where the network in the role of dependent network should be one-mode, and the dyadic covariate should be on the node set ('squared') of the dependent network;
- dyadBNetNetObjective is for combinations of two networks and a dyadic covariate, where the dyadic covariate should have the same first and second node sets as the explanatory network;
- behaviorOneOneModeObjective is for dependent behavioral variables and two explanatory one-mode directed networks;
- behaviorSymSymObjective is for dependent behavioral variables and two explanatory one-mode non-directed networks;
- behaviorOneModeSymObjective is for dependent behavioral variables and two explanatory one-mode networks, the first directed, the second non-directed.
- behaviorBipBipObjective is for dependent behavioral variables and two explanatory two-mode networks with identical actor sets;
- dyadBehaviorNetObjective is for dependent behavioral variables, an explanatory one-mode network and an explanatory dyadic covariate.

If you wish to add an effect with a combination of variables that is not yet implemented, you have also to treat it in effects.r (which will require some figuring out) and mention it in effectsDocumentation.r.

### A Appendix: List of Functions in Order of Execution

This appendix provides a description of the functions that constitute the RSiena package. This is intended as a quick reference or catalogue for the user to employ Stochastic Actor Oriented Models (SAOM) to analyze network dynamics in R.

The functions are presented in execution order (more or less as they would be used in practice). A list of useful R functions to read and prepare the data set is also included at the beginning. In all cases examples on how to use these functions are provided. In the 'syntax' column, when arguments of functions are followed by = and a single option, this is the default option.

The descriptions provided are suitable for beginner and intermediate R and Siena users. For the advanced specifications of the functions the user should refer to the help by typing ''funName' in the R console, where 'funName' is the name of the function.

We consider that the model estimation is composed by 6 stages:

- 1. Getting started
- 2. Get the data the right format or check that it is in the correct format
- 3. Data specification
- 4. Model specification
- 5. Model estimation
- 6. Working with the results

Tables 5 and 6 present the list of useful R functions and the list of RSiena functions in execution order, respectively.

$\mathbf{Stage}$	Name	$\mathbf{Syntax}$	$\mathbf{Examples}$	Description
1	getwd	getwd()		Returns the name of the current working directory. Does not require arguments
-	list.files	list.files(dir)	list.files ("C:/User/	Returns a character vector with the names of the files in the directory 'dir'. If
			MyDocuments/	no argument is provided, 'dir' is the current working directory.
	*		Mysiena )	
$\vdash$	$ \operatorname{setwd}^* $	$\operatorname{setwd}(\operatorname{dir})$	setwd("C:/MyDocum	setwd("C:/MyDocumests the working directory to 'dir'. In this context the working directory should
			MySiena")	be where the data is saved
	install.packages*	install.packages()		It is used to install packages. If no arguments are provided it opens a GUI
				to select a mirror site and the packages that we want to download and install.
				This is not necessary if the package has already been installed.
	library*	library(package)	library(RSiena)	Loads the library named 'package'.
1	read.table	read.table(file,	net1 <- read.table(	Reads a file in table format and creates a data frame from it. The argument
		header = FALSE,	"network1.dat",	'file' is the file containing the data. In the case of adjacency matrices, the
		sep = "",	header = FALSE	file should have the same number of columns and rows. 'header' is a logical
		quote="``",)		argument indicating whether the first row of the data contains the column
				names. 'sep' is the field separator character (such as space, comma, etc.). See
				the help on the function to specify other arguments
2	as.matrix	$\operatorname{as.matrix}(x,)$	net1 < -	Transforms an object 'x' into a matrix. Siena works with matrices and not
			as.matrix(net1)	with data frames
2	class	class(x)	class(net1)	Returns the type of object that 'x' is
2	dim	$\dim(x)$	dim(net1)	Returns the dimension of object 'x'
4	fix*	f(x)	fix(effects)	Allows editing the object 'x' by opening a window and it replaces the old object
				by the edited 'x'

Table 5: Useful functions from R in execution order

\* Also available via a menu option

Table 6: List of RSiena Functions in order of Execution

Stage	Name	Syntax	Fyamples	Description
	installGu	installGui()		Starts the installer for the standalone version of RSiena. Only for Windows. Does not require arguments
က	sienaNodeSet	sienaNodeSet (n, nodeSetName = "Actors", names = NULL)		Creates a Siena node set which can be used as the nodes in a siena network. 'n' is the number of actors or nodes; 'nodeSetName' is a character string to name the node set (defaults to 'Actors') and 'names' is a string vector with length n with the names of each node (optional)
က	sienaDependent	sienaDependent (netarray, type = c("oneMode", "bipartite", "behavior"), nodeSet = "Actors", sparse = is.list (netarray))	sienaDependent(array(c(net1,net2,net3), dim = c(dim(net1),3)))	Creates a Siena network object by forming an array of network observations represented as matrices, arrays or sparse matrices. 'netarray' is a matrix (type = "behavior" only) or array of values or list of sparse matrices of type 'dgTMatrix'; 'type' is either 'one mode' (default), 'bipartite' or 'behaviour'; 'nodeSet' is the name of the node set. It is a vector with two strings for a bipartite network; 'sparse' is logical and it is set to TRUE if the data is in sparse matrix format, FALSE otherwise
က	coCovar	coCovar(val, nodeSet = "Actors")	cons <- as.matrix( read.table ("cons.DAT")) cons1 <- coCovar (cons[,1])	Creates a constant covariate object, where val is the vector of covariate values and nodeSet is the name of the actors' set. The dimension of val should be (1, # Actors)
က	varCovar	varCovar(val, nodeSet = "Actors")	chan <- as.matrix (read.table ("chan.DAT")) chan <- varCovar (chan[,1])	Creates a changing covariate object where 'val' is a matrix with the covariate values with one row for each actor and one column for each period; 'nodeSet' is the name of the set of actors
				Continued

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i	1	i	0	
$\mathbf{Stage}$	Name	$\mathbf{Syntax}$	$\mathbf{Examples}$	$\operatorname{Description}$
3	coDyadCovar	coDyadCovar(val,		Creates a constant dyadic covariate object where
		nodeSets =		'val' is a matrix of the same dimension as the
		c("Actors", "Actors"))		network observations and nodeSets are the sets
				of actors with which the constant covariate is as-
				sociated
3	varDyadCovar	varDyadCovar(val,		Creates a changing dyadic covariate object where
		nodeSets =		'val' is an array of matrices. Each matrix has
		c("Actors", "Actors"))		the same dimension of the actor set and 'val' has
				one less matrices than observations of the net-
				work; 'nodeSets' are the sets of actors to which
				the varying covariate object is associated
က	sienaCompositionChange	sienaCompositionChange(		Creates a list of events describing the moments
		changelist,		in which each actor is present in the network:
		nodeSet = "Actors",		'changelist' is a list with an entry for each ac-
		option = 1)		tor in the node set. Each entry is a vector in-
				dicating intervals in which an actor is present
				in the network. 'nodeSet' is the name of the
				set of actors corresponding to these composi-
				tion changes and 'option' (defaults to 1) is an
				integer controlling the processing of the net-
				work entries for the actors not currently present.
				See help(sienaCompositionChange) for details on
				this
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	page
	previous
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objects of class 'siena'; 'singleOK' is a boolean variable to indicate if it is OK to have just one object; 'getDocumentation' is a flag to allow documentation of internal functions, not for use by users  Prints a html or IATEX table with the effects details		getDocumentation = FALSE) effectsDocumentation()	effectsDocumentation	4
Creates an object of class 'sienaGroup' from a list of Siena data objects: 'objlist' is a list of objects of class 'siena'; 'singleOK' is a boolean	sienaGroupCreate (list( MyData1, MyData2))	(objlist,	sienaGroupCreate	က
Creates a siena object from networks, covariates, composition and behaviour objects: .'' represents the objects of class 'sienaDependent', 'co-Covar', 'varCovar', 'coDyadCovar', 'varDyadCovar', 'compositionChange'. 'nodeSets' is a list of Siena node sets. Default is a single set named 'Actors' with length equal to the number of rows in the first object of class 'SienaNet', it has to match the nodeSet supplied when the arguments are created; 'getDocumentation' is a flag to allow documentation for internal functions, not for use by users	MyData < sienaDataCreate (net, cons1, cons2, cons3, chan, dyad)	sienaDataCreate(, nodeSets = NULL, getDocumentation = FALSE)	sienaDataCreate	က
Description Creates a siena object from networks, covariates, composition and behaviour objects:' repre-	Examples MyData <- sienaDataCreate (net,	Syntax sienaDataCreate(, nodeSets = NULL,	Name sienaDataCreate	Stage 3
Description	${f Examples}$	$\mathbf{Syntax}$	$\mathbf{Name}$	$\mathbf{Stage}$

	page
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Stage	Name	Syntax	Examples	Description
4	getEffects	getEffects(x, nintn = 10,	$\mid \text{MyEff} < - \text{getEffects}$ (	Creates a siena effects objects (a data frame)
		behNintn = 4,	MyData, nint = $2$ ,	that contains a list of the effects that can be in-
		getDocumentation =	behNint = 1)	cluded in the model. Type fix(MyEff) to edit the
		FALSE)		effects through a GUI (e.g. Including them or ex-
				cluding them, changing their names, initial val-
				ues, fixing them, etc.) The arguments are a siena
				or a siena group object 'x', the number of lines
				for user defined network interactions 'mint' and
				the number of lines for user defined behaviour
				interactions 'behNintn'. 'getDocumentation' is
				a flag to allow documentation for internal func-
				tions, not to be used by users
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Stage	Name	Syntax	Examples	Description
4	includeEffects	includeEffects(myeff,,	MyEff<-	The function is a way to select the effects to be
		include = TRUE,	includeEffects(MyEff,	included. 'myeff' is an effects object, as created
		name = $myeff \$name[1]$ , $type$	transTrip, balance)	by getEffects. It is necessary to indicate the short
		= "eval",		names to identify the effects to be included (ar-
		interaction1 = "",	$\mathrm{MyEff}<-$	gument). Use myeff \$ shortName to get a list
		interaction 2 = ""	include Effects(MyEff,	of the short names of possible effects to include
			sameX,	and myeff \$effectName to get the full name of the
			sameXRecip,	effects. This information can also be found in
			interaction1 = "gender"	the documentation created by effectsDocumen-
				tation. The "include = TRUE" indicates that
				we want to include the '' effects in the model,
				it can be set to FALSE to exclude effects from
				the model. 'name' is the name of the network
				for which effects are being included. 'type' is
				to include 'eval' (evaluation function effects) or
				'endow' (endowment function effects). 'interac-
				tion1' and 'interaction2' are names of siena ob-
				jects (where needed) to completely identify the
				effects e.g. covariate name or behavior variable
				name. Use myeff \$effectName [myeff \$include] to
				get the names of the included effects. It returns
				a new effects object, so it is important to assign
				it to a name
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Stage	$\mathbf{Name}$	$\mathbf{Syntax}$	$\mathbf{Examples}$	Description
4	includeInteraction	includeInteraction(myeff,,   MyEff<-	MyEff<-	This function provides an interface to allow easy
		include = TRUE,	teraction(	My- update of an unspecified interaction row in a
		name = myeff \$name[1],	Eff,	Siena effects object. 'myeff' is a Siena effects
		type = "eval",	transTrip, egoX,	object as created by getEffects. To specify the
		interaction $1 = rep("", 3),$	interaction1	= effects to interact, list their short names in-
		interaction 2 = rep("", 3))	$\mathrm{c}("","\mathrm{beh"}))$	stead of ''; 'include' is a boolean variable, de-
				fault TRUE to include the interaction, it can be
				switched to FALSE to turn off an interaction.
				'name' is the name of network for which interac-
				tions are being defined. Defaults to the first in
				the effects object. 'type' is the type of effects to
				be interacted: currently only 'eval' or 'endow'.
				'interaction1' is a vector of siena objects where
				needed to completely identify the effect e.g. co-
				variate name or behavior variable name. Trailing
				blanks may be omitted. 'interaction2' is a vector
				of siena objects where needed to completely iden-
				tify the effect e.g. covariate name or behavior
				variable name. Trailing blanks may be omitted.
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Description	Interface to change the attributes of a particular effect. The required arguments are an effect object ('myeff'), the short name of the effect to modify ('shortName') and a required integer value that defaults to zero ('parameter'). Depending on what it is desired to be modified we can supply: "fix = TRUE", if we wish to fix that parameter; "test = TRUE" if we wish to test that parameter; "initialValue = 2" (or any desired number) to modify the effect's initial value (Defaults to zero); "include = TRUE or FALSE" depending on whether we want to include/exclude the effect (defaults to TRUE). The arguments 'name', 'type' and 'interaction1' and 'interaction2' are defined as in includeInteraction and includeEffects.	Prints a report of a Siena data object and its default effects. We need to supply a Siena data object ('data') a siena effects object ('myeff') and a model name ('modelname') that defaults to 'Siena'. It creates and saves a file named 'modelname.txt' (Siena.txt) that contains preliminary information on the data.	Continued
${\bf Examples}$	MyEff <- setEffect(MyEff, transTrip, initialValue = 3, include = TRUE)	print01Report(MyData) Prin defa defa obje a m 'Sier nam info	
$\mathbf{Syntax}$	setEffect(myeff, shortName, parameter = 0, fix = FALSE, test = FALSE, initialValue = 0, include = TRUE, name = myeff \$name[1], type = "eval", interaction1 = "", interaction2 = "")	print01Report(data, model- name = "Siena", session = NULL, getDocumentation = FALSE)	
$\mathbf{Name}$	set Effect	print01Report	
$\mathbf{Stage}$	7	က	

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page
previous
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9
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Stage		Sympa	Evamples	Describing
5	sienaAlgorithmCreate	siena Algorithm Create (	MyAlgorithm <- siena-	Creates a siena algorithm object that can be used
		$f_{n} = simstats0c,$	AlgorithmCreate	to call siena07. 'fn' is function to do one simu-
		projname = "Siena",	(projname =	lation in the Robbins-Monro algorithm. 'proj-
		MaxDegree = 0,	"MyProject")	name' is character string name of project. No
		useStdInits = FALSE, $n3 =$		embedded spaces. 'MaxDegree' is a named vec-
		1000,		tor of maximum degree values for corresponding
		nsub = 4, maxlike = FALSE,		networks. 'useStdInits' is a boolean variable, if
		diag = lmaxlike,		TRUE, the initial values in the effects object will
		condvarno = 0,		be ignored and default values used instead. 'n3'
		condname = "", firstg = $0.2$ ,		is the number of iterations in phase 3 (defaults
		cond = NA, findiff = FALSE,		to 1000). 'nsub' is the number of subphases
		seed = NULL		in phase 2 (defaults to 4). 'maxlike', boolean
				to indicate whether to use maximum likelihood
				method or straightforward simulation (defaults
				to false). 'diag' is boolean to indicate if the com-
				plete estimated derivative matrix should be used;
				'condvarno', if conditional estimation is used the
				parameter is the sequential number of the net-
				work or behaviour variable on which to condi-
				tion. 'condname' is the name of the dependent
				variable on which to condition (only use cond-
				name or condvar, not both). 'firstg' initial value
				of gain parameter in the Robbins-Monro proce-
				dure. 'cond' is boolean, If TRUE, use condi-
				tional simulation. If missing, decision is deferred
				until siena07 is called, when it is set to TRUE
				if there is only one dependent variable, FALSE
				otherwise. 'findiff' is boolean, if TRUE, estimate
				derivatives using finite differences and if FALSE,
				use scores. 'seed' is an integer referring to the
				starting value of random seed. Not used if par-
				allel testing.
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Stage	lvarne	Symax	Examples	Description
5	siena07	siena $07(x, batch = FALSE,$	ans <-	Estimates parameters using Robbins-Monro al-
		verbose = FALSE, silent =	siena07(MyModel, data =	gorithm. Note that the particular model to be
		•	MyData, effects = $MyEff$ ,	used is passed on as the algorithm object, and
		useCluster = FALSE,	batch = FALSE,	data for the model must be passed by using
		nbrNodes = 2,	verbose = TRUE,	named arguments. 'x' is a model object; 'batch'
		initC = FALSE,	useCluster = $TRUE$ ,	is a boolean variable to indicate if it is desired to
		clusterString =	nbrNodes = 2)	open the GUI of Siena simulation; 'verbose' is a
		rep("localhost", nbrNodes),		boolean variable to produce output on the con-
		tt = NULL,		sole; 'silent' is also a boolean variable, if true, no
		parallelTesting = FALSE,)		output is printed to the console; 'useCluster' is a
				boolean variable to indicate if it is desired to use
				a cluster of processes; 'nbrNodes' is the number
				of processes to use if useCluster is TRUE; ' 'clus-
				terString' is the definition of clusters, default set
				up to use the local machine only;
				siena07 returns an object of class sienaFit
				(let's say ans). The main attributes are
				ans \$theta, which are the estimated coefficients;
				ans \$covtheta is the estimated covariance matrix
				of theta; ans \$dfra is the matrix of estimated
				derivatives; ans \$targets and ans \$ targets2 are
				the observed statistics and the observed statistic
				by wave, respectively; ans \$ssc are the score func-
				tion contributions for each wave for each sim-
				ulation in phase 3: ans \$sims is the simulated
				networks as edgelists. Use names(ans) to obtain
				more characteristics; only recommended if you
				are proficient in RSiena.
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	page
•	previous
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	Table

$\mathbf{Stage}$	$\mathbf{Name}$	$\mathbf{Syntax}$	$\mathbf{Examples}$	Description
9	print.sienaFit	print(x, tstat = TRUE,)	print(ans)	The function prints a table containing the estimated parameter values, standard errors and (optionally) t-statistics for convergence. If 'x' is a summary(sienaFit) it prints on the console all the summary elements. 'tstat' is a boolean argument, set to TRUE if it is desired for the t-statistics for convergence to be added to the
				report
9	summary.sienaFit	$\operatorname{summary}(x,)$	summary(ans)	Prints a table containing the estimated parameter values, standard errors and t-statistics for convergence together with the covariance matrix of the estimates, the derivative matrix of expected statistics D by parameters, and the covariance matrix of the expected statistics D. The only required argument is a 'sienaFit' object 'x', as produced by siena07.
9	xtable.sienaFit	xtable(x, caption = NULL, label = NULL, align =	sienaxtab <- xtable(ans,	Creates an object of class xtable.sienaFit which inherits from class xtable and passes an extra
		NULL,	caption = "My Table",	arguments to the print.xtable. The argument is
		digits = NULL,	digits = 2).	a sienaFit object 'x'.
		display = NULL,)		

# B Appendix: Changes compared to earlier versions

This presents the main changes, especially those directly visible to the user, in reverse time order from the current version until version 1.0.6 of end October 2009. For more complete information about changes, consult the NEWS.md file in the source code, and the ONWS ('old news') and ONWS\_gh ('old news in GitHub') files, which contain more complete listings. The NEWS.md file can also be consulted during an R session if you installed the package RSiena, provided you have also attached the packages commonmark and xml2 to render the Markdown format.

After installing these two packages, to get the NEWS about any package that you installed (such as RSiena), give the command

## news(package="packageName")

(where you can fill in the package name).

- 2024-07-31 GitHub, package version 1.4.14 Changes in RSiena:
  - New effects:
    - \* New gmm effects for co-evolution of multiple networks: crprod\_gmm, to\_gmm, from\_gmm.
  - Bug corrections:
    - \* Bug corrected in sienaGOF which occurred for models with tested effects if iterations is less than sienaFitObject\$n3.
- 2024-06-06 GitHub, package version 1.4.13 Changes in RSiena:
  - New effects:
    - \* New effect homXOutAct2.
  - Improvements of functionality:
    - \* The handling of internal effect parameters in the columns effectName and functionName of sienaEffects objects is improved. This was achieved by changes in functions setEffect and includeInteraction. Effectnames as reproduced by sienaFit objects should now contain the correct values of internal effect parameters.
    - \* New parameter thetaBound for siena07, which has the effect of stopping the estimation process if some parameters become too large (which would signal divergence).
    - \* Display of deviations in siena07gui modified so that numbers larger than or equal to 1e5 in absolute value are displayed in exponential format (and use only one line in the gui).
  - Bug corrections:
    - \* Function setEffect corrected (it did not give the proper internal parameters in the effectName).

- \* Function updateSpecification corrected (it did not work properly for including interactions).
- 2024-04-23 GitHub, package version 1.4.11

- Effects:
  - \* Effect name outOutDist2AvIntn changed to avAlt.2M.tot.
  - \* New effects avAlt.2M.tie, avAlt.2M.tot, avAltU.2M.tie, dist2OutInActIntn, nDist2ActIntn, sharedToU.
  - $* \ \, \textbf{Changed sqrt treatment in outOutDist2ActIntn} \ and \ outOutDist2AvIntn} \, / \, avAlt.2M. \, tot.$
- Improvements of functionality:
  - \* Method print.sienaEffects has an extra parameter includeShortNames to do what the name of this parameter suggests.
- $\bullet~2024\text{-}03\text{-}25$  GitHub, package version 1.4.10

Changes in RSiena:

- New effects:
  - \* New effects crossXOutAct, outOutDist2ActIntn, outOutDist2AvIntn, inPopOutW.
  - \* New effect group doubleCovarNetObjective.
  - \* New effects sameXV and sameXVInPop for bipartite and symmetric networks.
  - \* sameXCycle4 added for one-mode and symmetric networks.
  - \* sharedTo gets default internal effect parameter p=3.
- Improvements of functionality:
  - \* Function updateSpecification now also updates interaction effects and initialValues.
  - \* New parameter silent in sienaAlgorithmCreate activated.
- $\bullet~2024\text{-}02\text{-}29$  GitHub, package version 1.4.8.

Changes in RSiena:

- Bug corrections:
  - \* Correction of memory leak in siena07setup.ccp for ML estimation.
- New functionality:
  - \* New parameter silent in sienaAlgorithmCreate.
- 2024-02-20 GitHub and CRAN (2024-02-22), package version 1.4.7 Changes in RSiena:
  - Bug corrections:
    - \* An error in a .cpp file was corrected.
- $\bullet~2024\text{-}02\text{-}19$  GitHub, package version 1.4.6

Changes in RSiena:

- Improvements of functionality:

- \* New parameter targets in siena07, used to supersede the targets calculated from the data (not for use in estimation for regular data sets, see the help file for siena07).
- \* effectsDocumentation() reports to the console the name of the file that was written.
- \* sienaDataCreate stops with an error message if there is a bipartite network before a one-mode network.
- \* If the data contains a continuous dependent behavioral variable and the algorithm specifies conditional estimation, the estimation stops with a clear error message.

### - Bug corrections:

- \* Various corrections in the cpp code.
- \* Allow sienaDataCreate to work with a single variable defined as a dependent network given as a list of sparse matrices.
- \* Allow getEffects to construct effects of more than one dependent network on continuous behavior dependent variables.
- \* Changed the check for constant dyadic covariates for sparse matrices (GitHub issue #88).
- \* Some of the recently added effect groups were missing from effectsDocumentation. This led to an incomplete listing of the effects. They are now included.

## - Dropped functionality:

- \* sienaRI temporarily disabled because of a memory leak.
- \* The option prML=2 temporarily disabled because of a memory leak in the move proposal distribution (sienaAlgorithmCreate).
- Other changes:
  - \* List of changes in the code before 2022, but since the source code was hosted at GitHub (i.e., since September, 2020) moved from NEWS.md to ONEWS\_gh.
- 2023-12-14 GitHub, package version 1.4.2.

Changes in RSiena:

- New effects:
  - \* New effects outXMore, outMore3.
  - \* Corrected effect outMore.
  - \* Interactiontype of altLThresholdX and altRThresholdX is dyadic.
    Interactiontype of degAbsDiffX, degPosDiffX, and degNegDiffX is "" (blank).
- Improvements of functionality:
  - \* New parameter iterations in sienaGOF to allow shorter computations.
- Bug corrections:
  - \* bxeffects initialized to 0 in ContinuousVariable::accumulateScores.
  - \* Improved initialization, may have corrected some unknown bugs.
- 2023-11-01 CRAN and GitHub, package version 1.4.1.

- Meta-data:
  - \* citation for RSiena improved.

- \* Authors of RSiena now are listed as Tom A.B. Snijders, Ruth M. Ripley, Krist Boitmanis, Christian Steglich, Nynke M.D. Niezink, Felix Schoenenberger, and Viviana Amati.
- Corrections of documentation:
  - $\ast\,$  Various small improvements in help files, in accordance with "Guidelines for Rd files" at CRAN.
- 2023-10-11 GitHub, package version 1.3.28
  - Changed effect:
    - \* threshold, threshold2, threshold3 and threshold4 changed to work with non-centered parameter (not backward-compatible).
  - Improved documentation:
    - \* Descriptions of effects altInDist2W and totInDist2W added to the manual (the effects had been there since a long time, but not documented).
- 2023-09-29 GitHub, package version 1.3.27

- New effects:
  - \* avInSimDist2, totInSimDist2, sameEgoDist2, sameEgoInDist2, outMore2, divOutEgoIntn, divInEgoIntn, divOutAltIntn, divInAltIntn.
  - \* avTAltDist2 and totTAltDist2 also implemented for behavior co-evolving with symmetric networks.
- Improvements of functionality:
  - \* The slowness of siena07 that occurred since version 1.3.18 was ended (by correcting one line in siena07models.cpp).
- Improved documentation:
  - \* Some explanation is given in the manual about internal effect parameters for interactions created by includeInteraction.
- 2023-08-15 GitHub, package version 1.3.26

- Coding:
  - \* Improved Phase 1 derivative matrix computation for basic SDE parameters (for continuous behavior).
  - \* Added continuous behavior to returned simulated data.
- Corrections:
  - \* Period/groupwise tests in sienaTimeTest corrected for the case of non-saturated sets of dummy variables.
  - \* plot.sienaTimeTest for pairwise=TRUE changed so that the warning is avoided.
  - \* sienaGOF corrected so that again it can handle auxiliary functions referring to more than one varName (such as in mixedTriadCensus).
- 2023-08-01 GitHub, package version 1.3.24 Changes in RSiena:

#### - Corrections:

- \* In getEffects, the effects object was given an attribute version (was not correct in version 1.3.23).
  - (This led in that version to always giving a warning if any interaction effects were specified.)
- \* Corrections of implementation of acceptance by sienaGOF of a list of sienaFit objects (was not correct in version 1.3.23).
- 2023-06-29 GitHub, package version 1.3.23.

Changes in RSiena:

- New effects:
  - \* New effects diffWXClosure, sameWWClosure, diffWWClosure, diffXWClosure, sameXWClosure, unequalX.
  - \* JoutMix made available for bipartite dependent networks.
  - \* For continuous behavior variables depending on a bipartite dependent network, the effect group continuousBipartiteObjective was created, with effects outdeg, outdegSqrt, and isolateOut.
  - \* sameXOutAct and diffXOutAct now have a parameter 2 for  $\sqrt{\ }$ .
- Improvements of functionality:
  - \* The model for continuous behavior variables seems to work now.
  - \* sienaGOF now also accepts a list of sienaFit objects.
  - \* In sienaDataCreate, the warning message that there is at least one "upOnly" period now is made for each dependent variable instead of only the last.
  - \* A warning is given if the used version of the effects object is not current and it contains interaction effects (then it is possible that the interacting effects are chosen incorrectly, even though the effectName of the interaction seems OK).
  - \* Various coding improvements and better texts of warnings and error messages.
  - \* In the violin plot for sienaGOF, the default bandwidth selection method is used (the use of "nrd" sometimes led to absent plots because of negative bandwidth).
- Bug corrections:
  - \* The error was corrected that occurred if useStdInits = TRUE in sienaAlgorithm-Create and the effects object includes interaction effects.
- 2023-05-11 GitHub, package version 1.3.22.

- New functionality:
  - \* For one-mode networks, new options 7, 8, 9, 10 for modelType set in siena-AlgorithmCreate; see Section 5.10.
- Bug corrections:
  - \* Coding of some "generic effects" was cleaned up and corrected. This led to changes in coding of several distance-2 covariate-related network effects such as altDist2, totDist2 and altInDist2; some of these <u>may</u> have been incorrect in earlier versions.

- \* In sienaAlgorithmCreate, changed default prML=2 back to prML=1; stop if Maximum Likelihood estimation is attempted for a data set containing more than one dependent variable with prML=2.
- $\bullet$  2023-04-22 GitHub, package version 1.3.20.

- Improvements of functionality:
  - \* The keyword parameter in includeInteraction was dropped because it did not have any consequences. The help page for includeInteraction now explains how internal effect parameters for user-defined interactions are determined.
  - \* The column dimnames of the Simulations array returned by sienaGOF are set to the names of the elements of the auxiliary function.
  - \* Standard deviations added to output of descriptives.sienaGOF.
  - st Improved error message in initializeFRAN in the case of mismatch between effects objects.
  - \* Warning in sienaAlgorithmCreate if maxlike is chosen and MaxDegree is specified. This is now also mentioned in the help page for sienaAlgorithmCreate.
- Bug corrections:
  - st updateSpecification (in effectsMethods) now also updates internal parameter values
  - \* In TriadCensus, the empty network will not lead to an error but be reported with the correct triad census.
  - \* For reciAct, check whether internal parameter is equal to 2 replaced by check whether absolute difference from 2 is less than 0.001.
  - \* In phase2.r, z\$sd is calculated using sqrt(pmax(..., 0)) to avoid the extremely rare case of a negative calculated variance.
  - \* In sienaDataCreate, handling of structurally determined values in checkConstraints corrected (thanks to issue raised by Jos Elkink).
- Improvements of documentation:
  - \* In the help page for sienaDependent, it is mentioned that if there are one-mode as well as two-mode dependent networks, the one-mode networks should come first.
- $\bullet~2023\text{-}02\text{-}07$  GitHub, package version 1.3.19.

- New effects:
  - \* New effect inPop\_dya .
  - \* Parameter 2 allowed for sameXInPop, diffXInPop.
- Improvements of functionality:
  - \* coCovar and varCovar now accept vectors with at least one non-missing value, but will stop with an error message if all values are missing.
- Coding:
  - \* siena07internals.cpp adapted to be compatible with new clang 16 C++ compiler (thanks to Brian Ripley).
- Corrections of documentation:

- \* Help page for siena07 corrected with respect to x\$lessMem.
- 2023-01-29 GitHub, package version 1.3.18.

- Improvements of functionality:
  - \* Additional step type move for MH proposal distribution for likelihood estimation (thanks to Charlotte Greenan).
  - \* Accordingly, parameters changed that are used in sienaAlgorithmCreate for probabilities of moves, now summarized in prML; with a new default.
  - \* List elements accepts, rejects, aborts for sienaFit objects produced by ML estimation improved/corrected by reorganizing them in C++.
  - \* List element ac3 (auto-correlations of statistics during Phase 3) added to sienaFit object produced by siena07 for ML estimation.
- Corrections of documentation:
  - \* Extended explanation of maximum likelihood estimation.
- 2023-01-06 GitHub, package version 1.3.17

Changes in RSiena (including those for version 1.3.16):

- Effects:
  - \* inPopIntnX, inActIntnX, outPopIntnX, outActIntnX, sameXInPopIntn, sameXOutPopIntn, sameXInActIntn, sameXOutActIntn restored (these had got lost in some way...).
- Improvements of functionality:
  - \* sienaGOF now accepts simulated auxiliary statistics containing missing values. If there are any, this will be reported with a warning if giveNAWarning is TRUE.
  - \* sienaDataCreate now also accepts, as "...", a list of such objects.
- 2022-11-27 GitHub, package version 1.3.15

Changes in RSiena:

- Correction:
  - \* Correction: siena08: correct p-value pTsq for overall test statistic Tsq.
- Improvements of functionality:
  - \* Minor improvements in meta.table.
- $\bullet$  2022-10-06 CRAN / GitHub, package version 1.3.14

Changes in RSiena:

- Some changes for increasing the compatibility with various hardware platforms.
- 2022-10-06 GitHub, package version 1.3.12

- Updates:
  - \* Changes to comply with new version of Matrix package.
- New effects:

- \* Several new effects related to primary setting: nonPCompress, primCompress, primary, primDegAct, primDegActDiff, primDegActDiffSqrt, primDegActSqrt, primDegActLog, primDegActInv.
- \* gwdspFB effect added for two-mode networks.
- \* New effects outAct\_ego, inAct\_ego,reciAct\_ego, toAny.
- \* For effects to, toBack, toRecip, mixedInXW, internal effect parameter 3 now specifies truncation of the number of twosteps (change to MixedTwoStepFunction).
- Improvements of functionality:
  - \* sigmas and meansigmas added to signaRI object.
  - \* Print of standard deviations in the sienaRl object for printSigma=TRUE changed to using averages at the variance level.
  - \* If returnThetas in the call of siena07, also simulated estimation statistics during Phase 2 (deviations from targets) are returned.
- Bug corrections:
  - \* universalOffset initialized as 0; it was earlier initialized as the maximum real number (NetworkLongitudinalData.cpp).
- Corrections of documentation:
  - \* Modified help page for sienaRI.
  - \* Small modifications of help page for sienaGOF.
- 2022-05-30 GitHub, package version 1.3.11

- Corrections:
  - \* Correction in effects.r of error that led to warning in R 4.2.0 for multivariate networks.
  - \* Correction of help page for sienaGOF (groupName).
  - \* Correction of igraphNetworkExtraction in the help page for sienaGOF-auxiliary.
- Improvements of functionality:
  - \* Further explanation of mixedTriadCensus in the help page for sienaGOF-auxiliary.
- 2022-04-28 GitHub, package version 1.3.10

Changes in RSiena:

- Effects:
  - \* New effects avInAltW, avWInAlt, totInAltW, totWInAlt (help from Robert Krause).
  - \* Corrected implementation of sharedTo.
- Corrections:
  - \* Bug corrected that occurred when several two-mode networks were included in the dependent variables, with an order restriction between them (HigherFilter, DisjointFilter).
- $\bullet~2022\text{-}03\text{-}18$  GitHub, package version 1.3.9.

- Effects:
  - \* Corrected implementation of simAllNear and simAllFar.
- Corrections:
  - \* Small correction of summary.sienaGOF.
  - \* Small correction of sienaTimeTest.
- 2022-03-07 GitHub, package version 1.3.8.

- Effects:
  - \* New effect avDeg.
  - \* Changed default internal effect parameter for simAllNear to 2 and for simAllFar to 4.
- Improvements of functionality:
  - \* Function sienaRl was made available also for two-mode networks (provided the number of second-mode nodes is smaller than the number of actors = first-mode nodes), and corrected for behavioral dependent variables.
  - \* In sienaTimeTest, added warn = FALSE to varCovar to avoid warnings.
  - \* Small improvements to help pages for sienaGroupCreate and sienaGOF.
- $\bullet~2022\text{-}02\text{-}16$  GitHub, package version 1.3.6.

Changes in RSiena:

- Effects:
  - \* New effects simAllNear, simAllFar, absOutDiffIntn, avDegIntn.
  - \* New effects recipRateInv, recipRateLog (Steffen Triebel).
  - \* Default internal effect parameter for outOutActIntn, outOutAvIntn, and both changed from 2 to 1.
- Improvements of functionality:
  - \* Function includeInteraction now also can modify the initialValue of an effect; and the order of parameters for this function was changed, bringing it in line with setEffect.
  - \* Small clarifications of help pages for includeInteraction and setEffect.
- $\bullet~2021\mbox{-}12\mbox{-}15$  GitHub, package version 1.3.5.

Changes in RSiena:

- Bug corrections:
  - \* Corrected the check for effects in initializeFRAN.r which led to errors if interaction effects were included, because of the changes to includeInteraction in version 1.3.4.
- $\bullet~2021\text{-}12\text{-}08$  GitHub, package version 1.3.4.

- New effects:
  - \* New effects inRateInv, inRateLog (thanks to Steffen Triebel).

- Improvements of functionality:
  - \* When an effects object with interaction effects is printed, the names of the interacting effects are mentioned, and prefixes int. and i3. were dropped.
  - \* The check of whether an interaction effect is allowed now is done immediately when creating the interaction effect instead of waiting for its use in siena07.
  - \* For function sienaGroupCreate some changes were made: if it is applied to a list of length 1, attributes of the single group are not recomputed; if it is applied to a list of length larger than 1, the attributes range and range2 of behavioral variables of individual groups are computed as the range of the unions of the ranges of all the groups. The same is done for covariates.
  - \* print.sienaGroup slightly extended.
  - \* Creation of covariates gives a warning (optional) if all values are missing, and also if all non-missing values are the same.
- Bug corrections:
  - \* Some bug corrections referring to the use of sienaBayes, which will be integrated later with RSiena.
- 2021-10-08 GitHub, package version 1.3.3.

- New effects:
  - \* internal effect parameter for diffusion rate effects ('at least p').
  - \* New effect outOutAvIntn.
  - \* outOutActIntn also made available for non-directed explanatory networks and for two-mode dependent networks.
- Improvements of functionality:
  - \* toggleProbabilities added to output of sienaRI.
  - \* Trial values of theta used during Phase 2 of siena07 added to the object ans produced by sienaFit as ans\$thetas.
  - \* warnings if a data object contains only missings, or only the same value.
- Bug corrections:
  - \* if a data set contains a constant covariate, the simX effect will not run into an error any more; this is obtained by defining the range attribute of the constant covariate as 1 to prevent division by zero, and the simMean attribute as 0 instead of NaN (sienaDataCreate). This is relevant especially for sienaGroup data sets, where covariates might be constant for some of the groups.
- Corrections of documentation:
  - \* In sienaAlgorithmCreate, the default values of diagonalize for MoM is 0.2; this was corrected in the help file.
- 2021-07-29 GitHub, package version 1.3.2.

- New effects: crprodInActIntn (thanks to Nynke Niezink), XXW.
- Improvements of functionality:
  - \* updateTheta also accepts sienaBayesFit objects as prevAns.

- \* Effects of type creation or endow represented in the output of siena.table by creation and maintenance, respectively.
- Bug corrections:
  - \* If upOnly or downOnly, the (out)degree (density) effect is also excluded for symmetric networks (this was reported by printO1Report, but not carried out).
  - \* Message corrected in sienaDataCreate if there is an attribute higher.
- 2021-07-13 R-Forge, RSienaTest version 1.2-30.

- sienaBayes: adaptation of nwarm and nmain at the end of initializeBayes dropped.
- 2021-05-02 CRAN/GitHub, package version 1.3.0.

The cumulative importance of all changes in the past two years (including Generalized Method of Moments and continuous behavioral variables) justifies that the version number now proceeds from 1.2.x to 1.3.0.

Changes in RSiena:

- Drop testthat in tests.
- 2021-04-30 GitHub, package version 1.2.34.

Changes in RSiena:

- New function testSame.RSiena to test equality of parameters.
- New effects: avInSim (thanks to Steffen Triebel), totInSim, avInSimPopAlt, totInSimPopAlt, constant, avAttHigher, avAttLower, totAttHigher, totAttLower.
- Changed effects: endowment and creation types for avInSim (brought in line with these types for avSim).
- Improvements of functionality:
  - \* funnelPlot adapted to lists of sienaFit objects containing some missing estimates or standard errors.
  - \* plot.sienaGOF: new parameter position.
  - \* Small improvements (constant length of effect names) in meta.table and siena.table.
- Bug corrections:
  - \* Restore backward compatibility with respect to checks of x\$gmm.
  - \* Correct names reported for tested effects (avoid wrong names being given if there are interactions without main effects).
- 2021-04-14 R-Forge, RSienaTest version 1.2-29.

Changes in RSienaTest:

- The pre-warming phase of sienaBayes is interrupted (and followed by the warming phase) if no proposals were accepted during two batches of iterations.
- 2021-03-19 GitHub, package version 1.2.33.

Changes in RSiena:

 configure.ac changed (thanks to Alvaro Uzaheta). This gives wider possibilities for installation at a variety of platforms.

- 2021-03-18 R-Forge, RSienaTest version 1.2-28. Changes in RSienaTest:
  - glueBayes corrected.
- 2021-03-16 GitHub, package version 1.2.32.

- Generalized Method of Moments implemented (Viviana Amati): see
   https://github.com/stocnet/rsiena/blob/main/docs/manual/Changes\_RSiena\_GMoM.
   pdf; new function includeGMoMStatistics, extended functionality of siena07.
- effectsDocumentation now also includes gmm effects (at the bottom).
- dyadicCov made to accept also changing dyadic covariates.
- new arguments plotAboveThreshold and verbose for funnelPlot.
- Resolved issue with continuous dependent behavior variables (Nynke Niezink).
- New effects: homXTransRecTrip, toU.
- This implied creation of a new effect class dyadANetNetObjective.
- sqrt versions for parameter 2 for the effects to, toBack, toRecip, from, fromMutual.
- Reinstated effect MixedInXW, also with sqrt version for parameter 2.
- Dropped effect to.2 (identical to 'to') and MixedInWX (identical to 'toBack').
- Description of toBack and toRecip in manual.
- Correction in phase 3.2 of a bug that sometimes led to an error message if simOnly.
- Display of deviations from targets changed to after subtraction of targets.
- Stop if no parameters are estimated and simOnly is FALSE.
- Vignette dropped; it is available on the website.
- Require R >= 3.5.0.
- xtable added to Imports (used to be in Suggests).
- 2020-12-10 GitHub, package version 1.2.29.

Integration of GitHub and what was earlier at R-Forge.

- New effects (due to Christoph Stadtfeld): transtrip.FR, transtrip.FE, transtrip.EE,
   WWX.EE, WWX.FR, WXX.FE, WXX.ER, XWX.ER, XWX.FE, to.2, toBack, toRecip.
- New functions meta.table and funnelPlot.
- In sienaAlgorithmCreate, use the definitions for projname = NULL also if any environment variable \_R\_CHECK\* is set.
- Adapted filter disjoint so that it operates correctly also when the network is symmetric.
  - Consequence: constraint that two networks are disjoint operates correctly also when one of the networks is symmetric and the other is not.
- Adapted filter higher so that it operates correctly also when the other network is symmetric.
  - Consequence: constraint that one network is at least as high as another network operates correctly also when the higher networks is symmetric and the other is not.

- In CheckConstraints, used in sienaDataCreate, the requirement was dropped that the
  two networks have the same symmetry property; and for higher it is required that if
  the lower network is symmetric, the higher network is also symmetric.
- In sienaDataConstraint, if type is disjoint or atLeastOne, the constraint is also implemented for the pair (net2, net1).
- New parameter tested in sienaGOF.
- 2020-09-16 R-Forge Revision 350, package version 1.2-27.

## Changes in RSiena and RSienaTest:

- tcltk no longer a requirement; if not available, batch mode is followed for siena07 (with help by James Hollway).
- New effect transTripX.
- For effect from.w.ind, option parameter = -1 added.
- The to effect is an ego effect.
- For siena.table, some of the effectNames changed to nice strings, so that LATEX can run without errors on the table produced if type = "tex".
- The sienaMeta object produced by siena08 now has IWLS estimates more easily accessible, as object\$muhat and object\$se.muhat.
- Error message in sienaTimeTest for sienaFit objects produced with lessMem=TRUE.
- More extensive error message for error in named vectors in algorithm object (checkNames in initializeFRAN).
- For sienaDataCreate: more extensive error message, and class(...) replaced by class(...) [1].
- multiplication factor mult added to print.sienaAlgorithm if maxlike.
- sienaAlgorithmCreate: requirements for mult corrected in help page.

## Changes in RSiena:

 Vignette basicRSiena added (was earlier available as a script); thanks to James Hollway.

## Changes in RSienaTest:

- Improved initialisation in sienaBayes: initial scale factors (squared) and initial parameter estimate ('Kelley').
- Allow parameter targetMHProb of sienaBayes to have length 2, applying separately to the random parameters and the fixed parameters.
- Test on class() == replaced by inherits in sienaBayes.
- Another attempt to discard the objects zn and zsmall remaining after operation of sienaBayes.
- 2020-04-08 R-Forge Revision 348, package version 1.2-25.

## Changes in RSiena and RSienaTest:

- Correction of sienaDataCreate for actor covariates (gave a warning).

- Auxiliary function dyadicCov for sienaGOF now can handle missings, does not return a named value for frequency of zeros, and returns the frequencies without any division. The help page indicates how this function can be used to check fit for ego-alter combinations of monadic variables.
- descriptives.sienaGOF also shows the exceedance probabilities of the observed statistics.
- The NULL default values for modelType and behModelType are implemented already at the R side instead of waiting until the information comes to C++.
- Some extra information in help page for siena07.

- Error corrected that occurred in sienaBayes, extract.sienaBayes, and extract.posteriorMeans for data with multiple periods.
- 2020-02-16 R-Forge Revision 347, package version 1.2-24.

Changes in RSiena and RSienaTest:

- New auxiliary function dyadicCov for sienaGOF.
- Correction of error in modelType when one-mode as well as two-mode networks are used.
- Added startingDate in siena08 to the object produced.

#### Changes in RSienaTest:

- Corrected siena.table for sienaBayesFit objects; added startingDate in sienaBayes to the object produced.
- 2020-01-12 CRAN Siena version 1.2-23.

Changes in RSiena and RSienaTest:

- Changed extension names of output files from .out to .txt.
- Option projname = NULL in sienaAlgorithmCreate. This option is used for all examples in RSiena where siena07 is called, to restrict file writing for examples to the temporary directory.
- 2020-01-16 R-Forge Revision 345, package version 1.2-22.

Changes in RSiena and RSienaTest:

- Tested effects reported in print of Multipar.RSiena and score.Test.
- Reordered effects object so that reciAct comes after instead of before inAct and inAct.c.
- 2019-12-15 R-Forge Revision 344, package version 1.2-19.

Changes in RSiena and RSienaTest:

- New effects avGroupEgoX, outIso.
- Starting date of estimation reported in siena.table(..., type = "tex", ...).
- Handling of missings in changing covariates corrected (?).

- Changes of RSienaTest version 1.2-18 (see below) ported to RSiena. The main changes are the introduction of the option of continuous dependent behavior variables and allowing imputationValues also in sienaDependent (both by Nynke Niezink).

## Changes in RSienaTest:

- In sienaBayes: more precise check that prevAns has same specification as effects.
   priorSigEta added to sienaBayesFit object. dimnames(ThinParameters)[[3]] defined as effect names; ridge for prior variance of rate parameters if priorRatesFromData=1 or 2 decreased from 0.5 to 0.01.
- sienaBayes: zm and zsmall are removed at the end.
- plotPostMeansMDS also shows a title, and coordinates show the dimensions of the MDS solution.
- The value returned by plotPostMeansMDS contains not only the plot coordinates, but also the similarities (correlations) between the groups.
- glueBayes: added z\$varyingInEstimated, z\$varyingNonRateInEstimated, z\$ratesInVarying
  to the object produced. (Else extract.sienaBayes is not going to work properly.)
- glueBayes: if one of the sampling parameters is different, give a warning instead of a stop.
- Corrected error in names of array returned by extract.posteriorMeans.
- New parameter excludeRates in extract.posteriorMeans, plotPostMeansMDS.
- Use parameter pmonly also in plotPostMeansMDS.
- 2019-10-16 R-Forge Revision 341, RSienaTest version 1.2-18.

## Changes in RSienaTest:

- Continuous dependent behavior variables implemented (Nynke Niezink). This implies new effect types continuousFeedback, continuousIntercept, continuousOneModeObjective, continuousRate, continuousWiener, unspecifiedContinuousInteraction.
- imputationValues allowed in sienaDependent (Nynke Niezink).
- New effect outMore.
- component startingDate added to sienaFit object; this date is reported in siena.table(..., type = "tex", ...).
- Object names are given in sienaFit.print if simOnly.
- Speeded up calculation of IndegreeDistribution and OutdegreeDistribution for sienaGOF if there are no missings or structurals.
- regrCoef and regrCor added to the sienaFit object also when not dolby.
- Immediate stop if useCluster and returnChains both are used (in this case, no chains would be returned anyway).
- sienaDataCreate: more informative message in case of constraints.
- Small improvements in many help pages.
- Corrected error in names of array returned by extract.posteriorMeans.
- New parameter excludeRates in extract.posteriorMeans, plotPostMeansMDS.
- Use parameter pmonly also in plotPostMeansMDS.

- 2019-05-20 R-Forge Revision 340, package version 1.2-17.
   Changes in RSiena and RSienaTest:
  - New effects outAct.c, inAct.c, outPop.c, inPop.c, degPlus.c. These are centered versions, which might have better convergence properties than their namesakes without the .c.
  - Effects antiInIso and antiInIso2 implemented also for non-directed networks.
  - Effects outTrunc, outTrunc2, outSqInv, isolateNet, and degPlus got endowment effects.
  - For inPopIntn, outPopIntn, inActIntn, and outActIntn, added 'centered' to the statistic name.
  - plot.sienaGOF has a new parameter fontsize; the dots ... will also accept parameters
     cex, cex.main, cex.lab, cex.axis, as used more generally for plotting in R.
  - In case of simulation as indicated by x\$simOnly = TRUE, the object produced by sienaO7 now also contains the standard errors of the mean, called estMeans.sem (see the help page for sienaO7).
  - On various places diagnostic output messages were relabeled to 'message' or 'warning',
    which implies that they can be turned off by commands such as suppressMessages or
    suppressWarnings.

- In sienaBayes:
  - prevAns now is allowed to have a different specification of random effects than effects; it is not allowed to use prevAns and prevBayes objects simultaneously; some further small algorithm changes (see CHANGELOG).
- $\bullet~$  2019-02-26 R-Forge Revision 339, package version 1.2-16.

#### Changes in RSiena and RSienaTest:

- New effects maxAalt, minXalt (thanks to Per Block, Marion Hoffman, Isabel Raabe, and Kieran Mepham), outIsolate.
- Effect name of isolate effect (not its shortName) changed to in-isolate.
- New parameter thetaValues in siena07, allowing simulations in Phase 3 with varying parameters according to matrix input. Corresponding changes in print.sienaFit.
- sienaRI: earlier check for bipartite dependent variables; if so, stop.
- Argument verbose added to includeEffects and setEffect. (It existed already for includeInteraction.)
- Wald.RSiena, Multipar.RSiena and score. Test also produce one-sided test statistic if df
   1.
- objects produced by Wald.RSiena, Multipar.RSiena and score. Test have class siena Test.
   These now also have a print method.

#### Changes in RSienaTest:

- Allow arbitrary number of interactions in sienaBayes (via getEffects).
- simpleBayesTest and multipleBayesTest corrected; they were wrong for models with userdefined interactions.

- glueBayes extended (to allow good use for models with fixed parameters) and check for prior rates relaxed (to allow priorRatesFromData=2 leading to different prior rates.)
- New default prevBayes\$nwarm >= 1 for newProposalFromPrev in sienaBayes.
- siena.table for sienaBayes results extended with between-groups s.d.
- Argument verbose added to extract.posteriorMeans.
- sienaBayes: check that prevAns is a sienaFit object, and not sienaBayes; totally skip initial multi-group estimation if prevAns is given with ((prevAns\$n3 >= 500) & (!usePrevOnly)).
- 2019-01-15 R-Forge Revision 338, package version 1.2-15.

- New effects sameXReciAct and diffXReciAct.
- Increased burn-in for ML simulations in siena07.
- siena.table adapted for sienaBayes results.
- New functions shortBayesResults and plotPostMeansMDS.
- extract.posteriorMeans works also for an object saved as PartialBayesResult (which has NA results for as yet unfinished runs).
- Examples added to help page for print.sienaBayesFit (in dontrun...).
- 2018-12-05 R-Forge Revision 337, packages version 1.2-14.

#### Changes in RSiena and RSienaTest:

— In the help page of sienaAlgorithmCreate, the possibility is mentioned that the multiplication factor (parameter mult) can be specified as a vector. This allows the possibility for ML estimation to specify the multiplication factor separately for periods × waves.

#### Changes in RSienaTest:

#### – sienaBayes:

the possibility now exists to specify prior variances for non-varying parameters, which can be meaningful especially for effects of group-level variables; a new, hopefully better, initial value is used, defined as a precision weighted mean of the multi-group MoM estimate and the prior mean; number of subphases for initial global parameter estimation by MoM decreased to 2; prewarming phase introduced before the first <code>improveMH</code>; checks for dimensions of prior mean and covariance matrix put earlier in the initialization phase.

- extract.sienaBayes corrected.
- 2018-10-30 R-Forge Revision 336, packages version 1.2-13.

- Correct error in sienaGroupCreate for non-centered actor covariates.
- Correct error in print01Report that occurred for changing dyadic covariates given as lists of sparse matrices.
- Also get simulated dependent behavior variables for siena07(..., returnDeps = TRUE, ...) for ML estimation (see help page).
- siena.table corrected for data sets with several dependent variables.

- For siena08: new parameters which and useBound in plot.sienaMeta; new parameter reportEstimates, allowing to reduce the output produced.
- updateSpecification: also update randomEffects column.
- More explanation of sparse matrix input in the help page for sienaDependent.

sienaBayes: new parameters proposalFromPrev, which allows taking proposal distributions from prevBayes object;

incidentalBasicRates resuscitated;

allow fixed rate parameters, with priorRatesFromData = −1;

changed initial values of scale factors for proposal distributions;

new parameters target and usePrevOnly;

in case prevBayes is used, parameters nrunMHBatches, nSampVarying, nSampConst, and nSampRates in the function call of sienaBayes supersede those in the prevBayes object:

correct bug for three-effect interactions;

copy parameters modelType, behModelType, MaxDegree, Offset, initML, from parameter algo to the algorithms created within sienaBayes;

more extensive checking of smallest eigenvalue of covariance matrix;

for priorRatesFromData = 1 or 2, the resulting prior Covariance matrix for priorKappa != 1 was incorrect; this was corrected;

reported timing changed to elapsed system time.

- 2018-05-13 CRAN, RSiena version 1.2-12.
- 2018-05-06 R-Forge Revision 335, packages version 1.2-11.

Changes in RSiena and RSienaTest:

- New effects gwdspFF and gwdspFB.
- Effects simEgoInDist2 and simEgoInDist2W for two-mode networks were corrected.
   Perhaps simEgoDist2 and simEgoDist2W were broken in a previous version; if so, that was now corrected.
- Added sqrt version for reciAct, obtained for parameter = 2.
- Allowed the value parameter = NULL for setEffect and includeInteraction, meaning that no change is made for the internal effect parameter. For setEffect this is the new default, implying that when starting the default values from allEffects.csv are used, just like in includeEffects (where no parameter can be given).
- New auxiliary functions for sienaGOF: Triad Census and mixedTriadCensus (contribution by Christoph Stadtfeld).
- triad census from igraph added to help page of sienaGOF-auxiliary.
- improved signatable output for type = "html": added rules = none, frame = void to general options; changed minus to – ; added column for asterisks to have better alignment for estimates.
- Stop cluster also for a user interrupt in siena07.
- Extension of help page for siena07 by mentioning functions for accessing simulated networks for ML.

### Change in RSienaTest:

- extract.sienaBayes: error corrected that occurred if called with extracted = "all" but there are no varying, or no non-varying parameters.
- 2018-03-24 R-Forge Revision 334, packages version 1.2-10.

Change in RSiena and RSienaTest:

- Example in help file siena07 for accessing generated networks in case of ML estimation/simulation.
- 2018-03-21 R-Forge Revision 332, packages version 1.2-9.

Changes in RSiena and RSienaTest:

- Added effects avExposure, totExposure, infectDeg, susceptAvCovar, infectCovar for symmetric networks, in new effect group covarBehaviorSymmetricRate.
- New effects degAbsDiffX, degPosDiffX, degNegDiffX, degAbsContrX, degPosContrX, degNegContrX.
- Effects XWX, XWX1, and XWX2 enabled for bipartite networks, in new effect group dyadBipartiteObjective.
- Added parameter = 2 for FFDeg, BBDeg, FBDeg, FRDeg, BRDeg.
- Corrected altInDist2, totInDist2 for two-mode networks.
- Dropped some duplications in effectsDocumentation.
- Modification of effect outTrunc2. Perhaps this was superfluous.
- Export of last simulated state from ML simulations if returnDeps.
- Added endowment and creation effects for maxAlt and minAlt.
- Error messages for non-character nodeSets in functions sienaDependent, coCovar, varCovar.
- Improved way of handling missings for distance-2 network effects.
- Added components requestedEffects, theta, and se to siena08 (theta and se are the ML estimates).
- Error in summary of sienaMeta object corrected: for the estimated mean parameter
  the two-sided p was announced but a one-sided p was given. Also the ML results under
  normality assumptions were copied from print.sienaMeta to print.summary.sienaMeta.
- For non-directed networks, the initial model now contains only basic rates and degree effect.
- Added reference to DESCRIPTION and citation info in \inst, so citation("RSiena")
  now gives useful information.
- The dropping of the exclusion of effects if the variance of a covariate is 0, or a covariate has only two values, of version 1.1-306, was taken further (was incomplete).
- Message if attributes higher, disjoint, or atLeastOne are TRUE at the end of sienaDataCreate.

#### Changes in RSienaTest:

- new function extract.posteriorMeans for sienaBayes results.
- Restrict check in sienaBayes of maximum estimated parameter value after initialization to non-fixed effects.

- Correct construction of groupwise effects object in sienaBayes so that this will work also when evaluation effects for density and/or reciprocity are not included; and when there are interaction effects for which the main effects are not included.
- Corrections for print and summary of sienaBayes objects. In print.sienaBayesFit, include fixed parameters and give credibility intervals for rate parameters; include variance parameters; allow shorter ThinParameters; print objects returned through partialBayesResult.RData (by adding na.rm = TRUE to quantile).
- multipleBayesTest corrected (there was an error for testing 2 or more linear combinations simultaneously) and adapted for cases with fixed parameters; adapted help file text.
- All remaining parts of rsiena01gui removed.
- As a compensation of this, sienaDataCreateFromSession exported, and a more informative help page written.
- 2017-09-09 R-Forge Revision 318, packages version 1.2-4. Changes in RSiena and RSienaTest:
  - Longer Description field in DESCRIPTION, as per CRAN suggestions.
  - Correction of print.sienaFit, repairing the option include = FALSE in setEffect and includeEffects.

- Extra line in example for sienaBayes.Rd.
- 2017-09-08 R-Forge Revision 316, CRAN package version 1.2-3.

Changes in RSiena and RSienaTest:

- totAlt also for non-directed networks.
- Further corrections to make RSiena acceptable for CRAN.
- 2017-09-04 R-Forge Revision 312.

- Score test and sienaGOF corrected for the case that some parameters are fixed and not tested.
- New function score. Test. This allows, for a siena Fit object in which some effects were tested with test = TRUE, to get the results of the score-type test for some or all of the parameters tested. When some effects are tested with test = TRUE, the results of the score test are also presented when printing the estimation result.
- Argument varName added to updateTheta. This allows the update to be restricted to one or more of the dependent variables.
- Operation of option 'absorb' (behModelType = 2) corrected.
- In sienaAlgorithmCreate, for parameters Offset, MaxDegree, modelType, and behModelType, require that these are named vectors with the names of the dependent variables, or NULL; this is checked in initializeFRAN.
  - The use of non-named vectors for these, in the case of using multiple processes may have led to errors in earlier versions.
  - Note that the default for MaxDegree now is not MaxDegree = 0 but MaxDegree = NULL.
- New effects sameXInPopIntn, sameXOutPopIntn, sameXInActIntn, sameXOutActIntn.

- For networkModelType 3 (Initiative, AAGREE), an offset is added to the confirmation model; this is taken from Offset in the algorithm object.
- For effects inPopX, outPopX, inActX, outActX, sameXInPop, sameXOutAct, diffXInPop, diffXOutAct, homXInPop, homXOutAct, altXOutAct, diffXOutAct, changed interaction2 to '(was '1', erroneously) and default parameter to 1 (AllEffects.csv).
- Additional parameter dropRates in print.sienaEffects (useful for sienaBayes with many groups).
- In print.sienaEffects, omit last remark about random effects if only one line is printed.
- Corrected use of modeltype (initializeFRAN.r, CInterface.r),
- Change in print.sienaAlgorithm for modelType.
- Added an example for multiple processes in sienaGOF.Rd, and took out verbose reporting from sienaGOF in case of multiple processes.

- New parameter OffSet in sienaAlgorithmCreate.
- Registration of native routines (requirement for CRAN).

### Changes in RSienaTest:

- Parameter UniversalOffset of sienaAlgorithmCreate renamed to Offset.
- Added posterior variances to print.sienaBayesFit.
- Correction of error in sienaBayes that could lead to errors in results of estimations with data sets for multiple dependent variables (networks and/or behavior) with model specifications that contain an interaction effect for a dependent variable that is not the last of the dependent variables.
- Various changes in sienaBayes for less memory use and avoiding crashes in some situations.

## • 2017-05-08 Revision 306.

- New function updateSpecification to includes in an effects object a set of effects that are included in another effects object.
- New effects inPopX, outPopX, inActX, outActX, sameWXClosure, degPlus, absDiffX, avAltPop, totAltPop, egoPlusAltX, egoPlusAltSqX, egoRThresholdX, egoLThresholdX, altRThresholdX, altLThresholdX.
- outOutAss dropped for symmetric networks; only outInAss remains.
- egoX effect has interactionType = "ego" also for symmetric networks.
- The exclusion of effects if the variance of a covariate is 0, or a covariate has only two values, is dropped (these effects have no meaning, but their exclusion was a potential nuisance for meta-analyses).
- Description of gwespFB and gwespBF corrected.
- includeInteraction has an additional parameter random.
- siena08 now also accepts a list for ...

- Argument behModelType added to sienaAlgorithmCreate. For behModelType = 2, the
   'absorbing option' is chosen in the model for behavioral dependent variables; see Section 5.11 of this manual.
- Indication of effect parameters dropped in names for altInDist2 and totInDist2 (they have no effect parameters).
- ModelType now is specific to the dependent variable: given as a named integer vector in sienaAlgorithmCreate.
- sienaAlgorithmCreate, siena07: new option lessMem, reducing storage in siena07 by leaving out z\$ssc and z\$sf2 from the object produced; but these are used by sienaTimeTest and sienaGOF, so running those functions will be impossible for sienaFit object obtained with lessMem = TRUE.
- extended information in print.sienaAlgorithm.
- Modified check for singular covariance matrix after Phase 3.
- Warning if includeEffects is used with parameter random.
- Warning for impossible or zero changes if maximum likelihood (see Section 6.11, and News page of Siena website).
- Some parts dropped from the object produced by siena07 to reduce memory use.

- sienaBayes: various changes to save memory (thanks to Ruth Ripley); improved reporting of groups with no changes; warning for impossible changes, see Section 11.4.1; if priorRatesFromData = 2, change to different robust covariance matrix estimator when this is necessary (i.e., for small number of groups); in print.summary, also report nImproveMH; a few lines added to help file.
- print.sienaEffects gives dimensions of priorMu and priorSigma if includeRandoms.
- 2016-10-13, 14 Revision 303, 304.

Changes in RSiena and RSienaTest (George Vega Yon):

- New parallellization option -cl- for siena07.
- 2016-10-09 Revision 301.

- New effects: sameXCycle4, homCovNetNet, contrastCovNetNet, covNetNetIn, homCovNetNetIn, contrastCovNetNetIn, inPopIntnX, inActIntnX, outPopIntnX, outActIntnX.
- Changes permitting the 4-cycles effects for larger and denser networks.
- Dropped cl.XWX effect from two-mode one-mode coevolution (did not belong).
- egoSqX is an ego effect.
- Added cycle4 for one-mode networks.
- Added outAct, outInAss for symmetric networks.
- SienaRI: Structural zeros and ones are excluded from the calculations; added option getChangeStats; row names given to matrices that have rows corresponding to effects; adapted so that it runs for models with only 1 parameter; adapted so that for a bipartite dependent variable it does not crash.

- Warning if includeEffects is used with parameter parameter.
- Small additions to print.sienaAlgorithm.
- Clearer output for MaxDegree in print.sienaFit.
- Correction of how effect parameter for outInAss for 2-mode networks is reported.

#### • 2016-08-17 Revision 296.

Changes in RSiena and RSienaTest:

- Warning if includeInteraction is used for more interactions than available given parameters nintn and behNintn.
- Deleted session parameter from print01Report.
- Corrected cycle4 effect for parameter = 2 (sqrt version).
- Additional auxiliary function CliqueCensus in help page sienaGOF-auxiliary.
- Error corrected in DyadicCovariateAvAltEffect.cpp.

#### • 2016-05-28 Revision 294.

Changes in RSiena and RSienaTest:

- The manual was taken out of the installation; it still is in the source code distribution as RSienaTest\doc\RSiena\_Manual.tex. It still is publicly it can be downloaded from http://www.stats.ox.ac.uk/~snijders/siena/RSiena\_Manual.pdf.
- New effects totAltEgoX, totAltAltX, egoSqX, diffX, diffSqX, egoDiffX, avAltW, totAltW, avSimW, totSimW, jumpFrom, jumpSharedIn, mixedInXW, mixedInWX, avWalt, totWAlt.
- $-\,$  in ActIntn also implemented for two-mode dependent networks.
- Endowment and creation effects were added for inAct, inActSqrt, outAct, outActSqrt.

#### Changes in RSiena:

- (siena01Gui) and sienaDataCreateFromSession dropped.

## Changes in RSienaTest:

- Change in endowment effect estimation for avalt effect.
- New effects simmelian, simmelianAltX, avSimmelianAlt, totSimmelianAlt.
- 2016-02-03 Revision 291.

Identical to 290, but now there is a Mac version.

• 2016-02-01 Revision 290.

- New effects FBDeg, FRDeg, BRDeg (RFDeg was mentioned earlier but was not implemented; its place is now taken by FRDeg), gwespFFMix, gwespBBMix, gwespBFMix, gwespFBMix, gwespRRMix, gwespMix.
- Corrected the omission of the check for positive derivative matrix at the end of Phase 1 for effects with fixed parameters.
- Added parameters fix, test, parameter to includeInteraction.

- More helpful error message for incorrect nodesets in sienaDataCreate; extended help pages for sienaDataCreate and sienaDependent.
- Correction to allow estimation for a one-dimensional parameter.
- fromBayes bug corrected.

- sienacpp added (programmed by Felix Schönenberger); this includes gmm estimation (Amati - Snijders).
- sienaBayes: option initML added; check for zero distances; corrected function improveMH in initialization.
- print.multipleBayesTest: option descriptives added.
- glueBayes: added p1 and p2 to created object.

### • 2015-09-10 Revision 289.

#### Changes in RSiena and RSienaTest:

- New defaults for siena07 in sienaAlgorithmCreate: doubleAveraging = 0, diagonalize = 0.2 (for MoM).
- Improved one-step approximations to expected Mahalanobis distances in sienaGOF (control variates for score function).
- Permit 3-way interactions with one ego and two dyadic effects (initializeFRAN.r) (this was erroneously not allowed).
- New effects Jin, Jout, JinMix, JoutMix, altXOutAct, doubleInPop, doubleOutAct.
- print01Report now reports in-degrees also for two-mode networks.
- Better error handling for sienaTimeTest and scoreTest.
- inOutAss is dyadic.
- Corrected effectName and functionName of inPopIntn, outPopIntn, inActIntn, and outActIntn ('in' and 'out' were missing).
- Check for positive derivative matrix at the end of Phase 1 (non-positive estimated derivatives lead to repeating a prolonged Phase 1) omitted for effects with fixed parameters.

## Changes in RSiena:

- New effects homXOutAct, FFDeg, BBDeg, RFDeg, diffXTransTrip (ported from RSiena-Test).
- sameXInPop and diffXInPop also added for two-mode networks; but they are not dyadic!
- In names of behavior effects and statistics dropped the (redundant) parts "behavior" and "beh.".

## Changes in RSienaTest:

- New function extract.sienaBayes.
- sienaBayes: options diagonalize = 0.2, doubleAveraging = 0 for estimation of initial models in initialization phase; save initial results in case of divergence during initialization phase; check for large initialEstimates done only for non-rate parameters.

• 2015-07-18 Revision 288.

Changes in RSiena and RSienaTest:

- plot.sienaRI: new parameter actors; proportions with piechart improved (hopefully);
   effect of parameter radius changed.
- siena.table does no more produce the double minus sign in html output.

#### Changes in RSiena:

Correction of error for two-mode networks in sienaGOF.

#### Changes in RSienaTest:

- New effects homXOutAct, FFDeg, BBDeg, RFDeg, diffXTransTrip.
- sameXInPop and diffXInPop also added for two-mode networks; but they are not dyadic!
- In names of behavior effects and statistics dropped the (redundant) parts behavior and beh..
- sienaBayes: new parameter nSampRates; correction in use of prevBayes; more efficient calculation of multivariate normal density.
- Small changes in HowToCommit.tex.
- 2015-05-21 and 2015-06-02 Revision 286.

Changes in RSiena and RSienaTest:

- SienaRIDynamics dropped from RSiena, it still has an error (retained in RSienaTest).

## Changes in RSienaTest:

- Correction of error for two-mode networks in sienaGOF.
- 2015-05-20 Revision 285.

- tmax added to signafit objects and tconv.max mentioned in print.signafit.
- sienaAlgorithmCreate has new arguments n2start, truncation, doubleAveraging, standardizeVar; this leads to various changes in phase2.r.
- Diagonalization corrected (matrix transpose) (phase2.r).
- When there are missings in constant or changing monadic covariates, and centered=FALSE for their creation by coCovar or varCovar, the mean will be imputed (used to be 0, which was an error). For changing covariates, this is the global mean.
- In coCovar and varCovar there is a new argument imputationValues, which are used (if given) for imputation of missing values. Like all missings, they are not used for the calculation of the target statistics in the Method of Moments.
- New effects: outOutActIntn, toDist2, from.w.ind.
- In the target statistic for the **higher** effect, contributions for value(ego)=value(alter) are now set appropriately at 0.5 (was 0).
- New effectGroup tripleNetworkObjective (allEffects.csv, effects.r) (see earlier in this manual for its characteristics).

- Decent error message when there are (almost) all NA in the dependent behavioural variable (effects.r, function getBehaviorStartingVals).
- The centering within effects for similarity variables at distance 2 now is done by the same similarity means as for the simX effect (attr(mydata\$cCovars\$mycov, "simMean"), etc.).

#### • 2015-04-02 Revision 284.

Changes in RSiena and RSienaTest:

- New effects: simEgoDist2, simEgoInDist2, simEgoInDist2W, simEgoInDist2W, sameXOutAct, diffXInPop, diffXOutAct. The centering for the similarity measure in effects such as simEgoDist2 and simDist2 is not yet clear (this affects only the outdegree parameter).
- Relevant for creating new effects: New effect groups covarABNetNetObjective, covarANetNetObjective, and covarBNetNetObjective. See SienaSpec.tex/pdf, section 4.9: covarNetNet.
- Bug corrected that occurred in print01Report for a sienaGroup object where the component objects have constant dyadic covariates.
- When a statistic is not plotted in plot.sienaGOF because its variance is 0, a note about this is printed to the screen.
- Minimum and maximum of plotted region in plot.sienaGOF is calculated without taking into account non-plotted statistics.
- Bug corrected with includeTimeDummy for timeDummy greater than or equal to 10 (sienaTimeTest.r).
- In case of collinear parameter estimates, standard errors are reported as NA.
- Arguments main and ylab dropped from plot.sienaGOF; they did not work, and their functionality now is covered by the ... argument (so using main and ylab as arguments now should work). (Thanks to David Kavaler.)

### Changes in RSienaTest:

sienaBayes: correction in initialization of truncation rate parameters based on prior;
 error corrected for sampling constant parameters.

## Changes in RSiena:

- Parameter reduceg added in sienaAlgorithmCreate for use in siena07, like in RSienaTest.
- 2014-12-11 R-Forge Revision 282.

- Effects cl.XWX and cl2.XWX corrected (thanks to Christoph Stadtfeld).
- interactionType of gwesp.. effects was made dyadic.
- Some layout changes in warning message in siena07.
- New effects reciPop, reciAct, in3Plus, maxAlt, minAlt, transTrip1, transTrip2.
- Effect antiInIsolate2 got alias in2Plus.
- inPop is dyadic effect (except for non-directed networks) (it is  $\sum_i x_{ij} \{ \sum_{h \neq i} x_{hj} + 1 \}$ ).

- egoX added as effect for non-directed networks (can be important for representing effects of group-level covariates in multi-group analyses).
- Components IActors and expected added to signal and print.signaRl.
- The check for MaxDegree when running siena07 now works properly also for sienaGroup objects.
- Manual introduces the term elementary effects.

- sienaBayes: the stop caused by singularity of the precision matrix after the multi-group estimation now is circumvented; still a warning is printed to the screen.
- sienaBayes: option priorRatesFromData changed to values 0-1-2, with 0 = former FALSE, 1 = former TRUE, 2 = robust estimation of prior for rate parameters from estimates at the end of initialization phase.
- Correction of print.summary.sienaBayesFit for models with more than one dependent variable.
- 2014-11-19 R-Forge Revision 280. Changes in RSienaTest:
  - Small changes in help pages for sienaGOF and for sienaCompositionChange.
  - New parameters nSampVarying and nSampConst in sienaBayes.
- 2014-11-13 R-Forge Revision 279.

- Effect AltsAvAlt renamed to avXAlt.
- Effects object no longer used as argument for print01Report.
- A lot of new effects: sameXInPop, transRecTrip2, totAlt, avInAlt, totInAlt, totRecAlt, totXAlt, avXInAlt, totXInAlt, avAltDist2, totAltDist2, avTAltDist2, totAAltDist2, avXAltDist, totXAltDist2, avTXAltDist2, totAXAltDist2, avInAltDist2, totInAltDist2, avTInAltDist2, totAInAltDist2, avXInAltDist2, totXInAltDist2, avTXInAltDist2, totAXInAltDist2, XWX1, XWX2, cl.XWX1, cl.XWX2.
- Endowment and creation effects added for gwesp... effects.
- Some meaningless effects for two-mode networks dropped.
- For non-invertible covariance matrices at the end of siena07, give diagnostic for the linear combination that gives trouble.
- Correction of igraphNetworkExtraction in the help page for sienaGOF-auxiliary (the earlier version dropped isolated nodes from simulated networks).
- In the help page for sienaGOF-auxiliary.Rd, the example of constraint is replaced by the example of eigenvector centrality (because constraint is undefined for isolated nodes, leading to computational problems).
- Set diagonal of observed networks to 0 in sparseMatrixExtraction.
- sienaRIDynamics restored, after corrections.
- 'file" parameter of sienaRI dropped (implied platform dependence).
- Section in manual about user-defined interaction effects updated.
- Parameter showAll added to descriptives.sienaGOF.

- Small correction of print.siena (reporting uponly/downonly).
- Some changes in print.sienaAlgorithm.
- Check in siena07 for incorrect MaxDegree specification.
- Correction of printing errors arising when result of score-type test is NA.
- maxRatio checked for NA or NaN in phase2.r.
- Siena\_algorithms4.tex renamed Siena\_algorithms.tex; this document now is made available as a pdf file at the SIENA website.
- Some improvement of error messages for sienaTimeTest.
- p-value for goodness of fit (sienaGOF) rounded to 3 decimal places.
- File effects.pdf dropped from \inst\doc (it can be created by effectsDocumentation).

- sienaBayes: new parameters nImproveMH and priorRatesFromData; these give the possibility to truncate initial rate parameters depending on prior.
- glueBayes corrected so that it can be applied sequentially.
- multipleBayesTest now allows matrix parameter to test linear combinations.
- Improved plot.multipleBayesTest (to show truncation at 0).

#### • 2014-07-08 Revision 278.

#### Changes in RSiena and RSienaTest:

- Added s50s to data set.
- Corrected se component of sienaFit objects (should be standard error, was its square).
- new effects totDist2, altInDist2, totInDist2, totDist2W, altInDist2W, totInDist2W.
- Some warnings for calculations of z\$regrCor and z\$regrCoef avoided in siena07.
- print01Report errors corrected, and slightly improved, for descriptives for changing dyadic covariates and for upOnly / downOnly cases.

#### Changes in RSienaTest:

- sienaBayes: Internally multiplied the data-dependent choice of priorSigma for rate parameters by priorKappa; changed z\$nwarm to 0 if prevBayes is used; dropped plotit functionality.
- sienaBayes, glueBayes, and print.sienaBayes: adapted to allow inclusion of interaction effects without the corresponding main effects.
- Added parameter nwarm2 to glueBayes. Checks of identical prior parameters in this function restricted to non-rate parameters.

## • 2014-06-22 R-Forge Revision 277.

## Changes in ${\sf RSiena}$ and ${\sf RSienaTest:}$

- Higher write-to-screen frequency for batch operation of siena07.
- Function include Effects now includes parameters fix and test.
- Various small bug corrections (see the ChangeLog, 1.1-275).

- Changes in sienaBayes and its print and summary methods.
- Corrected starting printing sienaBayesFit at nfirst.
- New function glueBayes for combining sienaBayesFit objects.
- Added functions simpleBayesTest and multipleBayesTest.
- 2014-04-26 R-Forge Revision 274.

Changes in RSiena and RSienaTest:

- Correction of effect homWXClosure.
- Small change in print01Report to improve reporting two files of composition change.
- sienaRI: require that file argument is not NULL for non-Windows operating systems.
- 2014-04-13 R-Forge Revisions 267-271.

Changes in RSienaTest:

Updates to let sienaBayes accept a wider range of data and models (e.g., user-defined interactions); and various corrections to sienaBayes.

### Changes in RSiena and RSienaTest:

- Added function sienaRI for assessment of relative importance of effects, with print and plot methods.
- In coDyadCovar and varDyadCovar, centering now also is optional by the new option centered (like it was done for coCovar and varCovar in revision 1.1-251).
- Corrected bug when printing siena object with a symmetric network; and in varDyad-Covar corrected a bug occurring when calling it with a named list.
- Added standard errors as component se to sienaFit objects.
- Internal changes in the code version 1.1-255 to 1.1-267 (see ChangeLog).
- No noticeable changes from version 1.1-251 to 1.1-254.
- 2014-02-13 R-Forge Revision 251

It should be noted that two changes were made that potentially have an influence on some results obtained.

First, the effects gwespFF, gwespBB, gwespFB, gwespBF, gwespRR were modified in RSiena-Test to bring them in accordance with the literature. This means that the 'old' parameter  $\alpha'$  was effectively replaced by  $\alpha = -\log\left(1 - \exp(\alpha')\right)$ ; here  $\alpha$  is the internal effect parameter divided by 100. For the default  $\alpha = \log(2)$  this means no difference.

Second, in the help page for sienaGOF-auxiliary, geodesic distances were changed to non-directed. This makes more sense usually and was done to avoid runtime errors that occurred very rarely.

- New effects cl.XWX, homXTransTrip, homWXClosure, and sharedPop.
- Effect cycle4 extended to non-directed one-mode networks (for directed one-mode networks this is sharedPop).

- Effect jumpXTransTrip ported to non-directed networks.
- gwesp.. effects modified and ported to non-directed networks.
- Also take account of behavior user-specified interactions in includeInteraction and set-Fffect
- Correction: Effect to is not a dyadic effect.
- Manual: added paragraph about how to import results from xtable and siena.table into MS-Word.
- sienaGOF: added the name of the sienaFit object as attribute sienaFitName to each of the sienaGofTest objects.
- Correction in sparseNetworkExtraction to avoid errors occurring when the extracted network has no edges.
- In the help page for sienaGOF-auxiliary, geodesic distances changed to non-directed;
   which avoids a further error when the extracted network has no edges.
- Correction of an error in print.siena for data sets including other types than oneMode.
- Changed bandwidth selector for violin plots in plot.sienaGOF to 'nrd", to avoid long violins in cases where all simulations have the same outcome.

Further work on SienaBayes.

#### Changes in RSiena:

- Ported effects outRateLog and outTrunc2 from RSienaTest.
- 2013-12-04 R-Forge Revision 250

Changes in RSiena and RSienaTest:

- New option centered in coCovar and varCovar.
- setEffect, updateTheta, and prevAns in siena07 now also work properly for user-specified interactions.
- Tests Wald.RSiena and Multipar.RSiena added.
- Error occurrence with message about cvalue in EvaluateTestStatistic corrected.
- Divergent parameters in siena07 get NA for their rows and columns in the resulting covariance matrix.

The following changes in revision 244 were ported from RSienaTest to RSiena:

- In siena08, also report Bonferroni combination of the two Fisher combinations.
- In siena07, rolled back change in truncation from version 1.1-227 to the earlier procedure.
- descriptives.sienaGOF added.
- Minor changes of output in siena.table, print.siena, siena07, and in error message for includeEffects.
- Change artificial results from 999 to NA in siena07.
- For ML estimation: added autocorrelations during phase 3 to print.summary.sienaFit.

• 2013-10-31 R-Forge Revision 246

Changes in RSiena and RSienaTest:

- New behavior objective function effects avSimAltX, totSimAltX and avAltAltX to differentiate sources of peer influence in directed networks.
- Added effect class covarBehaviorNetObjective to effectsDocumentation.R.
- Fix of a bug that occurred in the case of on average decreasing behavior variables.
- 2013-16-09 R-Forge Revision 245

Changes in RSienaTest:

- New structural rate effect outRateLog.
- Duplication of outTrunc effect: outTrunc2, allowing use with two different parameters.
- In siena08, also report Bonferroni combination of the two Fisher combinations.
- In phase 2 of siena 07, rolled back change in truncation from version 1.1-227 to the earlier procedure.
- Added function descriptives.sienaGOF with numerical results of plot.sienaGOF.
- Minor changes of output in siena.table and various reports, and in error message for includeEffects.
- Change artificial ('missing') results of siena07 from 999 to NA.
- Added autocorrelations during phase 3 to print.summary.sienaFit for ML estimation.
- Start of the manual reorganized and partially rewritten (with help from Zsófia Boda and András Vörös); instructions for siena01Gui separated in siena01gui.pdf.
- 2013-10-09 R-Forge Revision 244

Changes in RSienaTest:

- Repair bug that prevented compilation for Mac.
- 2013-09-17 R-forge revision 243

Changes in  ${\sf RSiena}$  and  ${\sf RSienaTest:}$ 

- Correct bug in EffectFactory for isolatePop effect.
- Improved plotting of sienaGOF objects so that observed values outside of the range of simulated values don't run off the chart.
- Improve treatment of structural values in sienaGOF.

Changes in RSiena:

- Add functions AntilsolateEffect.h and AntilsolateEffect.cpp which were forgotten to include in revision 242.
- 2013-08-27 R-forge revision 242

Changes in RSiena as well as RSienaTest:

 Correction to Dolby option for the case of more than 2 waves: in phase1.r and phase3.r, scores are added (instead of averaged) over waves. (Averaging was wrong, because in phase 2 they are added.)

- New effects: anti isolates, anti in-isolates, and anti in-near-isolates.
- Effect inIsolatePop dropped (it was shortlived).
- Improved printing of results of siena07 in the case simOnly.
- Prettier response printed to console for includeEffects and setEffect.
- z\$estMeans added to sienaFit objects z: vector of estimated expected values of statistics; this is colMeans(z\$sf) + z\$targets but if dolby, the regression on the scores is subtracted.
- 2013-08-23 R-forge revision 241

Changes in RSiena as well as RSienaTest:

- Corrected bug (leading to error message) that occurred if there was only one option
  for choice of alter. It appeared mainly in cases where all changes were upward only
  (or downward only); in practice, it only was observed yet for two-mode networks with
  upward changes only.
- Drop the unintended multiplication of the target statistic for the inPop effect by n.
- Trapped some execution errors (mainly associated with inversion of singular matrices) and allowed the functions to end properly with a warning message.
- Added various degree-related effects to bipartite networks.
- New effect inIsolatePop.
- 2013-08-08 R-forge revision 240

Changes in RSienaTest:

- Added the parameter reduceg to siena07.

Changes in RSiena and RSienaTest:

- Added effects crprod and inPopIntn for two-mode networks.
- 2013-06-18 R-forge revision 232

Changes in RSiena:

- The possibility to use the obsolete packages snow and recuyer for R versions older than 2.14.0 was dropped; their functionality was replaced by package parallel.
- The DESCRIPTION file was corrected to satisfy CRAN requirements.
- 2013-06-15 R-forge revision 231

Changes in RSiena as well as RSienaTest:

- Make the 'cumulative' option operational in BehaviorDistribution for sienaGOF.
- Correct bug in treatment of missing values in sparseMatrixExtraction for sienaGOF.
- Allow sparse observed data matrices, and structural zeros and ones, in sparseMatrix-Extraction and networkExtraction, and bipartite networks in networkExtraction for sienaGOF.
- Report correct centering (by overall means) of individual covariates for multi-group objects in print01Report.
- If there is a composition change object, MoM estimation is forced to be non-conditional.
   This is reported in the help file for sienaCompositionChange.

• 2013-05-10 R-forge revision 230

Changes in RSiena as well as RSienaTest:

- Check whether the maximum observed degree is not higher than maxDegree.
- Fix error in implementation of maxDegree.
- Fix bug in print.siena and extend print.siena.
- Make print method for class sienaDependent.
- 2013-04-19 R-forge revision 227

Changes in RSiena as well as RSienaTest; both now are very similar; sienaBayes, algorithms, and profileLikelihoods are the only functions in RSienaTest not in RSiena. Available effects now are the same in both packages.

Main changes visible to users:

For Siena only:

- function bayes was removed (still under development in RSienaTest).
- Attributes allowOnly and simOnly ported from RSienaTest.
- Improved error messages in includeEffects ported from RSienaTest.
- sienaGOF ported from RSienaTest.
- siena.table ported from RSienaTest.

### For RSienaTest only:

- bayes renamed to signa Bayes and considerably changed, with print option.

#### For RSiena and RSiena Test:

- Changes to sienaGOF: new use structure with extraction functions sparseMatrixExtraction, networkExtraction, behaviorExtraction, allowing the testing of any dependent variable; commented out some superfluous lines.
- The function sienaModelCreate is now called sienaAlgorithmCreate, but the earlier name is still retained as an alias; the class name of the object created by this function is now called sienaAlgorithm.
- The function sienaNet is now called sienaDependent, but the earlier name is still retained as an alias; the class name of the object created by this function is now siena Dependent.
- The function effectsDocumentation now has an extra argument effects; if this points to an effects object, all available effects in this effects object are listed with shortName, with a variety of other often used characteristics.
- Added effects (some existed already in RSienaTest): average exposure effect on rate xxxxxx, avExposure susceptibility to av. exp. by indegree effect on rate xxxxxx, susceptAvIn total exposure effect on rate xxxxxx, totExposure, infection by indegree effect on rate xxxxxx, infectIn, infection by outdegree effect on rate xxxxxx, infectOut, susceptibility to av. exp. by zzzzzz effect on rate xxxxxx, susceptAvCovar infection by zzzzzz effect on rate xxxxxx, infectCovar, WW=>X cyclic closure of xxxxxx, cyWWX

```
WW=>X shared incoming xxxxxx, InWWX
WW=>X shared outgoing xxxxxx, OutWWX
xxxxxx alter at distance 2 (#), altDist2
xxxxxx similarity at distance 2, simDist2
transitive triplets xxxxxx similarity, simXTransTrip
transitive triplets same xxxxxx, sameXTransTrip
transitive triplets jumping xxxxxx, jumpXTransTrip
transitive reciprocated triplets, transRecTrip
GWESP I - > K - > J (#), gwespFF
GWESP I < - K < - J (#), gwespBB
GWESP I < - K - > J (#), gwespFB
GWESP I -> K < - J (#), gwespBF
GWESP I \ll K \ll J (#), gwespRR
isolate - popularity, isolatePop
in-isolate Outdegree, inIsDegree
network-isolate, isolateNet
outdegree \wedge (1/\#) xxxxxx popularity, outPopIntn
closure jumping yyyyyy, jumpWWClosure
mixed xxxxxx closure jumping yyyyyy, jumpWXClosure
cyclic closure of xxxxxx, cyClosure
shared incoming xxxxxx, sharedIn
```

- Outdegree-popularity effect: multiplication by n dropped.
- GWESP effects: default parameter changed from 25 to 69 (corresponding to  $\alpha = \log(2)$ .) See earlier in this manual.
- Added to siena07 (defined by sienaAlgorithmCreate): option Dolby for variance reduction. Correlations between scores and statistics are reported in output file; this is a measure for the amount of variance reduction.
- Added to siena07: option diagonalize for having more possibilities for tuning the algorithm (extent of diagonalization of matrix D in Robbins-Monro update).
- sienaTimeTest updated; now also contains effect-wise tests, groupwise tests (for group objects), automatic exclusion of collinear effects, and has prettier output and improved summary.
- Overall maximum convergence ratio, x\$tconv.max (maximum value of t-ratio for convergence, for any linear combination) added to result of siena07.
   This is a very severe convergence criterion, and not meant as a default criterion to judge convergence in practical cases.
- The printmethod for objects of class siena (created by sienaDataCreate) has been extended with printing uponly and downly attributes, if these are TRUE.
- A bug in the starting values for two-mode networks was corrected.
- Small bug fixed in print01Report for reporting of uponly and downonly, in the case where this does not affect all periods.
- Changed almost all .Rd documentation files: sometimes to make them better understandable or complete, sometimes to make more appropriate examples, sometimes only minor prettifications.

- Updated scripts:
   Rscript01DataFormat.R, Rscript02VariableFormat.R, Rscript03SienaRunModel.R,
   Rscript04SienaBehaviour.R. (of RSienaDescriptives only the date was changed.)
- 2012-12-24 R-forge revision 222
  - Changed example on sienaNet help page to stop it masking data files in the package.
  - Example on sienaGOF page now runs. (RSienaTest only)
  - profileLikelihood (RSienaTest only) returns its object invisibly (i.e. it does not print when not assigned but can be assigned).
- 2012-12-23 R-forge revision 221,RSienaand RSienaTest: changed version check to cope with R version 3.0.0.
- 2012-07-05 R-forge revision 219, for RSienaTest only:
  - Further changes to bayes.
  - Additional effects connected with triadic closure interacting with covariates.
  - Some networks from Chris Baerveldt's data set added as data objects (N34\* and HN34\*).
- 2012-06-11 R-forge revision 217, for RSienaTest only:
  - Preliminary updates to sienaGOF to put more power in the hands of the user. A user may now extract more than one dependent variable from the dataset.
- 2012-06-07 R-forge revision 216, for RSienaTest only:
  - New effects connected with isolates; and with mixed WWX triadic closure (in various patterns) for dyadic covariates as well as multiple dependent networks.
  - Modifications to bayes (seems to run OK now, except that multiple groups option does not work for dyadic covariates; still not documented for general use).
- 2012-05-18 R-forge revision 213, for RSienaTest only:
  - Allow observed networks to have density 0 or 1 (not that it is generally advisable to use such data sets).
  - Incorporated argument simOnly in sienaModelCreate to facilitate simulation without estimation.
  - Incorporated argument allowOnly in sienaNet to permit ignoring monotonicity in data and its consequences for upOnly and downOnly.
  - Some new effects: interactions between reciprocity and transitivity.
- 2012-03-29 R-forge revision 211 Fixed bug in effectsDocumentation.
- 2012-03-29 R-forge revision 210
   Altered ML code in hope of fixing intermittent ML bug. Just might cause different answers.
- $\bullet~2012\text{-}03\text{-}25$  R-forge revision 208
  - fix bug in bipartite network endowment and creation effect scores.

- Rationalise behavior/network effects for symmetric networks in allEffects.csv, effects.r (partially).
- Fix bug which caused crash creating starting values for sparse matrices with movements only in one direction.
- 2012-03-16, 2012-03-21, R-forge revisions 206/207: new behavior rate effects.
- 2012-03-07 R-forge revision 205
  - Bug fix for effects AvSimEgoX, totSimEgoX, avAltEgoX with changing covariates.
  - Minor alterations to altDist2, simDist2 and the multi network versions.
  - Bug fix in probability in chain for symmetric networks type b models.
- 2012-02-29 R-forge revision 204
  - Fixed bugs in endowment and creation effect statistics for behavior Similarity effects.
  - Fixed bug causing occasional failure in bayes routine
  - Fixed bug causing occasional failure in maximum likelihood with constraints.
  - Added error message if try to use maximum likelihood with composition change.
  - Fixed bug in endowment and creation effect score calculation for symmetric network pairwise models.
  - File cluster.out is now removed before recreation.
  - Meta analysis summary now does not contain a list of NULLs at the end.
  - Minor changes to print and messages formats.
- 2012-02-19 R-forge revision 203
  - Fixed minor bug in ML initialisation: will alter results slightly.
  - New check for updated version when using multiple processes.
  - Amended code in manual for making sparse matrices.
- 2012-02-07 R-forge revision 200
  - Bug fix to scores for behavior variable rate effects.
  - siena07 now stops if cannot get a derivative matrix in phase 1.
- 2012-01-29 R-forge revision 198 Fix bug in initializing rate parameters for bipartite and behavior variables in maximum likelihood. Will change results slightly.
- 2012-01-29 R-forge revision 197
  - Fix bug in effFrom with changing covariates. The targets depended on the compiler.
  - Fix bug in creation effects in Maximum likelihood.
- 2012-01-20 R-forge revision 195
  - NaN's in covariates were causing problems: now treated as though NA in C++, as they always were in R.
  - New file 'arclistdata.dat' added to examples directory
  - New maintainers address: rsiena@stats.ox.ac.uk

- Print method for sienaFit objects now includes values of fixed parameters, rather than NA.
- 2012-01-17 R-forge revision 194
  - fix to prtOutMat to stop crash with null matrix
  - relaxed restrictions on behavior interactions in line with the manual
  - changes to validation of bipartite networks: should now be consistent
- 2012-01-17 R-forge revision 192.
  - minor but extensive changes to manual
  - minor changes to scripts
- 2011-12-15 R-forge revision 191. Some of these may alter results slightly.
  - Altered calculations of probabilities to avoid overflows
  - Fixed bug in storage of MII in bayes
  - Removed endowment effect for IndTies for symmetric networks.
  - Removed code for storing change contributions on ministeps: not functioning
  - Set random number type to 'default' at start of siena 07.
- 2011-12-14 R-forge revision 190 Fixed bug in Bayes left over from R 189.
- 2011-12-14 R-forge revision 189 Fixed minor bugs in reports and error messages.
- 2011-12-04 R-forge revision 186 Fixed some bugs in ML estimation procedure which will alter the results slightly. Added algorithm functions to RSienaTest package.
- 2011-11-27 R-forge revision 185
  - Bayes and algorithm code now uses parallel package
  - Fixed memory leaks in ML estimation. Less space needed!
  - Other minor changes to ML with missing values (still incomplete)
- 2011-11-14 R-forge revision 184: Fix memory leaks in calculation of rate statistics.
- 2011-11-11 R-forge revision 183:
  - Fix bug stopping interruption in phases 1 and 3 (since recent change)
  - Check whether dfra from maxlike or not when using prevAns
- 2011-11-11 R-forge revision 182:
  - fix bug in ML/Bayes returning acceptances.
  - fix bug in sienaTimeFix with multi groups and differing actor set sizes
- 2011-11-04 R-forge revision 181: reset random number type after using parallel package.
- 2011-10-28 R-forge revision 179: fix bug in forking processes
- 2011-10-27 R-forge revision 177/8:
  - Change to covariance matrix for effects which have been fixed

- Added new package for parallel running to be used from R 2.14.0. New option to use forking processes on non-Windows platforms.
- Changes from revision 175 copied to RSiena
- Updates to maximum likelihood estimation: NB this is still under development, and should not be used with missing data.
- Added bayes, updateTheta functions to RSiena
- sienaTimeTest for finite differences or ML now in RSiena
- Space saving matrices used for derivatives in RSiena now, and optional by wave in ML.
- 2011-10-14 R-forge revision 176 (RSienaTest only) bug fix in diffusion effects, altered scripts in manual a little
- 2011-10-06 R-forge revision 175
  - Fix bug with multiple symmetric networks.
  - Limit constraints to be between both symmetric or non-symmetric networks
  - Added scripts to package (RSienaTest only)
  - siena07 called with batch = FALSE no longer crashes if called on mac or linux with no X11 available. (RSienaTest only)
- 2011-09-19 R-forge revision 172: (RSienaTest only) Diffusion rate effects.
- 2011-09-07 R-forge revision 171
  - Fix bug in siena08: crashed if underlying effects for interaction were not selected. (Or possibly with time dummies!).
  - Fix bug where print from siena08 was not produced if a previous display to the screen had occurred.
  - New parameter in siena08 to control number of iterations.
  - RSienaTest only: added validation to updateTheta
- 2011-08-08 RSienaTest only R-forge revision 168/9
  - More work on maximum likelihood.
  - When using finite difference derivative estimation or maxmimum likelihood estimation, return of derivatives by wave is optional, controlled by parameter by Wave to siena07.
  - Format of derivatives by wave has altered: sienaTimeTest will be incompatible with older objects which used finite differences or maximum likelihood.
  - New function updateTheta to copy theta values from a fit to an effects object.
  - Time dummies in siena07 are created before the initial values are updated from any prevAns, so values may be copied to time dummies also.
  - Amended headings in print and summary for siena fit objects.
  - New function bayes is now fully available with a help page and no need to use RSiena::: when calling it.
- 2011-08-03 R-forge revision 167
  - Fix another display of manual in siena01Gui

- Added network names to relevant behavior effects if there is more than one network.
- Altered names of interaction effects to remove duplicate network names
- Renamed avSimX, totSimX, avAltX to avSimEgoX, totSimEgoX, avAltEgoX
- Trapped error caused by omitting Actors node set when specifying others.
- 2011-07-27 R-forge revision 164:
  - Include quadratic shape effect by default unless range is less than 2.
  - Shorten behavior interaction effectnames by removing the repeated variable name.
  - Fix bug when displaying manual from siena01Gui.
- 2011-07-23 R-forge revision 163: Fix bug in effectsDocumentation, reduce memory size needed for non-ML, non-finite-difference models.
- 2011-07-02 R-forge revision 161:
  - Fix problem with bayes routine with single data object only.
  - Fix problems with getRSienaRDocumentation: internal functions within internal functions now work (but still not automatically) and function now runs on non-Windows too.
- 2011-06-24. R-forge revision 160: behavior endowment effects are now all defined consistently as current value less previous one, as in the manual.
- 2011-06-22, 2011-06-23. R-forge revision 158/159: behavior interactions. Minor bug fixes to correct effects object and inclusion of non-requested underlying effects. Replace influence interaction effects by the three options. Time dummies for behavior effects.
- 2011-06-18 R-forge revision 157: Fixed minor bug in siena07: code controlling maximum size of move was incorrect if using prevAns for an exactly equivalent fit.
- 2011-06-13 R-forge revision 156: fixed bug removing density effect for bipartite networks with some only waves up or down only.
- 2011-06-12 R-forge revision 155:
  - Fixed bug with behavior variables with values 10 or 11: the 10th value in the matrix had 10 subtracted from it.
  - Fix for short name for egoXaltX effect for non-directed networks. Now matches the name for directed networks
- 2011-06-04 R-forge revision 153:
  - Maximum likelihood: this is still under development. Correction for bipartite networks and networks with constraints. Variable length of permutations will change results (slightly) compared with previous versions.
  - Creation effects: not yet complete.
  - Requested time dummies should now appear on the effects object print.
  - Bayesian routine (still very much under development) has altered.
- 2011-05-27 R-forge revision 150: Removed effect for an absolutely constant covariate with two networks.

- 2011-05-26 R-forge revision 148: Improvements to signaGOF, added script to manual.
- 2011-05-16 R-forge revision 146:
  - Documentation improvements
  - Can read (some) non-directed network from Pajek files
- 2011-04-19 R-forge revision 144:
  - New effects: out trunc effect
  - Enhancements to siena08
- 2011-03-13 R-forge revision 142/3: GWESP effects (RSienaTest only)
- 2011-02-24 R-forge revision 140: adds functionality to sienaGOF for plotting image matrices of the simulations, cumulative tests based on the Kolmogorov- Smirnov test statistic, and conforms to coding standards.
- 2011-02-24 R-forge revision 139: fixes for bipartite networks with ML. ML is still incomplete, and will not work correctly with missing data or endowment effects.
- 2011-02-22 R-forge revision 137: (RSienaTest only) Additional work on the goodness of fit functionality (see ?sienaGOF)
- 2011-02-21 R-forge revision 136:
  - Fixed bug in bipartite network processing. Diagonal (up to number of senders) was being zeroed.
  - siena01Gui: corrected test for maximum degree in display.
- 2011-02-05 R-forge revision 134:
  - Enhanced features (and minor bug fixes) in siena08 report. (in revision 133 in RSiena-Test)
  - Bug fix in iwlsm
  - sienaDataCreateFromSession, sienaTimeTest: improved error messages.
  - ML support for bipartite networks (still work in progress, particularly for missing data)
- 2011-01-17 R-forge revision 131/2: (RSienaTest only) New goodness of fit functions: work in progress.
- $\bullet~$  2011-01-16 R-forge revision 130:
  - Fix bug for bipartite networks which usually crashed, but could have given incorrect answers.
  - Fix bug with multiple processes and sienaTimeTest.
  - Default value of siena 07 argument in it C is now TRUE.
- 2011-01-08 R-forge revision 129:
  - fix to sienaTimeFix for time dummies on covariate effects etc.
  - Suppressed warning message when loading snow package.
- 2010-12-02 R-forge revision 128:

- Corrections to scores for symmetric pairwise models
- ML now runs with missing data. Not yet sure it is correct!
- New multiple network effects: To, altDist2W, simDist2W.
- Can now use setEffects to update basic rate initial values
- multiplication factor for ML now a parameter in sienaModelCreate.
- Fixed bug meaning that covariate multiple network effects did not appear if the covariates was a behavior variable.
- Can now run sienaTimeTest on fits from finite differences and maximum likelihood.
- User defined interactions (and time dummies) can be expanded when printing the effects object, use parameter expandDummies = TRUE.

# • 2010-11-25 R-forge revision 126:

- Changed version of RSienato be 1.0.12 and copied all new features which were only in RSienaTest to RSiena.RSienaand RSienaTest are functionally the same at this time.
- New version of sienaTimeTest and sienaTimeFix.
- Bayesian routine (experimental) can be used with multiple dependent variables.

### • 2010-11-05 R-forge revision 125:

- Corrected bug in report from siena 07 about detailing network types.
- Networks appear in data object before behavior variables regardless of order of submission to sienaDataCreate
- RSienaTest only: new effect: in structural equivalence
- RSienaTest only: new models for symmetric networks
- bug fixed to sienaTimeTest: non included underlying effects for user defined interactions, multiple dependent networks and multiple groups.

### • 2010-10-22 R-forge revision 124:

- Fixed bug in sienaTimeTest when only one effect
- Removed standalone siena01Gui. Still available within R.
- 2010-10-09 R-forge revision 122:
  - Distance two effects: added parameter
  - Bug in calculation of starting values for behavior variables. (RSiena only)
- 2010-09-20 R-forge revision 120: Bug fixes:
  - Multiple groups with 2 dyadic covariates had incorrect names
  - Multiple processes failed (RSiena only)
  - Minor print format corrections
  - Bug in calculation of starting values for behavior variables. (RSienaTest only)
- 2010-08-20 R-forge revision 117: RSienaTest only. Documentation updates, algorithms may work again!
- 2010-08-20 R-forge revision 116: forgotten part of change for print of sienaFit (RSiena only)

- 2010-08-20 R-forge revision 115: fixed bug in siena08 p-values on report, and minor corrections to layout of print of sienaFit.
- 2010-07-19 R-forge revision 114: fix a bug in initial report: names of multiple behavior variables were incorrect.
- 2010-07-10 R-forge revision 113: fix bugs
  - 1. endowment effect unless using finite differences failed
  - 2. could not return bipartite simulations
- 2010-07-04 R-forge revision 112: fix bug in groups with constant dyadic covariates and only 2 waves. (Introduced in revision 109).
- 2010-07-03 R-forge revision 111: bipartite networks now have a no-change option at each ministep of simulation.
- 2010-06-25 R-forge revision 110: updated manual pages for dyadic covariates.
- 2010-06-25 R-forge revision 109:
  - Dyadic covariates may have missing values and sparse input format.
  - Removed some inappropriate dyadic covariate effects for bipartite networks.
  - Score test output now available via summary on a fit.
  - Corrected conditional estimation for symmetric networks.
  - Now do not need to specify the variable to condition on if it is the first in sienaModel-Create
- 2010-06-21 R-forge revision 108:
  - effects print method with no lines selected no longer gives error, new argument includeOnly so you can print lines which are not currently included.
  - effectsDocumentation was failing due to timeDummy column
  - New average alter effects
  - Corrected format of error message if unlikely to finish epoch/
  - Corrected print report for multiple groups via the GUI, and for 8 waves.
  - Fixed names for used defined dyadic interactions.
  - Fixed bug where sienaTimeTest dummies with RateX would not work with changing covariates.
- 2010-06-21 R-forge revision 107: RSienaTest only: reinstated includeTimeDummy.
- 2010-06-18 R-forge revision 106: new version numbers: 1.0.11.105 and 1.0.12.105 for RSiena and RSiena Test respectively.
- 2010-06-18 R-forge revision 105: Fixed siena01Gui bug when trying to edit the effects. Problem was introduced in revision 81.
- 2010-06-10 Updated time heterogeneity script for Tom
- 2010-06-08 R-forge revision 102: RSienaTest only. Removed includeTimeDummy.
- 2010-06-08 R-forge revision 101: RSienaTest only. Fixed RateX so that it works with changing actor covariates as well.

- 2010-06-08 R-forge revision 100: corrected revision numbers in ChangeLog.
- 2010-06-08 R-forge revision 99 Fix to bug introduced in revision 98: bipartite networks could not have 'loops'
- 2010-06-08 R-forge revision 98
  - Fix to bug in constant dyadic covariates with missing values.
  - Changes to treament of bipartite networks. The processing of these is still under development: we need to add the possibility of 'no change' to the ministeps. Code to deal with composition change has been added, and the treatment of missing values in sparse matrix format networks has been corrected further (the change in revision 96 was not quite correct).
- 2010-06-04 R-forge revision 97RSienaincludeTimeDummy not exported so not available to the user.
- 2010-06-04 R-forge revision 96 RSiena
  - bug fixes as in revisions 92, 93.
  - Changes and bug fixes to sienaTimeTest etc. as in revisions 85–89,
  - includeInteractions now will unInclude too.
- 2010-06-04 R-forge revision 93 (RSienaTest only)
  - New algorithms function (not in package: in the examples directory).
  - Progress on maximum likelihood code.
  - Bug fixes: print empty effects object, misaligned print.sienaFits, crash in print.sienaEffects with included interactions.
  - silent parameter now supresses more.
  - Added time dummy field to setEffects and removed from includeEffects.
  - includeInteractions now will unInclude too.
  - includeTimeDummy now sets or unsets the include flag, and prints the changed lines.
  - Using composition change with bipartite networks will give an error message until this is corrected.
  - Separate help files for sienaTimeTest, plot.sienaTimeTest, includeTimeDummy.
  - Bug fix to treatment of missing data in sparse format bipartite networks.
  - Change to error message if an epoch is unlikely to terminate.
- 2010-06-04 R-forge revision 92 (RSienaTest only) New average alter effects. Bug fix to effects object for more than two groups.
- 2010-05-29 R-forge revision 89 (RSienaTest only) New option to control orthogonalization in sienaTimeTest, changes to includeEffects and sienaDataCreate (NB changes reverted in revision 93).
- 2010-05-28 R-forge revision 88 (RSienaTest only) Time dummies for RateX effects
- 2010-05-27 R-forge revision 87 (RSienaTest only) bug fix to plot.sienaTimeTest
- 2010-05-23 R-forge revision 86 (RSienaTest only) Bug fix to plot.sienaTimeTest, new function includeTimeDummy

- 2010-05-22 R-forge revision 85 (RsienaTest only) fixed bug in sienaTimeTest with unconditional simulation.
- $\bullet$  2010-04-24 R-forge revision 81 New print, summary and edit methods for Siena effects objects
- 2010-04-24 R-forge revision 80
  - fixed bug causing crash with rate effects and bipartite networks.
  - added trap to stop conditional estimation hanging
  - new functions (INCOMPLETE) for maximum likelihood and Bayesian estimation (one period (two waves) only, no missing data, one dependent variable only for Bayesian model).
- 2010-04-13 R-forge revision 79 new function: sienaTimeTest.
- 2010-04-12 R-forge revision 78 fix minor bugs in reports, allow character input to effect utility functions, include effect1-3 etc on display of included effects in siena01Gui.
- 2010-04-12 R-forge revision 77 (RSiena only) As for RSienaTest revision 76
  - Report of 0 missings corrected
  - display of effect1-effect3 in siena01Gui
  - allow entry of character strings or not in includeEffects etc.
- 2010-04-12 R-forge revision 76 (RSienaTest only) Various bug fixes
  - Memory problems when calculating derivatives with many iterations and parameters.
  - Occasional effects not being included correctly due to trailing blanks
  - Some minor details of reports corrected.
- 2010-03-31 R-forge revision 75 fixed bug with dyadic covariates and bipartite networks.
- 2010-03-27 R-forge revision 71 (RSienaTest only)
  - Fixes as for RSienain revision 68/69/70 for RSiena
  - New version number 1.0.12
- 2010-03-27 R-forge revision 70 (RSiena only)
  - Fix to crash at end of phase 3 with multiple processes and conditional estimation
  - Correct carry forward/backward/use mode for behavior variables
  - Fix bug causing crash in Finite Differences with only one effect
- 2010-03-24 R-forge revision 69 (RSiena only)
  - New features and bug fixes as for revision 63 in RSienaTest.
  - 4-cycles effect has new shortName: cycle4.
  - some percentages on reports were proportions not percentages
  - Sped up treatment of missing values in sparse format networks.
  - Fix: now allows more than one value to indicate missing in covariates.

- 2010-03-12 R-forge revision 68 new version number for RSiena. In siena01Gui, allow waves for SienaNet inputs to be numbered arbitrarily, rather than insisting on 1-n. Change simply allows this, the actual wave numbers are not yet used on reports etc.
- 2010-03-17 R-forge revision 66 Corrected processing of user-specified interaction effects with multiple processes. This had originally worked but failed when one no longer had to include the underlying effects.
- 2010-03-16 R-forge revision 64 covarBipartite ego effect had been given type dyadic rather than ego.
- 2010-03-16 R-forge revision 63 (RSienaTest only)
  - new functions siena08 and iwlsm, for meta analysis
  - can now use different processes for each wave. Not recommended: usually slower than
    by iteration, but will be useful with ML routines when they are completed.
  - No longer crashes with missing dyadic covariates.
- 2010-02-27 R-forge revision 61 (RSiena only) bug fix: random numbers used with multiple processes were the same in each run. Now seed is generated from the usual R random number seed. Also fixed a display bug if running phase 3 with few iterations.
- 2010-02-16 R-forge revision 60 (RSienaTest only) added average indegrees to reports. Also constraints.
- 2010-02-12 R-forge revision 59 (RSienaTest only) Fix to bugs in printing version numbers and in using multiple processes (would revert to RSiena package.) Added a skeleton MCMC routine.
- 2010-02-11 R-forge revision 57 Fix to bug in siena01Gui where in conditional estimation, the estimated values were not remembered for the next run.
- 2010-02-11 R-forge revision 56 (RSiena only) Multiple network effects, constraints between networks.
- 2010-02-11 R-forge revision 55 (RSienaTest only) New silent option forsiena07.
- 2010-02-11 R-forge revision 54 (RSienaTest only) Fix to covariate behavior effect bug.
- 2010-02-11 R-forge revision 53 Fixed bug in siena01 GUI which ignored changes to all effects
- 2010-02-07 R-forge revision 52 (RSiena only) New silent option forsiena07.
- 2010-02-04 R-forge revision 51 (RSiena only)
  - Fix to covariate behavior effect bug.
  - Fix to default effects with multiple networks.
- 2010-02-01 R-forge revision 49 (RSienaTest) only Fixes to bugs in constraints.
- 2010-01-28 R-forge revision 48 Fix to bug in sorting effects for multiple dependent variables.
- 2010-01-26 R-forge revision 47 (RSienaTest only)
  - New version: 1.0.10
  - Multiple networks

- Constraints of higher, disjoint, at Least One between pairs of networks.
- 2010-01-19 R-forge revision 45 (RSiena), 46 (RSienaTest) New documentation for the effects object.
- 2010-01-18 R-forge revision 43 (RSiena)
  - new behavior effects
  - user specified interactions
  - new utilities to update the effects object
- 2010-01-15 R-forge revision 41 (RSienaTest only)
  - new effect: Popularity Alter, and altered effect1-3 to integers to correct bug in fix(myeff)
  - new utility functions to update effects object
  - no longer necessary to include underlying effects for interactions.
  - user parameter for number of unspecified behavior interactions
  - remove extra sqrt roots in standard error of rates for conditional estimation (see revision 31)
- 2010-01-15 R-forge revision 40:RSienaonly remove extra sqrt roots in standard error of rates for conditional estimation (see revision 32)
- $\bullet~2010\text{-}01\text{-}02$  R-forge revision 34

Corrected layout of printand xtable for SienaFit objects with both behavior and network variables.

 $\bullet$  2010-01-01 R-forge revision 33

Updated change log and manual in RSiena and ChangeLog in RSienaTest.

- 2010-01-01 R-forge revision 32 print07report.r: corrected standard errors for rate estimate for conditional estimation: needed square roots. RSiena
- 2009-12-31 R-forge revision 31
  - print07report.r: corrected standard errors for rate estimate for conditional estimation: needed square roots. RSienaTest only
  - $-\,$  more behavior effects in RSienaTest.
- 2009-12-17 R-forge revision 30

Fixed bug in dyadic interactions in RSienaTest

• 2009-12-17 R-forge revision 29

Fixed bug in 3-way interactions in RSienaTest

• 2009-12-14 R-forge revision 28

Fixed bug in use of multiple processes for RSiena.

• 2009-12-14 R-forge revision 27

Fixed bug in use of multiple processes for RSienaTest.

 $\bullet~2009\text{-}12\text{-}01$  R-forge revision 26

Created RSienaTest which includes user specified interactions.

### • 2009-11-20 R-forge revision 25

- version number 1.0.8
- The default method for estimation is conditional if there is only one dependent variable.
- Movement of behavior variable restricted if all observed changes are in one direction.
   In this case, linear change effects removed.
- If all observed changes in a network are in one direction, density effects are removed.
- If a behavior variable only takes two values the quadratic effects are not selected by default.
- t-statistics appear on print of sienaFit object.
- easier to use xtable method
- warning if behavior variables are not integers
- Fixed bug in editing all effects in the GUI.
- Fixed a bug in effect creation for changing dyadic covariates
- Fixed a bug in returning simulated dependent variables
- Now fails if there are only two waves but you have a changing covariate. In the GUI, can just change the type.

## • 2009-11-08 R-forge revision 24

- version Number 1.0.7
- 2009-11-08 R-forge revision 23
  - corrected bug in creation of effects data frame for multi group projects and for changing covariates
  - added effect numbers to the Estimation screen
- 2009-11-08 R-forge revision 22
  - new option to edit effects for one dependent variable at a time. Model options screen layout altered slightly.
- 2009-11-08 R-forge revision 21
  - Fixed a bug causing crashes (but not on Windows!) due to bad calculation of derivative matrix.
- 2009-10-31 R-forge revision 17
  - version Number 1.0.6
  - xtable method to create L<sup>A</sup>T<sub>F</sub>Xtables from the estimation results object.
  - added support for bipartite networks
  - structural zeros and 1's processing checked and amended
  - use more sophisticated random number generator unless parallel testing with siena3.

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