This problem set has 5 questions, for a total of 110 points. Please carefully read the guidelines below:

- Provide your final answers only within the designated spaces on each of the questions. We use automatic grading for some questions – answers outside these spaces will not be graded.
- Generally, for short questions, no partial credit is given. To facilitate the evaluation of any regrading requests, you are encouraged to "show your work" using the empty spaces provided, however this is not a requirement.
- You may annotate your answers digitally on the PDF, or alternatively, you can print the PDF and write your answers by hand. If you choose the second option, please ensure that your handwriting is legible, and the software/hardware used for scanning the document does not change the original format of the PDF, keeping the same structure, orientation, and page size. Once you are done, submit your file through Gradescope.
- We expect you to focus on fully understanding each solution. This is important not only for your learning, but also for your performance in the course, as similar questions may appear in your exams. Do not hesitate to ask for help if you have any questions.

Your Name:

1. Rewrite the following expressions in *closed-form* (i.e., without summation notation). You may use the identities presented in class or refer to the "Theoretical Computer Science Cheat Sheet" available on the course website.

   (a) (5 points) $\sum_{i=1}^{n}(3+i) =$ _____-

   (b) (5 points) $\sum_{i=1}^{n^2} i =$ _____-

   (c) (5 points) $\sum_{i=1}^{\log_2 n} i^2 =$ _____-

2. For each of the following questions, indicate $T(n)$ – the exact number of times the line `// op` is executed. Separately, determine the rate of growth of $T(n)$ as $n \to \infty$, using Θ-notation. For example, if $T(n) = 5n^3 + n^2 + 2^8 n$, then its rate of growth is $\Theta(n^3)$. For this latter part, you may simplify $T(n)$ by discarding lower-order terms and constant factors.

   (a) (10 points)

```
for (int i = 0 ; i < 4*n ; i ++) {
    // op
}
```

   $T(n) =$ _____ and growth rate: _____

   (b) (10 points)

```
for (int i = 0 ; i < n ; i++) {
    for (int j = 0 ; j < 100*100 ; j++) {
        // op
    }
}
```

   $T(n) =$ _____ and Growth rate: _____

   (c) (10 points)

```
for (int i = 0 ; i < 4*n ; i++) {
    for (int j = 0 ; j < i ; j++) {
        // op
    }
}
```

   $T(n) =$ _____ and Growth rate: _____

(d) (10 points)

```
for (int i = 0 ; i < n*n ; i++) {
    for (int j = 0 ; j < i ; j++) {
        // op
    }
}
```

$T(n) =$ _____ and Growth rate: _____

(e) (10 points)

```
// your formula should assume n can be even or odd
// hint: can use the floor or ceiling function
for (int i = 0 ; i < n ; i += 2) {
    // op
}
```

$T(n) =$ _____ and Growth rate: _____

(f) (10 points)

```
for (int i = 0 ; i < 4 * n ; i++) {
    // op
}
for (int i = 0 ; i < 16 * 16 ; i++) {
    // op
}
```

$T(n) =$ _____ and Growth rate: _____

(g) (10 points)

```
// assume n is a power of 2
for (int i = 1 ; i <= n ; i *= 2) {
    // op
}
for (int i = n ; i > 1 ; i /= 2) {
    // op
}
```

$T(n) =$ _____ and Growth rate: _____

3. (5 points) Rank the following functions by their growth rate, from the slowest to the fastest.

$$\log\log n \qquad 2^{\log_2 n} \qquad 2^{100} \qquad 4^n \qquad n^2 \log n \qquad 4^{\log_2 n}$$

_____ _____ _____ _____ _____ _____

4. (5 points) Mark each of the following as true or false.

$\frac{n^2}{10} + 10n \log n$	$O(n \log n)$		$\Omega(n \log n)$		$\Theta(n \log n)$	
$2n^2 + n \log n$	$O(n^2)$		$\Omega(n)$		$\Theta(\log n)$	
$\frac{n}{2} \log n + 4n$	$O(2^n)$		$\Omega(n \log n)$		$\Theta(n \log n)$	
$10\sqrt{n} + 2 \log n$	$O(\log n)$		$\Omega(n)$		$\Theta(\log n)$	
$3\sqrt{n} + 10 \log n$	$O(\sqrt{n})$		$\Omega(1)$		$\Theta(\sqrt{n})$	

5. Consider the following function:

```
int foo(int x, int *y) {
    x = x + 10;
    *y = x * 2;
    return x;
}

int *bar(int x) {
    int y = 50 + x;
    return &y;
}
```

For each question below, determine the values of `x` and `y` after executing the given code. If you believe the code may cause an error at any point, explain the reason. Do not use a computer to solve this question.

(a) (5 points)

```
int x = 2, y = 3;
x = foo(x, &y);
```

(a) _____

(b) (5 points)

```
int x = 0, y = 0;
x = foo(x, &y);
```

(b) _____

(c) (5 points)

```
int x = 1, y = 0;
int *z = bar(y);
x = *z;
```

(c) _____