CSC 212: Data Structures and Abstractions

Hash Tables

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THINK BIG WE DO℠

Data Structure	Worst-case			Average-case			Ordered?
	insert at	**delete**	**search**	**insert at**	**delete**	**search**	
sequential (unordered)	O(n)	O(n)	O(n)	O(n)	O(n)	O(n)	No
sequential (ordered) binary search	O(n)	O(n)	O(log n)	O(n)	O(n)	O(log n)	Yes
BST	O(n)	O(n)	O(n)	O(log n)	O(log n)	O(log n)	Yes
2-3-4	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes
Red-Black	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes

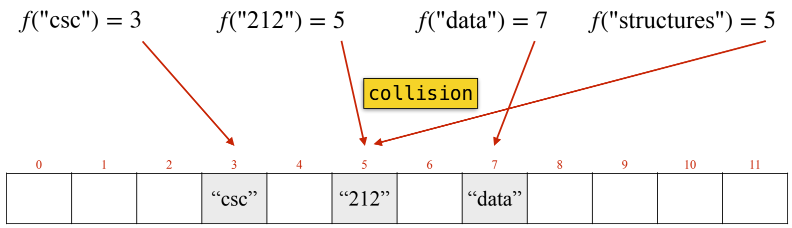
Can we do better?

Random access memory

- Random Access Memory (RAM) represents a fundamental principle in computer science
  - ✓ it allows the retrieval of any element in constant time $O(1)$, regardless of its position within the memory block
  - ✓ this principle is most commonly observed through arrays
- Arrays in C++
  - ✓ contiguous memory allocation
  - ✓ homogeneous elements (same data type)
  - ✓ fixed-length (traditional arrays have predetermined size)
  - ✓ zero-based indexing

Hash tables

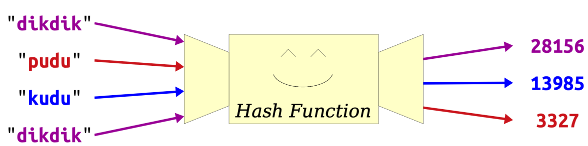
- A hash table is a data structure that implements an associative array
  - ✓ the array can store keys (set), or key-value pairs (map)
  - ✓ a function is used to compute an index, that can be used to find a desired key in the array
  - ✓ provides an efficient way to implement sets or dictionaries



Hash function

Hash function

- A hash function is a function that maps an input key to some integer value
  - ✓ must be deterministic (same input produces the same output)
  - ✓ should be well-distributed (the numbers produced are as spread out as possible)



- Hash tables
  - ✓ store keys in an array and use a hash function to map keys to indices
  - ✓ potentials issues:
    - may require a large array
    - array could be very sparse (wasting memory)

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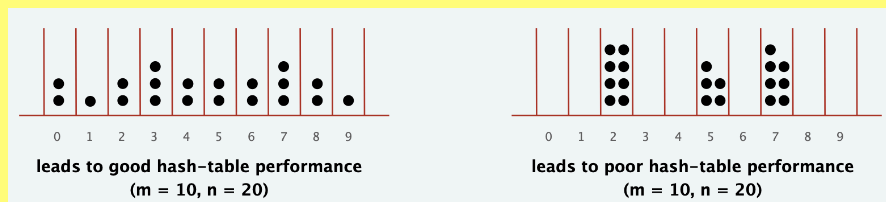
Practice

- Which of the following tables is a better choice?
- What is the load factor?



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Hash functions

- A hash function is a function that maps an input key to some integer value
- Properties:
  - must be **deterministic** (same input produces the same output)
  - should be **well-distributed** (the numbers produced are as spread out as possible)
  - should be **fast** to compute
- Hash tables (idea)
  - store keys in an array and use a hash function to map keys to indices
  - potentials issues:
    - may require a large array
    - array could be very sparse (wasting memory)

Hash functions

- Space efficiency
  - making all keys equally possible requires a huge array, even if we only have a couple of elements
  - idea: use a hash function, but modify the result to be within a smaller range (the size of the array)

```
// if hash() returns non-negatives
index = hash(key) % capacity

// if hash() returns any integer
index = abs(hash(key) % capacity)
```

- Collision
  - occurs when two different keys hash to the same index in the hash table
  - collisions can be resolved using:
    - separate chaining: each slot in the hash table contains a collection of all the keys that hash to that index
    - open addressing: if a collision occurs, the algorithm searches for the next available slot in the hash table
  - open addressing is more space-efficient than chaining, but it can be slower

Hash functions

- Hash functions can be used on any data type
  - integers: use the integer value as the hash value
  - floats: convert to binary and use the integer value as the hash value, or manipulate the bits (e.g. XOR the mantissa and exponent)
  - strings: use $31x + y$ rule or apply other variants
  - compound objects: use $31x + y$ rule or apply other variants
- Mapping hash values into a smaller “table size”
  - M is prime: helps distributing keys more uniformly, minimizing collisions
  - M is a power of two: modulo operation can be replaced with a faster bitwise AND operation

Hash functions (other uses)

- Storing passwords
  - hash the password and store the hash in the database
- File verification
  - hash the file (checksum) and compare the hash with the stored hash
  - e.g. when downloading a file, vendors publish a hash value, client checks whether hash matches, otherwise file is corrupted
- Examples (one-way hashes)
  - hash functions that are difficult to reverse or to find two keys that map to the same value
  - MD5, SHA-1, SHA-256, SHA-512, SHA3-512, …

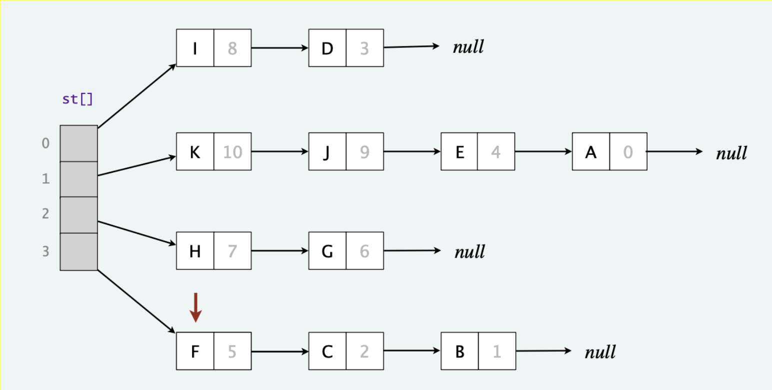
Separate chaining

Separate chaining

- Idea
  - ✓ solve collisions by storing a linked list at each index
  - ✓ assume duplicated keys are not allowed
- Operations
  - ✓ insert: if a collision occurs, add the new key to the linked list at that index
    - no need to keep the keys on each list in order
  - ✓ search: search the linked list at that index
  - ✓ delete: search the linked list at that index and removes the key from the list
- Comments
  - ✓ linked list can be replaced by a balanced tree or another data structure to improve access time

Practice

- Perform the following operations
  - ✓ insert(L, 11), delete(D), insert(M, 12), delete(E), search(C)
    - assume: insertions occur at front of the lists, hash(L)=3, hash(M)=0



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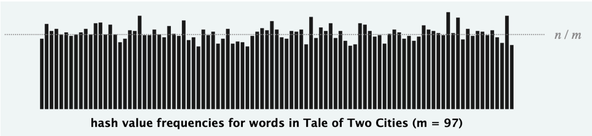
Analysis

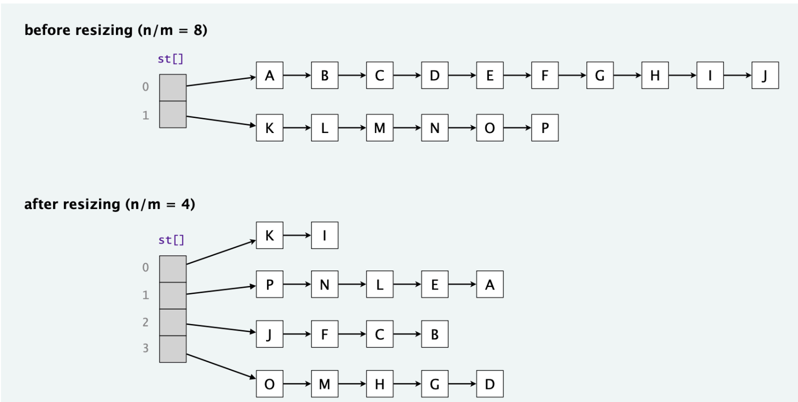
- Uniform hashing assumption
  - ✓ the hash function is a good one, and all keys are uniformly distributed



- Load factor (α)
  - ✓ the ratio of the number of keys (N) to the number of slots (M) $\alpha = \dfrac{N}{M}$
- Time complexity
  - ✓ average case for search, insert, and delete is $O(c + \alpha)$, where c is the time taken by the hash function
  - ✓ worst case for search, insert, and delete is $O(c + N)$ — all the keys hash to the same index
- Space complexity
  - ✓ the space needed is $O(N + M)$

Resizing a hash table

- Growing to a larger array when α exceeds a threshold
  - ✓ create a new table with larger capacity and rehash all the keys



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Considerations

- Choices for α
  - ✓ too small, the hash table will be too large and waste space
  - ✓ too large, the hash table will be too small and cause collisions
- Typical values
  - ✓ between 0.5 and 1.0 often provide a reasonable balance of space efficiency and lookup performance
  - ✓ higher load factors (>1.0) remain functional but with degraded performance characteristics
  - ✓ for performance-critical applications, implementers should conduct benchmarks with representative data sets to determine the optimal load factor for their specific use case

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Red-Black	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes
Hash table (separate chaining)	O(n)	O(n)	O(n)	O(1)	O(1)	O(1)	No

Open addressing

Open addressing

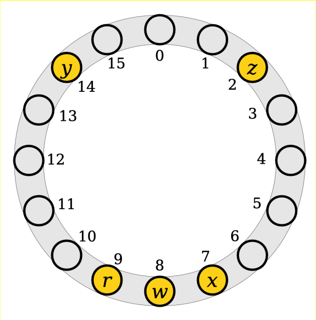
- Idea
  - solve collisions by "*probing*"
    - searching for the next available slot in the hash table
    - each slot holds a single element
    - if using *key-value* pairs, maintain two separate arrays
  - assume duplicated keys are not allowed and $M \geq N$
- Operations
  - insert: if a collision occurs, search for the next available slot in the hash table
  - search: search for the key in the hash table
  - delete: search for the key in the hash table and mark the slot it as deleted
- Comments
  - open addressing is more space-efficient than chaining, but it can be slower
  - works better with $\alpha \approx 0.5$

Probing

- Linear probing
  - moves to the next available index
  - use index i if free, otherwise try $i+1, i+2, i+3, \ldots$ (wrap around if necessary)
- Quadratic probing
  - moves to the next available index using a quadratic function
  - use index i if free, otherwise try $i+1^2, i+2^2, i+3^2, \ldots$ (wrap around if necessary)
- Double hashing
  - moves to the next available index using a second hash function
  - use index i if free, otherwise try $i + \text{hash2(key)}, i + 2 * \text{hash2(key)}, i + 3 * \text{hash2(key)}, \ldots$(wrap around if necessary)

Practice

- Perform the following operations (assume linear probing)
  - search(w), delete(z), delete(w), search(r), insert(c), insert(d), insert(e)
    - assume: hash(z)=2, hash(x)=7, hash(r)=7, hash(w)=7, hash(y)=14, hash(a)=12, hash(c)=8, hash(d)=15, hash(e)=14



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Hash table (separate chaining)	O(n)	O(n)	O(n)	O(1)	O(1)	O(1)	No
Hash table (open addressing)	O(n)	O(n)	O(n)	O(1)	O(1)	O(1)	No