CSC 212: Data Structures and Abstractions

Binary search trees (part 2)

Prof. Marco Alvarez

Department of Computer Science and Statistics
University of Rhode Island

Spring 2025

THINK BIG WE DO℠

Problems from lab

- Problem A
- Problem B
- Problem C

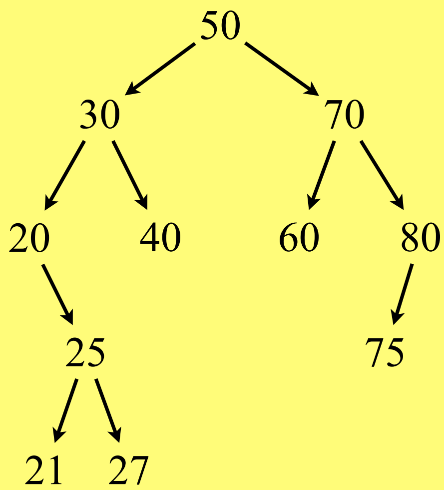
Operations

Contains

- Algorithm
  - ✓ start at root node
  - ✓ if the search key matches the current node's key then found
  - ✓ if search key is greater than current node's key
    - search on right child
  - ✓ if search key is less than current node's
    - search on left child
  - ✓ stop when current node is nullptr (not found)
- Time complexity
  - ✓ $O(h)$, where h is the height of the tree

Practice

- Search the following keys:
  - ✓ 25, 77, 18, 40, 75



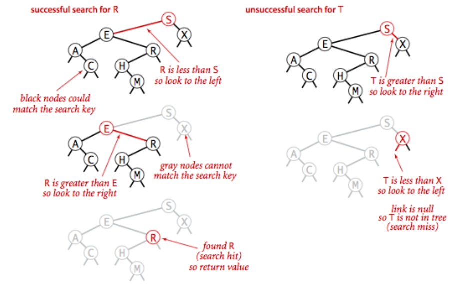
Recursive contains

```
bool BST::contains(Node* p, const T& key) {

    if ( !p ) {
        return false;
    }

    if (key < p->data) {
        return search(p->left, key);
    } else if (key > p->data) {
        return search(p->right, key);
    } else {
        return true;
    }

}
```



Successful (left) and unsuccessful (right) search in a BST

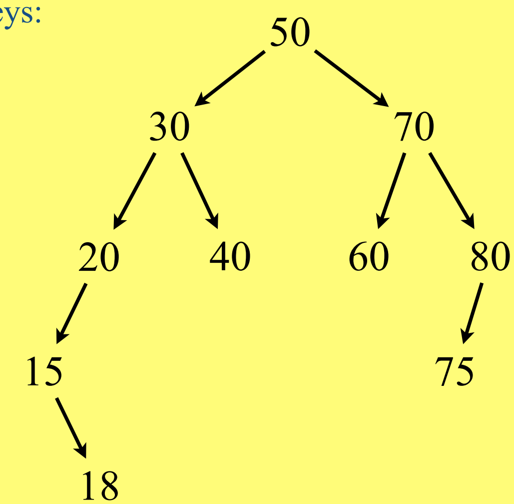
https://algs4.cs.princeton.edu/32bst/

Insert

- Algorithm
  - ✓ if tree is empty, create a new node as root, done
  - ✓ if not, start node p at the root, repeat:
    - compare the key to insert with the key of p, if equal, done
    - if the new key is less than the key of p, set p to the left subtree
      - if p is empty, create new node here, done
      - else continue
    - if the new key is greater than the key of p, set p to the right subtree
      - if p is empty, create new node here, done
      - else continue
- Time complexity
  - ✓ $O(h)$, where h is the height of the tree

Practice

- Insert the following keys:
  - ✓ 65, 27, 90, 11, 51



Repeated keys

- We assume the tree contains unique keys
  - ✓ no repeated keys are allowed
- Dealing with repeated keys
  - ✓ if key is in the tree, do nothing, just return
  - ✓ depending on the task being solved, may add a counter on each node, and it can be increased every time a repeated key is inserted
  - ✓ if the tree is used as a map or dictionary, may want to update the value of a repeated key

Recursive insert

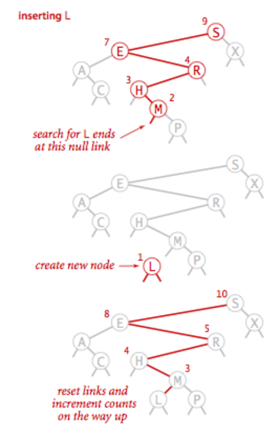
```
Node* BST::insert(Node* p, const T& key) {

    if ( !p ) return new Node(key);

    if (key < p->data) {
        p->left = insert(p->left, key);
    } else if (key > p->data) {
        p->right = insert(p->right, key);
    }

    return p;

}
```



Insertion into a BST

https://algs4.cs.princeton.edu/32bst/

Remove

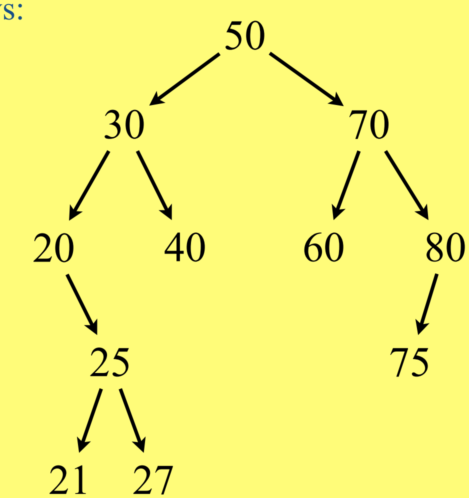
- Approach
  - ✓ find node to be removed, then apply one of the cases below
  - ✓ **case 1**: node is a leaf
    - trivial, delete node and set parent's pointer to nullptr
  - ✓ **case 2**: node has 1 child
    - set parent's pointer to the only child and delete node
  - ✓ **case 3**: node has 2 children
    - find successor (smallest node in the right subtree)
    - copy successor's data to node
    - delete successor

`can also use predecessor`

- Time complexity
  - ✓ $O(h)$, where h is the height of the tree

Practice

- Remove the following keys:
  - ✓ 27, 40, 80, 20, 30, 50



Analysis

Practice

- Starting from an empty BST, insert the following keys in the order given
  - 20 10 30 5 15 25 35
  - 10 20 5 15 30 35 25
  - 5 10 15 20 25 30 35
  - How is the order of insertion related to the shape of the tree?
  - How is the height of the tree related to the number of nodes?

Practice

- Complete the following table with rates of growth
  - as a function of the number of nodes

Operation	Best case	Average case	Worst case
Insert			
Remove			
Search			

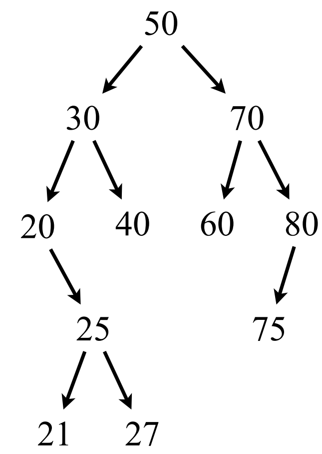
Average case

- Proposition
  - if n distinct keys are randomly inserted into a BST, the expected number of compares is $\sim c \log n$
    - can be formally justified through probabilistic analysis (not covered in this class)
- Implications
  - even without explicit balancing mechanisms, randomly built BSTs provide reasonably efficient operations

Traversals

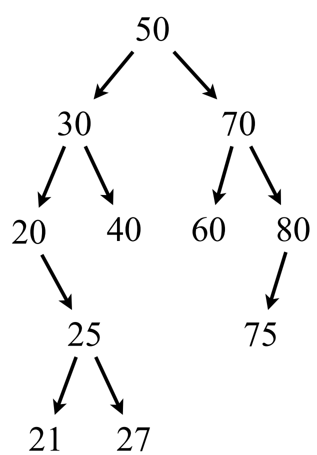
Preorder traversal

- Depth-first traversal that visits the root node first, then recursively visits all subtrees
  - ✓ visit the root node
  - ✓ recursively visit the left subtree
  - ✓ recursively visit the right subtree



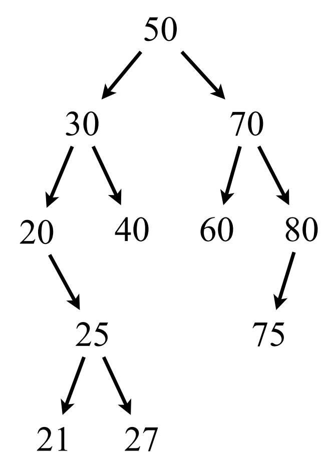
Postorder traversal

- Depth-first traversal that recursively visits all subtrees first, then visits the root node
  - ✓ recursively visit the left subtree
  - ✓ recursively visit the right subtree
  - ✓ visit the root node



Inorder traversal

- Depth-first traversal that recursively visits the left subtree first, then visits the root node, and finally recursively visits the right subtree
  - ✓ recursively visit the left subtree
  - ✓ visit the root node
  - ✓ recursively visit the right subtree



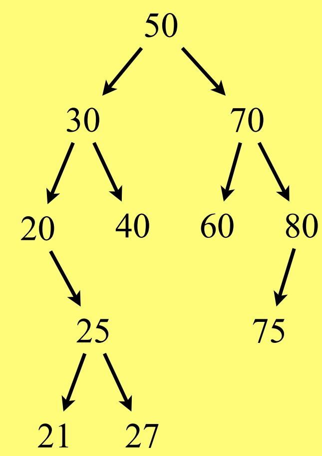
Practice

- Which traversal is best for printing all values in sorted order?
- Which traversal is best for deleting all nodes in a tree?
- What is the time complexity of each traversal?

Practice

- Trace the following algorithm and explain what it does

```
algorithm mistery(root) {
    queue q
    q.enqueue(root)
    while not q.isEmpty() {
        node n = q.dequeue()
        print(n.value)
        if n.left
          q.enqueue(n.left)
        if n.right
          q.enqueue(n.right)
    }
}
```



Collections

Operation	Description	Sequential (unordered)	Sequential (ordered)	BST
search	search for a key	O(n)	O(log n)	O(h)
insert	insert a key	O(n)	O(n)	O(h)
delete	delete a key	O(n)	O(n)	O(h)
min/max	find smallest/largest key	O(n)	O(1)	O(h)
floor/ceiling	find predecessor/successor	O(n)	O(log n)	O(h)
rank	count number of keys less than key	O(n)	O(log n)	O(h)