This problem set has 9 questions, for a total of 100 points. Please carefully read the guidelines below:

- Provide your final answers only within the designated spaces on each of the questions. We use automatic grading for some questions – answers outside these spaces will not be graded.
- You may annotate your answers digitally on the PDF, or alternatively, you can print the PDF and write your answers by hand. If you choose the second option, please ensure that your handwriting is legible, and the software/hardware used for printing the document does not change the original format of the PDF. Once you are done, print the assignment and bring it to class on April 24.
- We expect you to focus on fully understanding how to solve each problem, not just on obtaining the final answer. This is important not only for your learning, but also for your performance in the course, as similar questions may appear in your exams. Do not hesitate to ask for help if you have any questions.

Your Name:

Recurrences

1. Consider the C++ code below.

```
int foo(int n) {
    int result = 0;
    if (n == 1) {
        return result;
    }
    for (int i = 0 ; i < n ; i++) {
        result += i;
    }
    return foo(n/2) + result + foo(n/2);
}
```

(a) (5 points) Indicate the recurrence relation $T(n)$ for the time complexity of `foo`.

(a) _____

(b) (5 points) Solve the recurrence relation $T(n)$ using the *iteration method* and indicate the final result in **big-O** notation.

(b) _____

2. Solve the following recurrence relations using the *iteration method* presented in class. Indicate the final result in **big-O** notation.

   (a) (5 points) $T(n) = T(n-1) + 5$ with $T(1) = 1$

   (a) _____

   (b) (5 points) $T(n) = 2T(n-1) + 3$ with $T(1) = c$

   (b) _____

Binary Trees

3. Considering the definition of *height* of a binary tree, give a formula $F(n)$ for:

   (a) (5 points) The maximum height of a binary tree with n nodes

   (a) _____

   (b) (5 points) The minimum height of a binary tree with n nodes

   (b) _____

   (c) (5 points) The maximum number of nodes in a binary tree of height h

   (c) _____

   (d) (5 points) The minimum number of nodes in a binary tree of height h

   (d) _____

4. Let T be a **full** k-ary tree with n nodes and h denote the height of T.

   (a) (5 points) What is the minimum number of leaves in T?

   (a) _____

   (b) (5 points) What is the maximum number of leaves in T?

   (b) _____

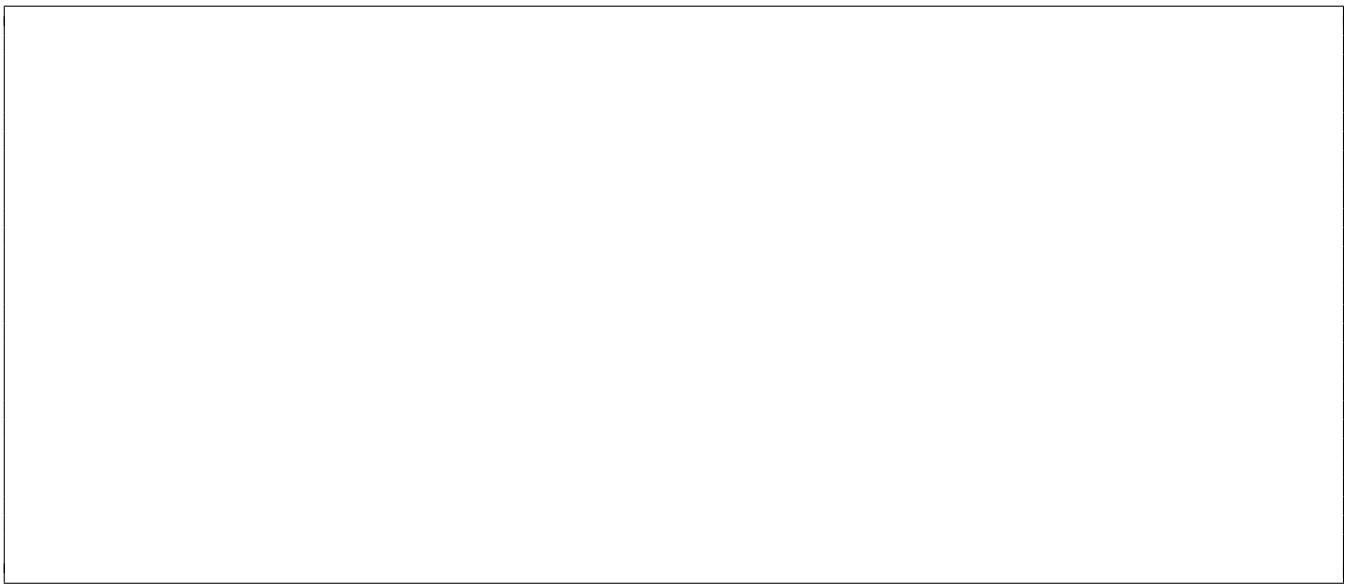
   (c) (5 points) What is the maximum number of internal nodes in T?

   (c) _____

   (d) (5 points) What is the minimum number of internal nodes in T?

   (d) _____

5. (10 points) Assume a *binary tree* `T`. A *p*ostorder traversal visits the nodes of `T` in the following order: $5, 9, 10, 18, 3, 4, 21, 15$. An *i*norder traversal visits the nodes of `T` in the following order: $5, 9, 15, 10, 3, 18, 21, 4$. Draw the tree `T`. Hint: note that this is a binary tree, not a binary search tree.



6. (10 points) Draw the resulting *b*inary search tree after inserting the following integers in the order given: $10, 5, 12, 8, 19, 6, 11, 15, 9$, and then removing the following integers in the order given: $19, 10, 11, 8$.

2-3-4 Trees

7. Consider the following elements sequence of elements $22, 15, 18, 33, 10, 8, 21, 17$

   (a) (5 points) Draw the resulting balanced multiway search tree of order $b = 3$ (a.k.a. 2-3 tree) after inserting the elements in the order given.

   (b) (5 points) Draw the resulting balanced multiway search tree of order $b = 4$ (a.k.a. 2-3-4 tree) after inserting the elements in the order given.

8. Consider a 2-3-4 tree with n keys.

   (a) (5 points) What is the minimum height of the tree as a function of n? hint: maximize the number of keys in the tree.

   (a) _____

   (b) (5 points) What is the maximum height of the tree as a function of n? hint: minimize the number of keys in the tree.

   (b) _____

9. Draw the corresponding $r$ed-black tree for the 2-3-4 tree you obtained in question 7.b.