AE353 (Spring 2021) Day 11 - Stability

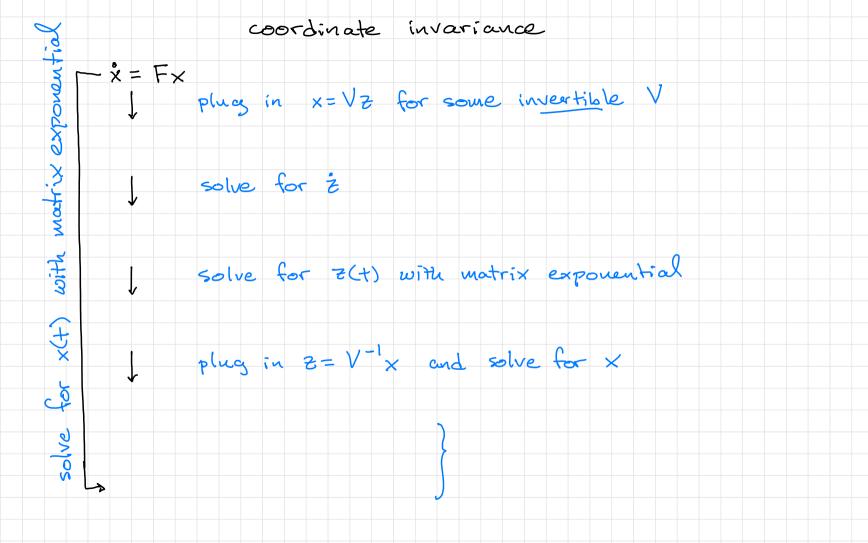
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nonlinear ODEs put in standard form m= s(m,n) 1 = y(m,n) linearize x=m-me x=Ax+Bu 0 = f(me, ne) u=n-ne y= Cx+ Du y=l-g(me, ne) choose input u = -Kxfind closed-loop system $\dot{x} = (A-BK) \times = F_{\times}$ $\dot{y} = (C-DK) \times = G_{\times}$ $\dot{y} = (C-DK) \times = G_{\times}$ $\dot{y} = (C-DK) \times = G_{\times}$ Solve by matrix exponential $\dot{y} = (C-DK) \times = G_{\times}$ $\dot{y} = (C-DK) \times =$ for which choices of X(+) -> 0 as + -> 00 K does for which choices of K does X(t) > 0 as t > 0 ???

STRATEGY

- (1) Answer this question in the special case when F is diagonal
- (2) Show how to rewrite (almost) any F as diagonal
- 3) Answer this question for (almost)
 any F

suppose F is diagonal eFt = I + Ft + 1/2 (Ft)2 + ... eft is easy to find



who cores? eft = Ve V-1FV+V-1 this is easy to find if VFV is diagonal so let's choose V so this is true our goal - find invertible V such that V'FV = diag(s, ..., sn) & this is the same as for example, suppose F is Z×Z: FV= Vdiag(s, ..., sn) $V = [v, v_2]$ diag(s, sz) = [s, 0] L columns of V (both are 2x1) then: V diag(s1, s2) = [FV = [

then
$$x(t) = e^{-(t-t_0)}x(t_0) = Ve^{-diag(s_1,...,s_m)(t-t_0)}V^{-1}x(t_0)$$

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×(t) -> 0 ces + -> 00 The system ×=F× is asymptotically stable if and only if all of F have