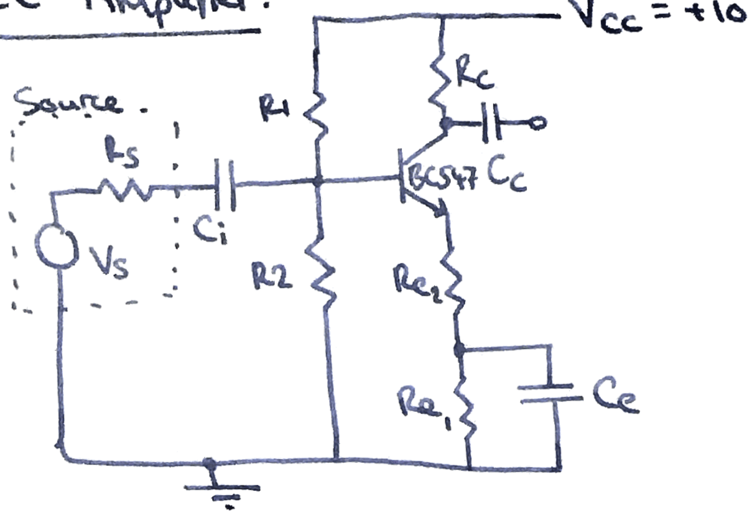
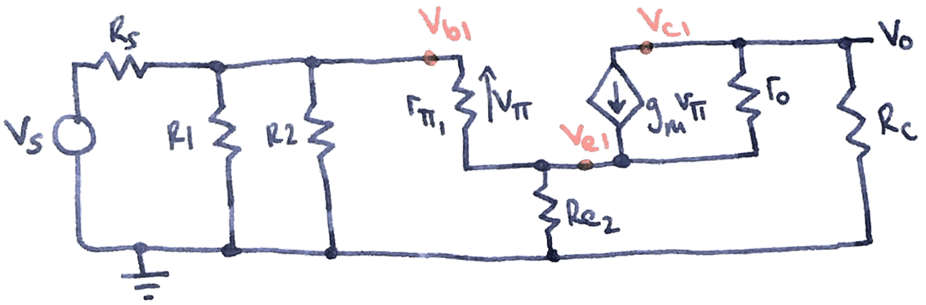
CE Amplifier:



Small Signal Model: (capacitors become short circuits).



$$V_e = \left(g_m v_\pi + \frac{V_{be}}{r_{\pi 1}}\right) R_{e2}$$

$$= V_\pi \left(g_m + \frac{1}{r_{\pi 1}}\right) R_{e2}$$

let $\frac{1}{r_{\pi 1}} = g_\pi$

$$= V_\pi (g_m + g_\pi) R_{e2}$$

$$\therefore V_e = V_\pi R_{e2} (g_m + g_\pi)$$

$$V_{in} = V_\pi + V_e$$

$$= V_\pi + V_\pi (g_m + g_\pi) R_{e2}$$

$$\therefore V_\pi = \frac{V_{in}}{1 + (g_m + g_\pi) R_{e2}}$$

$$V_o = \frac{-g_m R_c V_{in}}{1 + (g_m + g_\pi) R_{e2}}$$

As $\frac{V_{in}}{1 + (g_m + g_\pi) R_{e2}} = V_\pi$

$$\therefore \quad V_o = -g_m v_\pi R_c$$

$$\frac{V_o}{V_{in}} = \text{Gain} = \frac{-g_m R_c}{1 + (g_m + g_\pi) R_{e_2}}$$

but we said $g_\pi = \frac{1}{r_{\pi_1}}$

also $r_{\pi_1} = \frac{\beta}{g_{m_1}}$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{-g_m R_c}{1 + (g_m + \frac{1}{r_{\pi_1}}) R_{e_2}}$$

$$= \frac{-g_m R_c}{1 + R_{e_2}(g_m + \frac{g_m}{\beta})}$$

$$= \frac{-g_m R_c}{1 + g_m R_{e_2}(1 + \frac{1}{\beta})}$$

$$= \frac{-g_m \beta R_c}{\beta + g_m R_{e_2}(\beta + 1)}$$

as $\beta = g_m r_\pi$

cancel g_m

$$= \frac{-\beta R_c}{r_{\pi_1} + R_{e_2}(\beta + 1)}$$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{-\beta R_c}{r_{\pi_1} + R_{e_2}(\beta + 1)} = \text{Gain of CE Amplifier.}$$

CE Amplifier Input Impedance:

Resistance at base terminal: $R_b = \frac{v_{b1}}{i_{b1}} = \frac{v_{b1}}{(v_{b1}-v_{e1})/r_{\pi 1}}$

$$= \frac{r_{\pi 1}}{1 - \frac{v_{e1}}{v_{b1}}}$$

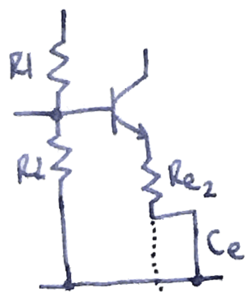
$$= r_{\pi 1}\left(1 + g_m R_{e2}\left(1 + \frac{1}{\beta}\right)\right)$$

As $g_m r_{\pi 1} = \beta$

$$\therefore R_b = r_\pi + \beta R_{e2}\left(1 + \frac{1}{\beta}\right)$$

$$= r_\pi + R_{e2}(\beta + 1)$$

Input impedance R_{in_1} is given by the parallel combination of R1, R2 and Rb



$\Rightarrow R1 // R2 // Rb$

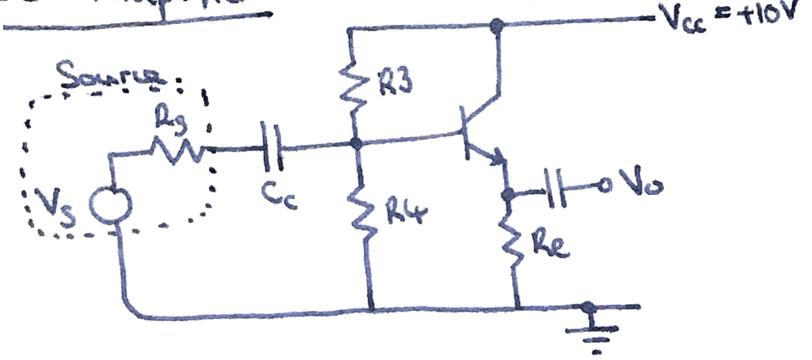
$$\therefore R_{in_1} = \left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{Rb}\right)^{-1}$$

$$R_{in_1} = \left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{r_\pi + R_{e2}(\beta+1)}\right)^{-1}$$

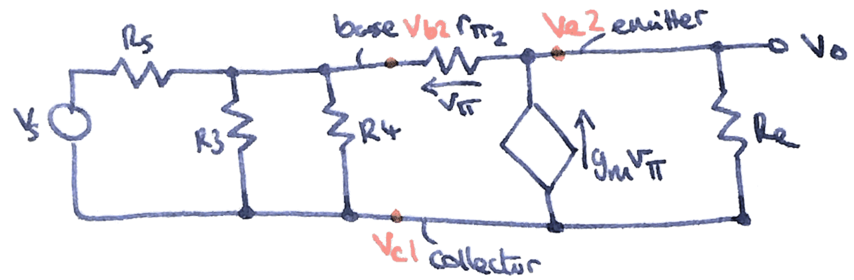
Output Impedance of CE Amplifier:

Simply the parallel combination $R_C // R_L$ where R_L is the load on the amplifier $R_{o_1} = \left(\frac{1}{R_C} + \frac{1}{R_L}\right)^{-1}$

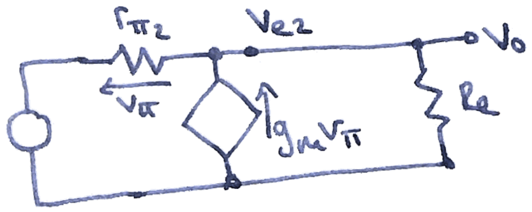
CC Amplifier:



Small Signal Equivalent:



Ignore R3, R4, Rs for gain calculation:



Applying ~~the~~ KCL at V_{e2} - emitter node:

$$g_m(V_e - V_s) + \frac{V_e - V_s}{r_{\pi 2}} + \frac{V_e}{R_e} = 0$$

using $g_m = \frac{\beta}{r_{\pi 2}}$

$$\frac{\beta}{r_\pi}(V_e - V_s) + \frac{V_e - V_s}{r_\pi} + \frac{V_e}{R_e} = 0$$

$$\beta V_e - \beta V_s + V_e - V_s + \frac{V_e r_{\pi 2}}{R_e} = 0$$

$$V_e\left(1 + \beta + \frac{r_{\pi 2}}{R_e}\right) - V_s(1+\beta) = 0$$

$$\frac{V_e}{V_s} = \frac{1+\beta}{1+\beta+\frac{r_{\pi 2}}{R_e}} = \frac{R_e(1+\beta)}{R_e(1+\beta) + r_{\pi 2}} = \text{Gain}$$

No voltage gain!

CC Amplifier Input Impedance:

By examing the input current:

$$i_{b2} = \frac{V_s - V_{e2}}{r_{\pi 2}} = \frac{V_s}{r_{\pi 2}} - \frac{V_{e2}}{r_{\pi 2}}$$

from our gain equation we can put this in terms of V_s

$$= \frac{V_s}{r_{\pi 2}} - \frac{V_s}{r_{\pi 2}}\left(\frac{R_e(1+\beta)}{R_e(1+\beta)+r_{\pi 2}}\right)$$

$$= \frac{V_s}{r_{\pi 2}}\left(1 - \frac{R_e(1+\beta)}{r_{\pi 2}+R_e(1+\beta)}\right)$$

$$= \frac{V_s}{r_{\pi 2}+R_e(1+\beta)}$$

Resistance of base input:

$$R_{b2} = \frac{V_{b2}}{i_{b2}} = \frac{V_s}{i_{b2}} = r_{\pi 2} + R_e(1+\beta)$$

Resistance at input to amplifier has a source resistance as well.

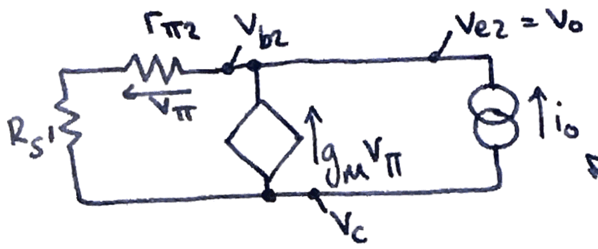
$$\therefore \; R_{in_2} = R_{b_2} \,//\, R_s$$

$$R_{in_2} = \left(\frac{1}{R_s} + \frac{1}{r_{\pi 2}+R_e(1+\beta)}\right)^{-1}$$

n.b. R_s is also in parallel with R3 and R4 but dominates the resistance. i.e. $R_s // R3 // R4 \simeq R_s$

CC Amplifier Output Impedance:

By small signal analysis we get this model:



current injected at the output.

R_s' is the effective source resistance $R_s // R3 // R4$

Applying KCL:

$$① \quad i_o + g_m v_\pi - \frac{V_o}{R_s' + r_{\pi_2}} = 0$$

$$② \quad v_\pi = -\frac{V_o r_{\pi2}}{R_s' + r_{\pi2}}$$

from ① & ②: $$i_o = V_o \left(\frac{g_m r_{\pi2}}{R_s' + r_{\pi2}} + \frac{1}{R_s' + r_{\pi2}} \right)$$

$$= \frac{(1+\beta) V_o}{R_s' + r_{\pi2}}$$

$$\therefore R_{out_2} = \frac{V_o}{i_o} = \frac{R_s' + r_\pi}{1+\beta}$$

Typically very low output impedance for a CC amplifier
↳ used as a buffer regularly as a result.