# Lab 3

# Recursion

## Recursive Functions

Recursion in C++ is a technique in which a function calls itself repeatedly until a given condition is satisfied. In other words, recursion is the process of solving a problem by breaking it down into smaller, simpler sub-problems.

### Syntax Structure of Recursion

```
return_type recursive_func {
    ....
       // Base Condition
       // Recursive Case
       ....
}
```

### Recursive Function

A function that calls itself is called a **recursive function**. When a recursive function is called, it executes a set of instructions and then calls itself to execute the same set of instructions with a smaller input. This process continues until a base case is reached, which is a condition that stops the recursion and returns a value.

### Base Condition

The base condition is the condition that is used to terminate the recursion. The recursive function will keep calling itself till the base condition is satisfied.

### Recursive Case

Recursive case is the way in which the recursive call is present in the function. Recursive case can contain multiple recursive calls, or different parameters such that at the end, the base condition is satisfied and the recursion is terminated.

**// C++ Program to calculate the sum of first N natural**

```
// numbers using recursion

#include <iostream>

using namespace std;
```

```
int nSum(int n)
{
    // base condition to terminate the recursion when N = 0
    if (n == 0) {
        return 0;
    }
    // recursive case / recursive call
    int res = n + nSum(n - 1);
    return res;
}
int main()
{
    int n = 5;
    // calling the function
    int sum = nSum(n);
    cout << "Sum = " << sum;
    return 0;
}
```

**Output**

```
Sum = 15
```

In the above example,

- **Recursive Function:** nSum() is the Recursive Function
- **Recursive Case:** The expression, **int res = n + nSum(n – 1)** is the Recursive Case.
- **Base Condition:** The base condition is **if (n == 0) { return 0;}**

## Example 1: Calculating a Factorial

Factorials are often used in statistical calculations. The factorial of n, written as n! is equal to the product of n(n-1)(n-2)...(1). For example 4! is equal to 4 × 3 × 2 × 1 = 24. There is an obvious way to do this calculation using a loop, but we can also do it recursively.

Let's consider the definition of a factorial in a slightly different way:

- if n = 0, n! is equal to 1.
- if n > 0, n! is equal to n × ((n-1)!)

An implementation of recursive factorial function

```
#include <iostream>
#include <conio.h>

using namespace std;

int fact(int n)
{  if (n == 0)
          return 1;
   else
          return n * fact(n - 1)
}

int main( )
  {
cout << fact(5) << endl;
getch(); //getch() is a function used in C/C++ programming to capture a single character input
from the user without waiting for the Enter key to be pressed.
          return 0;
  }
```

## Example 2: Reversing the String

This function takes a series of characters and outputs them in reverse order.

```
#include <iostream>
#include <conio.h>
using namespace std;

void rev( )
{

char ch;
cin.get(ch);

if (ch != '\n') // The condition if (ch != '\n') is used in C/C++ programming to check whether a
character variable ch is not a newline character (\n).

{

rev();
cout.put(ch);

}
}
int main( )
{
rev();

getch();

return 0;
}
```

```
String: Hello
Reversed String: olleh
```

## Example 3: Computing the Power

```
#include <iostream>
using namespace std;

int Power(int X, int N) {
    if (N == 0) {
        return 1;  // Base case: Any number raised to the power of 0 is 1
    }

Else
 {
        return Power(X, N - 1) * X;  // Recursive case: Multiply X with the result of
Power(X, N-1)

}
}

int main() {
    int base, exponent;
    cout << "Enter base number: ";
    cin >> base;
    cout << "Enter exponent: ";
    cin >> exponent;

    cout << base << "^" << exponent << " = " << Power(base, exponent) << endl;

    return 0;
```

- **Base Case**: If the exponent N is 0, the function returns 1 (as any number raised to the power of 0 is 1).

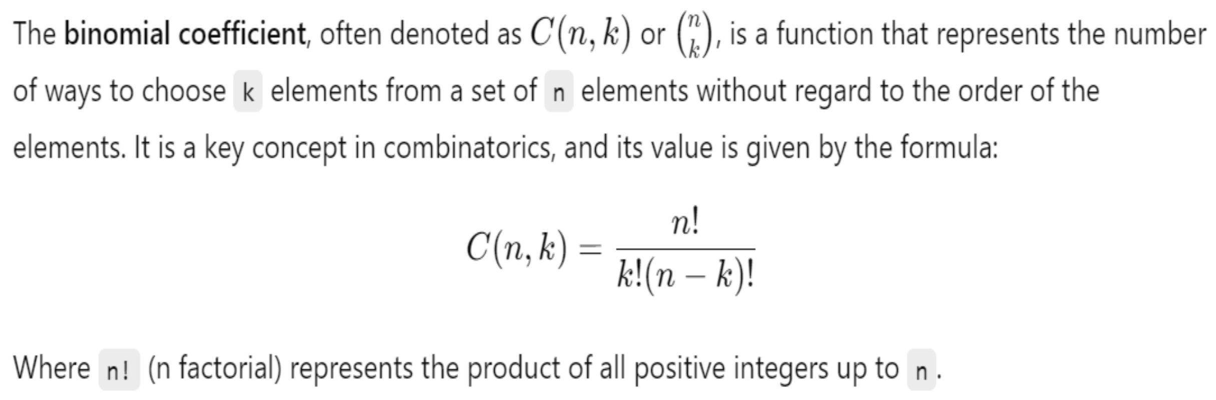
- **Recursive Case**: If N > 0, it calls itself with N - 1 and multiplies the result by X to get the result of X^N.

*For example, calling Power(2, 3) will result in 2 * 2 * 2 = 8*

If you input `2` for the base and `3` for the exponent, the recursive calls will look like this:

- `power(2, 3)` returns `2 * power(2, 2)`
- `power(2, 2)` returns `2 * power(2, 1)`
- `power(2, 1)` returns `2 * power(2, 0)`
- `power(2, 0)` returns `1` (base case)

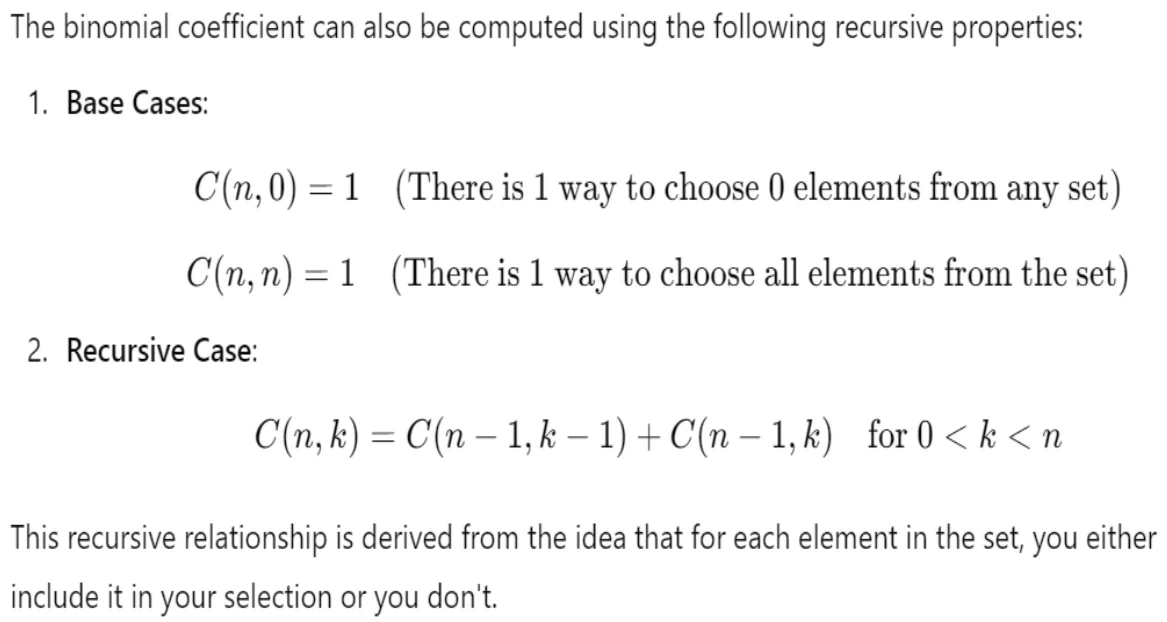
## Example 4:Recursion to compute binomial coefficients C(n, k)



The **binomial coefficient**, often denoted as $C(n,k)$ or $\binom{n}{k}$, is a function that represents the number of ways to choose `k` elements from a set of `n` elements without regard to the order of the elements. It is a key concept in combinatorics, and its value is given by the formula:

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

Where `n!` (n factorial) represents the product of all positive integers up to `n`.



The binomial coefficient can also be computed using the following recursive properties:

1. **Base Cases**:

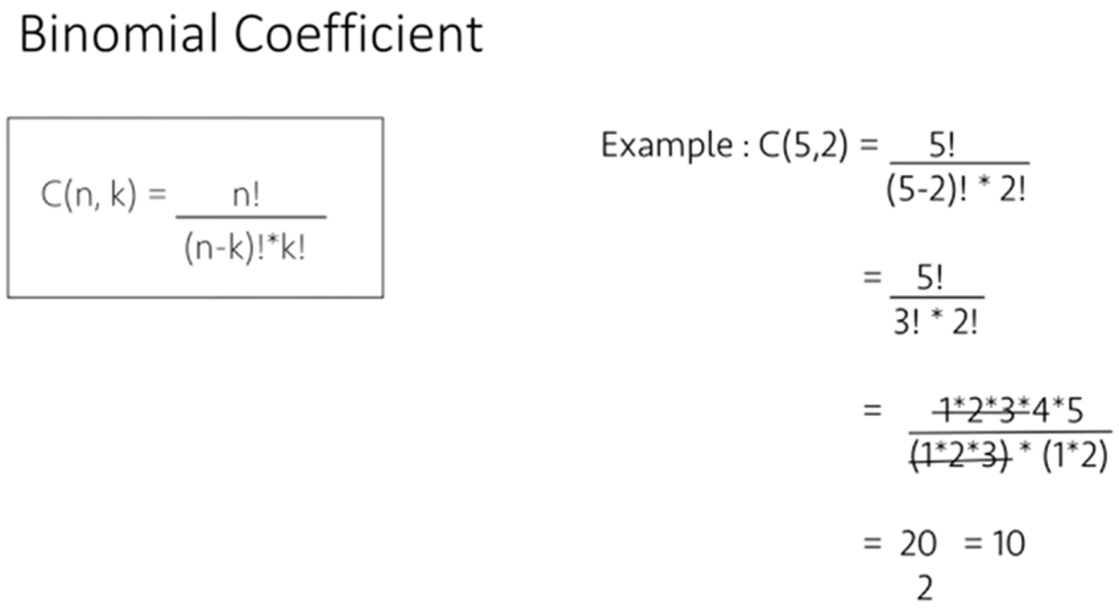
$$C(n,0) = 1 \quad \text{(There is 1 way to choose 0 elements from any set)}$$

$$C(n,n) = 1 \quad \text{(There is 1 way to choose all elements from the set)}$$

2. **Recursive Case**:

$$C(n,k) = C(n-1,k-1) + C(n-1,k) \quad \text{for } 0 < k < n$$

This recursive relationship is derived from the idea that for each element in the set, you either include it in your selection or you don't.



## Binomial Coefficient

$$\boxed{C(n, k) = \frac{n!}{(n-k)!*k!}}$$

Example : $C(5,2) = \frac{5!}{(5-2)! * 2!}$

$$= \frac{5!}{3! * 2!}$$

$$= \frac{\cancel{1*2*3*}4*5}{\cancel{(1*2*3)} * (1*2)}$$

$$= \frac{20}{2} = 10$$

If you input `n = 5` and `k = 2`, the recursive calls would be as follows:

- `C(5, 2) = C(4, 1) + C(4, 2)`
- `C(4, 1) = C(3, 0) + C(3, 1)`
- `C(4, 2) = C(3, 1) + C(3, 2)`
- And so on...

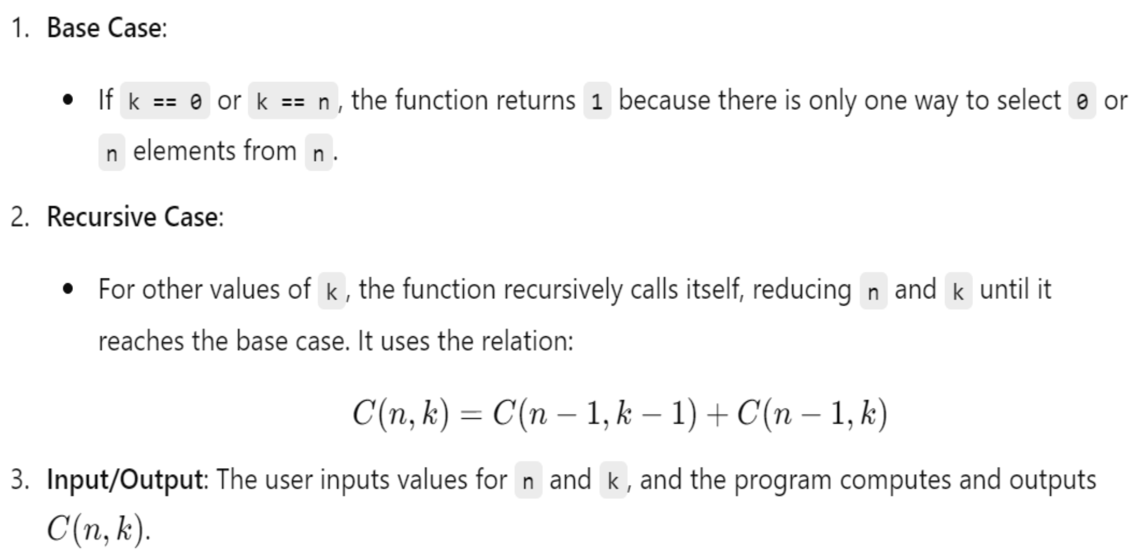
The final result for C(5,2)C(5, 2)C(5,2) would be `10`, since there are 10 ways to choose 2 elements from a set of 5.

## Lab Tasks:

1. **Write a function in C++ using Recursion to print numbers from *n* to 0.**
2. **Write a function in C++ using Recursion to compute binomial coefficients C(n,k) using the recursive definition:**

**C(n,n) = 1**
**C(n,0) = 1**
**C(n,k) = C(n-1, k-1) + C(n-1,k) for (0<k<n) and n>1**

**Hint:**



1. **Base Case**:
   - If `k == 0` or `k == n`, the function returns `1` because there is only one way to select `0` or `n` elements from `n`.
2. **Recursive Case**:
   - For other values of `k`, the function recursively calls itself, reducing `n` and `k` until it reaches the base case. It uses the relation:

$$C(n,k) = C(n-1,k-1) + C(n-1,k)$$

3. **Input/Output**: The user inputs values for `n` and `k`, and the program computes and outputs $C(n,k)$.

3. **Write a function in C++ using Recursion to check if a number *n* is prime. (You have to check whether *n* is divisible by any number below *n*)**

**Hint:**

• **Recursive Function:**

- **`n <= 0` check**: If the number is non-positive (i.e., less than or equal to 0), return `false` because prime numbers are positive integers greater than 1.
- **`i * i > n` check**: When the square of `i` exceeds `n`, it means all possible divisors have been checked up to the square root of `n`. Hence, the number is prime.
- **`n % i == 0` check**: If `n` is divisible by `i`, the number is not prime, and the function returns `false`.
- **Recursive call**: The function checks the next possible divisor (`i + 1`) until it reaches the base cases.

- **Main Function:**
  - Prompts the user to enter a number.
  - Calls the `isPrimeNumber` function to check if the number is prime.
  - Displays the result based on whether the function returns `true` or `false`.