MECH3750

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Tutorial – Focus on numerical theory.

Quizzes – Friday contacts W3, 5, 7, 9, 11

Resources → Blackboard, Github

Taylor Expansions.

- Making approximations for a function near to a known point.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^n(a)}{n!}(x-a)^n + R_n$$

know point : a

some point x near to a.

$$R_n = \frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1}$$

$$a \leq z \leq x$$

$\left.\begin{array}{l} O((x-a)^3) \\ \text{H.O.T} \end{array}\right\}$ Describe extra terms beyond expansion of interest.

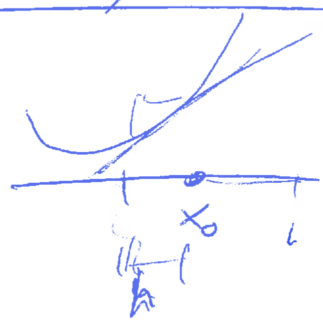
$f(x,y)$ or $f(\underline{x})$

$$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$+ \frac{1}{2}\Big(f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b)$$
$$+ f_{yy}(a,b)(y-b)^2\Big) + H.O.T.$$

$$f(\underline{x}) = f(\underline{x}^0) + \underline{\nabla} f^T(\underline{x}-\underline{x}^0) + \frac{1}{2}(\underline{x}-\underline{x}^0)^T H(\underline{x}-\underline{x}^0)$$
$$+ HOT$$

$$H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

Newton's Method. $\rightarrow f(x) = 0$



$$f(x_0+h) = f(x_0) + f'(x_0)h + \dots$$

for soln $= 0$ $\rightarrow$ $h = \dfrac{-f(x_0)}{f'(x_0)}$

$$x_1 = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)}$$

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$$f'(x_0) \rightarrow \underline{f}'(\underline{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \ddots & \vdots \\ \vdots & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

$$J \rightarrow$$

$$\underline{x}^{n+1} = \underline{x}^n - \left[J(\underline{x}^n)\right]^{-1} f(\underline{x}_n)$$

Stop iterating when $f(\underline{x}^{n+1}) < \varepsilon = \text{small}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \rightarrow \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

$$f(x,y) = f(a,b) + \dots + O(h^3).$$

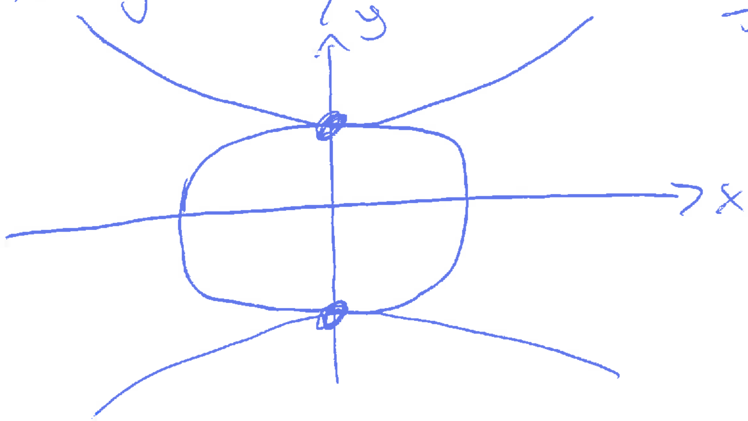
$$h = (x,y) - (a,b)$$

Error $\sim h^3$

$$h = \sqrt{(x-a)^2 + (y-b)^2}$$

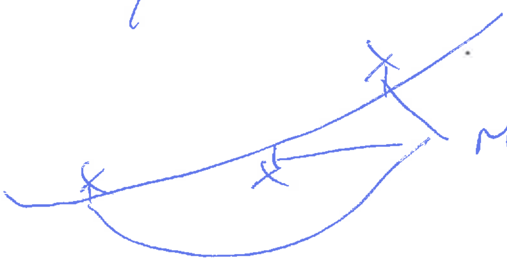
Newton's Method.

key: write your equation s.t. $f(\underline{x}) = 0$



Least Squares.

Principle — have some data → fit a known equation to it



Minimise sum of errors.

$$\text{Data} = (x_i, f_i)$$

$$y(x) = a_0 + a_1 x$$

$$E = \sum (a_0 + a_1 x_i - f_i)^2$$

Minimize E → derive & set $= 0$

$$\frac{\partial E}{\partial a_0} = \sum_i 2(a_0 + a_1 x_i - f_i) = 0$$

$$\frac{\partial E}{\partial a_1} = \sum_i 2x_i(a_0 + a_1 x_i - f_i) = 0$$

$$\sum 1(a_0 + a_1 x_i - f_i) = 0$$

$$\sum x_i(a_0 + a_1 x_i - f_i) = 0$$

$$\sum_i 1(a_0 + a_1 x_i) = \sum_i 1 f_i$$

$$\sum_i x_i(a_0 + a_1 x_i) = \sum_i x_i f_i$$

$$P\begin{pmatrix} a_0 + a_1 x_1 \\ a_0 + a_1 x_2 \\ a_0 + a_1 x_3 \\ \vdots \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & & x_n \end{pmatrix}}_{P} \underline{f}$$

$$P\begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = P\underline{f} \Rightarrow PP^T\underline{a} = P\underline{f}$$

$$\underline{a} = (PP^T)^{-1} P\underline{f}$$