MECH3750 - Tutorial 4 (week 5)

Question 1.
Show that:

$$\overline{\exp(iy)} = \exp(-iy)$$

$$\sin y = \frac{e^{iy} - e^{-iy}}{2i}$$

$$\cos y = \frac{e^{iy} + e^{-iy}}{2}$$

Question 2.

Using the complex inner product defined as:

$$(u, v) = \sum_{i=0}^{4} \overline{u_i} v_i$$

consider the vectors:

$$q_n^{(k)} = \exp\left(ik\frac{2\pi n}{M}\right) \qquad n = 0, 1, 2, 3$$

(a) Write $q^{(k)}$ explicitly for $k = 0, 1, 2, 3$.

(b) Use the inner product to find $\|q^{(k)}\|$ for $k = 0, 1, 2, 3$.

(c) Verify: $(q^{(0)}, q^{(1)}) = 0$; $(q^{(2)}, q^{(3)}) = 0$; $(q^{(0)}, q^{(2)}) = 0$

Question 3.

In our interpretation of the DFT, the values a_{p_k} represent the coefficients of the vector:

$$p_n^{(k)} = \frac{1}{N} \exp\left(i\frac{2\pi nk}{N}\right)$$

in the signal f_n for $k = 0, 1, \ldots, N-1$.

Verify that $p_n^{(N-1)} = p_n^{(-1)}$ and also $p_n^{(N-m)} = p_n^{(-m)}$. This is important for interpreting the values of the DFT for large k.

Question 4.

Find the DFT of:

(a) $\boldsymbol{f} = (1,\ 2,\ 0,\ 1)$

(b) $\boldsymbol{f} = (1,\ 1,\ \ldots,\ 1)$, for $N = 8$

Question 5.

Show that the DFT of: $\boldsymbol{f} = (f_0,\ f_1,\ \ldots,\ f_7)$ for:

$$f_n = \sin \frac{2\pi n}{8}$$

Is given by $(0,\ A, 0,\ 0,\ 0,\ 0, B)$, and determine $A,\ B$. You may use the orthogonality properties of $\boldsymbol{p}_n^{(k)}$.

Question 6.

The DFT of a signal $\boldsymbol{f}$ is: $(8,\ 4-8i,\ 2,\ -i,\ 0,\ i,\ 2,\ 4+8i)$

Determine the original signal $\boldsymbol{f}$.

Hint: Use the property that $p_n^{N-m} = p_n^{-m}$