MECH3750 - W3

Quiz 1 - Friday 9-10AM 8-139

→ Sample Quiz on BB.

→ 4Q @ 10marks each } 30 mins

Content: W1 & 2 material

→ Taylor Series } Multi-dimensional.

→ Newton's Method

→ Least Squares → data or function based

→ discrete / ctg.

- Theory not Practical (Python).

Least Squares - cont.

Last week → fit polynomial to discrete data.

1) define Error squared term

2) derive W.RT. coefficients → set to zero

3) Write in Matrix Form

4) Rearrange to solve for coefficients.

Solving → non-linear functions eg $\sin(ax+b)$

$\frac{\partial E}{\partial a}$ is non-linear → use Newton's Method.

Fitting also useful when approximating complex functions w/ simpler ones.

ie $E = \sum_a^b (\alpha_0 + \alpha_1 x_i - f_i)^2 \rightarrow \int_a^b (\alpha_0 + \alpha_1 x - f(x))^2 dx$

Discrete Continuous.

General $E = \int_a^b \left(\sum_0^N \alpha_i p_i(x) - f(x)\right)^2 dx$

Polynomials $p_0 = 1 \quad p_1 = x \quad p_2 = x^2 \ldots$

Trig $p_0 = \sin(0x) \quad p_1 = \sin(x) \quad p_2 = \sin(2x) \ldots$

→ rearrange to $A\underline{\alpha} = \underline{b}$

$$A = \begin{pmatrix} \int p_0 p_0 dx & \int p_0 p_1 & \int p_0 p_2 \cdots \\ \int p_1 p_0 dx & \int p_1 p_1 & \cdots \\ \vdots & \vdots & \ddots \\ & & \int p_N p_N \end{pmatrix}, b = \begin{pmatrix} \int f p_0 dx \\ \int f p_1 dx \\ \vdots \\ \int f p_N dx \end{pmatrix}$$

Trig $\int \sin(nx)\sin(mx)\,dx = \begin{cases} 0 & m \neq n \\ \pi/2 & m = n \end{cases}$

$m, n \in \mathbb{Z}$

Fourier Analysis → viz Vibrations & harmonics in MECH3200.

W3 lectures.

Inner product : operation giving a scalar.

(u, v)

Must hold
1) $(u+v, w) = (u, w) + (v, w)$
2) $(\alpha u, w) = \alpha (u, v)$
3) $(v, w) = (w, v)$
4) $(u, u) \geq 0$ $\quad u = 0$ only if $u = 0$.

Norm $\|u\| = \sqrt{(u, u)}$

Distance $d(u, v) = \|u - v\| = \sqrt{(u-v, u-v)}$

Orthogonal $(u, v) = 0$ $\rightsquigarrow$ $\|u\| = \|v\| = 1$