Fourier Series.

$$f(x) \simeq \frac{a_0}{2} + \sum_{\gamma=0}^{N} a_\gamma \cos(\gamma x) + b_\gamma \sin(\gamma x)$$

on $[-\pi, \pi]$.

$$\simeq a_0 \cdot \tfrac{1}{2} + a_1 \cos(x) + b_1 \sin(x) + \dots + a_N \cos(Nx) + b_N \sin(Nx).$$

$$= \alpha_0 P_0 + \alpha_1 P_1 + \alpha_2 P_2 + \dots + \alpha_{(2N-1)} P_{(2N-1)} + \alpha_{2N} P_{2N}$$

Normal eqns
$\hookrightarrow$

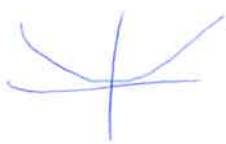
$$\begin{bmatrix} \pi/2 & & & \\ & \pi & & 0 \\ & & \pi & \\ 0 & & & \pi \end{bmatrix} \begin{bmatrix} \alpha \end{bmatrix} = \begin{bmatrix} \int_{-\pi}^{\pi} \tfrac{1}{2} f(x)\,dx \\ \int_{-\pi}^{\pi} \cos(nx) f(x)\,dx \\ \int_{-\pi}^{\pi} \sin(nx) f(x)\,dx \end{bmatrix}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(nx) f(x)\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(nx) f(x)\,dx.$$

ODD 

EVEN 

ODD $f(x) = -f(-x)$

EVEN $f(x) = f(-x)$.

$$\int_{-a}^{a} [\text{ODD}]\,dx = 0$$

$O \cdot E = O$

f even $\rightarrow b_n = 0$

f odd $\rightarrow a_n = 0$

Recall $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

F.S. $f(x) = \sum_{\gamma=-N}^{N} c_\gamma e^{i\gamma x}$

w/ $c_\gamma = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-i\gamma x}\, dx$

Discrete Fourier Transforms

& for $\underline{u}, \underline{v} \in \mathbb{C}$ $\quad (\underline{u},\underline{v}) = \bar{\underline{u}} \cdot \underline{v}$

$A = a+bi \; ; \; \bar{A} = a-bi$

• Fit F.S. to set of discrete data

$p_n^{(k)} = e^{ikx_n} / N \qquad x_n = \frac{2\pi n}{N}, \quad k = 0, \dots, N-1$

Data = $\underline{f} = (f_0, \dots, f_{N-1})^T$

$\underline{p}^{(k)} = \frac{1}{N}\left(e^{ikx_0}, \dots, e^{ikx_{N-1}}\right)$

Approx data w/ L.S. $y = \alpha_0 p_k^0 + \alpha_1 p_k^1 + \dots$

$$\begin{bmatrix} p_k^0 p^0 & & O \\ & p^1 p^1 & \\ O & & \ddots \end{bmatrix} \begin{bmatrix} \alpha \end{bmatrix} = \begin{bmatrix} (p^0, f) \\ (p^1, f) \\ \vdots \end{bmatrix}$$

DFT = $\underline{\alpha}$ & $\underline{\alpha_k = \sum_{n=0}^{N-1} e^{-ikx_n} f_n}$

if f_n is real.

$Re(\alpha_k)$ are symmetric about $N/2$

$Im(\alpha_k)$ are antisymmetric " ".

$$\alpha_{N/2} = 0$$