MECH 3750 Tutorial 3 17/08

Theory

Inner products:

	Discrete	Continuous.
i) $(u+v,w)=(u,w)+(v,w)$	$(u,v)=\sum_{i=1}^{N} u_i v_i$,	$(u,v)=\int_a^b u(x)v(x)\,dx$,
ii) $(\alpha v,w)=\alpha(v,w)$		
iii) $(v,w)=(w,v)$	$u,v \in \mathbb{R}^N$	$u,v \in [a,b]$
iv) $(u,u)\geq 0$ \& $(u,u)=0$ iff $u=\underline{0}$		

Norm: $\|u\| = \sqrt{(u,u)}$ $\quad\|u\| = \sqrt{\int_a^b u^2(x)\,dx}$

Distance: $d=\|u-v\|=\sqrt{(u-v,u-v)}$ $\quad d=\sqrt{\int_a^b [u(x)-v(x)]^2\,dx}$

Notes;

- vectors u,v are <u>orthogonal</u> if $(u,v)=0$
- vectors u,v are <u>orthonormal</u> if $(u,v)=0$ \& $\|u\|=1$, $\|v\|=1$

- when dealing with continuous functions, u,v can be orthogonal under a <u>given</u> norm $([a_1,b_1])$ but not orthogonal under another $([a_2,b_2])$

→ This gives us another method to approximate functions
- as before (least sq.) we want to minimize distance (error)
- use d^2, as allows us to use calculus.

ie. approximate f by L_0, L_1; $y=a_0L_0+a_1L_1$

minimize $d^2(y,f)=(y-f,y-f)=\int_a^b (a_0L_0+a_1L_1-f)^2\,dx$

$\dfrac{\partial d^2(y,f)}{\partial a_0}=0$, $\dfrac{\partial d^2(y,f)}{\partial a_1}=0$ $\Rightarrow$ separate equations, use matrices to solve simultaneous equations to get a_0, a_1.

Example use the first 3 Legendre Polynomials to approximate $\sin(x)$ on $[0, \pi/2]$.

Legendre Polynomials:

$$L_0 = 1$$
$$L_1 = 2x - 1$$
$$L_2 = 6x^2 - 6x + 1$$

$$y = a_0L_0 + a_1L_1 + a_2L_2$$
$$f = \sin(x)$$

distance; $d^2(y,f) = \int_0^{\pi/2} (a_0L_0 + a_1L_1 + a_2L_2 - f)^2 \, dx$

$$d^2(y,f) = \int_0^{\pi/2} \left[a_0 + a_1(2x-1) + a_2(6x^2-6x+1) - \sin(x)\right]^2 dx$$

$$\frac{\partial d^2(y,f)}{\partial a_0} = 0 = 2\int_0^{\pi/2} \left[a_0 + a_1(2x-1) + a_2(6x^2-6x+1) - \sin(x)\right] dx$$

$$\frac{\partial d^2(y,f)}{\partial a_1} = 0 = 2\int_0^{\pi/2}(2x-1)\left[a_0 + a_1(2x-1) + a_2(6x^2-6x+1) - \sin(x)\right] dx$$

$$\frac{\partial d^2(y,f)}{\partial a_2} = 0 = 2\int_0^{\pi/2}(6x^2-6x+1)\left[a_0 + a_1(2x-1) + a_2(6x^2-6x+1) - \sin(x)\right] dx$$

$$\Rightarrow \begin{bmatrix} \int_0^{\pi/2} 1 \, dx & \int_0^{\pi/2} (2x-1) \, dx & \int_0^{\pi/2} (6x^2-6x+1) \, dx \\ \int_0^{\pi/2} (2x-1) \, dx & \int_0^{\pi/2} (2x-1)^2 \, dx & \int_0^{\pi/2} (2x-1)(6x^2-6x+1) \, dx \\ \int_0^{\pi/2} (6x^2-6x+1) \, dx & \int_0^{\pi/2} (2x-1)(6x^2-6x+1) \, dx & \int_0^{\pi/2} (6x^2-6x+1)^2 \, dx \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$= \begin{bmatrix} \int_0^{\pi/2} \sin(x) \, dx \\ \int_0^{\pi/2} (2x-1)\sin(x) \, dx \\ \int_0^{\pi/2} (6x^2-6x+1)\sin(x) \, dx \end{bmatrix}$$

$\Rightarrow$ evaluate integrals

$\Rightarrow$ solve for a_0, a_1, a_2

$\Rightarrow$ sub back in to: $y = a_0L_0 + a_1L_1 + a_2L_2$