MECH3750 - Tutorial 1

Question 1.

Show that:

$$\frac{u(x+h)-2u(x)+u(x-h)}{h^2} = u^{''}(x) + \frac{u^4(\zeta_1)h^2}{24} + \frac{u^4(\zeta_2)h^2}{24}$$

For $x \leq \zeta_1 \leq x+h, \quad x-h \leq \zeta_2 \leq x$

Question 2.

(a) Find the Terms in the Taylor Series, up to quadratic order of the function $f(x,y) = \ln\left(x^2+y^2\right)$ about the point $(x,y) = (1,1)$.

(b) Evaluate both $f(x,y)$ and your Taylor series approximation at the points $(x,y) = (1.1, 0.9)$ and $(x,y) = (0.2, 0.2)$. Check the order of the error at each point. Is it what you expect?

Question 3.

Demonstrate the the following expression is a fourth order centered approximation of the second derivative $u^{''}(x)$

$$u^{''}(x) \approx \frac{-u_{i-2}+16u_{i-1}-30u_i+16u_{u+1}-u_{i+2}}{12h^2}$$

Question 4.

4.1

$$x^4 + y^4 = 1$$
$$x^2 - y^2 = -1$$

(a) Using the initial guess $\mathbf{x_0} = (x_0, y_0) = (1,1)$, what is the approxiation produced by one step of Newton's method?
(b) Using a graph in the xy plane, how many solutions do we expect to this problem?
(c) For what initial guesses $\mathbf{x_0}$ will Newton's method fail due to the Jacobian matrix $J(\mathbf{x_0})$ being singular?

4.2

$$x^2 + cos(y) = 5$$
$$sinh(x) + sin(y) = 4$$

(a) What is the vector function $F(x,y)$ for which we require roots?
(b) Evaluate the Jacobian ($J(\mathbf{x})$) for this system.
(c) Let $\mathbf{x_0} = (1,1)$ and $h = 0.1$, evaluate $J(\mathbf{x})$ using a forward difference.
(d) Using your answer from (c), compute $\mathbf{x_1}$ and $\mathbf{x_2}$.