The University of Manchester

School of Electrical and Electronic Engineering

Embedded Systems Project

DESIGN REPORT #1

Title: Electric Motor Characterisation and Gearbox Design

Group Number: 18

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Date: 15/10/2016

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1. Introduction

The overall task requested is the construction of a line-following robot buggy for a race around a white line track.

The aim of this experimental investigation is to acquire a comprehensive low-level understanding of the mechanical requirements and capabilities of a line following buggy by analysing each component and its mechanical role in the overall system.

The following measurements were necessary to ascertain component performance in simulated and real world environments. Motor constants were found, torque and speed characteristics measured, load readings taken, gearbox calculated and gear ratio specified.

Based on the data collected and post-processing calculations, final motor and gear assembly was designed in the goal of performance optimisation for a well-rounded embedded system.

2. Motor characterisation

2.1 Armature resistance measurements

In order to calculate armature resistance the following equations are considered:

$$I = \frac{V - V_b - K_E \times \omega}{R} \qquad -(1)$$

$$I = \frac{V - Vb}{R} \qquad -(2)$$

$$where\ I\ is\ current, V\ is\ voltage, V_b\ is\ brush\ voltage, K_E\ is\ emf\ constant, \omega\ is\ angular\ speed, R\ is\ resistance$$

At standstill $\omega = 0\,rpm$ so equation (1) can be simplified to (2) by substituting $\omega = 0$. From (2) we can find R by varying the supply voltage until the current limit is reached. The collected data is shown in table 1. To determine a more accurate value of resistance as well as estimate its uncertainty a standard VI plot is presented in figure 1. This will also later help deduce the value of voltage drop in the brushes.

Voltage (V)	Current (A)	Resistance calculated (Ω)
4.6	1.364	3.37
4.1	1.180	3.47
3.6	1.070	3.36
3.1	0.931	3.33
2.6	0.815	3.19
2.1	0.629	3.34
1.6	0.428	3.74
1.1	0.312	3.53

Table 1 - Table of results Motor Voltage vs Current

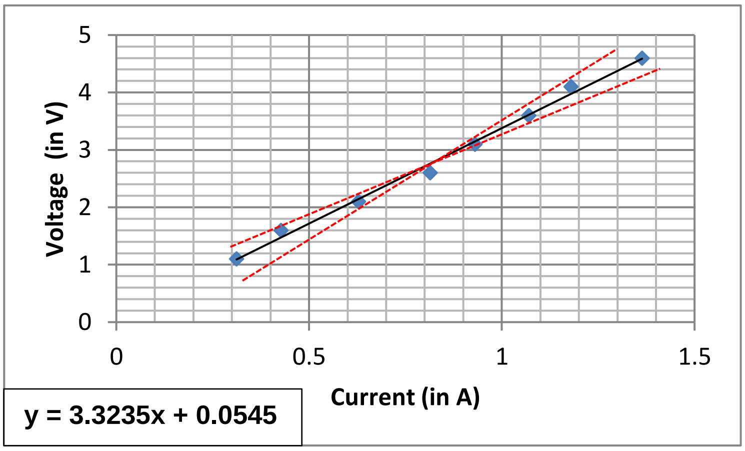

Figure 1 – Graph showing change in motor voltage over current (VI plot)

The relationship presented in figure 1 is linear with a high correlation as shown by a trendline within tight maximal and minimal gradient error lines.

The required resistance is given by the gradient of this line: $R \approx 3.3235\Omega$. The gradient error is calculated to be: $\sigma(R) \approx 7.7\%$. This gives a value of resistance of: $R = 3.3 \pm 0.3\ \Omega$. Voltage drop in the brushes (V_b) can be calculated from the y-intercept of the trend produced by the measured data. Per the equation of the line y: $V_b = 0.055 \pm 0.004$ V.

While the precision is limited by experimental uncertainty, the accuracy remains high as, with our uncertainty of 7.7%, the measured resistance falls within the datasheet's literature value of $R = 3\Omega$. [1]

Random sources of error reducing precision arose from the fluctuating current of the power supply, manufacturers uncertainty of the DMM and datasheet tolerances of the motor's performance.

Systematic sources of error reducing accuracy originated from heat generated by the immobilised motor. Since heat affects the physical characteristics of a given material the measured value of resistance is likely to be systematically altered by the experimental setup. Quantification of this error is possible by data analysis. With a supply voltage of 3V and a corresponding current is 0.951A, the multiplication of these values yields a power of 2.85W which reveals a significant heating effect.

These systematic setup-based errors were minimised by allowing time for the motor to cool as well as fine tuning of other apparatus for error minimisation.

2.2 Torque and speed measurements

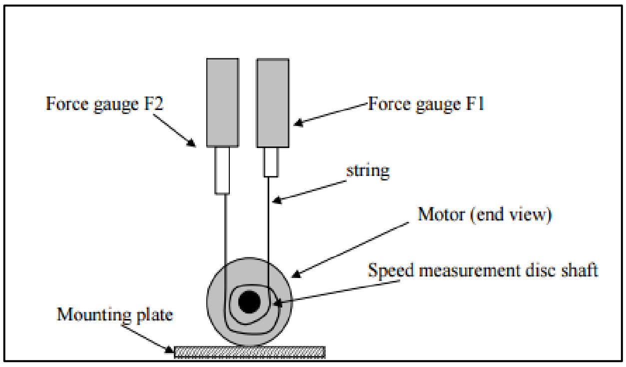

Figure 2 – Experimental setup diagram for torque measurements [2]

The measurement of the rotational force that produces torque was carried out using a string and two force gauges. The motor and the aforementioned components were set up as shown below.

The difference between the forces produced by the motor, read from force gauges F1 and F2, with a stalled motor were used to find the relation between the torque (T) and torque constant (K_T) using the following equation:

$$T = K_T I = (F_1 - F_2)\frac{d}{2} \qquad - (3)$$

$$where\ T\ is\ torque, K_T\ is\ torque\ constant, I\ is\ current, F\ is\ force, d\ is\ shaft\ diameter$$

Another setup was used to acquire the speed of the motor using an optical tachometer as shown aside.

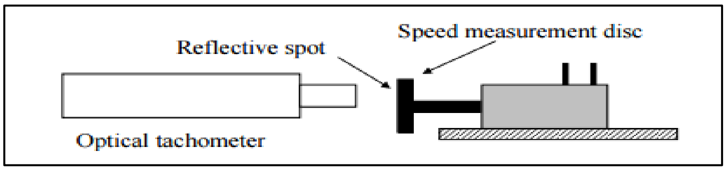

Figure 3 – Experimental setup diagram for speed measurements [2]

We know from Kirchhoff's Voltage Law [3] that motor emf is given by:

$$E = V - Vb - I \times R \qquad - (4)$$

$$where\ E\ is\ motor\ emf, V\ is\ volage, V_b\ is\ brush\ voltage, I\ is\ current, R\ is\ resistance$$

We know that shaft diameter $d \approx 0.01023$, motor armature resistance $R \approx 3.3\Omega$, voltage drop on brushes $Vb \approx 0.0545$, and from experiments in figures 2 and 3, we get:

Motor current I (A)	F1(N)	F2(N)	Motor supply voltage V(V)	Motor speed (RPM)	Motor speed (rads/s)	Torque $\times 10^3$ (kNm)	Motor emf (V)
1.4	2.5	0.6	5.2	1584	165.88	9.7	0.5255
1.2	2.1	0.5	5.2	2135	223.58	8.2	1.1855
1.0	1.65	0.3	5.2	2719	284.73	6.9	1.8455
0.8	1.3	0.3	5.2	3065	320.97	5.1	2.5055
0.6	0.9	0.2	5.2	3565	373.33	3.6	3.1655
0.4	0.5	0.1	5.2	4046	423.70	2.0	3.8255
0.2	0.1	0.0	5.1	4540	475.00	0.5	4.3855
0.1	0.0	0.0	5.0	4815	504.23	0.0	4.6155

Table 2 – Table of results for torque and speed measurements

As voltage was imposed to be within a given range, resistance (R) and brush voltage (V_b) are assumed to be the same as in the previous measurements (see Figure 1), thus the values of R and V_b remain unchanged.

The torque is related to torque constant by the following equation:

$$T = K_T I - T_{friction} \qquad -(5)$$

$$\textit{where } T \textit{ is torque}, K_T \textit{ is torque constant}, I \textit{ is current}, T_{friction} \textit{ is the back torque due to friction}$$

Note that $T_{friction}$ is so small that it can be neglected which means we can obtain K_T by plotting torque against current. In this case, K_T will be the gradient of the trendline.

From figure 4, the gradient of the graph gives us: $K_T = 0.0076 \pm 0.0008\ \mathrm{Nm/A}$.

Maximum stall torque and K_E are found from the y-intercept and gradient of the torque/speed in figure 5 as motor speed must be 0 to produce maximum torque.

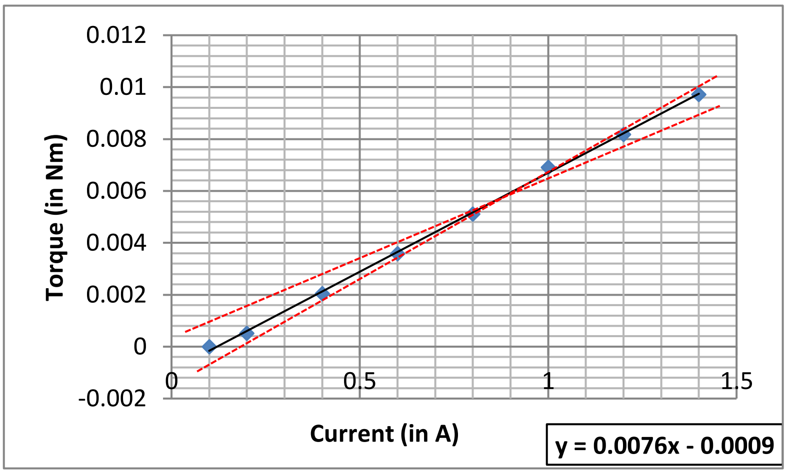

Figure 4 - Graph of motor torque over current

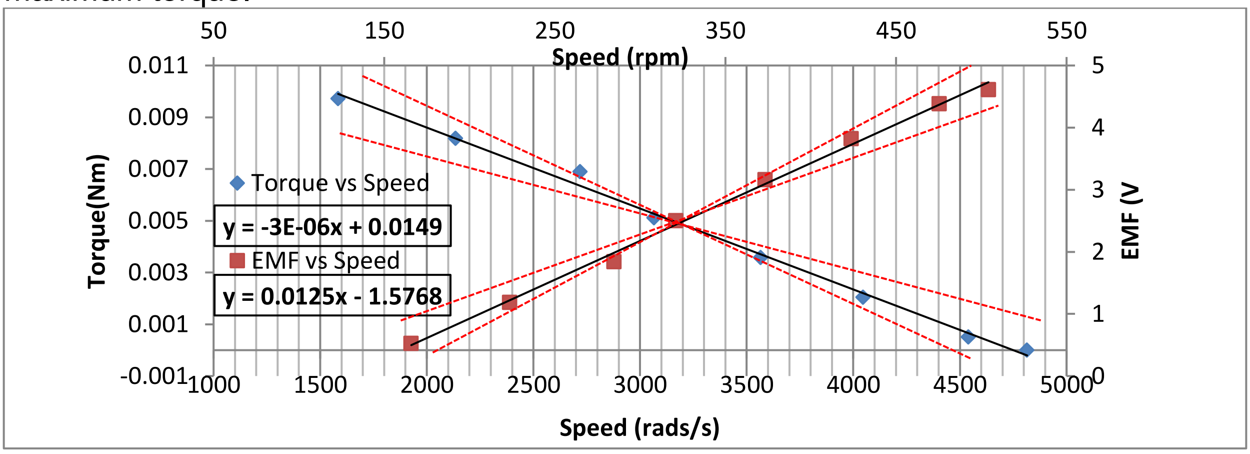

Figure 5 - Graph of motor torque over motor speed (in RMP and rad/s)

From figure 5, the y-intercept of speed in RPM gives a maximum stall torque of: $T = 0.015 \pm 0.002$ Nm while the gradient of speed in rad/s yields: $KE = 0.013 \pm 0.002$ Vs/rads. At this point it is important to discuss the discrepancy between K_T and K_E since for ideal motors these values are equal, but a difference of 70% was measured.

K_T expresses torque per unit current while K_E measures motor voltage over speed. Therefore, it is apparent that the losses will be much greater at high current as well as at high torque in comparison to when the motor spins freely. Another main reason for this difference is loss of power in the motor due the internal resistance of the motor causing heat. The effects of a rise in temperature cause an increase in the internal resistance of the motor resulting in further losses. Overall, there are two main losses which occur in the motor. The first is mechanical losses such as friction of the brushes, or losses occurring from the friction due to moving parts. The second is electrical losses such as power and voltage drop due to changing internal resistance.
The maximum motor torque is to be increased in later sections using a gearbox.

3. Load Measurements

3.1 Sloped load measurements

To establish torque requirements for the motor, load measurements are performed using a basic force experiment where a force gauge is used to manually pull the buggy up a ramp until force equilibrium occurs as shown in figure 6.

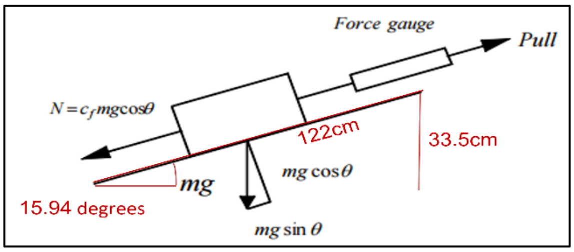

Figure 6 – Slope load measurements experimental setup [2]

The buggy was loaded with a range of weights to simulate possible weights of the final buggy's components. Measurements of the pull forces were taken producing the following data, shown in figure 7 and table 3:

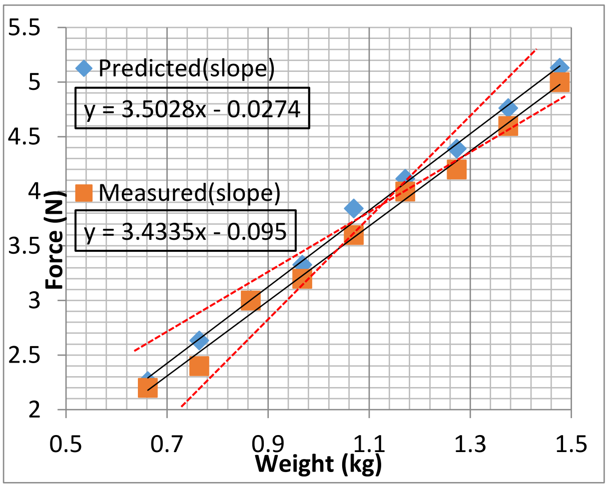

Figure 7 - Graph of force predicted vs measured results for slope

Mass m (kg)	Friction Coefficient F_c	Predicted Drive force (slope) (N)	Force measured (slope) (N)
0.66	0.077	2.26	2.20
0.76	0.080	2.63	2.40
0.87	0.082	3.00	3.00
0.97	0.079	3.33	3.75
1.07	0.100	3.84	3.60
1.17	0.090	4.12	4.00
1.27	0.080	4.39	4.20
1.38	0.080	4.76	4.60
1.48	0.080	5.13	5.00

Table 3 - Table of results for sloped load measurements

Predicted results were calculated via the use of the coefficient of friction found from an experiment of the buggy on a flat surface using the following equation:

$$C_f = \frac{N}{mg} \qquad - (6)$$

$$where\ C_f\ is\ friction\ coefficient, N\ is\ drag\ force, m\ is\ mass, g\ is\ gravitational\ acceleration$$

Figure 7 shows highly correlated linear behaviour for both predicted and measured plots, confirming larger weights require a larger force to be driven up the slope.

There are however some sources of random error, as indicated by the gradient error lines, mostly due to measurement precision and manufacturer's uncertainty of instruments' limited resolution.

As for systematic errors, the graph suggests an important discrepancy as measured forces are generally lower than predicted forces. This error may need to be considered when selecting the gear ratio as an allowance should be made for the gears to provide an extra safety margin for required torque. This accuracy error could have occurred for several reasons but it seems that it was mostly due to the predicted drag being assumed to be linear, omitting higher order drag and more complex and irregular behaviour of the non-perfect surface experimented on.

There is also a slight difference in the gradients of the lines showing the measured results to be roughly 0.07 less than predicted which is acceptably within experimental uncertainty and assumed to be insignificant.

3.2 Flat load measurements

Figure 8 below demonstrates the same method was used for flat and sloped ramps. The coefficient of friction for the flat ramp was again calculated from equation 5 and used to predict the drive force needed on the slope.

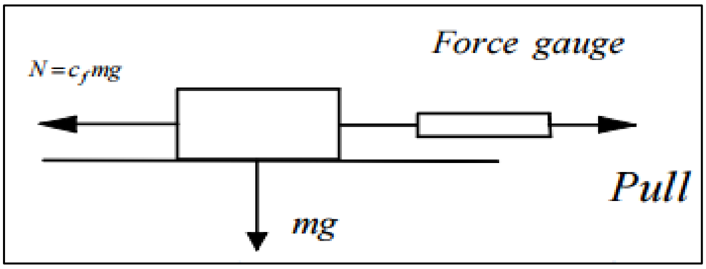

Figure 8 – Flat load measurements experimental setup [2]

Mass (kg)	Force measured (N)
0.66	0.50
0.76	0.60
0.87	0.70
0.97	0.75
1.07	1.00
1.17	1.00
1.27	1.00
1.38	1.10
1.48	1.20

Table 4 – Table of results for flat load measurements

These results are as expected and we notice a linear increase in force required to pull a range of masses across a flat surface. The values are then used to calculate torque in the following section. Random and systematic errors decrease precision and accuracy as seen in section 3.1.

3.3 Torque calculation

To determine the torque, the following equation is used:

$$T = F \times r \qquad - (7)$$

$$where\ T\ is\ torque, F\ is\ force, r\ is\ radius$$

Since we know the radius of the wheel is 0.0395m, and from previous data we can create the following table and graph:

Mass m (kg)	Forces measured (slope) (N)	Forces measured (flat) (N)	Calculated Torque (slope) (Nm)	Calculated Torque (flat) (Nm)
0.66	2.2	0.50	0.087	0.0198
0.76	2.4	0.60	0.095	0.0237
0.87	3.0	0.70	0.119	0.0277
0.97	3.2	0.75	0.126	0.0296
1.07	3.6	1.00	0.142	0.0395
1.17	4.0	1.00	0.158	0.0395
1.27	4.0	1.00	0.166	0.0395
1.38	4.6	1.10	0.182	0.0435
1.48	5.0	1.20	0.198	0.0474

Table 5 - Table of calculated torque based on measured forces

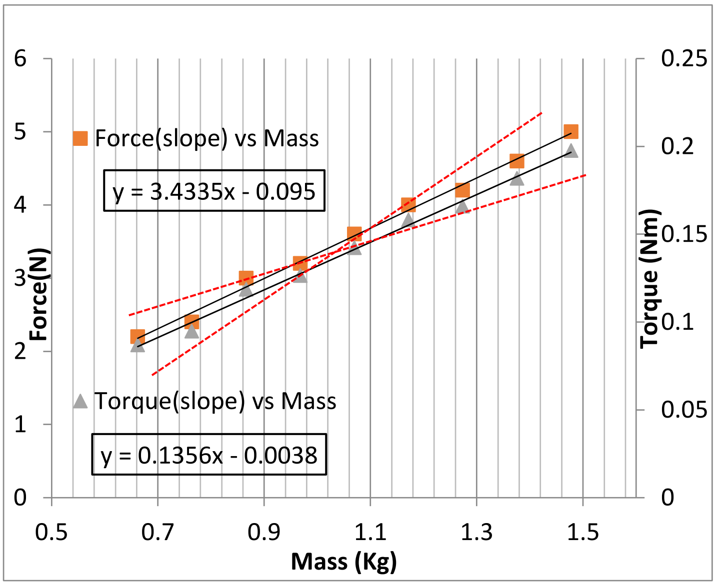

Figure 9 - Graph of force and torque required against mass (slope)

Figure 9 displays expected behaviour. The amount of force and torque required is highly linearly correlated to mass. The gradient for the measured force is steeper than the torque by a difference of roughly 3.3. This is expected as the torque is a multiple of the force and radius of the wheel (see equation 7). Assumptions were made during the calculations that the torque is directly proportional to the force, not considering any errors that might occur due to extra friction in the motor or working parts of the buggy.

4. Gear ratio selection

To calculate the gear ratio, we may use the following equation:

$$T_3 = n^2 T_1 \left(\frac{N_3 \times N_{2a}}{N_{2b} \times N_1}\right) \qquad -(8)$$

$$where\ T\ is\ torque, n\ is\ efficiency, N\ is\ cog\ number$$

Efficiency of n is assumed to be 0.85 [4]. We are aiming to achieve a gear ratio to allow each motor to provide enough torque to the wheels while keeping current draw to a minimum. To do this, we can plot designed torque and required torque over supply current:

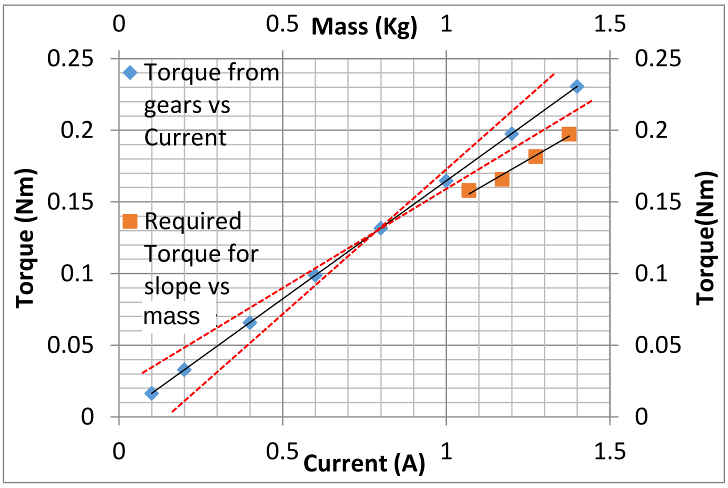

Figure 10 - Graph of designed and required torque over supply current

The data on figure 10 shows that for a gear ratio of 15:1 enough torque is provided for the highest four estimated masses of the buggy. These masses were selected as a worst-case scenario allow for the margin of safety identified earlier. A recommendation would be to keep the weight of the buggy low so that the supply of current to the motor can be kept low. This will allow a larger top speed for the buggy.

Top speed is calculated as follows:

$$\omega_3 = n^2\omega_1\left(\frac{N_{2b}\times N_1}{N_3\times N_{2a}}\right) \qquad -(9)$$

$$where\ \omega\ is\ angular\ speed$$

Wheel velocity is found from:

$$V = \omega\, r \qquad -(10)$$

$$where\ V\ is\ voltage, \omega\ is\ angular\ speed, r\ is\ radius\ = 0.0395\text{m}$$

These calculations are presented in table 6 and figure 11:

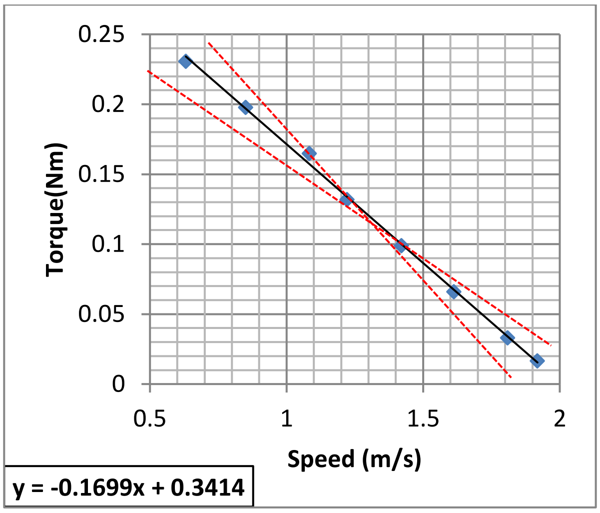

Figure 11 – Graph of torque over speed

Current (A)	Motor Emf (V)	Total torque with gears (Nm)	Total speed (m/s)
1.4	0.526	0.231	0.631
1.2	1.186	0.198	0.851
1.0	1.846	0.165	1.083
0.8	2.506	0.132	1.221
0.6	3.166	0.099	1.421
0.4	3.826	0.066	**1.612**
0.2	4.386	0.033	1.809
0.1	4.616	0.016	1.919

Table 6 - Table of results for calculations of max speed

Figure 11's data shows a highly correlated negative gradient as expected: when torque is decreased the speed of the buggy is increased. These results show the trade-off between torque and speed.

From figure 5 and if the buggy mass is at 1.48 kg, the largest predicted, the torque required for the slope will be approximately 0.198 Nm. This will give the buggy a max speed of around 0.59 m/s on the slope. Max speed for the flat approximated from the results will be 1.6 m/s. This demonstrates an excellent speed for the buggy at worse-case scenario in terms of weight. The full extent of this speed may however not be used for line-following reasons (sensor refresh rate may not allow an accurate tracing at high speeds). This must be investigated in a later report. We can now calculate the PCD using:

$$PCD = G_T \times MOD \qquad -(11)$$

$$where\ PCD\ is\ pitch\ circle\ diameter, G_T\ is\ gear\ teeth\ and\ MOD\ is\ module\ number$$

Gears	No. of teeth	MOD (mm)	PCD (mm)
Pinion	16	0.5	8
Free running	50	0.5	25
Free running	10	0.5	5
Press fit	48	0.5	24

Table 7 – PCD calculations

Gears combinations	PCD (A)	PCD (B)	Centre distance(mm)
Pinion & Free running	8	25	16.6
Free running & Press fit	5	24	14.6

Table 8 – Centre distance calculations

Using figures 16-20 the gear ratio is designed as shown aside in figure 21. A gear box ratio of 15:1 using 16 tooth pinion, 50/10 free running gear and 48/12 press fit gear is chosen. This provides an efficiency of 72.25%, with a torque large enough to deal with worst-case scenarios that could occur in the track's environment.

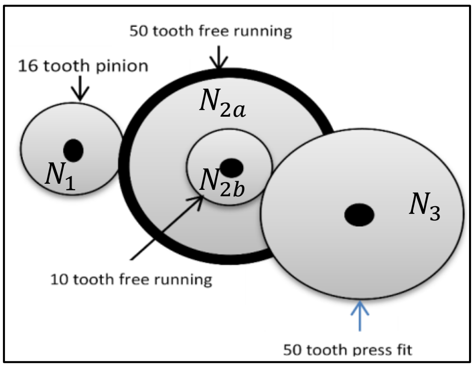

Figure 12 – Gears selected

5. Conclusions

5.1 Design recommendations

This investigation has allowed us to recommend the following specifications:

- Gear box ratio required to enhance the torque provided by the motor will be 15:1 using 16 tooth pinion, 50/10 free running gear and 48/12 press fit gear (ratio will cover torque required to climb slope and ensure max stall torque is not reached).
- Two stages to be used to assure relatively high efficiency of approximately 72.25%.
- Weight of the buggy should be kept to a minimum to allow for the option of a high maximum speed as mass/torque relationship and torque/speed trade-off revealed.
- Motor should not be run until overheating as derating will occur.
- Max speed may not be fully used as 1.6 m/s may be too fast for the sensors.

5.2 Summary of key results and assumptions

The main result of this investigation is a full working gearbox design with known motor performance specs to allow a thorough testing of the hardware once it is implemented.

Other deductions were reached in this report but many are dependent on the main assumptions (K_E = K_T at 0.0076, linear drag, $T_{friction}$ negligible…).

The main sources of error of the values produced are heat loss, non-linear drag, manufacturer's uncertainty & instrument resolution.

Other than for design purposes, the findings of this report are particularly important and relevant as in a race environment every single optimisation could have a large impact on final time. The results allow us to ascertain to what extent we can sacrifice speed for extra components or materials. It has also allowed us to get an idea of how fast we need to be able to sample our sensors to be able to run the buggy at its maximum speed.

6. References & acknowledgments

This investigation was considerably facilitated by the technical staff, lecturers' guidance and materials provided by The University of Manchester.

[1] RS Components (2016) *RE 385 Motor – Data Sheet.* Available at: uk.rs-online.com/web/p/dc-motors/2389737 (Accessed: 28/10/2016).

[2] Podd, F. and Apsley, J. (2016) *Embedded Systems Project: Technical and Management Manual.* The University of Manchester.

[3] Kirchoff, G. (1845) *unknown title.* Königsberg.

[4] Podd, F. and Apsley, J. (2016) *Embedded Systems Project: Project Handbook.* The University of Manchester.