



Machine Learning

Advice for applying  
machine learning

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Deciding what  
to try next

## Debugging a learning algorithm:

Suppose you have implemented regularized linear regression to predict housing prices.

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^m \theta_j^2 \right]$$

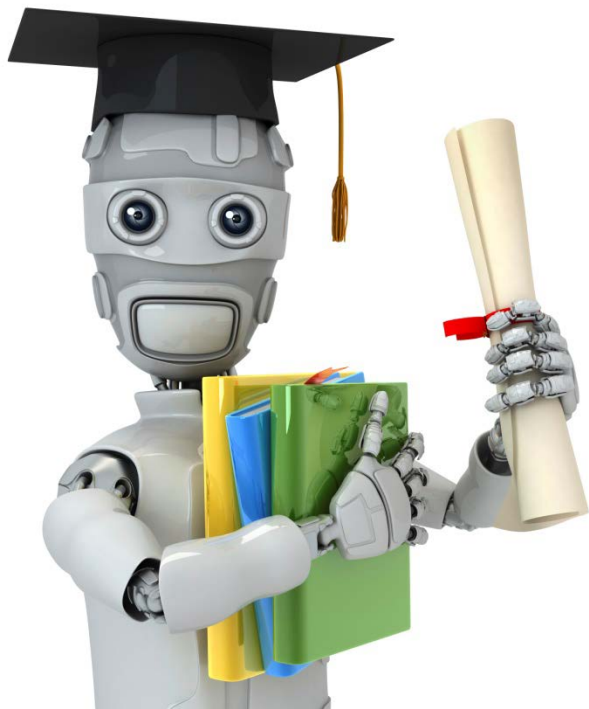
However, when you test your hypothesis on a new set of houses, you find that it makes unacceptably large errors in its predictions. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features ( $x_1^2, x_2^2, x_1x_2$ , etc.)
- Try decreasing  $\lambda$
- Try increasing  $\lambda$

## **Machine learning diagnostic:**

Diagnostic: A test that you can run to gain insight what is/Isn't working with a learning algorithm, and gain guidance as to how best to improve its performance.

Diagnostics can take time to implement, but doing so can be a very good use of your time.



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## Evaluating a hypothesis

# Evaluating your hypothesis



$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Fails to generalize to new examples not in training set.

$x_1$  = size of house

$x_2$  = no. of bedrooms

$x_3$  = no. of floors

$x_4$  = age of house

$x_5$  = average income in neighborhood

$x_6$  = kitchen size

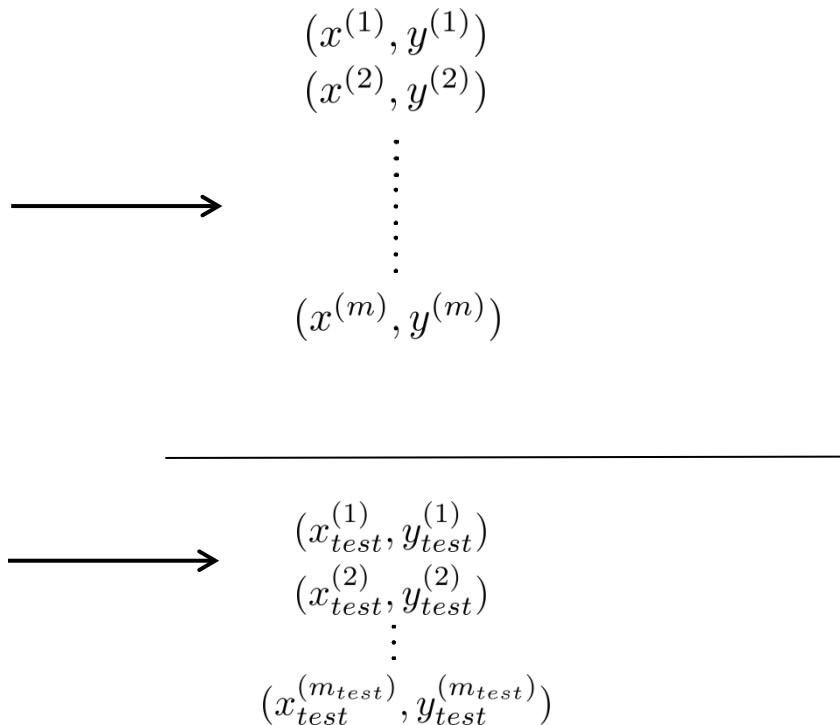
$\vdots$

$x_{100}$

# Evaluating your hypothesis

Dataset:

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243



# Training/testing procedure for linear regression

- Learn parameter  $\theta$  from training data (minimizing training error  $J(\theta)$ )
- Compute test set error:

## Training/testing procedure for logistic regression

- Learn parameter  $\theta$  from training data
- Compute test set error:

$$J_{test}(\theta) = -\frac{1}{m_{test}} \sum_{i=1}^{m_{test}} y_{test}^{(i)} \log h_{\theta}(x_{test}^{(i)}) + (1 - y_{test}^{(i)}) \log h_{\theta}(x_{test}^{(i)})$$

- Misclassification error (0/1 misclassification error):





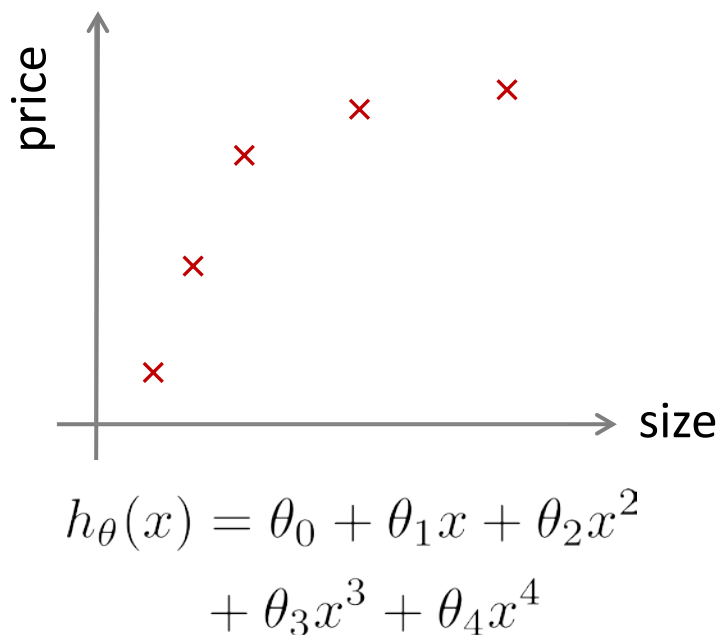
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Model selection and  
training/validation/test  
sets

## Overfitting example



Once parameters  $\theta_0, \theta_1, \dots, \theta_4$  were fit to some set of data (training set), the error of the parameters as measured on that data (the training error  $J(\theta)$ ) is likely to be lower than the actual generalization error.

## Model selection

1.  $h_{\theta}(x) = \theta_0 + \theta_1 x$
2.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$
3.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_3 x^3$
- $\vdots$
10.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{10} x^{10}$

Choose  $\theta_0 + \dots + \theta_5 x^5$

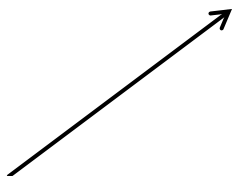
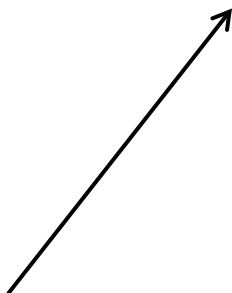
How well does the model generalize? Report test set error  $J_{test}(\theta^{(5)})$ .

Problem:  $J_{test}(\theta^{(5)})$  is likely to be an optimistic estimate of generalization error. I.e. our extra parameter ( $d$  = degree of polynomial) is fit to test set.

# Evaluating your hypothesis

Dataset:

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243



$$(x^{(1)}, y^{(1)})$$

$$(x^{(2)}, y^{(2)})$$

$$\vdots$$

$$(x^{(m)}, y^{(m)})$$

$$(x_{cv}^{(1)}, y_{cv}^{(1)})$$

$$(x_{cv}^{(2)}, y_{cv}^{(2)})$$

$$\vdots$$

$$(x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$$

$$(x_{test}^{(1)}, y_{test}^{(1)})$$

$$(x_{test}^{(2)}, y_{test}^{(2)})$$

$$\vdots$$

$$(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$$

## Train/validation/test error

Training error:

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

Test error:

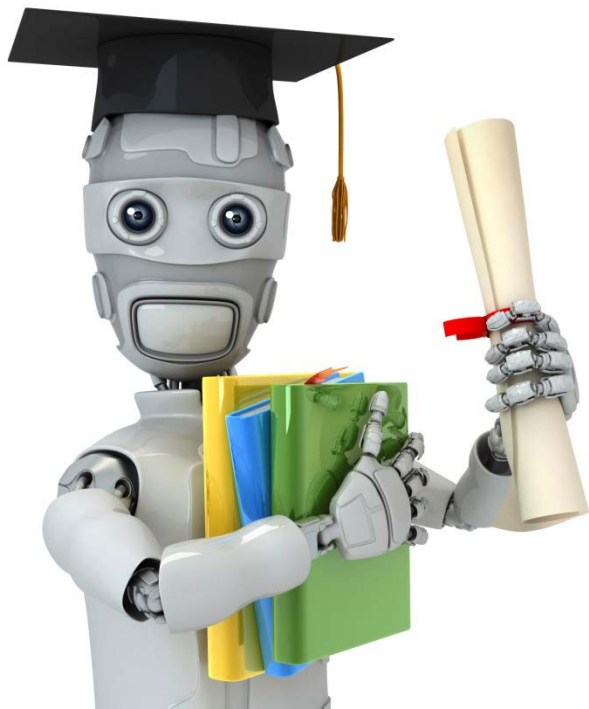
$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

## Model selection

1.  $h_{\theta}(x) = \theta_0 + \theta_1 x$
2.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$
3.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \cdots + \theta_3 x^3$
- $\vdots$
10.  $h_{\theta}(x) = \theta_0 + \theta_1 x + \cdots + \theta_{10} x^{10}$

Pick  $\theta_0 + \theta_1 x_1 + \cdots + \theta_4 x^4$

Estimate generalization error for test set  $J_{test}(\theta^{(4)})$



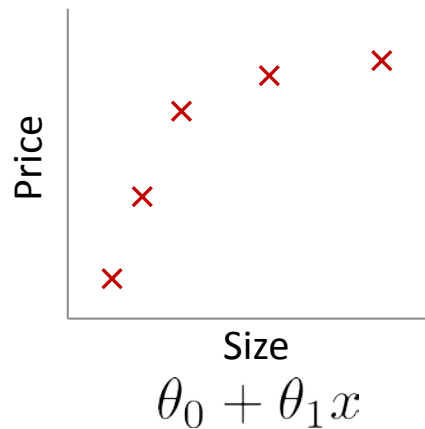
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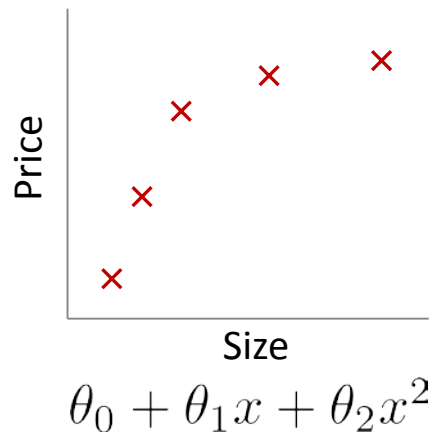
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## Diagnosing bias vs. variance

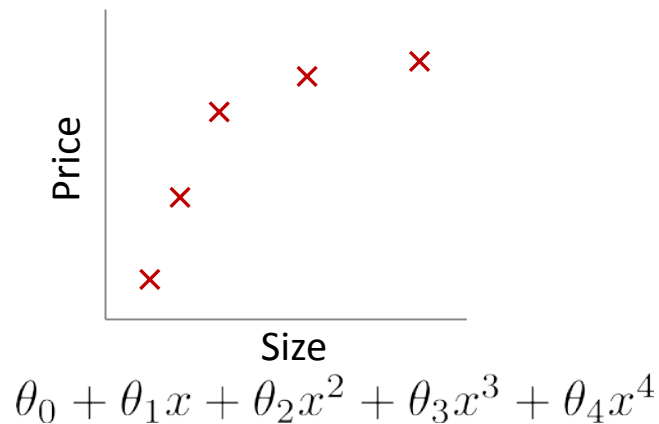
# Bias/variance



High bias  
(underfit)



“Just right”



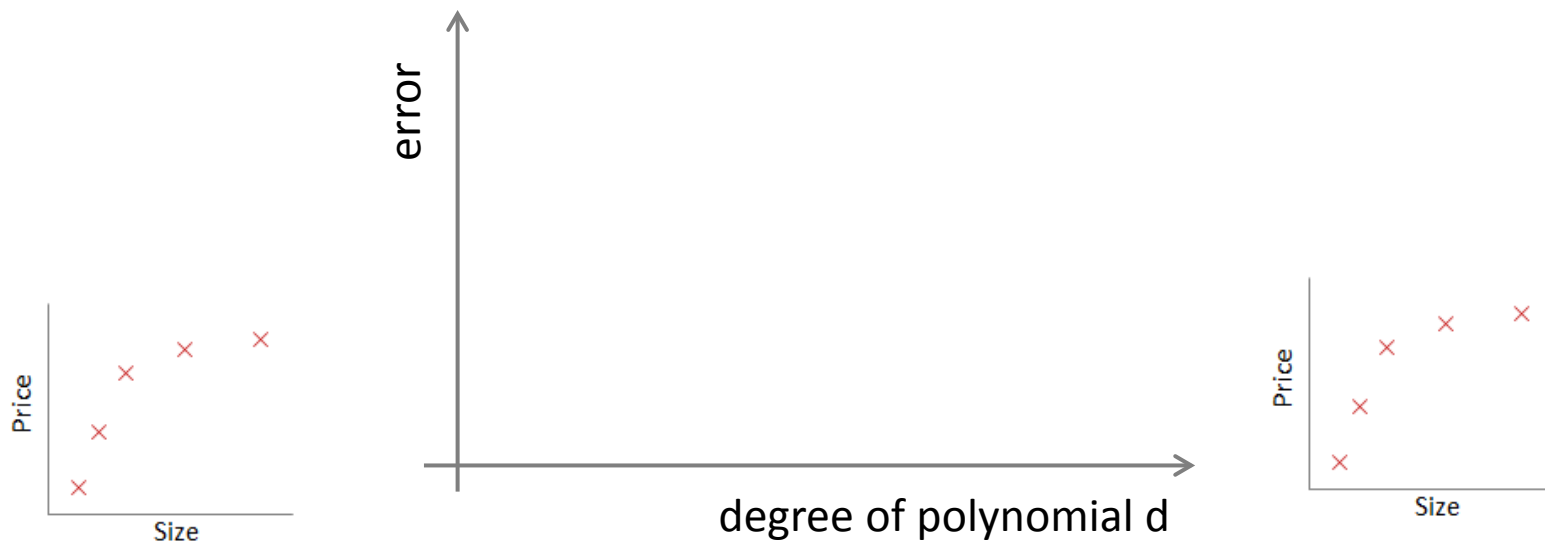
High variance  
(overfit)



# Bias/variance

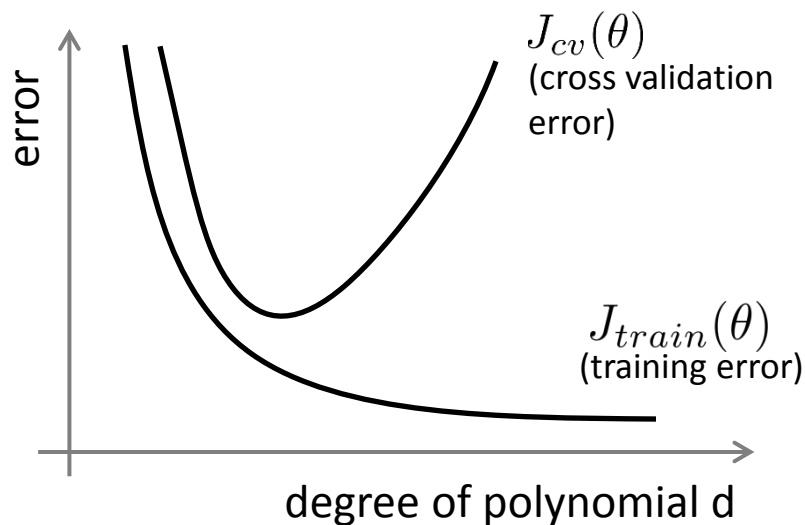
Training error:  $J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

Cross validation error:  $J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$



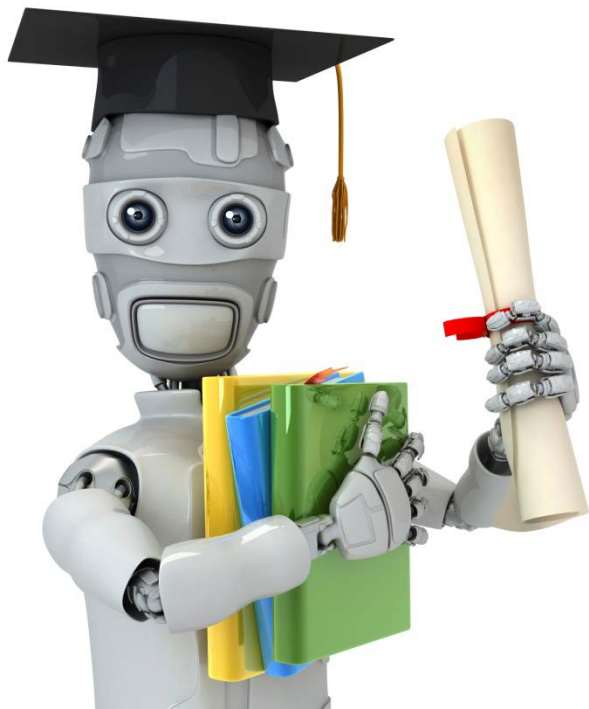
## Diagnosing bias vs. variance

Suppose your learning algorithm is performing less well than you were hoping. ( $J_{cv}(\theta)$  or  $J_{test}(\theta)$  is high.) Is it a bias problem or a variance problem?



Bias (underfit):

Variance (overfit):



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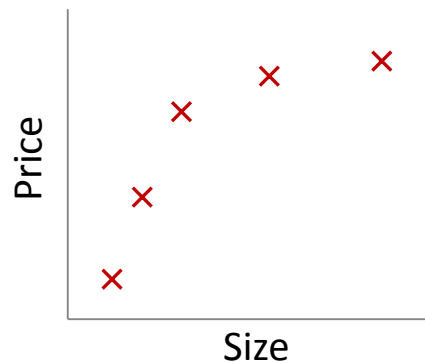
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## Regularization and bias/variance

# Linear regression with regularization

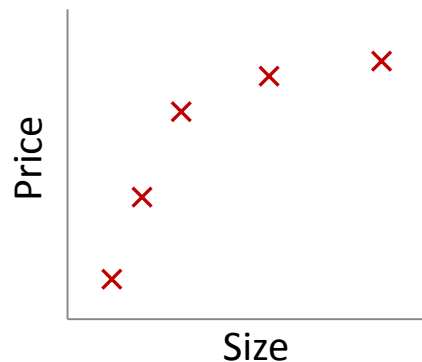
Model:  $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$



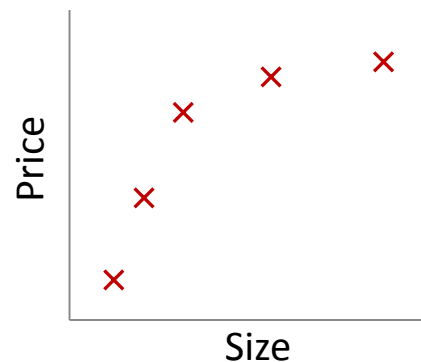
Large  $\lambda$

High bias (underfit)



Intermediate  $\lambda$

“Just right”



Small  $\lambda$

High variance (overfit)

$\lambda = 10000$ .  $\theta_1 \approx 0, \theta_2 \approx 0, \dots$

$h_{\theta}(x) \approx \theta_0$

## Choosing the regularization parameter $\lambda$

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

## Choosing the regularization parameter $\lambda$

Model:  $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$

1. Try  $\lambda = 0$
2. Try  $\lambda = 0.01$
3. Try  $\lambda = 0.02$
4. Try  $\lambda = 0.04$
5. Try  $\lambda = 0.08$
- $\vdots$
12. Try  $\lambda = 10$

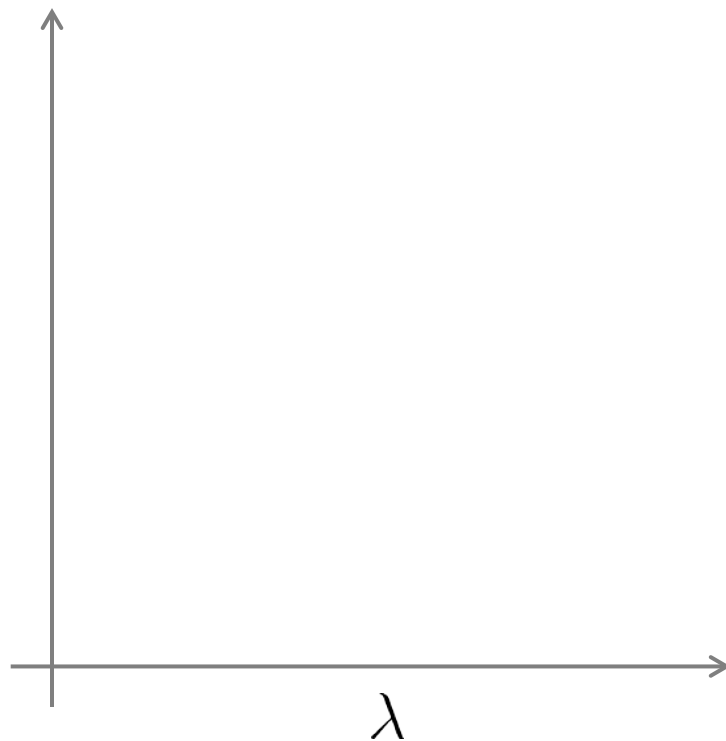
Pick (say)  $\theta^{(5)}$ . Test error:

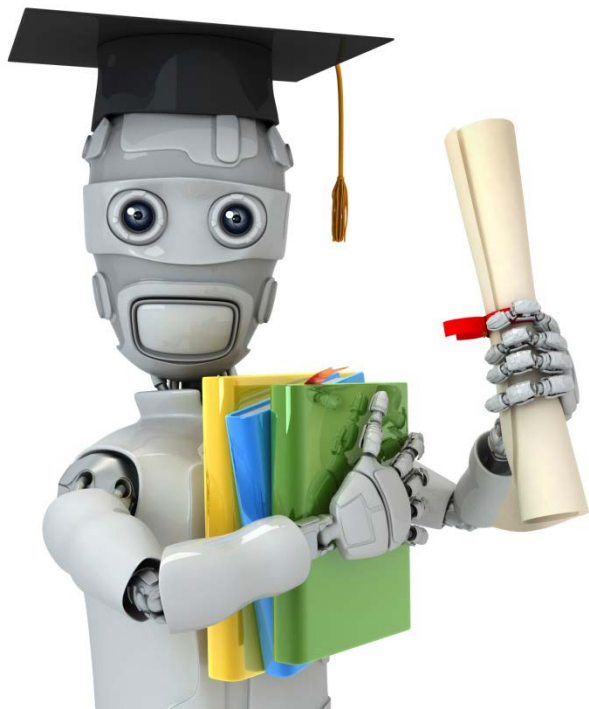
## Bias/variance as a function of the regularization parameter $\lambda$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$





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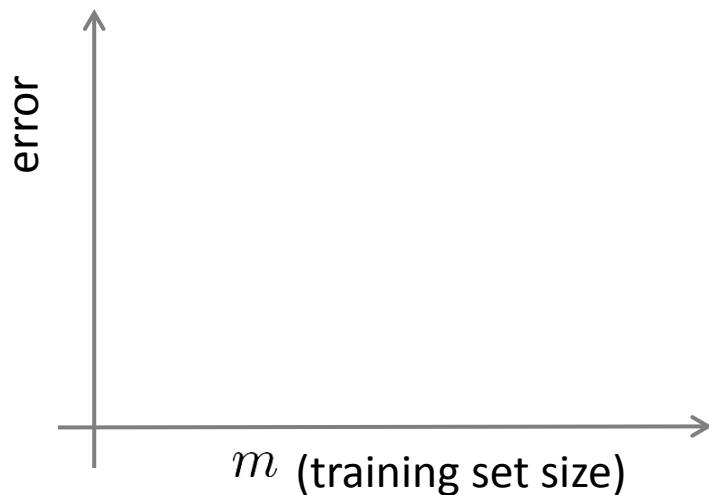
## Learning curves



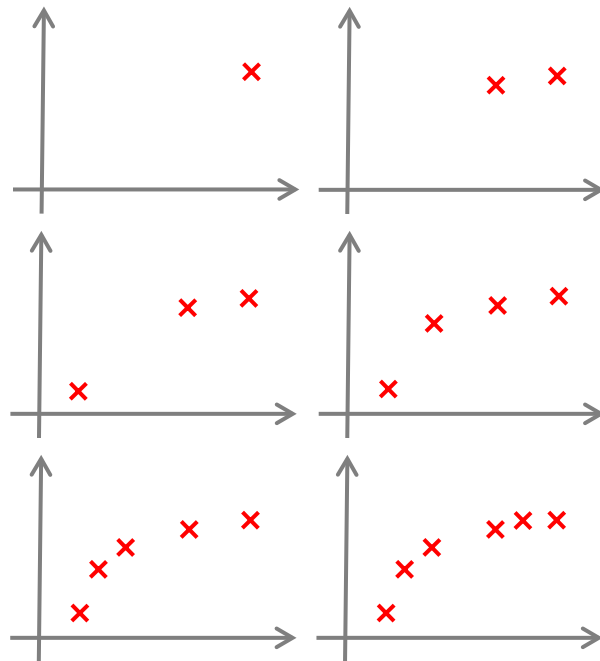
# Learning curves

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

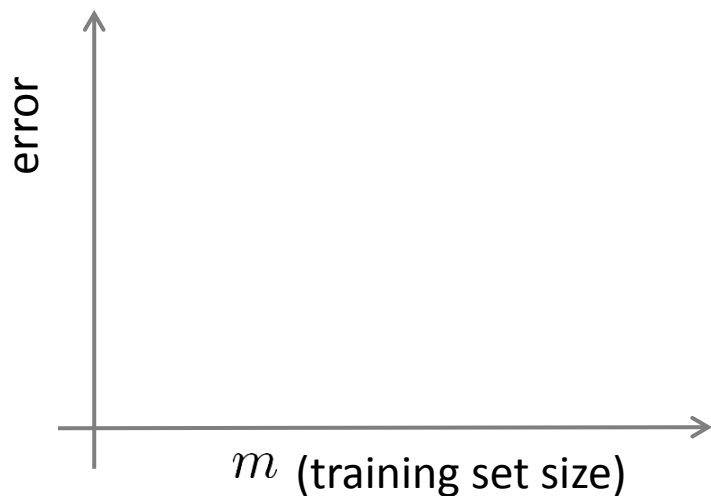
$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$



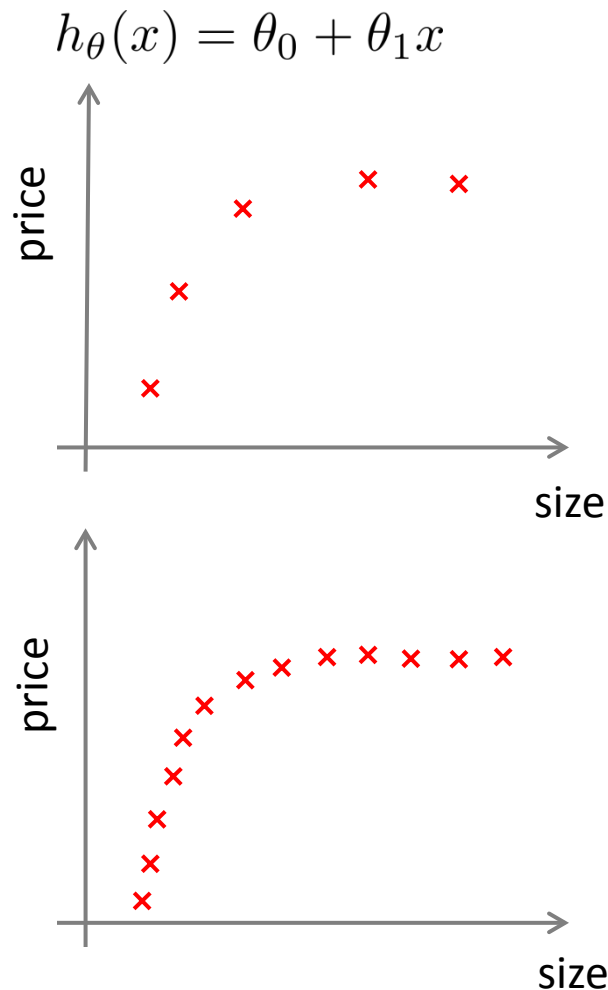
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$



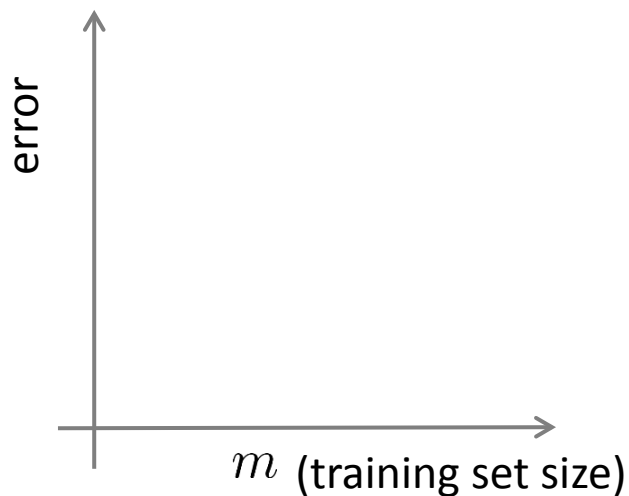
## High bias



If a learning algorithm is suffering from high bias, getting more training data will not (by itself) help much.



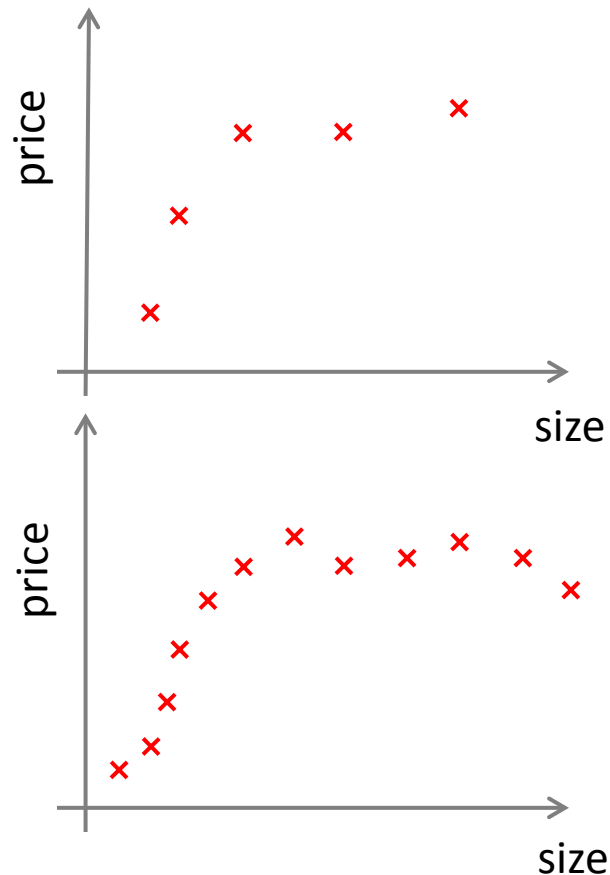
## High variance

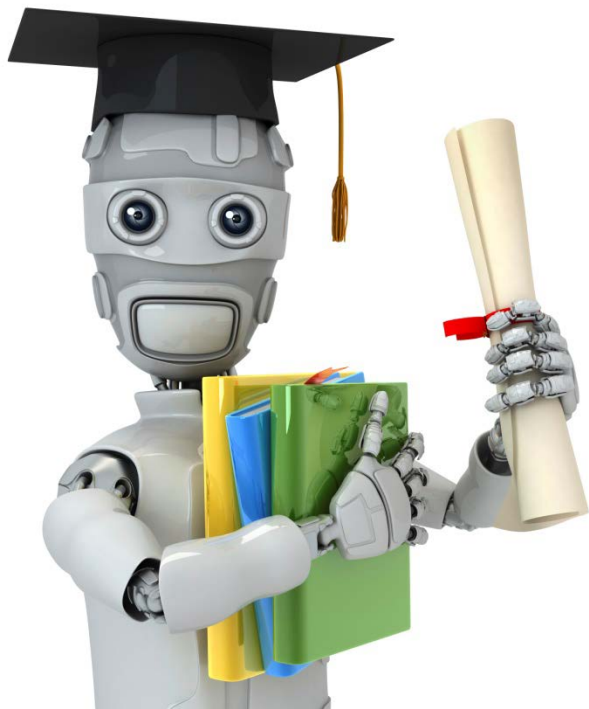


If a learning algorithm is suffering from high variance, getting more training data is likely to help.

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{100} x^{100}$$

(and small  $\lambda$ )





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## Deciding what to try next (revisited)

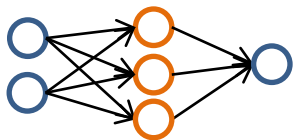
## Debugging a learning algorithm:

Suppose you have implemented regularized linear regression to predict housing prices. However, when you test your hypothesis in a new set of houses, you find that it makes unacceptably large errors in its prediction. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features ( $x_1^2, x_2^2, x_1x_2$ , etc)
- Try decreasing  $\lambda$
- Try increasing  $\lambda$

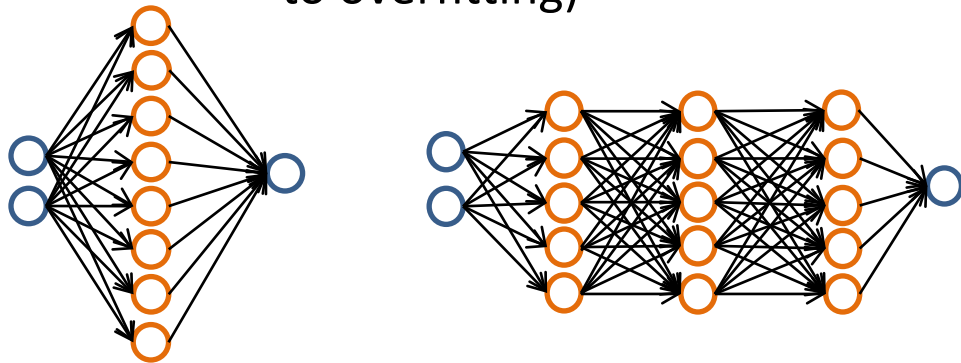
# Neural networks and overfitting

“Small” neural network  
(fewer parameters; more  
prone to underfitting)



Computationally cheaper

“Large” neural network  
(more parameters; more prone  
to overfitting)



Computationally more expensive.

Use regularization ( $\lambda$ ) to address overfitting.