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EE 521: Analysis of Power Systems

Lecture 7
State Estimation 2

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

- Recap – State Estimation Concepts
- Statistical Theory
  - The theoretical basis of state estimation
  - Weights of measurements
  - Bad data detection and identification

Definition of State Estimation

- Power System States:
  - *V*, *θ* at buses, same as those in the power flow problem
- State Estimation:
  - Estimates states from measured quantities:
    - Status, V_{rms}, I_{rms}, P_{line}, Q_{line}, P_{inj}, Q_{inj}
  - Fits measurements to a model by minimizing errors
  - Objective:
    - Filter noise
    - Identify bad data and missing data
    - Estimate unmeasured quantities such as *θ*

State Estimation Procedure

- Identify measurement variables and state variables (input and output)
  - *z* and *x*
- Formulate measurement equations
  - $z = h(x) + e$
- Derive Jacobian Matrix *H*: $H = \frac{\partial h(x)}{\partial x}$
- Solve for estimated states using Newton-Raphson method (*H* needs to be updated at every step)

$$\boxed{\hat{x}^{(k+1)} = \hat{x}^{(k)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(k)}\right)\right]}$$

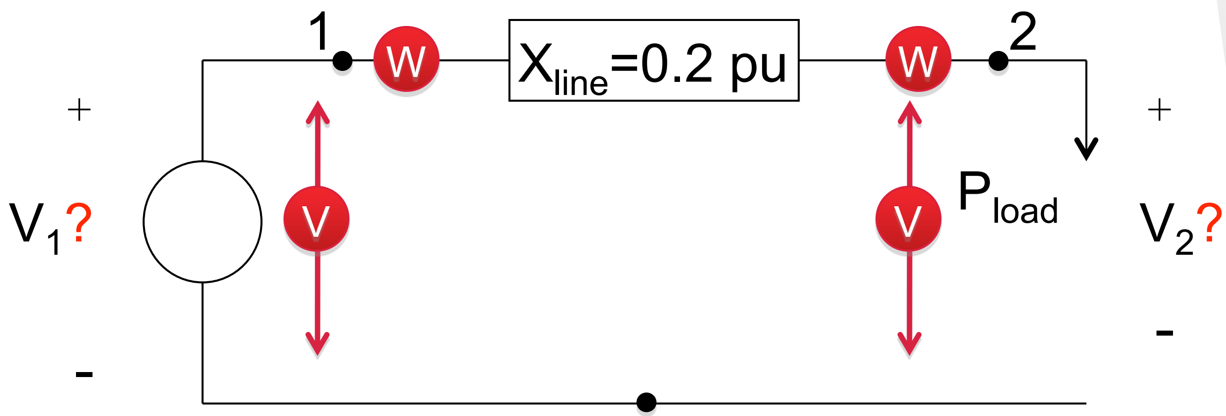
State Estimation vs. Power Flow

	Power Flow	State Estimation
Input	Given PV, PQ, Vθ	Measured $z = V_{rms}, I_{rms}, P_{line}, Q_{line}, P_{inj}, Q_{inj}$
Output	V and θ	x = V and θ
Formulation	P – P(V, θ) = 0 Q – Q(V, θ) = 0	z – h(x) = e
Objective	Drive ΔP, ΔQ towards 0.	Drive Δz towards a minimum.
Solution Method	$\begin{bmatrix} \theta^{n+1} \\ V^{n+1} \end{bmatrix} = \begin{bmatrix} \theta^{n} \\ V^{n} \end{bmatrix} - \left[J(x^n)\right]^{-1} \begin{bmatrix} \Delta P(x^n) \\ \Delta Q(x^n) \end{bmatrix}$	$\hat{x}^{(k+1)} = \hat{x}^{(k)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(k)}\right)\right]$
Jacobian Matrix	$J(x) = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$	$H = \left[\frac{\partial h(x)}{\partial x}\right]$ J is part of H.

Example – State Estimation

Problem:

V_1, V_2, P_{12}, Q_{21} are measured. *W* is given. Formulate the SE problem.



Solution:

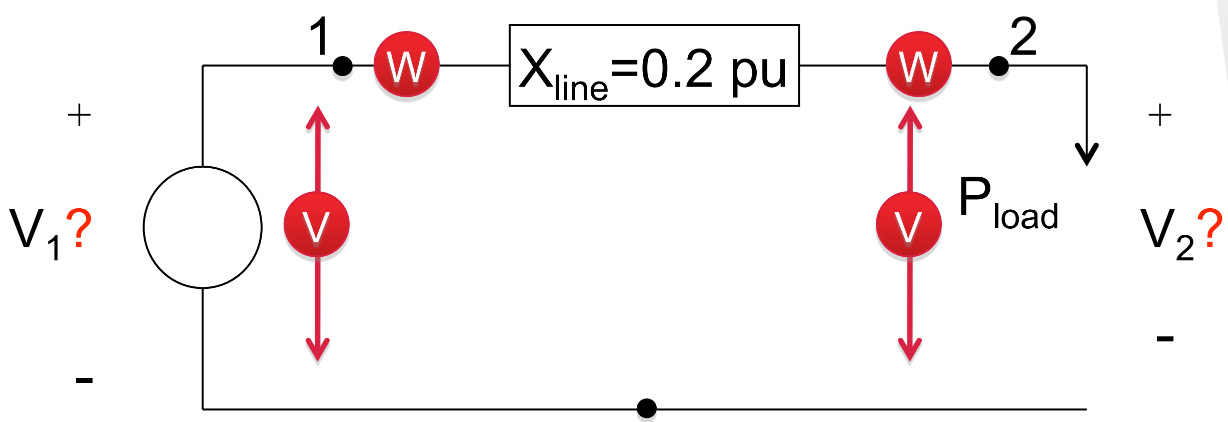
Define measurement variables and state variables: (θ_1 = 0, selected to be reference)

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ P_{12} \\ Q_{21} \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \theta_2 \end{bmatrix}$$

Example – State Estimation *cont'd*

Hint:

$$P_{ij} = -|V_i|^2 G_{ij} + |V_i||V_j|\left(G_{ij}\cos(\theta_j - \theta_i) - B_{ij}\sin(\theta_j - \theta_i)\right) \quad Q_{ij} = -|V_i|^2\left(\frac{B_{ij}'}{2} - B_{ij}\right) - |V_i||V_j|\left(G_{ij}\sin(\theta_j - \theta_i) + B_{ij}\cos(\theta_j - \theta_i)\right)$$



Measurement equations:

$$B_{12} = B_{21} = \frac{1}{X_{line}} = 5 \qquad h(x) = \begin{bmatrix} V_1 \\ V_2 \\ P_{12} \\ Q_{21} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 5V_1V_2\sin(\theta_1 - \theta_2) \\ 5V_2^{\,2} - 5V_1V_2\cos(\theta_1 - \theta_2) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 5x_1x_2\sin(-x_3) \\ 5x_2^{\,2} - 5x_1x_2\cos(-x_3) \end{bmatrix}$$

Example – State Estimation *cont'd*

Jacobian Matrix *H*:

$$h(x)=\begin{bmatrix} x_1 \\ x_2 \\ 5x_1x_2\sin(-x_3) \\ 5x_2{}^2-5x_1x_2\cos(-x_3) \end{bmatrix}$$

$$H=\left[\frac{\partial h(x)}{\partial x}\right]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5x_2\sin(-x_3) & 5x_1\sin(-x_3) & -5x_1x_2\cos(-x_3) \\ -5x_1\cos(-x_3) & 10x_2-5x_2\cos(-x_3) & -5x_1x_2\sin(-x_3) \end{bmatrix}$$

Iterative process using Newton-Raphson Method:

$$\hat{x}^{(k+1)}=\hat{x}^{(k)}+\left(H^TWH\right)^{-1}H^TW\left[z-h\left(\hat{x}^{(k)}\right)\right]$$

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Questions Remaining

- How do we know state estimation would give desired results?
- How to determine weights for measurements?
- What about bad measurements?

"Statistical Theory"

Expectation of State Estimation

- Expectation of Measurements (Observations)
  - y_1, y_2, y_3, … ➔ $E[y_i] = avg(y_i) = y_{mean} = y_{true}$
- For state estimation, we expect
  - Estimated *x* approaches its true value *x*.
  - Estimated *z* approaches its measured value *z*.

$$E(\hat{x}) = x$$

$$E(\hat{z}) = z$$

  - Let's prove these are true.

Revisit of State Estimation Equations

Weighted least square formulation:

$$\min_{x_1,x_2} f(x_1,x_2) = w_1 e_1^{\,2} + w_2 e_2^{\,2} + w_3 e_3^{\,2}$$

$$\frac{\partial f(x_1,x_2)}{\partial x_1} = 0 \qquad \frac{\partial f(x_1,x_2)}{\partial x_2} = 0$$

$$\Rightarrow \left.\frac{\partial f(x_1,x_2)}{\partial x_1}\right|_{\hat{x}} = 2\left.\left(w_1 e_1 \frac{\partial e_1}{\partial x_1} + w_2 e_2 \frac{\partial e_2}{\partial x_1} + w_3 e_3 \frac{\partial e_3}{\partial x_1}\right)\right|_{\hat{x}} = 0$$

$$\left.\frac{\partial f(x_1,x_2)}{\partial x_2}\right|_{\hat{x}} = 2\left.\left(w_1 e_1 \frac{\partial e_1}{\partial x_2} + w_2 e_2 \frac{\partial e_2}{\partial x_2} + w_3 e_3 \frac{\partial e_3}{\partial x_2}\right)\right|_{\hat{x}} = 0$$

$$\Rightarrow \left.\begin{bmatrix} \frac{\partial e_1}{\partial x_1} & \frac{\partial e_2}{\partial x_1} & \frac{\partial e_3}{\partial x_1} \\ \frac{\partial e_1}{\partial x_2} & \frac{\partial e_2}{\partial x_2} & \frac{\partial e_3}{\partial x_2} \end{bmatrix}\right|_{\hat{x}} \begin{bmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = 0 \quad \Rightarrow \quad H^T W \hat{e} = 0$$

$$\Rightarrow H^T W \left[z - h(\hat{x}) \right] = 0 \quad \Rightarrow \quad H^T W h(\hat{x}) = H^T W z$$

Expectation of Estimated *x*

Substitute *z* using measurement equations:

$$H^T W h(\hat{x}) = H^T W z = H^T W\left[h(x) + e\right] \Rightarrow H^T W\left[h(\hat{x}) - h(x)\right] = H^T W e$$

Linearize at x_0:

$$H^T W\left[\left[h\left(x^{(0)}\right) + H\left(\hat{x} - x^{(0)}\right)\right] - \left[h\left(x^{(0)}\right) + H\left(x - x^{(0)}\right)\right]\right] = H^T W e$$

$$\Rightarrow H^T W H(\hat{x} - x) = H^T W e \Rightarrow \hat{x} - x = \left(H^T W H\right)^{-1} H^T W e$$

Take expected value at both sides:

$$E(\hat{x} - x) = E(\hat{x}) - x = \left(H^T W H\right)^{-1} H^T W \; E(e)$$

Use Gaussian distribution assumption for measurements (*E(e)* = 0):

$$E(\hat{x}) - x = 0 \Rightarrow E(\hat{x}) = x \quad \blacksquare$$

Expectation of Estimated *z*

Start with measurement error:

$$\hat{e} = z - \hat{z} = z - h(\hat{x}) = h(x) + e - h(\hat{x}) = e + h(x) - h(\hat{x})$$

Linearize at x_0:

$$z - \hat{z} = e + \left[\left[h\left(x^{(0)}\right) + H\left(x - x^{(0)}\right)\right] - \left[h\left(x^{(0)}\right) + H\left(\hat{x} - x^{(0)}\right)\right]\right] = e - H(\hat{x} - x)$$

$$\Rightarrow \quad z - \hat{z} = e - H\left(H^T W H\right)^{-1} H^T W e = \left[I - H\left(H^T W H\right)^{-1} H^T W\right] e$$

Take expected value at both sides:

$$E(z - \hat{z}) = z - E(\hat{z}) = \left[I - H\left(H^T W H\right)^{-1} H^T W\right] E(e)$$

Use Gaussian distribution assumption for measurements (*E(e)* = 0):

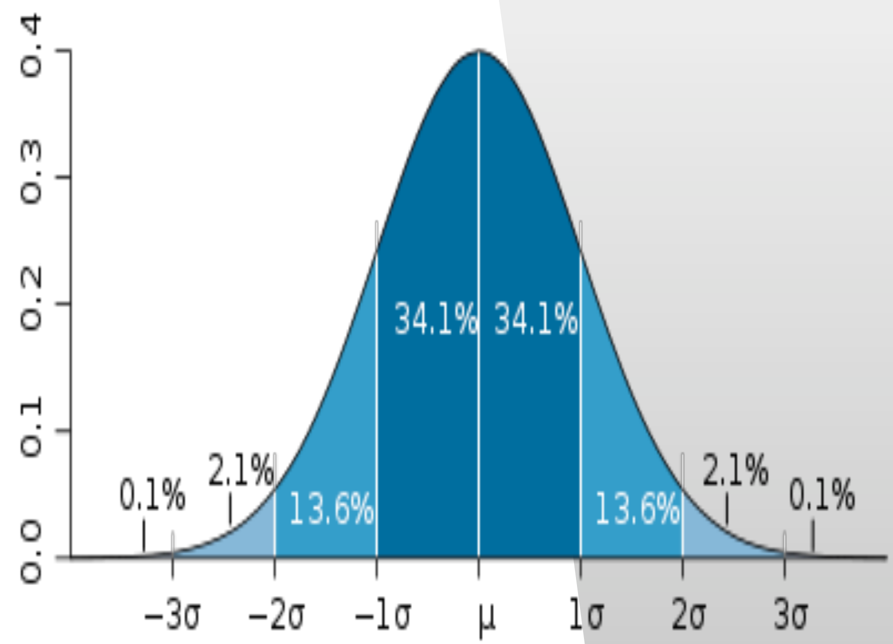
$$z - E(\hat{z}) = 0 \quad \Rightarrow \quad E(\hat{z}) = z \quad \blacksquare$$

Gaussian Distribution

- A bell-shape probability density function
  - Describe the probability of the occurrence of a measured value

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2\right]$$

- Mean Value: $\mu = E(z)$
- Standard Deviation: σ
- Variance: σ^2



Source: Wikipedia.

Expectation of Measurement Error

- Measurement Error is the deviation from true value μ

$$E(e) = E(z - \mu) = E(z) - \mu = 0$$

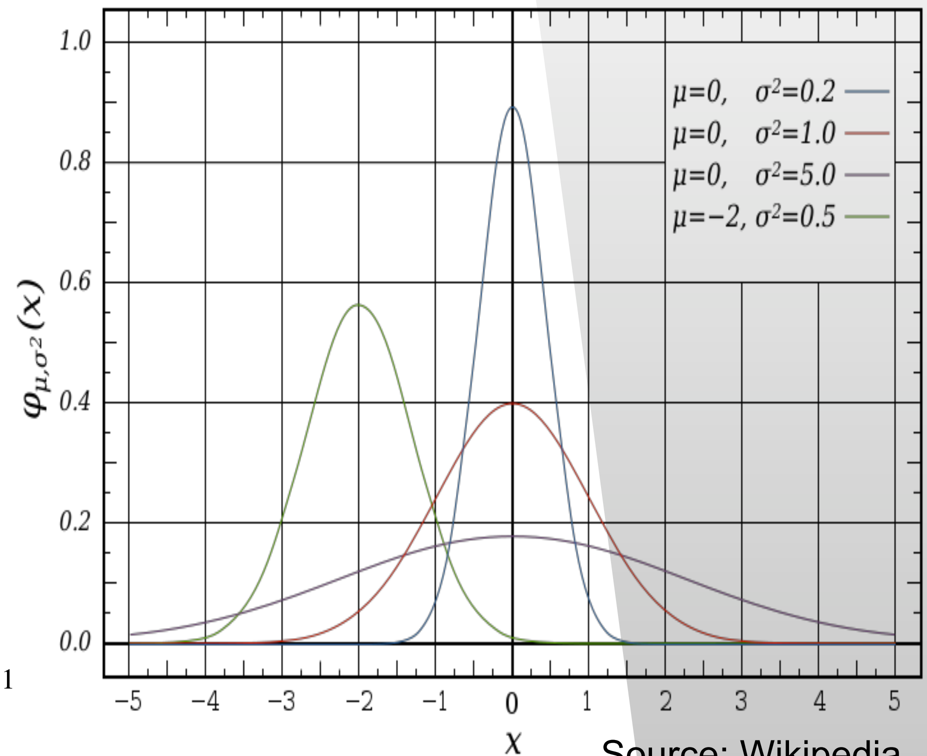
Expectation of the Square of Measurement Error

- This is defined as variance

$$E(e^2) = E\left[(z-\mu)^2\right] = \sigma^2$$

- Weights of measurement is chosen to the reciprocal of the variance

$$W = \begin{bmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma_1^{\ 2}} & & \\ & \frac{1}{\sigma_2^{\ 2}} & \\ & & \frac{1}{\sigma_3^{\ 2}} \end{bmatrix} = R^{-1}$$



Source: Wikipedia.

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Expectation of the Product of Measurement Error

$$E\left(ee^T\right) = E\left(\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}\begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}\right) = E\left(\begin{bmatrix} e_1^{\,2} & e_1e_2 & e_1e_3 \\ e_2e_1 & e_2^{\,2} & e_2e_3 \\ e_3e_1 & e_3e_2 & e_3^{\,2} \end{bmatrix}\right)$$

Assume measurements are statistically independent ($E(e_ie_j) = 0$ for $i \neq j$):

$$E\left(ee^T\right) = \begin{bmatrix} E\left(e_1^{\,2}\right) & 0 & 0 \\ 0 & E\left(e_2^{\,2}\right) & 0 \\ 0 & 0 & E\left(e_3^{\,2}\right) \end{bmatrix} = \begin{bmatrix} \sigma_1^{\,2} & 0 & 0 \\ 0 & \sigma_2^{\,2} & 0 \\ 0 & 0 & \sigma_3^{\,2} \end{bmatrix} = R$$

Expectation of the Product of Estimated Measurement Error

$$E\left(\hat{e}\hat{e}^T\right) = \left[I - H\left(H^T R^{-1} H\right)^{-1} H^T R^{-1}\right] E\left(ee^T\right)\left[I - R^{-1} H\left(H^T R^{-1} H\right)^{-1} H^T\right]$$

$$= R - H\left(H^T R^{-1} H\right)^{-1} H^T$$

$$= R'$$

(textbook section 15.3)

Variance of estimated measurement error:

$$E\left(\hat{e}_i^{\,2}\right) = R'_{ii} \quad \Rightarrow \quad E\left[\left(\frac{\hat{e}_i}{\sqrt{R'_{ii}}}\right)^2\right] = E\left[\left(\frac{z_i - \hat{z}_i}{\sqrt{R'_{ii}}}\right)^2\right] = 1$$

$$E\left(\hat{e}_i\right) = 0$$

Bad Data Detection

- Gaussian noise can be filtered out by the state estimation process ➔ not a concern
- The concerns are gross errors due to:
  - Malfunctioned sensors
  - Out-of-service sensors
  - Communication channel bias
  - Data loss
  - Manipulated data points (cyber security issues)
  - …

Revisit of the State Estimation Objective Function

- The state estimation process is to drive the objective function to a minimum

$$\min_{x} f(x) = \sum_{i=1}^{N_m} w_i e_i^2 \qquad \text{Estimated}: \hat{f}(x) = \sum_{i=1}^{N_m} w_i \hat{e}_i^2 = \sum_{i=1}^{N_m} \left[\frac{(z_i - \hat{z}_i)^2}{\sigma_i^2} \right]$$

- Question: what value of the objective function do we expect once the state estimation is converged?

$E(\hat{f})$?

Expectation of the Objective Function

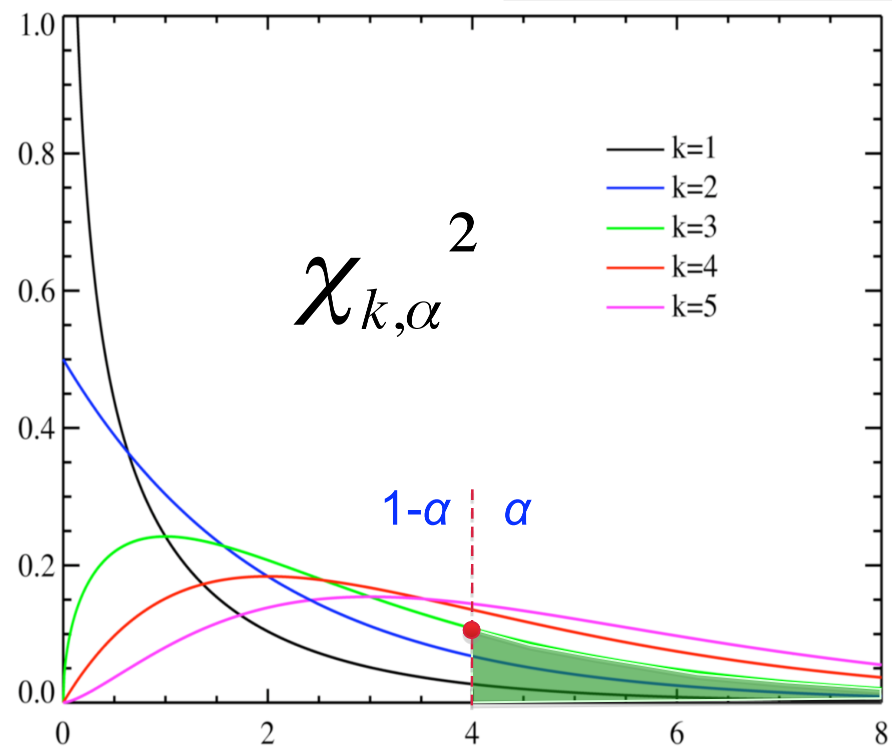
- Degrees of Freedom
  - Indication of redundancy of measurements
  - Larger than 1 as we usually have an over-determined system

$$E\left(\hat{f}\right) = \sum_{i=1}^{N_m} E\left[\frac{\left(z_i - \hat{z}_i\right)^2}{\sigma_i^{\,2}}\right] = \sum_{i=1}^{N_m} E\left[\frac{R'_{ii}}{\sigma_i^{\,2}} \frac{\left(z_i - \hat{z}_i\right)^2}{R'_{ii}}\right] = \sum_{i=1}^{N_m} \frac{R'_{ii}}{\sigma_i^{\,2}} E\left[\frac{\left(z_i - \hat{z}_i\right)^2}{R'_{ii}}\right] = N_m - N_s$$

  - Actual value of *f* hat will be in a range around (N_m-N_s) as it is a random variable.
  - What range is a good range?

Chi-Square Distribution

- Estimated *f* follows Chi-Square Distribution
- Determined by one parameter – expected value (degrees of freedom)
- Chi-Square Test
  - With a confidence of (1-*α*), *f hat* is less than 4.
  - If it is not less than 4, bad data exist.



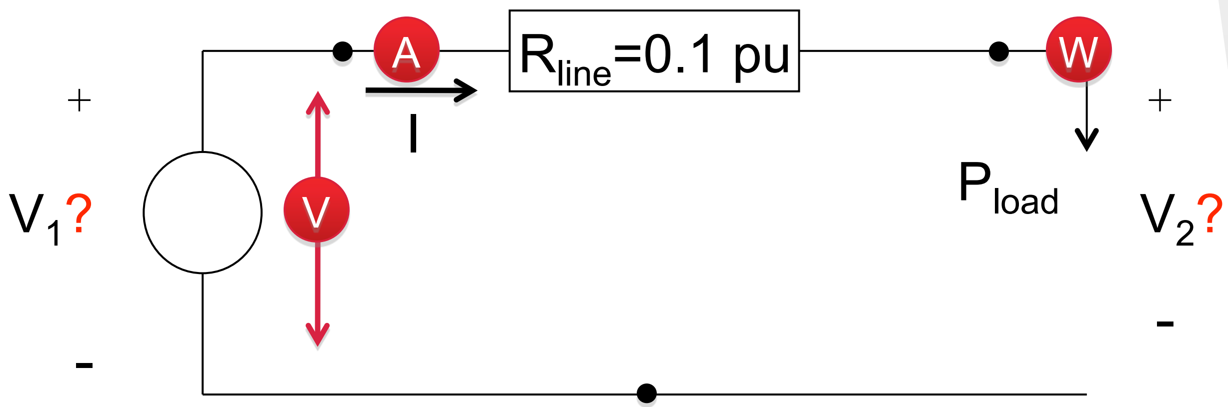
Bad Data Detection and Identification Procedure

- Run state estimation
- Estimate the objective function *f*
- Look up the Chi-Square number with degrees of freedom (N_m-N_s) and specified confidence
- Compare the Chi-Square number with estimated *f*
  - If *f* < Chi-Square number, no bad data
  - Else, bad data exist
- Take out the data with the largest error $\max_i \frac{z_i - \hat{z}_i}{\sqrt{R'_{ii}}}$
- Repeat

Example – Bad Data Detection and Identification

Problem:

Given Measurements: V_m = 0.9 pu, P_m = 2.6 pu, I_m = 4.5 pu, w = 100. Bad data?



Solution:

Define measurement variables and state variables:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} V_m \\ P_m \\ I_m \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Measurement equations:

$$z = h(x) + e = \begin{bmatrix} V_1 \\ V_2 \dfrac{V_1 - V_2}{R_{line}} \\ \dfrac{V_1 - V_2}{R_{line}} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 10x_1x_2 - 10x_2{}^2 \\ 10x_1 - 10x_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Example – Bad Data Detection and Identification *cont'd*

Converged results:

$$\hat{x} = [1.0217, 0.5714]^T$$

$$\hat{z} = [1.0217, 2.5730, 4.5033]^T$$

$$\hat{f} = 1.5548$$

$$\min_{x_1, x_2} \hat{f}(x_1, x_2) = w_1 \hat{e}_1^{\ 2} + w_2 \hat{e}_2^{\ 2} + w_3 \hat{e}_3^{\ 2}$$

27.000000000000000
1.642381656804740
1.554807088533027
1.554804815717737
1.554804815535279
1.554804815535258
1.554804815535259
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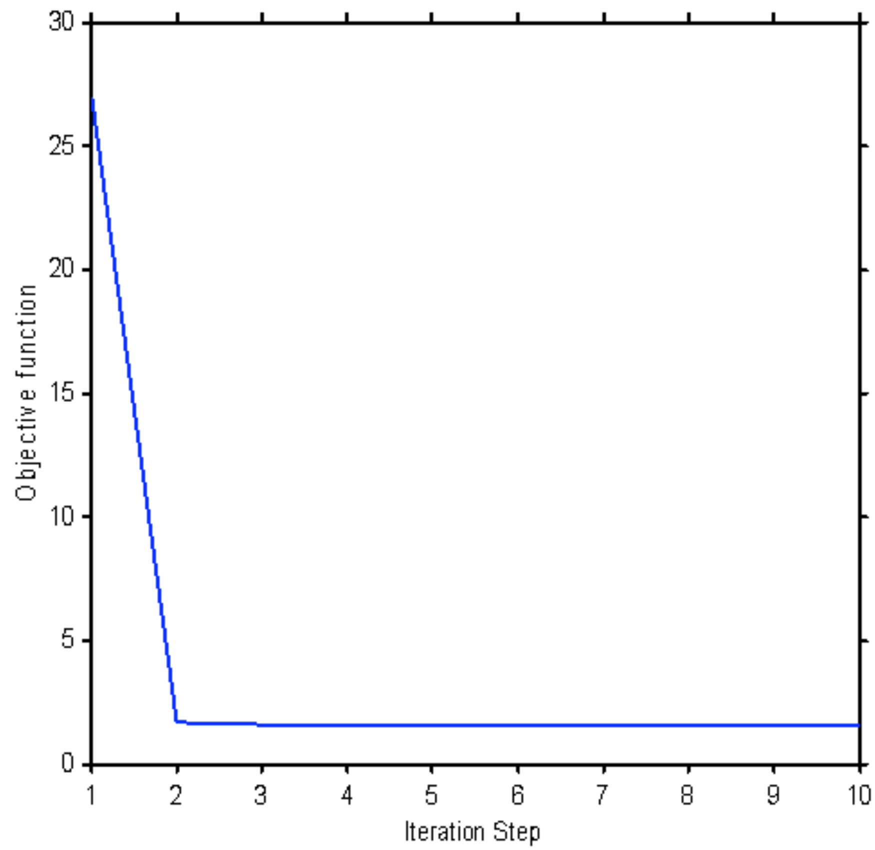

Example – Bad Data Detection and Identification *cont'd*

Find Chi-Square number:

Degrees of Freedom: $E(\hat{f}) = k = N_m - N_s = 3 - 2 = 1$

Specify a confidence level: $1 - \alpha = 99\% \Rightarrow \alpha = 0.01$

Chi-Square number: $\chi_{k,\alpha}{}^2 = \chi_{1,0.01}{}^2 = 6.64$

Perform Chi-Square test:

$\hat{f} = 1.5548 < \chi_{1,0.01}{}^2 = 6.64$ ⇒ No bad data exist! (99% confidence)

Compute estimated measurement errors:

$$R' = R - H\left(H^T R^{-1} H\right)^{-1} H^T = \begin{bmatrix} 0.0095 & -0.0021 & 0 \\ -0.0021 & 0.0005 & 0 \\ 0 & 0 & -6.86\text{e-}6 \end{bmatrix}$$

$$\frac{z_1 - \hat{z}_1}{\sqrt{R'_{11}}} = -1.2496 \qquad \frac{z_2 - \hat{z}_2}{\sqrt{R'_{22}}} = 1.2496 \qquad \frac{z_3 - \hat{z}_3}{\sqrt{R'_{33}}} = -1.2496$$

Questions?