Washington State University
Tri-Cities
World Class. Face to Face.

Instructor: Zhenyu (Henry) Huang
(509) 438-7235, h_zyu@yahoo.com

EE 521: Analysis of Power Systems

Lecture 18
Small Signal Stability

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

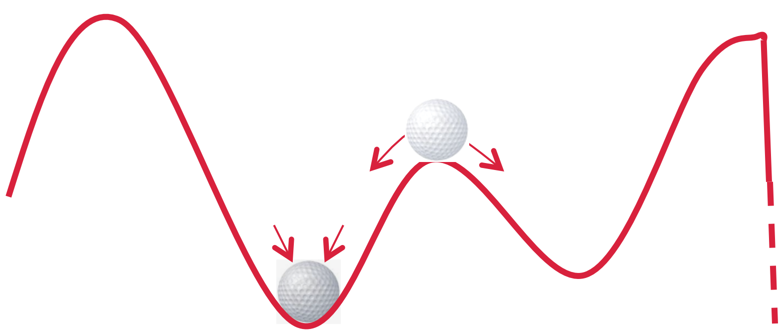
Test 216

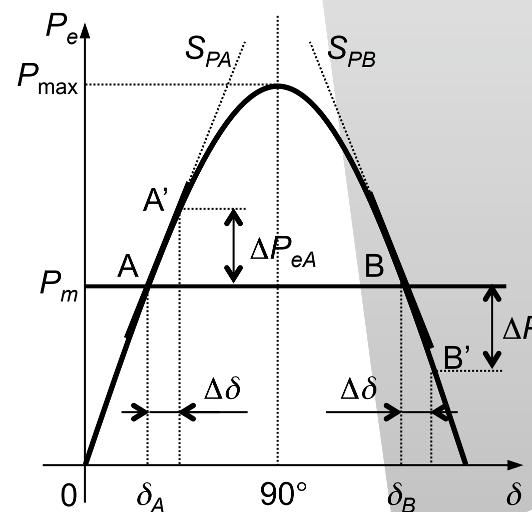
Topics

- Concept of Small Signal Stability
- Linearization of Swing Equations
- Eigenvalue Analysis
- Ways to Improve Small Signal Stability

Concept of Small Signal Stability

- Can the system remain at a steady state?
- Small Signal Stability at Points A and B
  - Point A: stable
  - Point B: unstable





WASHINGTON STATE UNIVERSITY
TRI-CITIES
World Class. Face to Face.

Analysis Method for Small Signal Stability

- Linearization
- Eigenvalue Analysis
  - Real part of the eigenvalue indicate the stability

$$\begin{cases} \dfrac{dx}{dt} = f(x, y, u) \\ y = g(x, u) \end{cases} \Rightarrow \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}$$

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_1}{\partial u} \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda = \sigma \pm j\omega$$

$$C = \begin{bmatrix} \dfrac{\partial y}{\partial x_1} & \dfrac{\partial y}{\partial x_2} \\ \dfrac{\partial y}{\partial x_1} & \dfrac{\partial y}{\partial x_2} \end{bmatrix} \qquad D = \begin{bmatrix} \dfrac{\partial y}{\partial u} \\ \dfrac{\partial y}{\partial u} \end{bmatrix}$$

Linearization of Swing Equations

D: Damping Coefficient, representing mechanical friction, air resistance, …

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e - D\omega = P_m - P_{\max}\sin\delta - D\omega$$

$$\begin{cases} \dfrac{2H}{\omega_s}\dfrac{d\omega}{dt} = P_m - P_{\max}\sin\delta - D\omega \\ \dfrac{d\delta}{dt} = \omega - \omega_s \end{cases}$$

Linearization:

$$\begin{cases} \dfrac{2H}{\omega_s}\dfrac{d\Delta\omega}{dt} = -P_{\max}\cos\delta\,\Delta\delta - D\Delta\omega = -S_P\Delta\delta - D\Delta\omega \\ \dfrac{d\Delta\delta}{dt} = \Delta\omega \end{cases}$$

S_P: Synchronizing Coefficient

State Space Representation and Eigenvalue analysis

Linearization:

$$\begin{cases} \dfrac{2H}{\omega_s}\dfrac{d\Delta\omega}{dt} = -S_P\Delta\delta - D\Delta\omega \\ \dfrac{d\Delta\delta}{dt} = \Delta\omega \end{cases}$$

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} -\dfrac{\omega_s}{2H}D & -\dfrac{\omega_s}{2H}S_P \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix}$$

Compare: $\Delta\dot{x} = A\Delta x$

Eigenvalue analysis:

$$\left| A - \lambda I \right| = 0$$

$$\begin{vmatrix} -\dfrac{\omega_s}{2H}D - \lambda & -\dfrac{\omega_s}{2H}S_P \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + \frac{\omega_s}{2H}D\lambda + \frac{\omega_s}{2H}S_P = 0$$

$$\lambda_{1,2} = \frac{-\dfrac{\omega_s}{2H}D \pm \sqrt{\left(\dfrac{\omega_s}{2H}\right)^2 D^2 - 4\dfrac{\omega_s}{2H}S_P}}{2}$$

$$= \frac{-KD \pm \sqrt{K^2D^2 - 4KS_P}}{2}$$

Example: Small Signal Stability

Problem: System and conditions shown in textbook example 16.7. Determine small signal stability for operating points for *D* = 0 and *D* = 0.05.

Solution:

$$\lambda_{1,2} = \frac{-KD \pm \sqrt{K^2D^2 - 4KS_P}}{2}$$

$$K = \frac{\omega_s}{2H} = \frac{2\pi f_0}{2 \bullet 5} = 37.699$$

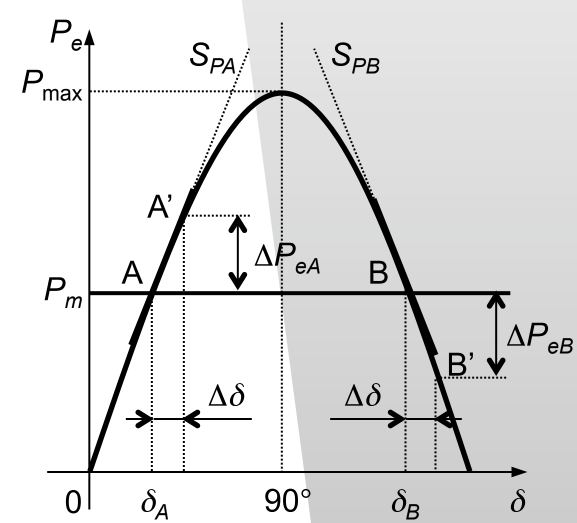
$$S_P = P_{\max} \cos\delta$$

$$P_e = P_{\max} \sin\delta = 2.1 \sin\delta$$

$$P_m = 1.0$$

$$\delta_A = 28.44° \qquad S_{P,A} = P_{\max} \cos\delta_A = 1.8466$$

$$\delta_B = 151.56° \qquad S_{P,B} = P_{\max} \cos\delta_B = -1.8466$$



Example: Small Signal Stability *cont'd*

Solution ***(cont'd)*****:**

$$\lambda_{1,2} = \frac{-KD \pm \sqrt{K^2D^2 - 4KS_P}}{2}$$

D=0:

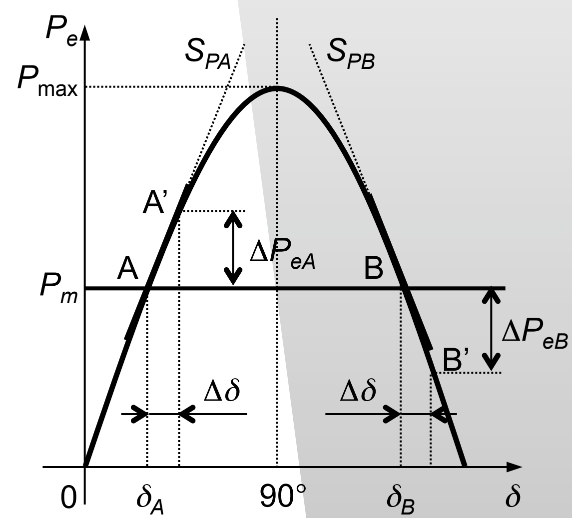
Point A: $\lambda_{1,2} = \pm\sqrt{-KS_P} = \pm j8.3436$

Point B: $\lambda_{1,2} = \pm\sqrt{-KS_{P,B}} = \pm 8.3436$

D=0.05:

Point A: $\lambda_{1,2} = -0.9425 \pm j8.2902$

Point B: $\lambda_{1,2} = 7.4542, \text{ and } -9.3391$



Example: Small Signal Stability *cont'd*

Solution ***(cont'd)*****:**

$$\lambda_{1,2} = \frac{-KD \pm \sqrt{K^2D^2 - 4KS_P}}{2}$$

Observation:

(1) S_P

When S_P <0, unstable

When S_P >0,

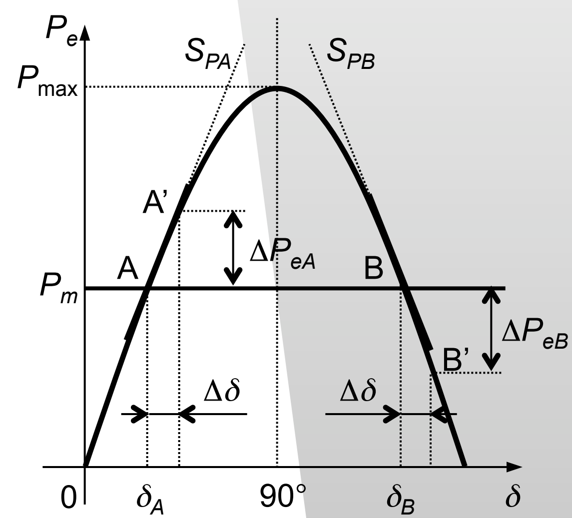
if $K^2D^2-4KS_P$>=0, stable

if $K^2D^2-4KS_P$< 0, stable

(2) D: helps to improve stability

(3) $\lambda = \sigma \pm j\omega$: damping and oscillation frequency

(4) The same approach and procedure are applicable for multi-machine system. Size of A matrix increase.



$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} = P_{ai}$$

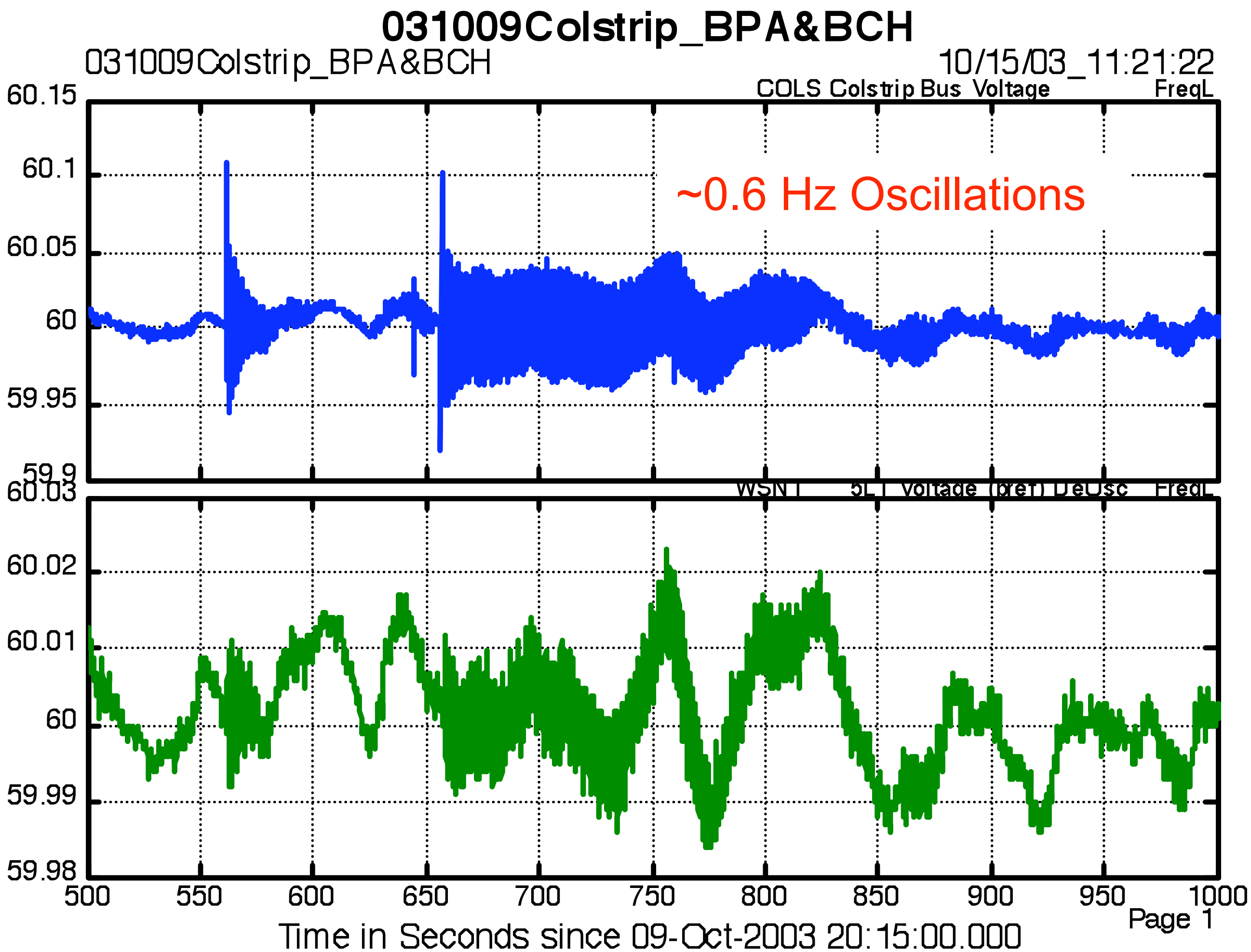
Ways to Improve Small Signal Stability

- Increase D
- …

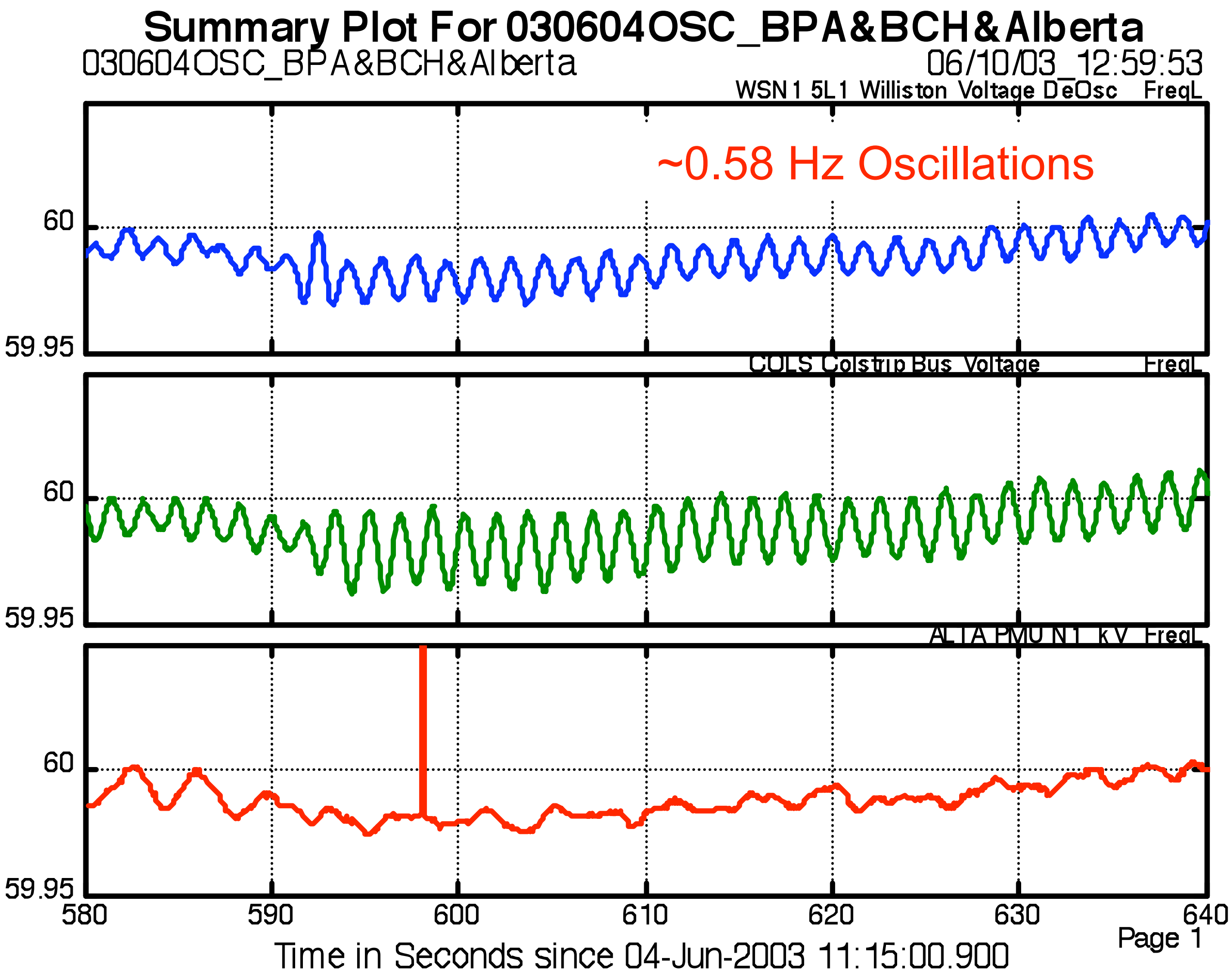
- AVR and PSS
  - A well-tuned PSS effectively increases D and thus improves small signal stability

WASHINGTON STATE UNIVERSITY
TRI-CITIES
World Class. Face to Face.

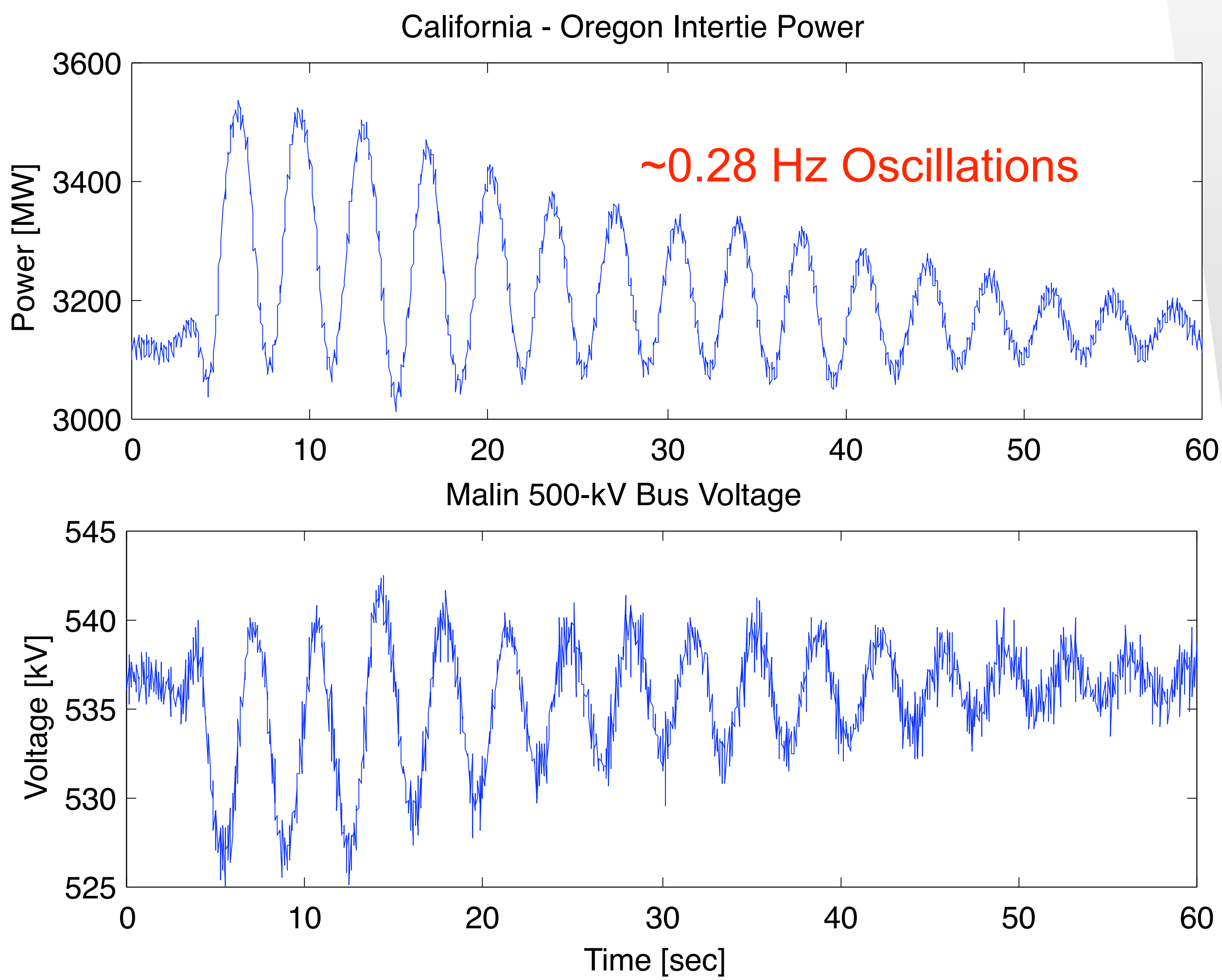
Past Oscillation Events – 2003/10/09



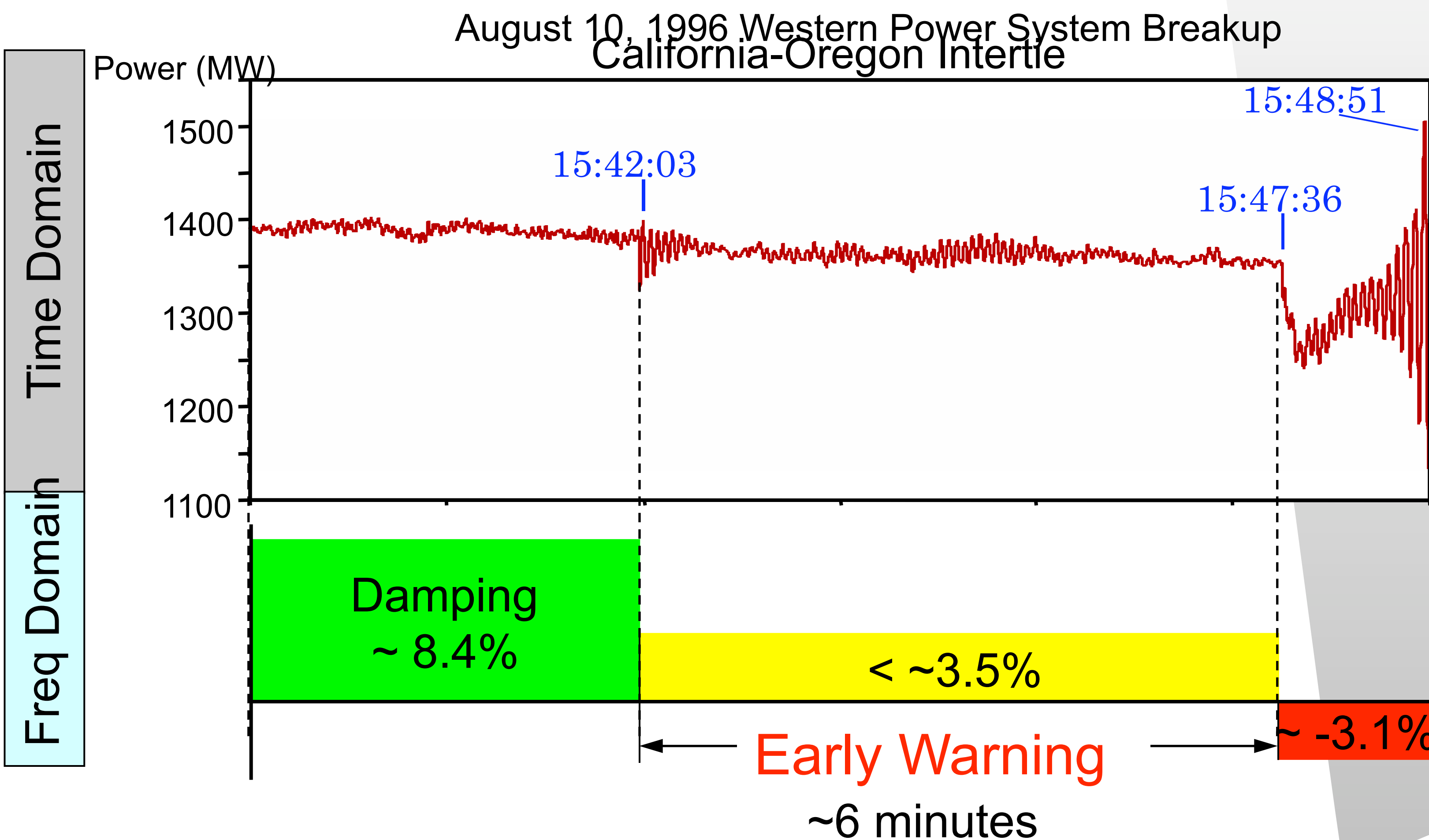
Past Oscillation Events – 2003/06/04



Past Oscillation Events – 2000/08/04



Past Oscillation Events – 1996/08/10



Questions?

WASHINGTON STATE UNIVERSITY
TRI-CITIES
World Class. Face to Face.