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EE 521: Analysis of Power Systems

Lecture 2
Power System Review

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

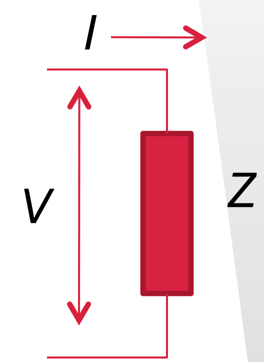
Topics to Be Reviewed

- Single-Phase vs. Three-Phase
- Per Unit System
  - Per Unit System with Transformers
- Steady-State Component Modeling
  - Transmission Line Modeling
  - Transformer Modeling
- Power Flow Basics
  - Network Matrix
  - Power Flow Equations
- Control Theory

Single-Phase vs. Three-Phase

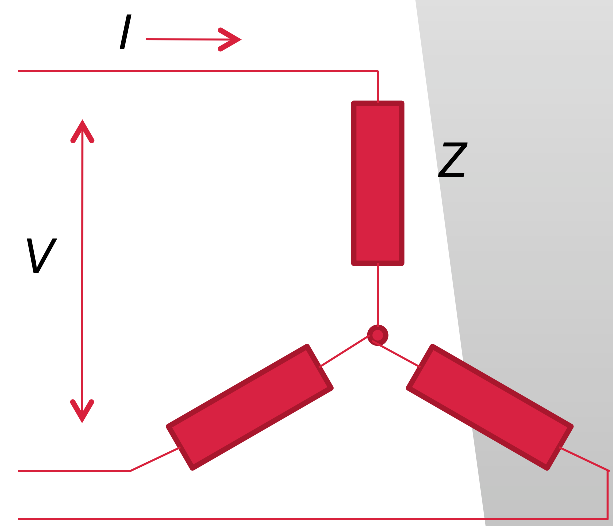
- Single-Phase AC Systems

$$S = P - jQ = VI^* = \frac{V^2}{Z^*}$$



- Three-Phase AC Systems

$$S = P - jQ = \sqrt{3}VI^* = \sqrt{3}\frac{V^2}{Z^*}$$



What is Per Unit System?

- Normalization by a base value
  - 120 V = 1 pu, 126 V = 1.05 pu
  - 500 kV = 1 pu, 525 kV = 1.05 pu
- Used extensively in power system analysis
- Simplifies calculations for large systems
- Especially true for systems with multiple voltage levels

Example: Importance of Per Unit System

- Voltage is usually required to be in a ±10% range of a nominal value = [0.9, 1.1] pu
- Different voltage levels have different nominal values
  - 500 kV, 230 kV, 138 kV, 60 kV, etc.

Actual Voltage (kV)	Per Unit Value (pu)	Base Value (kV)
510.00	1.02	500
257.60	1.12	230
230.00	1.00	230
460.00	0.92	500
139.38	1.01	138
51.00	0.85	60
131.10	0.95	138
250.70	1.09	230

Calculate Per Unit Values

- Select base values
  - Five important electrical quantities:
    - Power (*S*), Voltage (*V*), Current (*I*), Impedance (*Z*), Admittance (*Y*)
  - Only two are independent
    - *V* base is the nominal value of a voltage level
    - *S* base is the nominal capacity of an equipment, or a convenient number, e.g. 100 MVA
- Calculate other base values
  - For single phase systems

$$I_B = \frac{S_B}{V_B} \qquad Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B} \qquad Y_B = \frac{1}{Z_B}$$

Calculate Per Unit Values

- Calculate other base values
  - For three-phase systems

$$I_B = \frac{S_B}{\sqrt{3}V_B} \qquad Z_B = \frac{V_B}{\sqrt{3}I_B} = \frac{V_B^2}{S_B} \qquad Y_B = \frac{1}{Z_B}$$

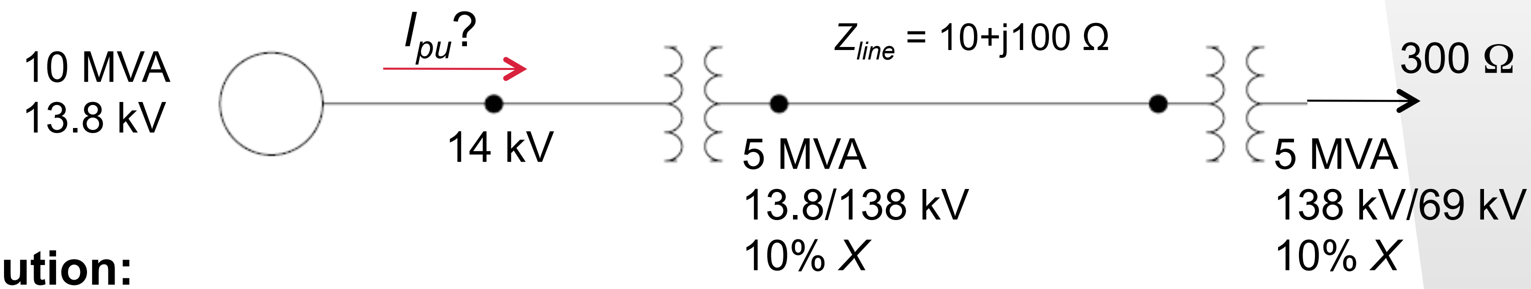
$$S = P - jQ = \sqrt{3}VI^* = \sqrt{3}\frac{V^2}{Z^*}$$

$$S_{pu} = P_{pu} - jQ_{pu} = V_{pu}I_{pu}^* = \frac{V_{pu}^2}{Z_{pu}^*}$$

- Calculate Per Unit Values by Normalization

Example:
Per Unit Calculation

Problem: Find I_{pu} in the diagram.



Solution:

$$S_B = 10MVA,\quad V_{B(1)} = 13.8kV,\quad V_{B(2)} = 138kV,\quad V_{B(3)} = 69kV$$

$$Z_{B(2)} = \frac{V_{B(2)}{}^2}{S_B} = \frac{\left(138\times10^3\right)^2}{\left(10\times10^6\right)} = 1904\Omega,\quad Z_{B(3)} = \frac{V_{B(3)}{}^2}{S_B} = \frac{\left(69\times10^3\right)^2}{\left(10\times10^6\right)} = 476.1\Omega$$

$$Z_{T1,pu} = j10\% \frac{V_{T1,high}{}^2}{S_{T1}} \Big/ Z_{B(1)} = j10\% \frac{V_{T1,high}{}^2}{S_{T1}} \frac{S_B}{V_{B(1)}{}^2} = j10\% \frac{S_B}{S_{T1}} = j10\% \frac{10MVA}{5MVA} = j0.2pu$$

$$Z_{line,pu} = \frac{Z_{line}}{Z_{B(2)}} = \frac{10 + j100\Omega}{1904\Omega} = 0.00525 + j0.0525pu$$

$$Z_{T2,pu} = j10\% \frac{S_B}{S_{T2}} = j10\% \frac{10MVA}{5MVA} = j0.2pu$$

$$V_{1,pu} = \frac{V_1}{V_{B(1)}} = \frac{14kV}{13.8kV} = 1.014pu$$

$$Z_{load,pu} = \frac{Z_{load}}{Z_{B(3)}} = \frac{300\Omega}{476.1\Omega} = 0.63pu$$

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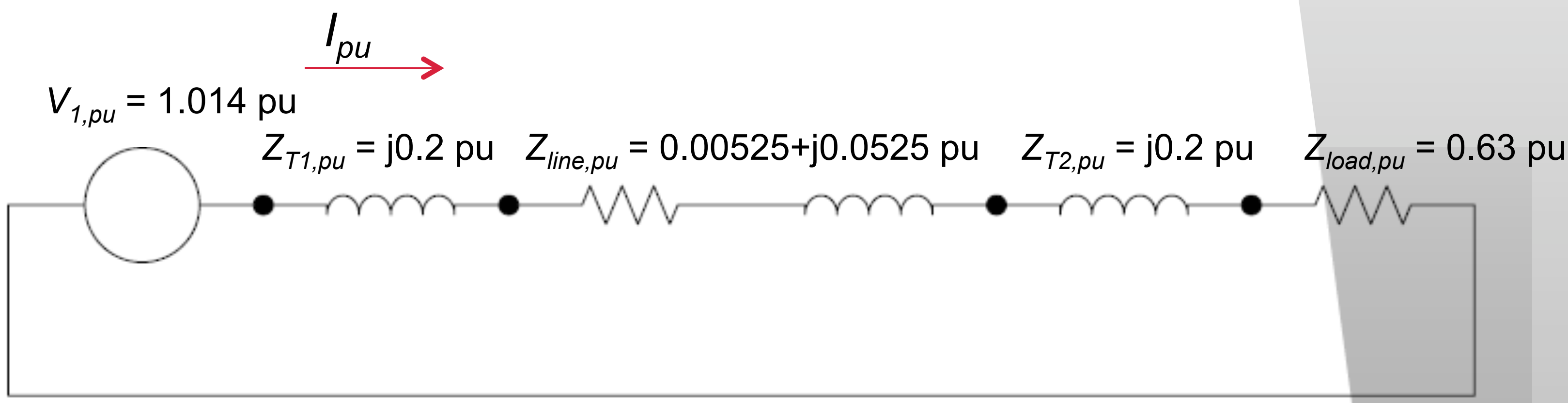
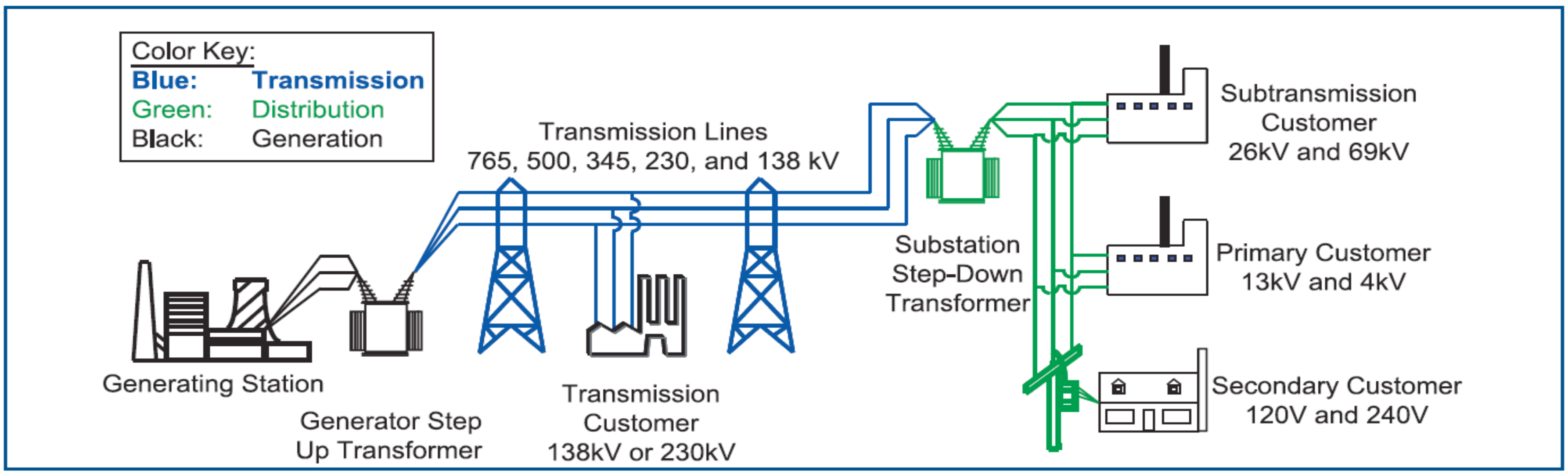
Example: Per Unit Calculation *cont'd*

$$
\begin{aligned}
I_{pu} &= \frac{V_{1,pu}}{Z_{total,pu}} \\
&= \frac{1.014}{j0.2 + 0.00525 + j0.0525 + j0.2 + 0.63} \\
&= 1.059 - j0.754\, pu \\
&= 1.3\angle -35^\circ\, pu
\end{aligned}
$$



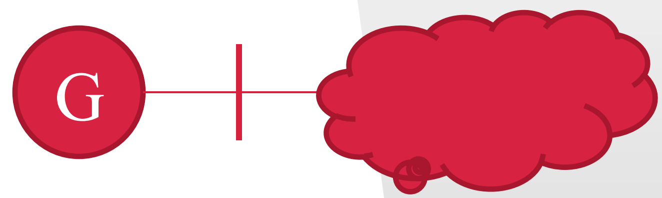
Steady-State Component Modeling

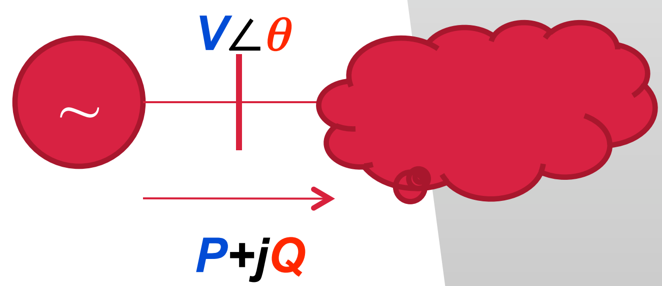
- Generators
- Transformers
- Transmission Lines
- Loads



Generators

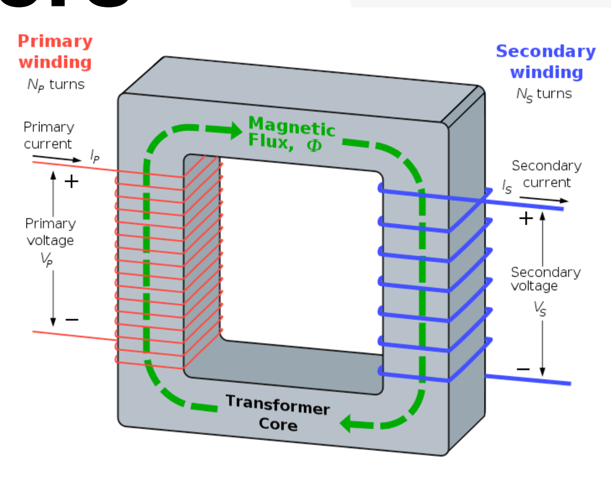
- Characteristics
  - Real power output and voltage are usually controlled by governor and exciter, respectively
- Modeled as a voltage source
  - with real power and voltage known
  - Phase angle and reactive power are unknown



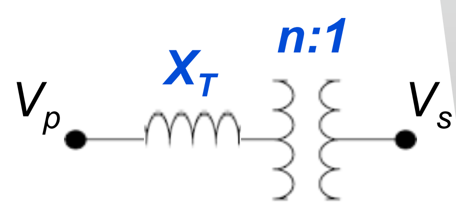


Transformers

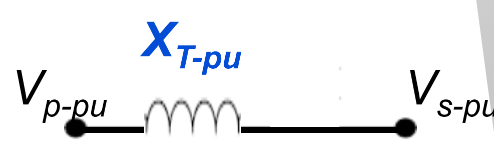
- Characteristics
    - Two different voltage levels at primary and secondary sides
- Modeled as an impedance with a voltage ratio
    - Usually ignore real power loss, only reactance is modeled
    - Ratio becomes 1 when using per unit values



Source: Wikipedia



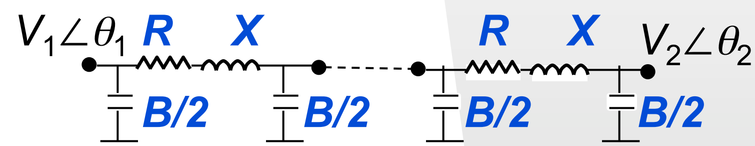
Actual Value

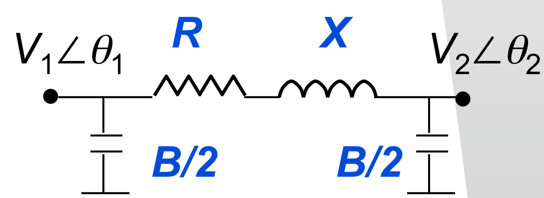


pu Value

Transmission Lines

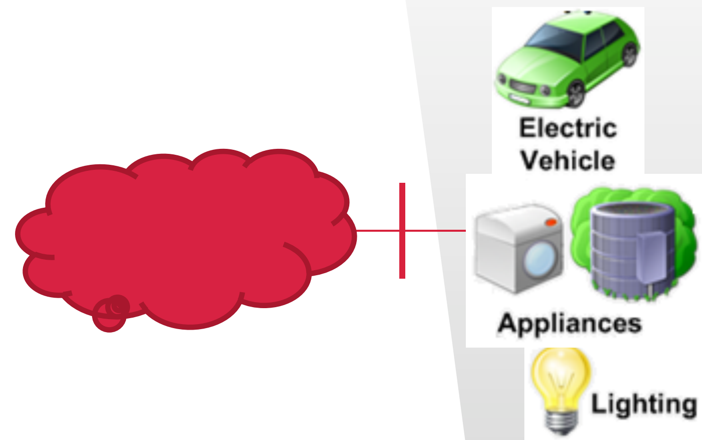
- Characteristics
  - Has losses
  - Has capacitor charging effect
- Modeled as impedance circuit
  - Long lines: cascading "π"
  - Medium lines: single "π"
  - Short lines: single impedance

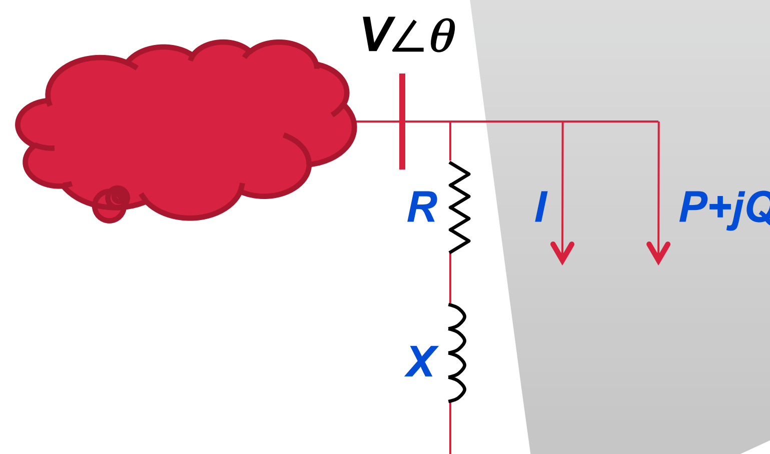




Loads

- Characteristics
  - Most are resistive and inductive
  - Most are not “smart”
  - Usually know the amount of power consumption
- Modeled as passive elements
  - Constant PQ load
  - Constant Z load
  - Constant I load
  - ZIP load

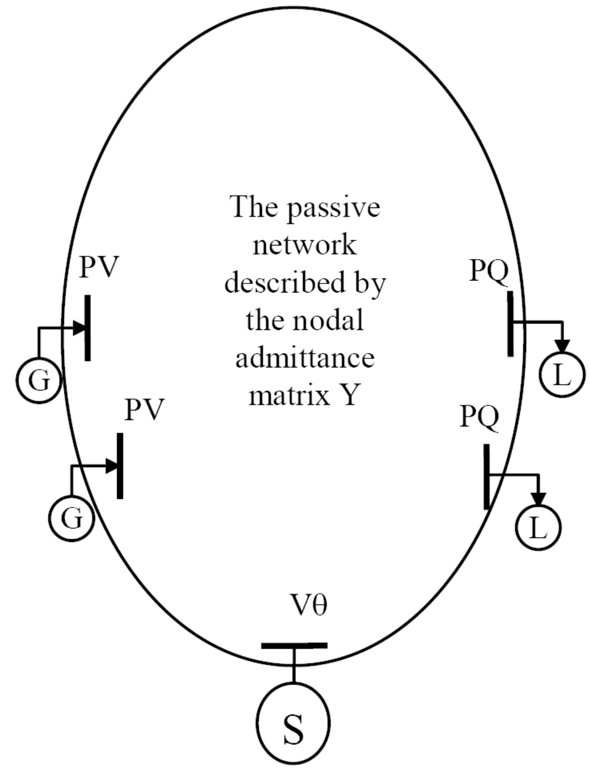




Power Flow Basics

- The problem: solve electrical circuits, but at a large scale

Bus Type	Boundary Conditions				Unknowns				Total Number of Buses of the Type
	P	*Q*	*V*	*θ*	*P*	*Q*	*V*	*θ*	
PV									*r*
PQ									*N-r-1*
Vθ									*1*



Power Flow Equations

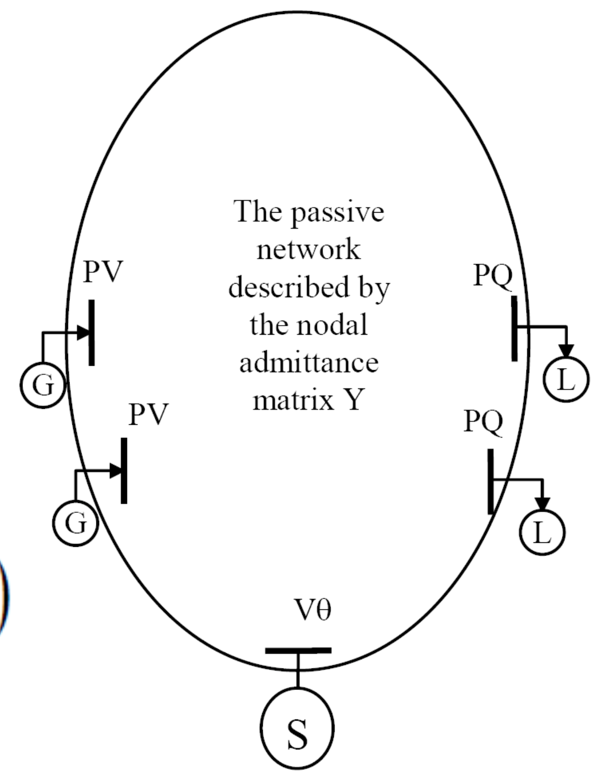
$$\mathbf{I} = \mathbf{YV}$$

$$\mathbf{E}^*\mathbf{I} = \mathbf{E}^*\mathbf{YV}$$

$$P_i - j\,Q_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j|\left(g_{ij}\cos(\theta_{ij}) + b_{ik}\sin(\theta_{ij})\right)$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j|\left(g_{ij}\sin(\theta_{ij}) + b_{ik}\cos(\theta_{ij})\right)$$



Nodal Admittance Matrix

$Z_l = .5i$

$$Y_{bus} = \begin{bmatrix} -2i & 2i \\ 2i & -2i \end{bmatrix}$$

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Nodal Admittance Matrix

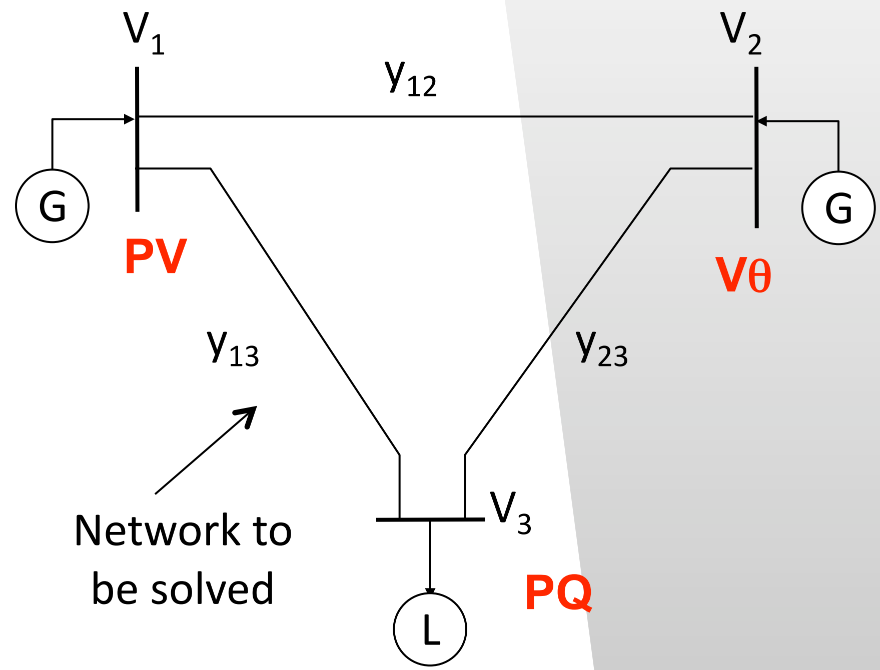
$$Y = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix}$$

$$P_1 - jQ_1 = |V_1|^2 (y_{12} + y_{13}) - V_1^* V_2 y_{12} - V_1^* V_3 y_{13}$$

$$P_2 - jQ_2 = |V_2|^2 (y_{12} + y_{23}) - V_2^* V_1 y_{12} - V_2^* V_3 y_{23}$$

$$P_3 - jQ_3 = |V_3|^2 (y_{13} + y_{23}) - V_3^* V_1 y_{13} - V_3^* V_2 y_{23}$$

$$\left\{ \begin{array}{l} \text{PV} \left\{ \begin{array}{l} P_1 = f_{P1}(V_1, V_2, V_3, \theta_1, \theta_2, \theta_3) \\ V_1 = known \end{array} \right. \\ \text{V}\theta \left\{ \begin{array}{l} V_2 = known \\ \theta_2 = known \end{array} \right. \\ \text{PQ} \left\{ \begin{array}{l} P_3 = f_{P3}(V_1, V_2, V_3, \theta_1, \theta_2, \theta_3) \\ Q_3 = f_{Q3}(V_1, V_2, V_3, \theta_1, \theta_2, \theta_3) \end{array} \right. \end{array} \right.$$



Power Flow Solution Methods

- Gauss-Seidel
- Newton-Raphson
- Decoupled
- DC

Gauss-Seidel

- First effective method to be implemented on digital computers
- In general has a linear convergence rate
- Can generally accept a “flat start”
- Has some issues with reactance reactance branches
- Each bus is treated individually

Newton-Raphson

- Based on the concept of driving an error function to zero
- Generally has a quadratic convergence rate
- Some issues exist with “flat start” values
- Each iteration requires more computations than for a Gauss-Seidel, but quadratic convergence results in fewer iterations
- Very robust, but the values of the Jacobian must be updated at each iteration

Decoupled Power Flow

- Exploits the decoupled nature of voltage and angle, thus reducing the number of non-zero entries in the Jacobian
- Assumptions are made that greatly improve the convergence time
- When the assumptions are not true, convergence issues may arise

DC Power Flow

- Linear/non-iterative solution to the power flow problem
- Will only calculate the MW flows in the system, not MVAR
- Effective for situations requiring an approximate solution, e.g. contingency analysis

Control Theory

- We will be dealing only with linear control theory
- This theory will facilitate numeric simulation of complex dynamic systems
- Eigen value analysis will be of particular importance

Control Theory

- Linearization

$$\begin{cases} \dfrac{dx}{dt} = f(x, y, u) \\ y = g(x, u) \end{cases}$$

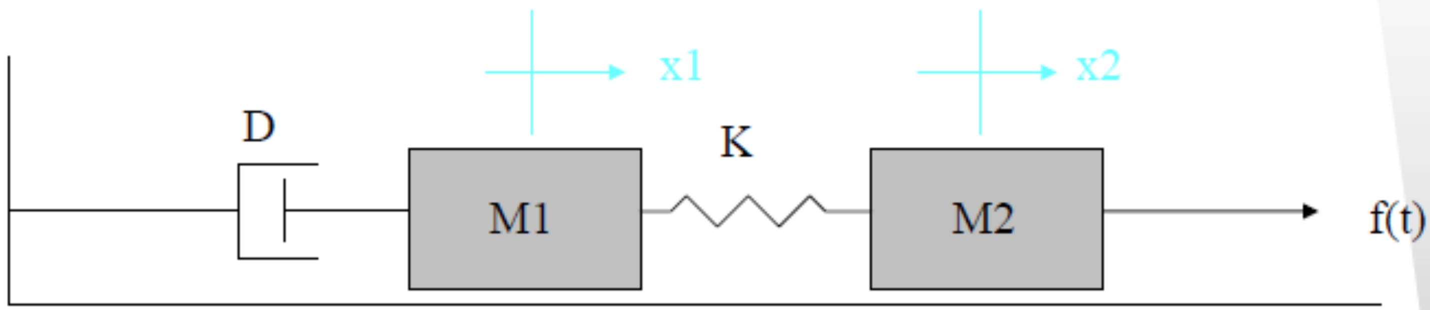
$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_1}{\partial u} \end{bmatrix}$$

$$C = \begin{bmatrix} \dfrac{\partial y}{\partial x_1} & \dfrac{\partial y}{\partial x_2} \\ \dfrac{\partial y}{\partial x_1} & \dfrac{\partial y}{\partial x_2} \end{bmatrix} \qquad D = \begin{bmatrix} \dfrac{\partial y}{\partial u} \\ \dfrac{\partial y}{\partial u} \end{bmatrix}$$

Control Theory Example



$$F_{net} = ma = m\frac{d^2x}{dx^2}$$

$$M_1\frac{d^2x_1}{dt^2} + D\frac{dx_1}{dt} + K(x_1 - x_2) = 0$$

$$-Kx_1 + M_2\frac{d^2x_2}{dt^2} + Kx_2 = f(t)$$

Control Theory Example

$$M_1 \frac{d^2 x_1}{dt^2} + D \frac{dx_1}{dt} + K(x_1 - x_2) = 0 \qquad - Kx_1 + M_2 \frac{d^2 x_2}{dt^2} + Kx_2 = f(t)$$

$$\frac{dx_1}{dt} = v_1$$

$$\frac{dv_1}{dt} = -\frac{K}{M_1} x_1 - \frac{D}{M_1} v_1 + \frac{K}{M_1} x_2$$

$$\frac{dx_2}{dt} = v_2$$

$$\frac{dv_2}{dt} = \frac{K}{M_2} x_1 - \frac{K}{M_2} x_2 + \frac{1}{M_2} f(t)$$

Control Theory Example

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

Example: Power System Simulation

- Model of a synchronous generator
- Physical model, automatic voltage regulator (AVR), and power system stablizer(PSS)

$$\begin{bmatrix} \Delta\dot{\omega}_r \\ \Delta\dot{\delta} \\ \Delta\dot{\psi}_{fd} \\ \Delta\dot{v}_1 \\ \Delta\dot{v}_2 \\ \Delta\dot{v}_s \end{bmatrix} = \begin{bmatrix} -1 & -.1092 & -.1236 & 0 & 0 & 0 \\ 376 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1938 & -.4229 & -27.3 & 0 & 27.3 \\ 0 & -7.3125 & 20.839 & -50 & 0 & 0 \\ 0 & -1.0372 & -1.1738 & 0 & -.7143 & 0 \\ 0 & -4.8404 & -5.477 & 0 & 26.967 & -30.303 \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \\ \Delta\psi_{fd} \\ \Delta v_1 \\ \Delta v_2 \\ \Delta v_s \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T_m$$

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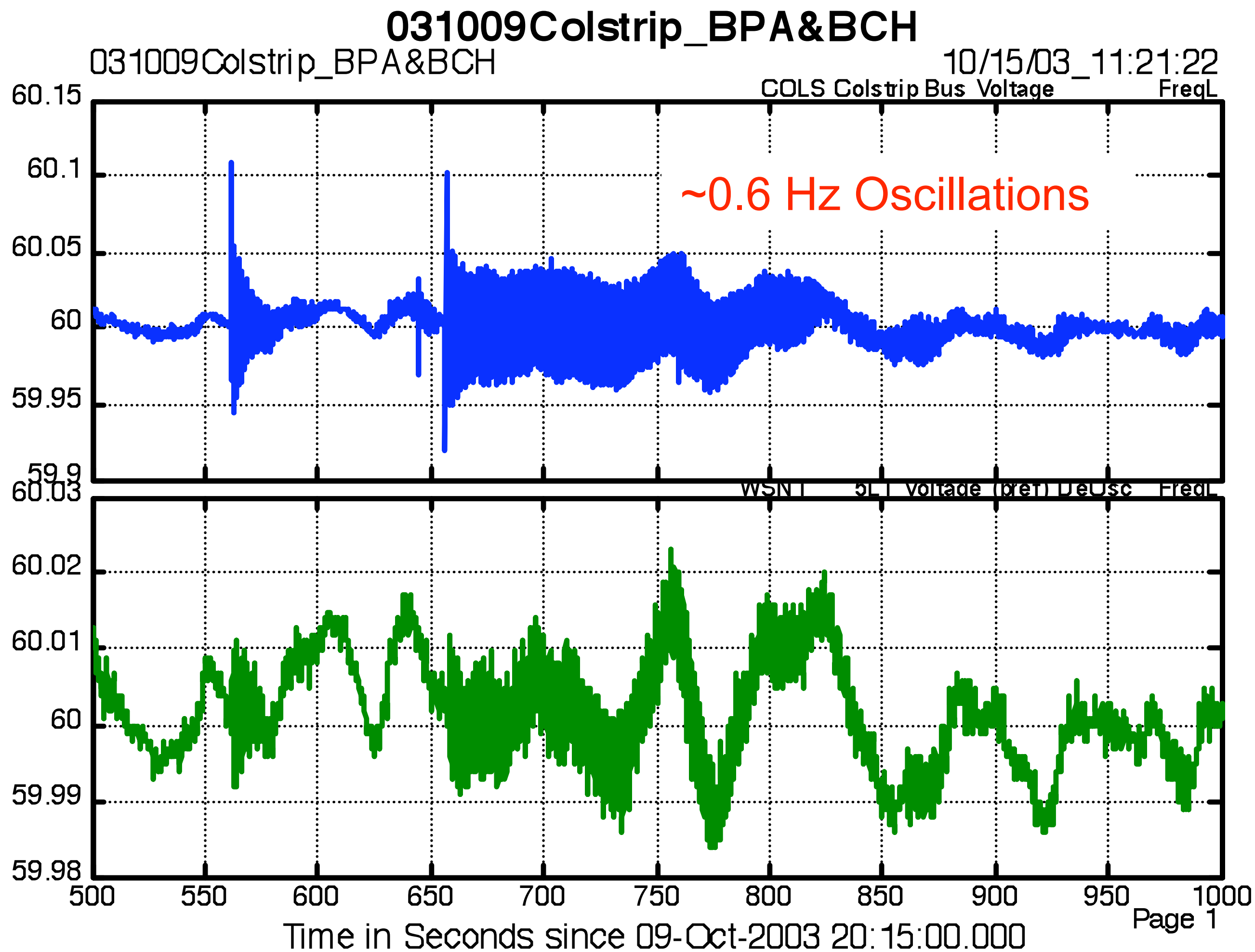
Past Oscillation Events – 2003/10/09



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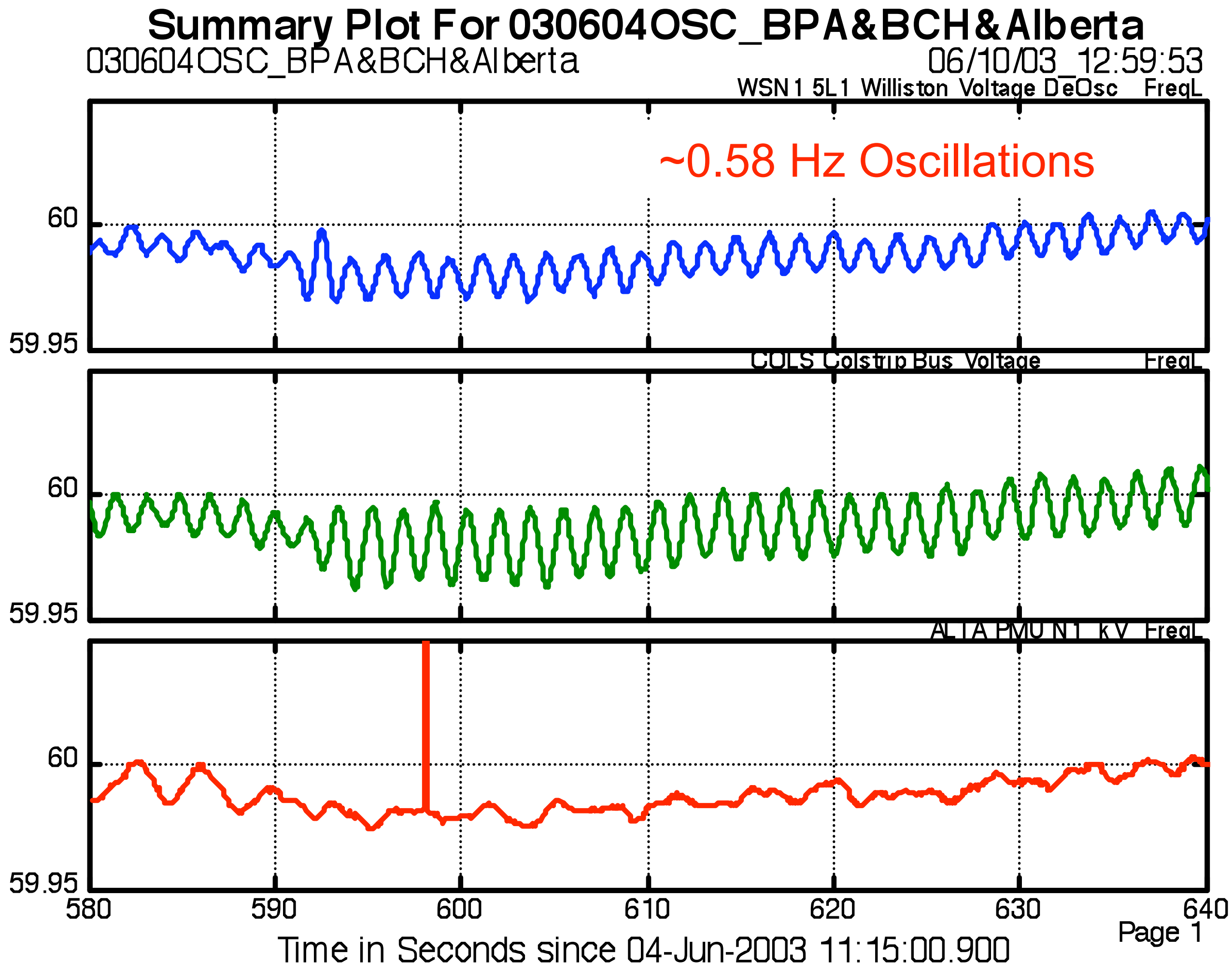
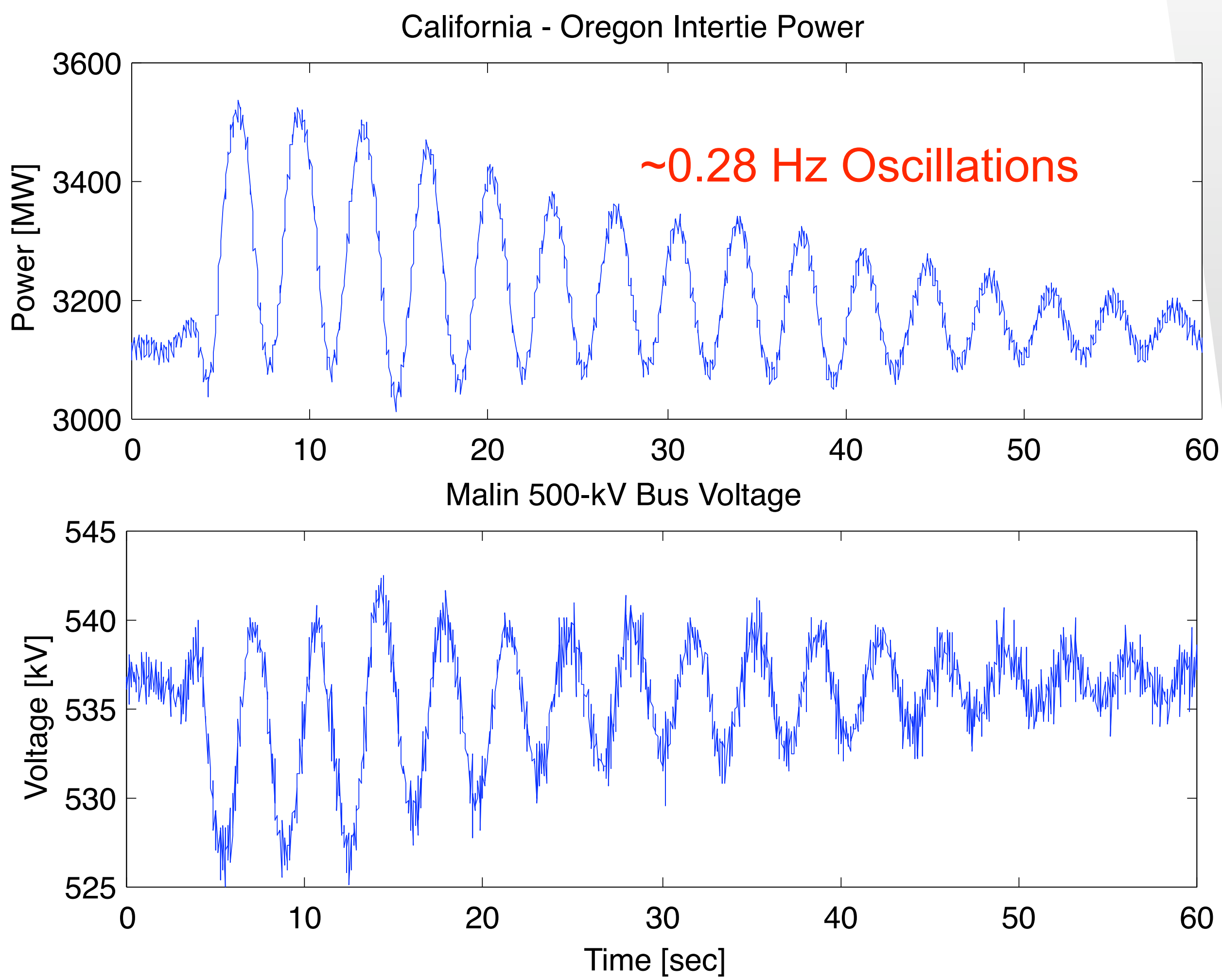
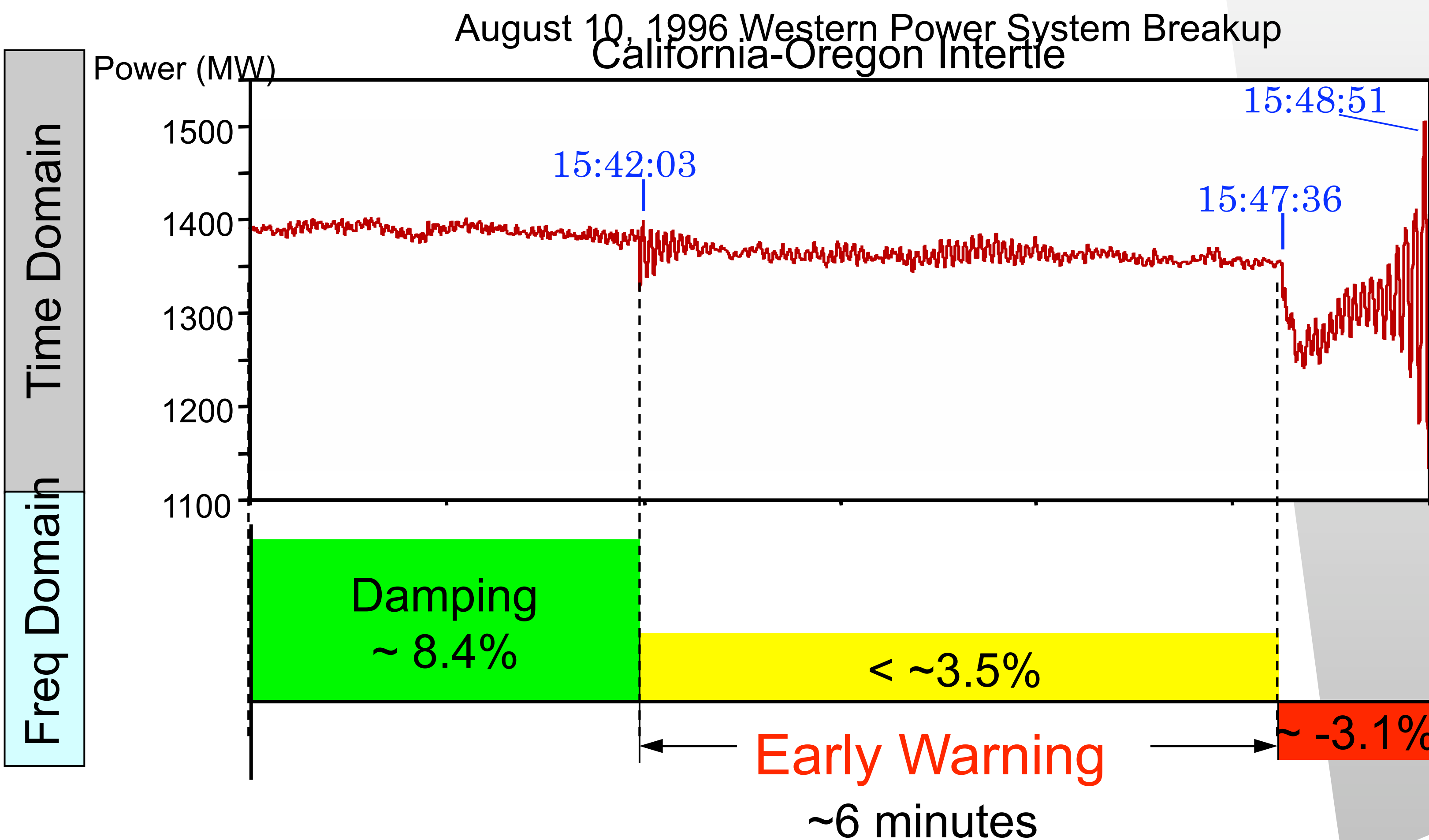
Past Oscillation Events – 2003/06/04



Past Oscillation Events – 2000/08/04



Past Oscillation Events – 1996/08/10



Questions?

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