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EE 521: Analysis of Power Systems

Lecture 17
Transient Stability II

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

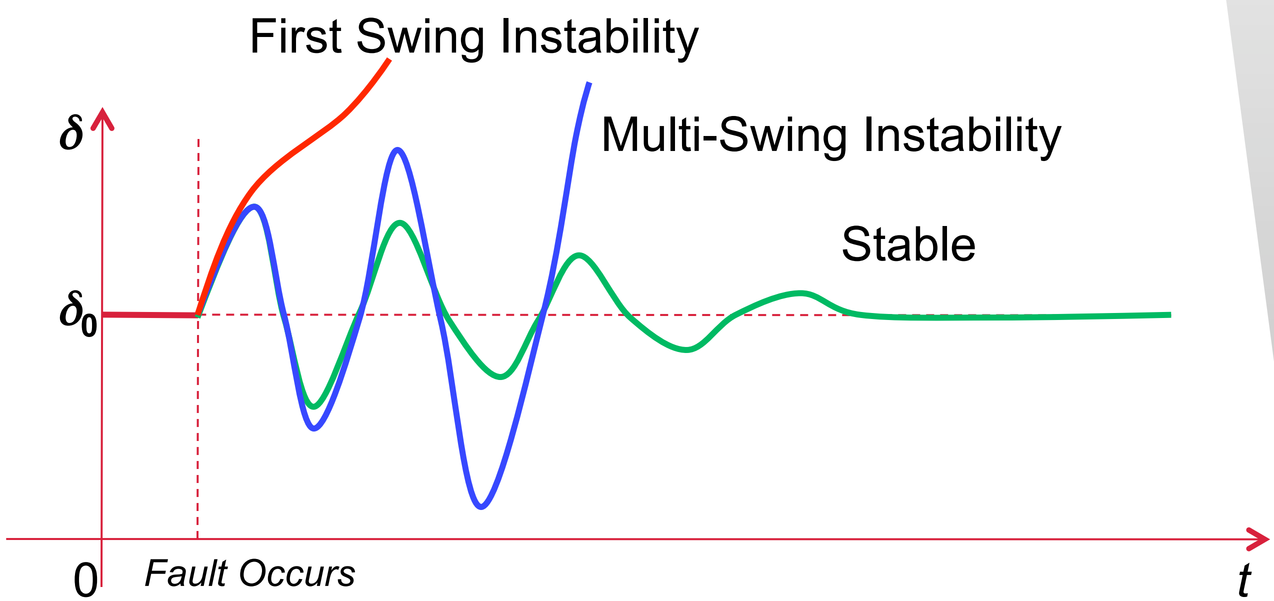
- Numerical Solution to Swing Equations
- Multi-Machine Systems
- Ways to improve transient stability
- Course Project

Limitation of Equal Area Criterion

- No need to know the trajectory of the system.
- It is effective for the single-machine-infinite-bus system or two-machine systems.
    - Multi-machine systems can be equivalent to a two-machine systems based on coherence groups.
- It can not be directly applied to multi-machine systems.
- It is not as effective when machines are not modeled as swing equations.
    - Exciters, governors, Power System Stabilizers (PSS), …

Numerical Solution

- The objective is to find the system trajectory with respect to time.
- So the system stability can be observed with time.



Problem Formulation

In a multi-machine system, we have a swing equation for each machine:

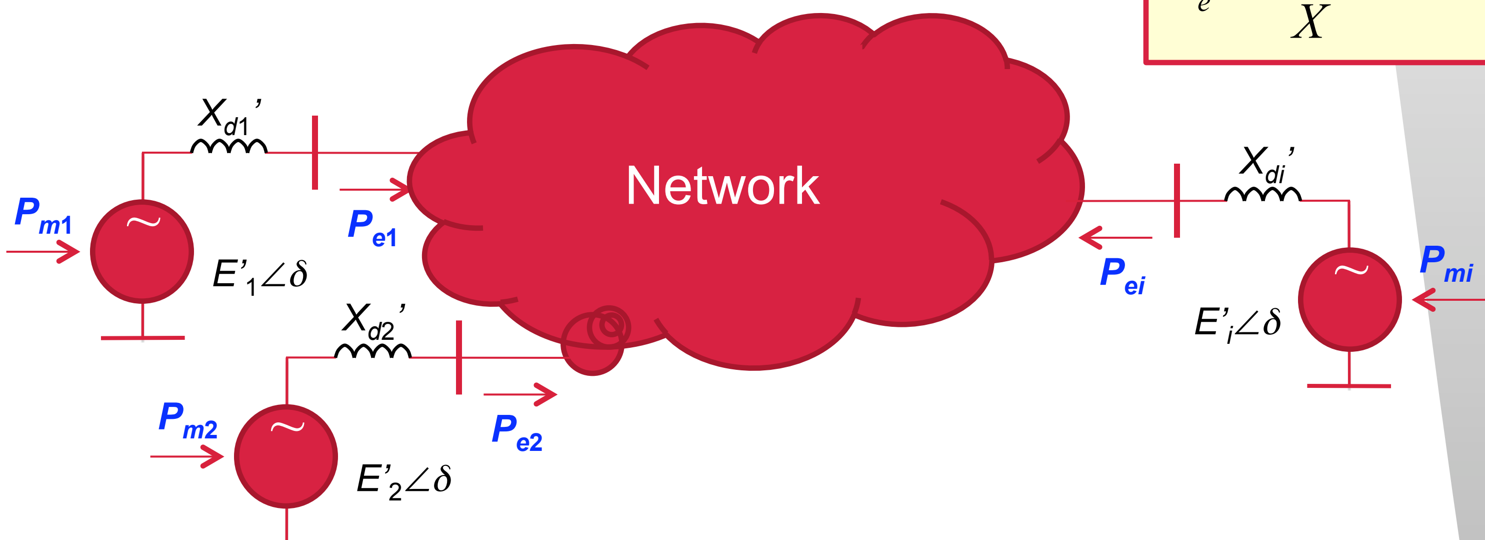
$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} = P_{ai}$$

$$P_{ai} = P_{ai}(\delta_1, \delta_2, \ldots, \delta_L) \quad L = \text{number of machines}$$

Recall the SMIB system:

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$P_e = \frac{E'V}{X}\sin\delta$$



Euler Methods

General differential equation:

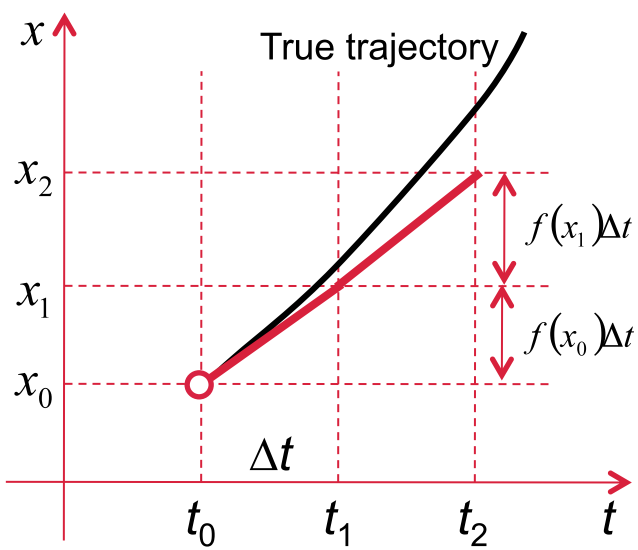
$$\frac{dx}{dt} = f(x)$$

Initial condition: $x = x_0$ at $t = t_0$

Other methods:
- *Trapezoidal*
- *Runge-Kuta*
- *…*

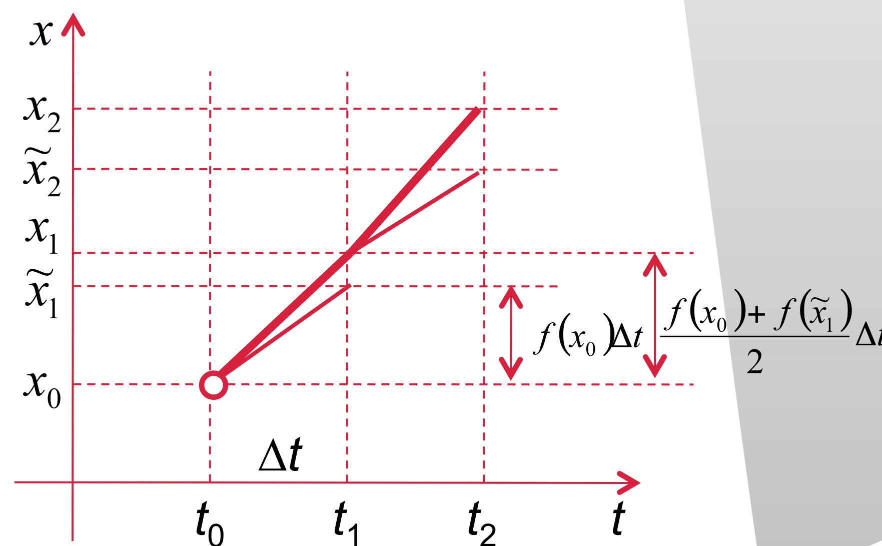
Euler Method:

$$x_1 = x_0 + f(x_0)\Delta t$$



Modified Euler Method:

$$\tilde{x}_1 = x_0 + f(x_0)\Delta t \Rightarrow x_1 = x_0 + \frac{f(x_0) + f(\tilde{x}_1)}{2}\Delta t$$



Application to Swing Equations

Swing equations:

$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} = P_{ai}$$

$$\Rightarrow \begin{cases} \dfrac{2H_i}{\omega_s}\dfrac{d\omega_i}{dt} = P_{mi} - P_{ei} = P_{ai} \\ \dfrac{d\delta_i}{dt} = \omega_i - \omega_s \end{cases}$$

$t = t_0$, initial conditions (power flow solution):

$$P_{ai0} = P_{mi0} - P_{ei0}$$

$$\delta_{i0}, \omega_{i0}$$

$t = t_1$

$$\begin{cases} \left.\dfrac{d\omega_i}{dt}\right|_{t_0} = \dfrac{2H_i}{\omega_s} P_{ai0} \\ \left.\dfrac{d\delta_i}{dt}\right|_{t_0} = \omega_{i0} - \omega_s \end{cases} \Rightarrow \begin{cases} \tilde{\omega}_{i1} = \omega_{i0} + \left.\dfrac{d\omega_i}{dt}\right|_{t_0} \Delta t \\ \tilde{\delta}_{i1} = \delta_{i0} + \left.\dfrac{d\delta_i}{dt}\right|_{t_0} \Delta t \end{cases}$$

$$\begin{cases} \left.\dfrac{d\omega_i}{dt}\right|_{\tilde{t}} = \dfrac{2H_i}{\omega_s} \tilde{P}_{ai1} \\ \left.\dfrac{d\delta_i}{dt}\right|_{\tilde{t}} = \tilde{\omega}_{i1} - \omega_s \end{cases} \Rightarrow \begin{cases} \omega_{i1} = \omega_{i0} + \dfrac{\left.\dfrac{d\omega_i}{dt}\right|_{t_0} + \left.\dfrac{d\omega_i}{dt}\right|_{\tilde{t}}}{2} \Delta t \\ \delta_{i1} = \delta_{i0} + \dfrac{\left.\dfrac{d\delta_i}{dt}\right|_{t_0} + \left.\dfrac{d\delta_i}{dt}\right|_{\tilde{t}}}{2} \Delta t \end{cases}$$

$t = t_2$

....

Example: Euler Methods

- Textbook example 16.11.

Multi-Machine Systems

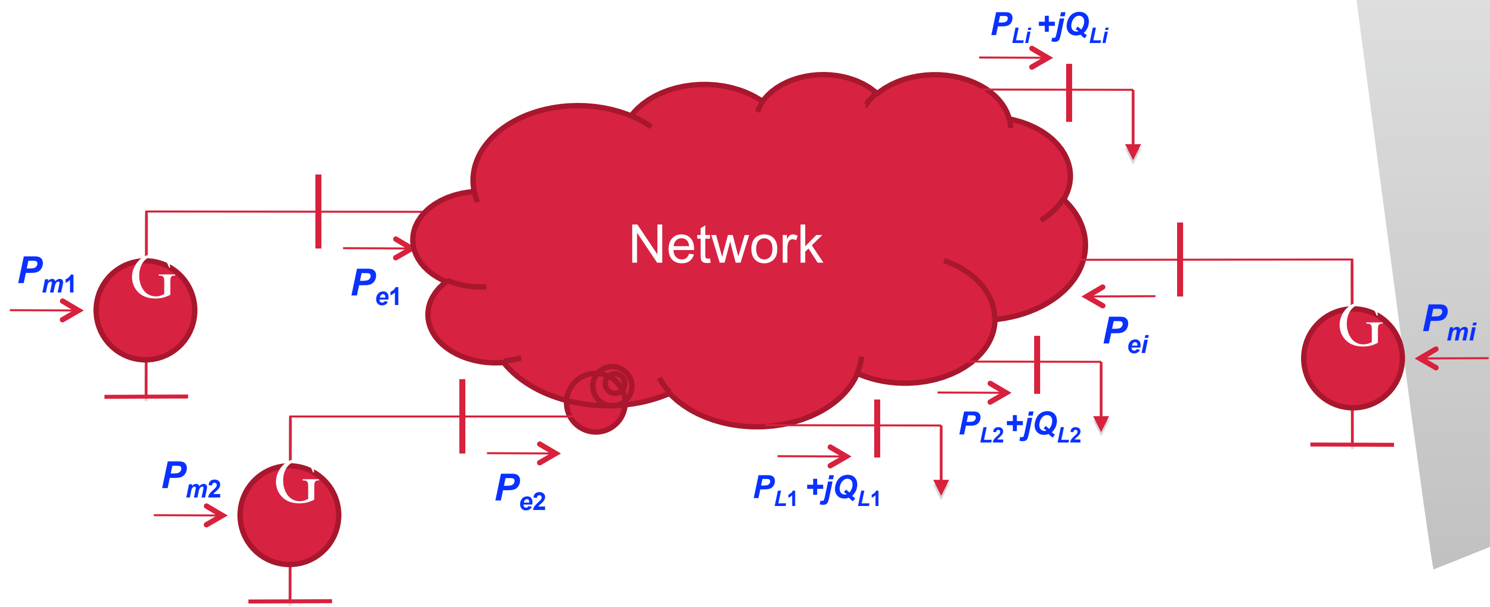
Modeling Approaches:

Classical model:

1. *P_m = const*
2. *Generator model: E' = const, X_d'*
3. *Machine dynamics: δ and ω only*
4. *Load model: $Z=V^2/S^*$=const*
5. *Two state variables: δ and ω*

Full model (simulation programs):

1. *Governor system: P_m != const*
2. *Excitation system: E' != const*
3. *Machine dynamics: δ, ω, E'', dq axis X, damping (friction)*
4. *Load model: ZIP*
5. *>3 state variables: δ, ω, E''*



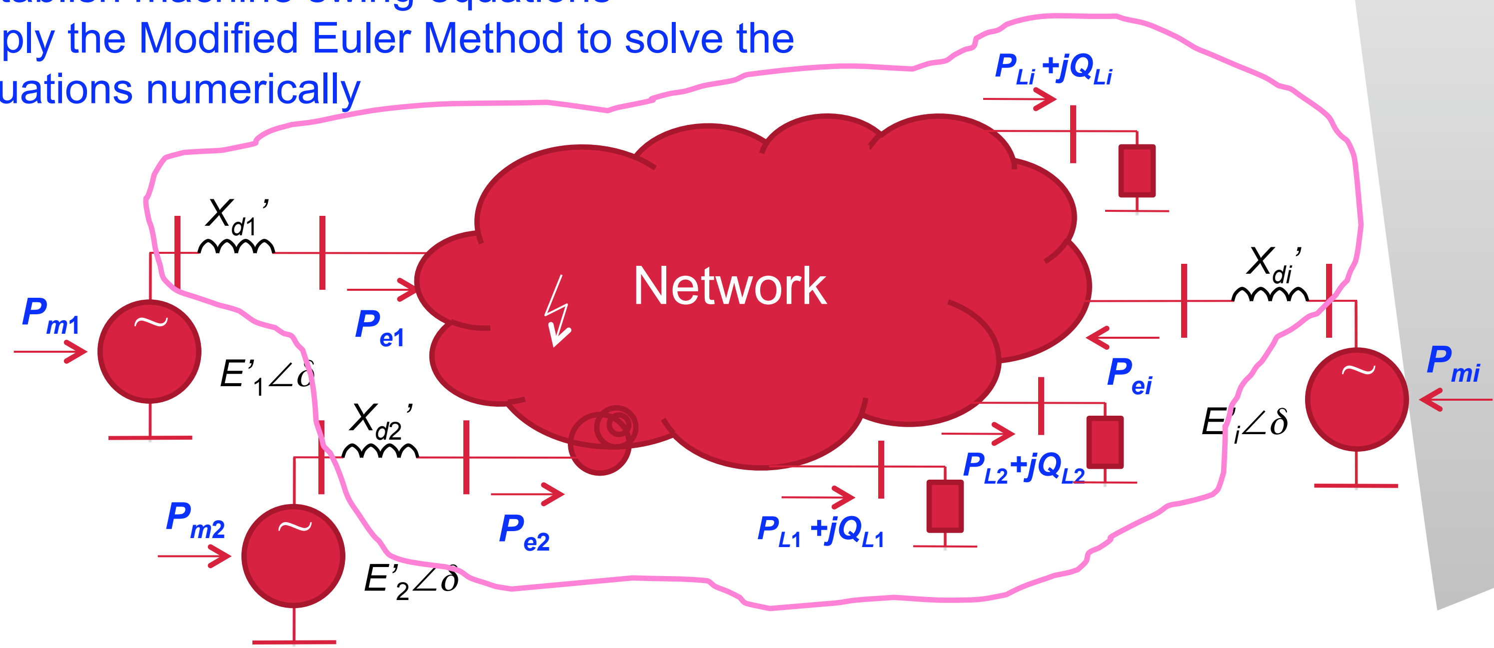
Solution Steps (Classical Model)

1. Solve pre-fault power flow
2. Convert loads to constant impedance
3. Expand Y matrix with load impedance and machine reactances
4. Reduce Y matrix to contain only machine buses
5. Calculate E' and initial δ
6. Derive power-angle equations for each machine for pre-fault, fault-on, and post-fault conditions (diff Y)
7. Establish machine swing equations
8. Apply the Modified Euler Method to solve the equations numerically

$$Y_{\exp} = \left[\begin{array}{c|c} Y + y_{load} & -y_g \\ \hline -y_g & y_g \end{array}\right]$$

$$P_{ai} = P_{ai}\left(\delta_1, \delta_2, \ldots, \delta_L\right)$$

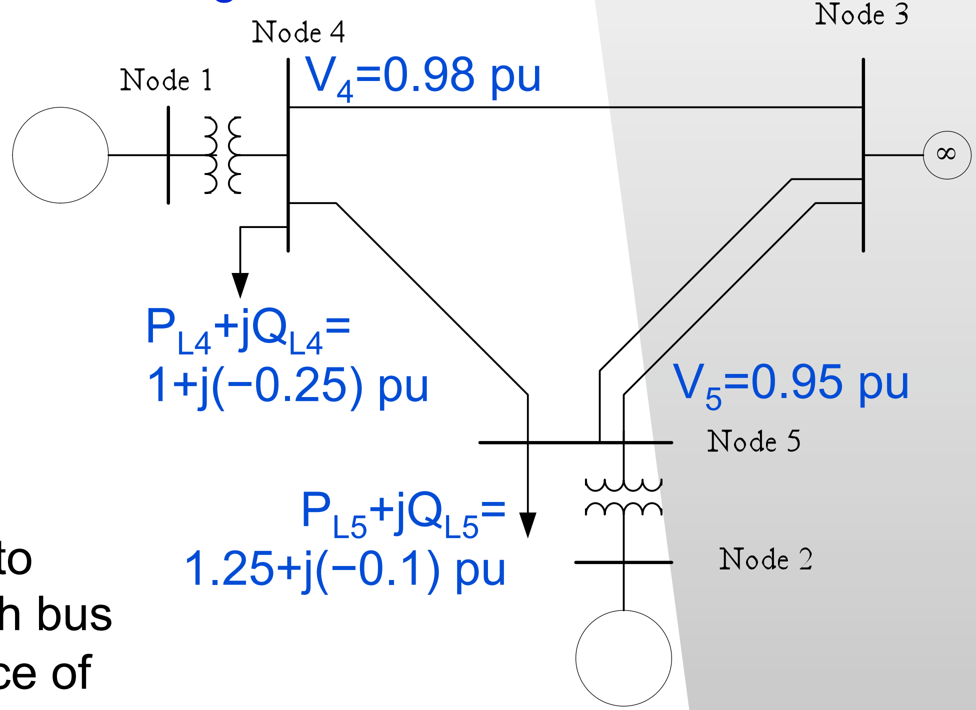
$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} = P_{ai}$$



Example: Gaussian Elimination (Kron Reduction)

Problem: Bus admittance matrix and power flow solution are given. Reduce the network to retain only generator buses 1 through 3

$$Y_{bus} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & -j21.5 & j5 \\ 0 & j12.5 & j10 & j5 & -j27.5 \end{bmatrix}$$



Solution:

Step 1: Perform a complete power flow to determine the voltage magnitude at each bus

Step 2: Calculate the effective impedance of the loads based on the voltage magnitude

Step 3: Perform a Gaussian elimination to remove bus 4 and 5

Review

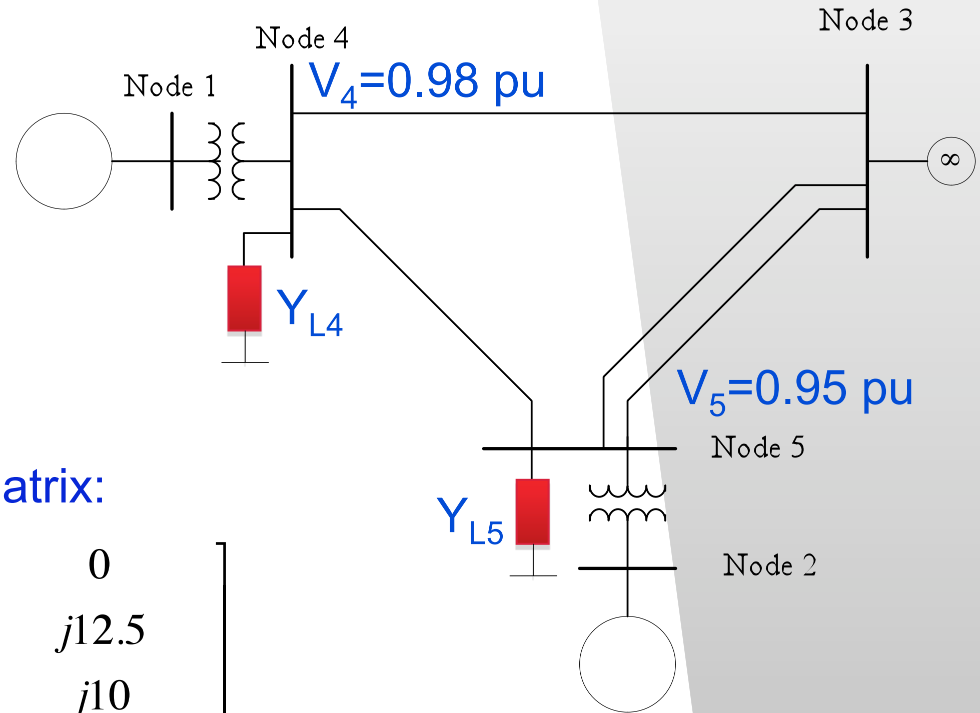
Example: Gaussian Elimination (Kron Reduction) *cont'd*

Effective load impedance:

$$Y_L = \frac{P_L - jQ_L}{|V_L|^2}$$

$$Y_{L4} = \frac{P_{L4} - jQ_{L4}}{|V_4|^2} = \frac{1 + j0.25}{0.98^2} = 1.04 + j0.26$$

$$Y_{L5} = \frac{P_{L5} - jQ_{L5}}{|V_5|^2} = \frac{1.25 + j0.1}{0.95^2} = 1.39 + j0.11$$



Include effective load impedance in Y matrix:

$$Y_{bus}^{(1)} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & 1.04 - j21.24 & j5 \\ 0 & j12.5 & j10 & j5 & 1.39 - j27.39 \end{bmatrix}$$

Review

Example: Gaussian Elimination (Kron Reduction) *cont'd*

Formulate new network equations:

$$\begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & 1.04-j21.24 & j5 \\ 0 & j12.5 & j10 & j5 & 1.39-j27.39 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \\ 0 \end{bmatrix}$$

Pivot around node 4:

$$\begin{bmatrix} 0.36-j5.16 & 0 & 0.11+j2.35 & 0 & 0.14+j2.94 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0.12+j2.35 & 0 & 0.04-j13.25 & 0 & 0.05+j10.94 \\ 0 & 0 & 0 & 0 & 0 \\ 0.14+j2.94 & j12.5 & 0.05+j10.94 & 0 & 1.45-j26.22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \\ 0 \end{bmatrix}$$

$$Y_{ij}^{(1)} = \left(Y_{ij} - \frac{Y_{ik} Y_{kj}}{Y_{kk}} \right)$$

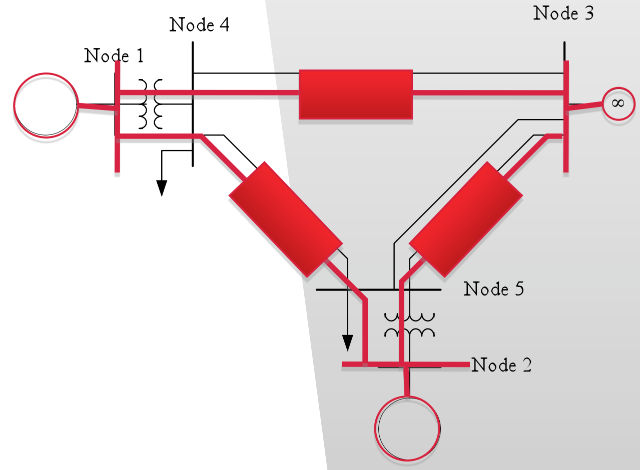
$$I_i^{(1)} = I_i - \frac{Y_{ik}}{Y_{kk}} I_k$$

Review

Example: Gaussian Elimination (Kron Reduction) *cont'd*

Pivot around node 5:

$$\begin{bmatrix} 0.41-j4.83 & 0.14+j1.39 & 0.24+j3.57 & 0 \\ 0.14+j1.39 & 0.33-j6.56 & 0.31+j5.20 & 0 \\ 0.25+j3.57 & 0.31+j5.20 & 0.33-j8.70 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \end{bmatrix}$$



The resulting reduced equations:

$$\begin{bmatrix} 0.41-j4.83 & 0.14+j1.39 & 0.24+j3.57 \\ 0.14+j1.39 & 0.33-j6.56 & 0.31+j5.20 \\ 0.25+j3.57 & 0.31+j5.20 & 0.33-j8.70 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

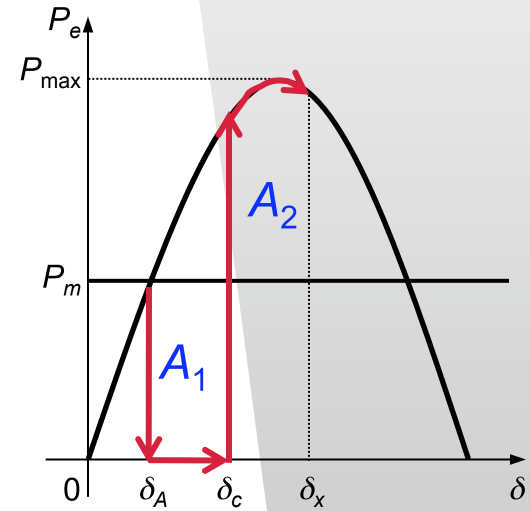
Tip: the resulting matrix should be symmetrical as the original matrix.

Course Project

- See handouts.
- Due December 2, 2009.
- You are welcome to discuss the project with me.

Ways to Improve Transient Stability

- Increase operation margin
- Reduce P_m: governor control
- Clear fault quicker: relay actions
- Increase E': excitation control
- Reduce X: line compensation or enforcement
- Reduce probability of short circuit: vegetation management, …
- ……



(Think about these when working on the course project)

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Questions?