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EE 521: Analysis of Power Systems

Lecture 5
Equivalent Networks

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

- Recap of Power Flow and Matrix Operations
- Matrix Inversion Update
- Gaussian Elimination (Kron Reduction)
- Ward Equivalent
- Ward Equivalent – PV

Recap of Power Flow and Matrix Operations

- Power Flow Equations
- Power Flow Solutions
  - Newton-Raphson Method
  - Decoupled Power Flow
  - DC Power Flow
- Matrix LU Decomposition
  - Crout's Method
  - Dolittle's Method

$$I = YV$$

$$E^{*}I = E^{*}YV$$

$$\begin{bmatrix} \theta^{n+1} \\ V^{n+1} \end{bmatrix} = \begin{bmatrix} \theta^{n} \\ V^{n} \end{bmatrix} - \left[J\left(x^{n}\right)\right]^{-1} \begin{bmatrix} \Delta P\left(x^{n}\right) \\ \Delta Q\left(x^{n}\right) \end{bmatrix}$$

$$Ax = b$$

$$LUx = b$$

$$\begin{cases} Lz = b \\ Ux = z \end{cases}$$

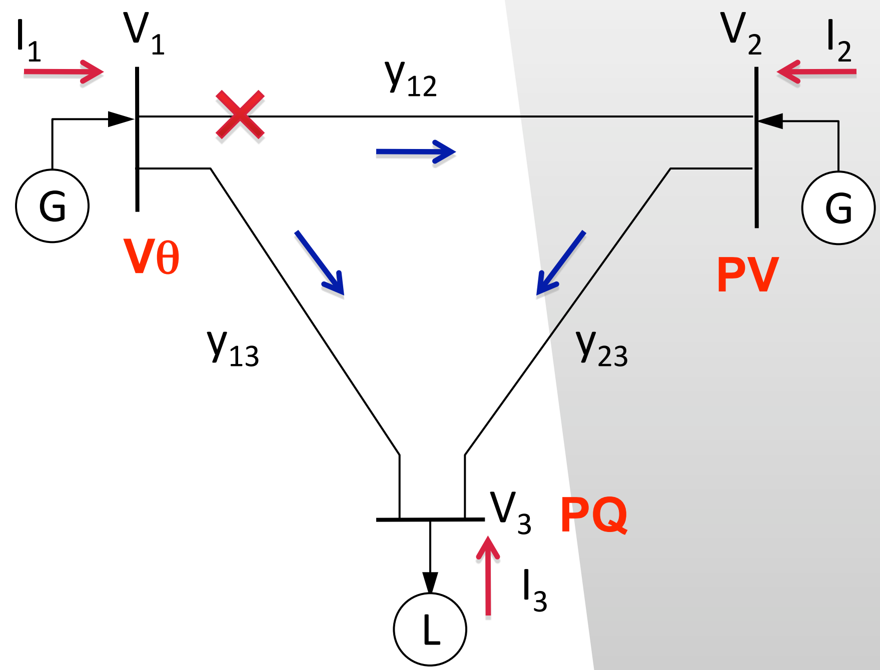
Matrix Update for Contingency Analysis

$$Y_{new} = A^T \begin{bmatrix} y_{12} - y_{12} & 0 & 0 \\ 0 & y_{23} & 0 \\ 0 & 0 & y_{13} \end{bmatrix} A$$

$$= A^T \begin{bmatrix} y_{12} & 0 & 0 \\ 0 & y_{23} & 0 \\ 0 & 0 & y_{13} \end{bmatrix} A + A^T \begin{bmatrix} -y_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} A$$

$$= Y + A^T y A$$

$$Y_{new}{}^{-1} ?$$



A more general problem:

$$\text{Known } A^{-1}, \; A_{new} = A + \Delta A, \quad A_{new}{}^{-1} ?$$

Sherman-Morrison-Woodbury

- When a network is changed the calculated inverse is no longer valid, e.g. topology reconfiguration
- Methods exist for the "updating" of a matrix inversion when there are changes to the source matrix without performing the matrix inversion again

$$(A+uv)^{-1} = A^{-1} - \frac{A^{-1}uvA^{-1}}{1+vA^{-1}u}$$

Simplification for a symmetric matrix

$$\left(A + uaa^T\right)^{-1} = A^{-1} - \gamma bb^T$$

$$b = A^{-1}a$$

$$\gamma = \left(u^{-1} + a^T b\right)^{-1}$$

Example: Sherman-Morrison-Woodbury

Problem: find A_{new}^{-1}

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A_{new} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{bmatrix} = A + 2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

Solution:

$$b = A^{-1}a = \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \qquad \gamma = \left(u^{-1} + a^T b\right)^{-1} = 0.4$$

$$A_{new}^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 3 \end{bmatrix} - 0.4\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}\begin{bmatrix} -1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 & 0.2 \\ -0.2 & 0.4 & -0.4 \\ 0.2 & -0.4 & 1.4 \end{bmatrix}$$

Equivalent Networks

- Solving the power flow problem for the entire electrical network is not always practical
- In general, an individual balancing authority has authority over only a small portion of the electrical network
- Individual nodes can be “collapsed” to form an equivalent network
- Large portions of a system can be represented as an equivalent network

Gaussian Elimination (Kron Reduction)

- For an arbitrary 4 node system the equations are

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 = I_1 \qquad 1.1$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2 \qquad 1.2$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 = I_3 \qquad 1.3$$

$$Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 = I_4 \qquad 1.4$$

- The first step is to pivot about the node to be reduced

$$V_1 + \frac{Y_{12}}{Y_{11}}V_2 + \frac{Y_{13}}{Y_{11}}V_3 + \frac{Y_{14}}{Y_{11}}V_4 = \frac{1}{Y_{11}}I_1 \qquad 1.5$$

Gaussian Elimination (Kron Reduction) *cont'd*

- Multiply (1.5) by Y_{21} , Y_{31} , Y_{41} and subtract from (1.2) to (1.4), respectively

$$\left(Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{23} - \frac{Y_{21}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{24} - \frac{Y_{21}Y_{14}}{Y_{11}}\right)V_4 = I_2 - \frac{Y_{21}}{Y_{11}}I_1$$

$$\left(Y_{32} - \frac{Y_{31}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{33} - \frac{Y_{31}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{34} - \frac{Y_{31}Y_{14}}{Y_{11}}\right)V_4 = I_3 - \frac{Y_{31}}{Y_{11}}I_1$$

$$\left(Y_{42} - \frac{Y_{41}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{43} - \frac{Y_{41}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{44} - \frac{Y_{41}Y_{14}}{Y_{11}}\right)V_4 = I_4 - \frac{Y_{41}}{Y_{11}}I_1$$

$$\boxed{\begin{aligned} Y_{ij}^{(1)} &= \left(Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}}\right) \\ I_i^{(1)} &= I_i - \frac{Y_{ik}}{Y_{kk}}I_k \end{aligned}}$$

$$Y_{22}^{(1)}V_2 + Y_{23}^{(1)}V_3 + Y_{24}^{(1)}V_4 = I_2^{(1)}$$

$$Y_{32}^{(1)}V_2 + Y_{33}^{(1)}V_3 + Y_{34}^{(1)}V_4 = I_3^{(1)}$$

$$Y_{42}^{(1)}V_2 + Y_{43}^{(1)}V_3 + Y_{44}^{(1)}V_4 = I_4^{(1)}$$

Gaussian Elimination (Kron Reduction) *cont'd*

- By pivoting around multiple nodes, the system can be reduced significantly

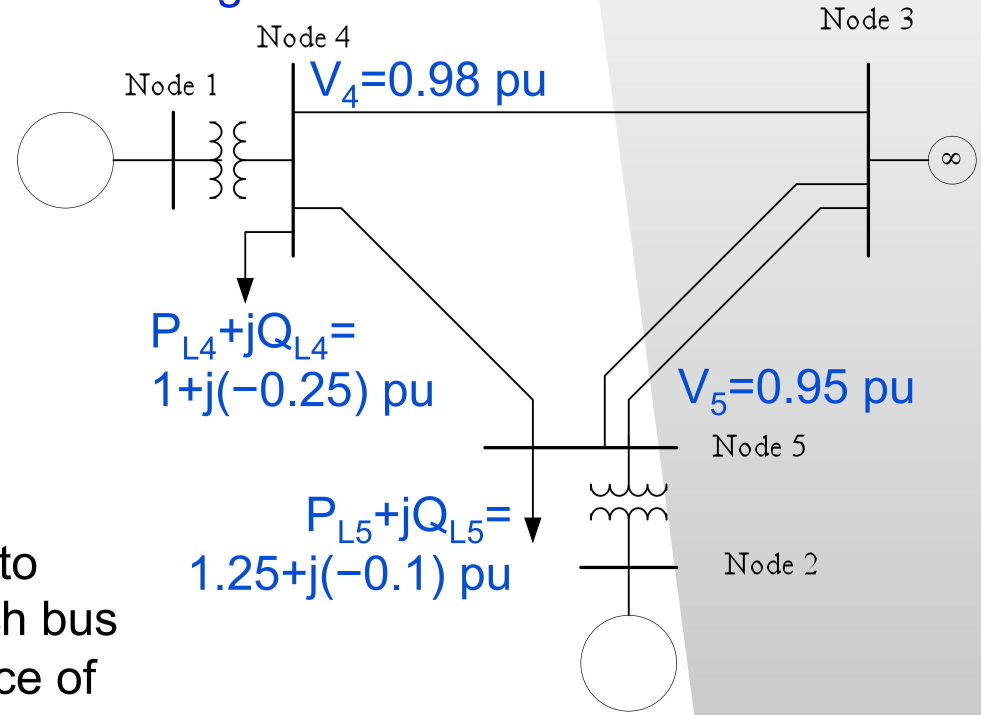
- For a large system, the model can be reduced to only the generator buses and their interconnections
  - Especially useful when studying dynamic stability of a power system (*a topic will be addressed later*)

- This requires the conversion of loads from constant power to constant impedance

Example: Gaussian Elimination (Kron Reduction)

Problem: Bus admittance matrix and power flow solution are given. Reduce the network to retain only generator buses 1 through 3

$$
Y_{bus} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & -j21.5 & j5 \\ 0 & j12.5 & j10 & j5 & -j27.5 \end{bmatrix}
$$



Solution:

Step 1: Perform a complete power flow to determine the voltage magnitude at each bus

Step 2: Calculate the effective impedance of the loads based on the voltage magnitude

Step 3: Perform a Gaussian elimination to remove bus 4 and 5

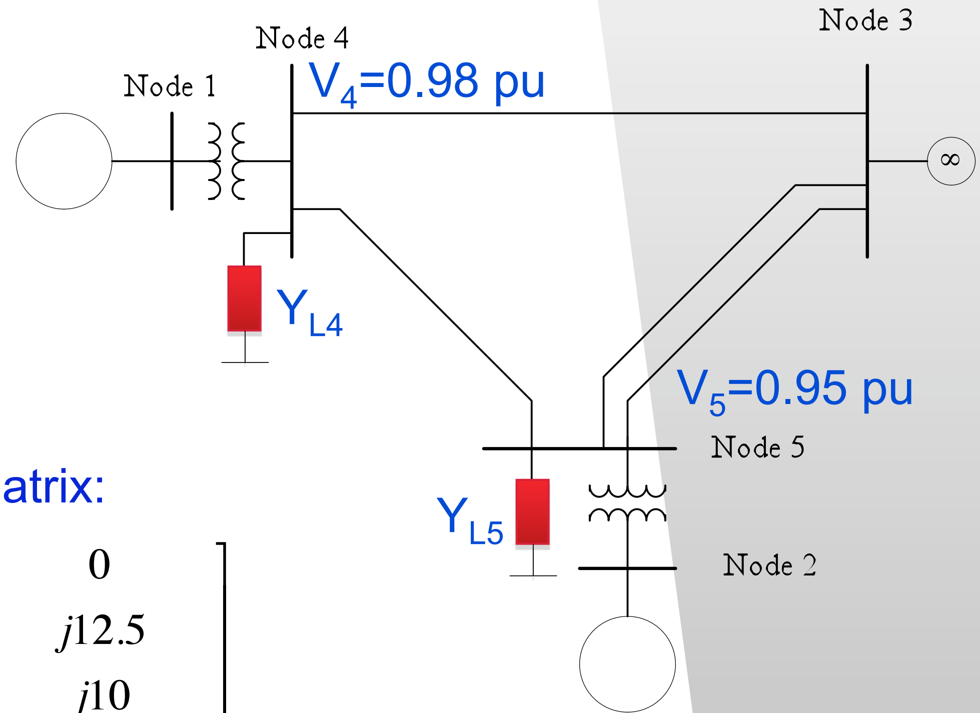
Example: Gaussian Elimination (Kron Reduction) *cont'd*

Effective load impedance:

$$Y_L = \frac{P_L - jQ_L}{|V_L|^2}$$

$$Y_{L4} = \frac{P_{L4} - jQ_{L4}}{|V_4|^2} = \frac{1 + j0.25}{0.98^2} = 1.04 + j0.26$$

$$Y_{L5} = \frac{P_{L5} - jQ_{L5}}{|V_5|^2} = \frac{1.25 + j0.1}{0.95^2} = 1.39 + j0.11$$



Include effective load impedance in Y matrix:

$$Y_{bus}^{(1)} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & 1.04 - j21.24 & j5 \\ 0 & j12.5 & j10 & j5 & 1.39 - j27.39 \end{bmatrix}$$

Example: Gaussian Elimination (Kron Reduction) *cont'd*

Formulate new network equations:

$$\begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ j12.5 & 0 & j4 & 1.04-j21.24 & j5 \\ 0 & j12.5 & j10 & j5 & 1.39-j27.39 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \\ 0 \end{bmatrix}$$

Pivot around node 4:

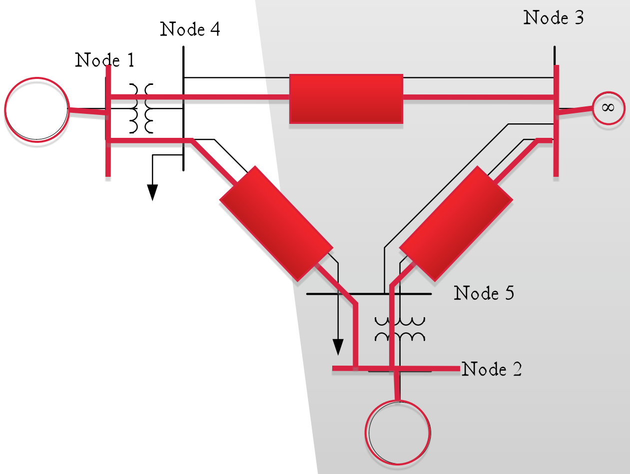
$$\begin{bmatrix} 0.36-j5.16 & 0 & 0.11+j2.35 & 0 & 0.14+j2.94 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0.12+j2.35 & 0 & 0.04-j13.25 & 0 & 0.05+j10.94 \\ 0 & 0 & 0 & 0 & 0 \\ 0.14+j2.94 & j12.5 & 0.05+j10.94 & 0 & 1.45-j26.22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} Y_{ij}^{(1)} &= \left(Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}} \right) \\ I_i^{(1)} &= I_i - \frac{Y_{ik}}{Y_{kk}} I_k \end{aligned}}$$

Example: Gaussian Elimination (Kron Reduction) *cont'd*

Pivot around node 5:

$$\begin{bmatrix} 0.41-j4.83 & 0.14+j1.39 & 0.24+j3.57 & 0 \\ 0.14+j1.39 & 0.33-j6.56 & 0.31+j5.20 & 0 \\ 0.25+j3.57 & 0.31+j5.20 & 0.33-j8.70 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \end{bmatrix}$$



The resulting reduced equations:

$$\begin{bmatrix} 0.41-j4.83 & 0.14+j1.39 & 0.24+j3.57 \\ 0.14+j1.39 & 0.33-j6.56 & 0.31+j5.20 \\ 0.25+j3.57 & 0.31+j5.20 & 0.33-j8.70 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Tip: the resulting matrix should be symmetrical as the original matrix.

Elimination of a Group of Nodes

- Matrix form of Gaussian Elimination (Kron reduction)

$$\begin{bmatrix} Y_{ee} & Y_{er} \\ Y_{re} & Y_{rr} \end{bmatrix} \begin{bmatrix} V_e \\ V_r \end{bmatrix} = \begin{bmatrix} I_e \\ I_r \end{bmatrix}$$

"*e*" = eliminated nodes
"*r*" = retained nodes

$$Y_{ee}V_e + Y_{er}V_r = I_e$$

$$\Rightarrow V_e = Y_{ee}^{-1}I_e - Y_{ee}^{-1}Y_{er}V_r$$

$$\boxed{\left(Y_{rr} - Y_{re}Y_{ee}^{-1}Y_{er}\right)V_r = I_r - Y_{re}Y_{ee}^{-1}I_e}$$

Example:
Elimination of a Group of Nodes

$$\boxed{\left(Y_{rr} - Y_{re} Y_{ee}^{-1} Y_{er}\right) V_r = I_r - Y_{re} Y_{ee}^{-1} I_e}$$

$$\left[\begin{array}{ccc|cc} -j12.5 & 0 & 0 & j12.5 & 0 \\ 0 & -j12.5 & 0 & 0 & j12.5 \\ 0 & 0 & -j14 & j4 & j10 \\ \hline j12.5 & 0 & j4 & 1.04-j21.24 & j5 \\ 0 & j12.5 & j10 & j5 & 1.39-j27.39 \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \\ V_5 \end{array}\right] = \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ \hline 0 \\ 0 \end{array}\right]$$

Y_{rr} $\quad Y_{re}$

Y_{er} $\quad Y_{ee}$

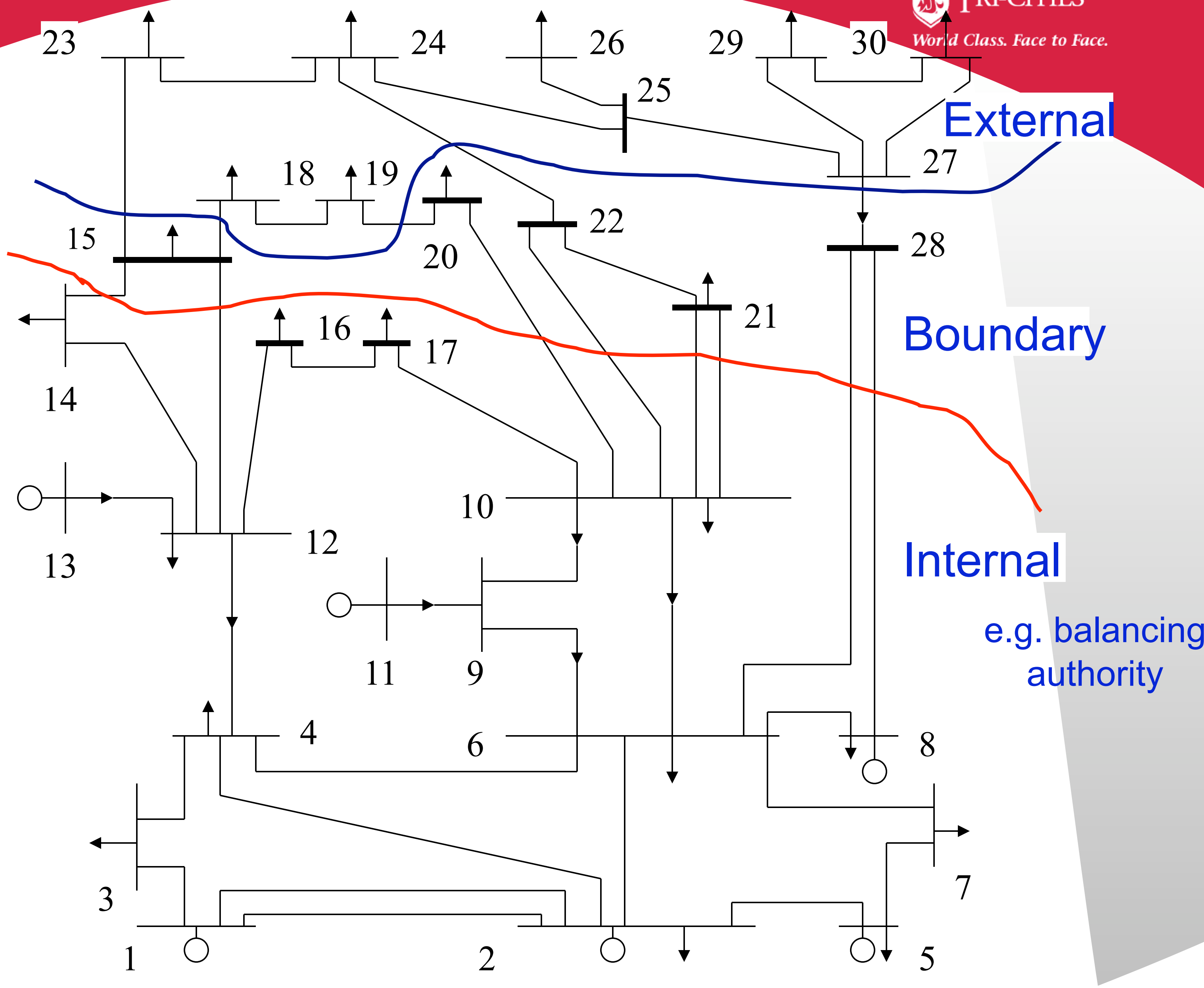
Elimination of a Group of Nodes – A Special Case

- External-Boundary-Internal System – Ward Equivalent

$$\begin{bmatrix} Y_{ee} & Y_{eb} & 0 \\ Y_{be} & Y_{bb}^{(e)} + Y_{bb}^{(i)} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} I_e \\ I_b \\ I_i \end{bmatrix}$$

"*e*" = external nodes
"*b*" = boundary nodes
"*i*" = internal nodes

External Nodes ↔ **Boundary Nodes** ↔ **Internal Nodes**


External
Boundary
Internal
e.g. balancing
authority
23
24
26
25
29
30
27
18
19
15
20
22
28
21
16
17
14
13
12
10
11
9
4
6
8
3
7
1
2
5

Ward Equivalent

Objective: eliminate external nodes

$$\begin{bmatrix} Y_{ee} & Y_{eb} & 0 \\ Y_{be} & Y_{bb}^{(e)} + Y_{bb}^{(i)} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} I_e \\ I_b \\ I_i \end{bmatrix}$$

Expand the equations:

$$Y_{ee}V_e + Y_{eb}V_b = I_e$$

$$Y_{be}V_e + \left(Y_{bb}^{(e)} + Y_{bb}^{(i)}\right)V_b + Y_{bi}V_i = I_b$$

$$Y_{ib}V_b + Y_{ii}V_i = I_i$$

Express V_e by V_b from the 1st equation:

$$V_e = -Y_{ee}^{-1}Y_{eb}V_b + Y_{ee}^{-1}I_e$$

Substitute V_e into the 2nd equation:

$$-Y_{be}Y_{ee}^{-1}Y_{eb}V_b + Y_{be}Y_{ee}^{-1}I_e + \left(Y_{bb}^{(e)} + Y_{bb}^{(i)}\right)V_b + Y_{bi}V_i = I_b$$

Re-arrange:

$$\left(Y_{bb}^{(e)} + Y_{bb}^{(i)} - Y_{be}Y_{ee}^{-1}Y_{eb}\right)V_b + Y_{bi}V_i = I_b - Y_{be}Y_{ee}^{-1}I_e$$

Define equivalent Y matrix and current injection:

$$Y_{eq} = Y_{bb}^{(e)} - Y_{be}Y_{ee}^{-1}Y_{eb}$$

$$I_{eq} = -Y_{be}Y_{ee}^{-1}I_e$$

Ward equivalent:

$$\boxed{\begin{bmatrix} Y_{bb}^{(i)} + Y_{eq} & Y_{bi} \\ Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_b \\ V_i \end{bmatrix} = \begin{bmatrix} I_b + I_{eq} \\ I_i \end{bmatrix}}$$

Ward Equivalent Procedure

- Determine the model of the whole network
- Solve power flow
- Convert constant power load to constant current load
- Determine the equivalent admittance matrix and current injection
- Derive Ward equivalent model based on the equations on the previous slide

Ward Equivalent

- Ward Equivalent gives reasonably accurate results for real power flows, whereas the accuracy for reactive power flow is relatively poor

- This is due to the fact that the change in reactive power injection to maintain constant voltage at external PV buses is not accounted for

Ward-PV Equivalent

- The Ward reduction process is applied only to external PQ buses
- The external PV buses are retained
- The external buses are separated into Q and V buses
- The Ward-PV equivalents give excellent results for contingency evaluation

Ward-PV Equivalent

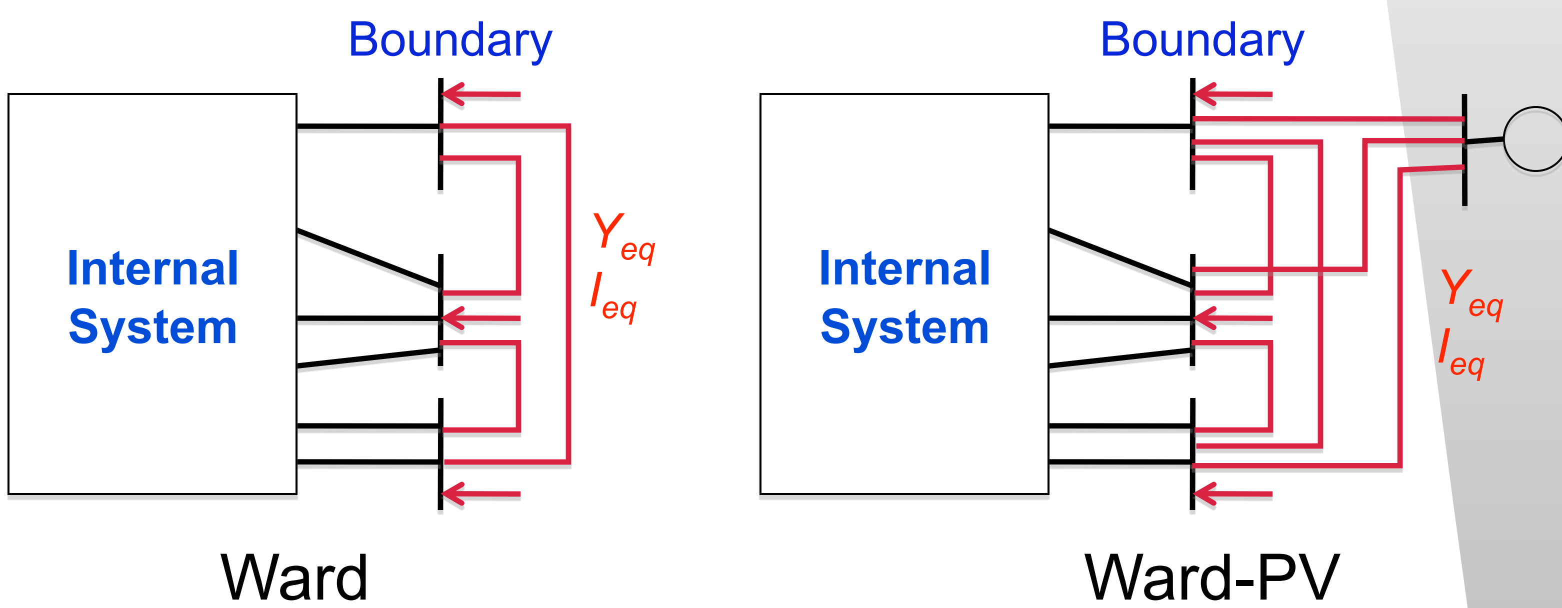
- Gaussian elimination is used to remove terms involving Q buses only

$$\begin{bmatrix} Y_{QQ} & Y_{QV} & Y_{Qb} & 0 \\ Y_{VQ} & Y_{VV} & Y_{Vb} & 0 \\ Y_{bQ} & Y_{bV} & Y_{bb}^{(e)} + Y_{bb}^{(i)} & Y_{bi} \\ 0 & 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_Q \\ V_V \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} I_Q \\ I_V \\ I_b \\ I_i \end{bmatrix}$$

$$\begin{bmatrix} Y_{VV}^{(1)} & Y_{Vb}^{(1)} & 0 \\ Y_{bV}^{(1)} & Y_{eq}^{(1)} + Y_{bb}^{(i)} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_V \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} I_V^{(1)} \\ I_b + I_{eq}^{(1)} \\ I_i \end{bmatrix}$$

Ward Equivalent Summary

- Ward: Pseudo injections at the boundary buses
- Ward-PV: PV buses of the external network are retained



Reading Assignment

Network Reduction and Equivalent Networks

- Chapter 7.6
- Chapter 7.7
- Chapter 14.6

State Estimation (next class's topic)

- Chapter 15

Questions?

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