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EE 521: Analysis of Power Systems

Lecture 6
State Estimation Concepts

Fall 2009

Mondays & Wednesdays 5:45-7:00

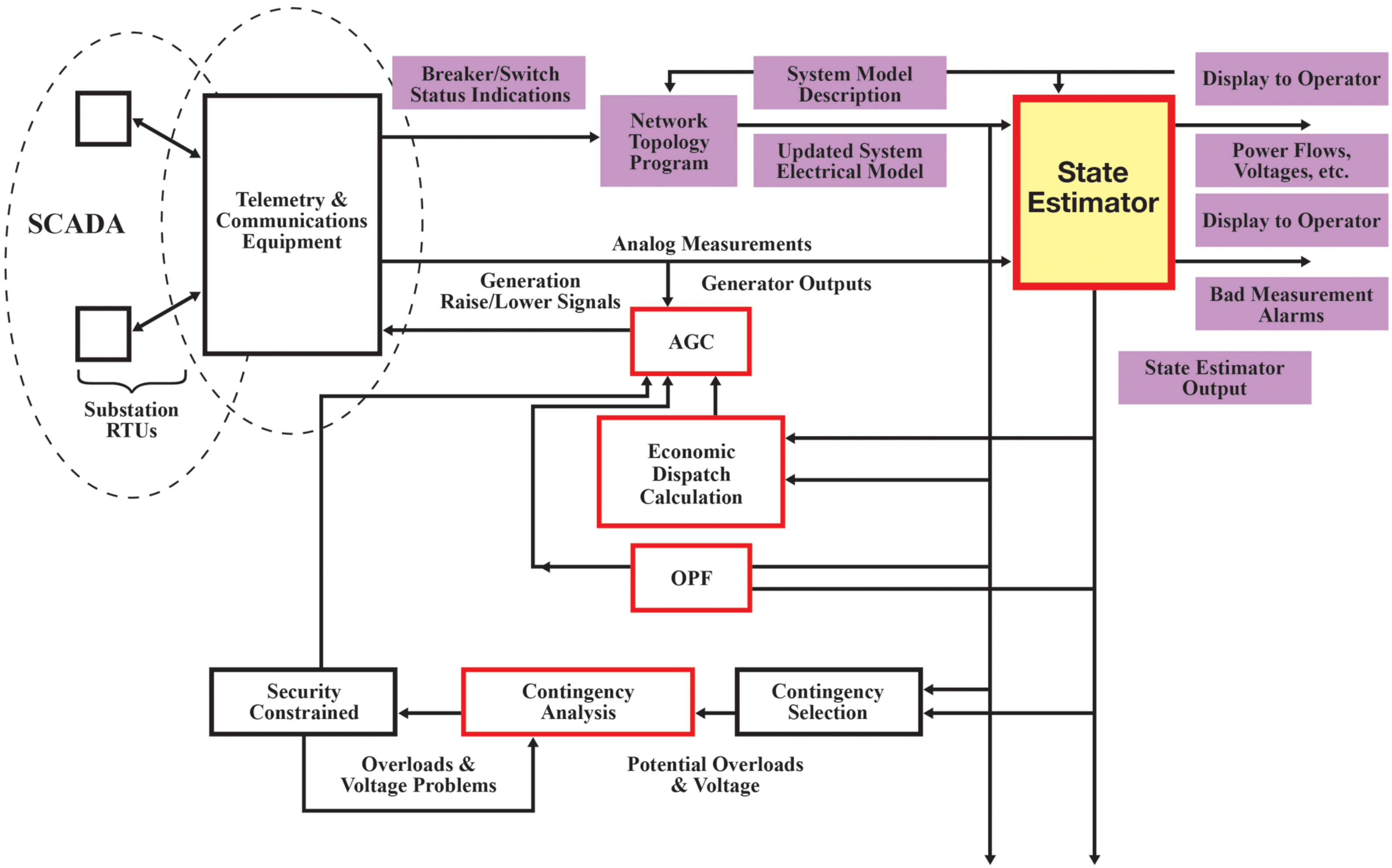
August 24 – December 18

Test 216

Topics

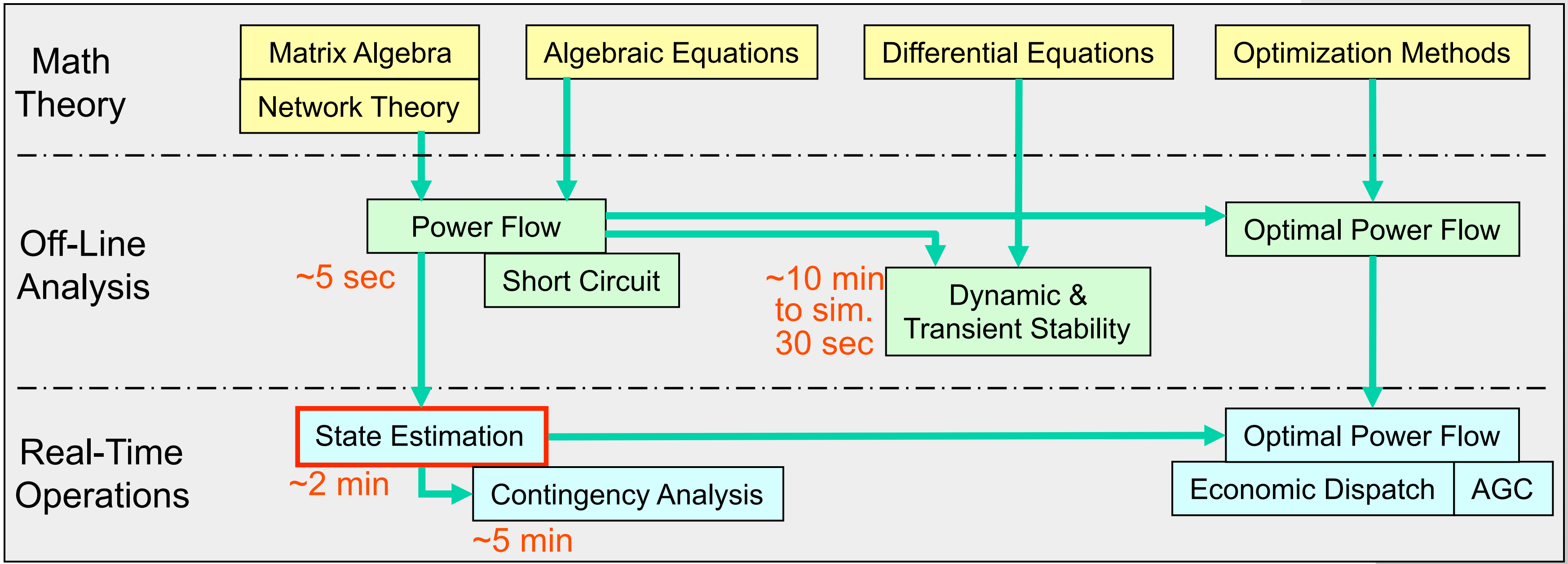
- Overview of Real-Time Power Grid Operation
- Why We Need State Estimation
- Formulation of State Estimation
  - Weighted Least Square
- Solution Methods for State Estimation
  - Newton-Raphson

Overview of Power Grid Operation



Mathematical Basis for Power Grid Operation

- Based on steady-state modeling (power flow model)
- Formulated using algebraic equations in matrix form

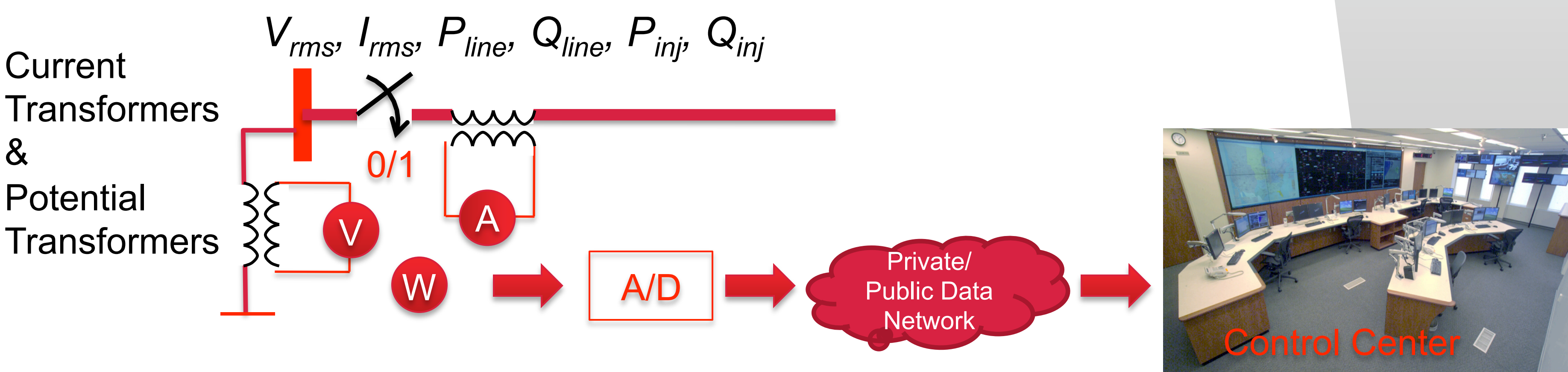


SCADA Systems
(Supervisory Control And Data Acquisition)

- Many uses:
  - Manufacturing, production, and fabrication processes
  - City water systems, oil and gas pipelines, electrical power grids, and large communication systems.
  - Facilities such as buildings, airports, and space stations.
- Components:
  - A human-Machine Interface
  - A computer system
  - Remote Terminal Units (RTUs) connecting to sensors
  - Communication infrastructure

Power Grid SCADA Systems

- Quantities Measured:
  - Status, V_{rms}, I_{rms}, P_{line}, Q_{line}, P_{inj}, Q_{inj}
- Issues with Measurements
  - Measurement Redundancy
  - Measurement Accuracy
  - Measurement Reliability

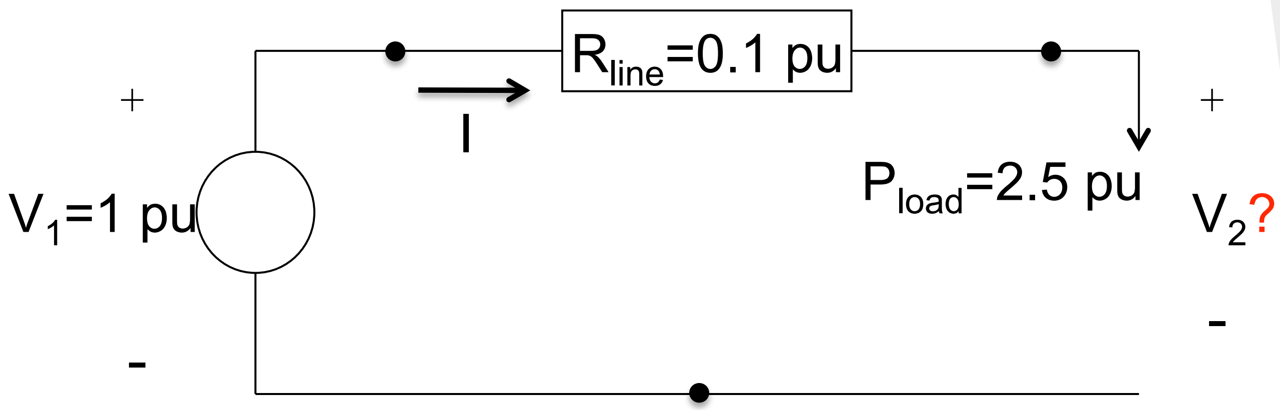


Definition of State Estimation

- Power System States:
  - *V*, *θ* at buses, same as those in the power flow problem
- State Estimation:
  - Estimates states from measured quantities:
    - Status, V_{rms}, I_{rms}, P_{line}, Q_{line}, P_{inj}, Q_{inj}
  - Fits measurements to a model by minimizing errors
  - Objective:
    - Filter noise
    - Identify bad data and missing data
    - Estimate unmeasured quantities such as *θ*

Power Flow Problem

Given: V_1 = 1 pu, P_{load} = 2.5 pu. Find V_2.



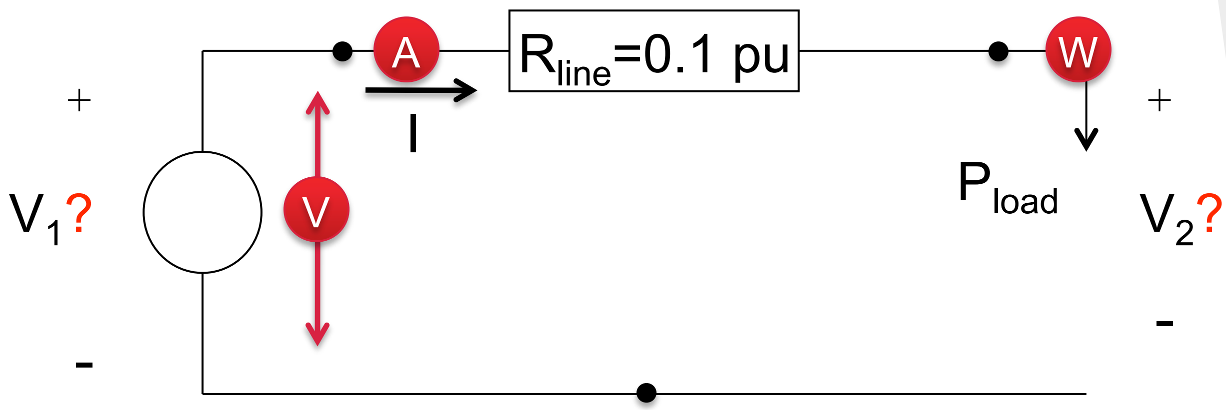
$$\begin{cases} I = \dfrac{V_1 - V_2}{R_{line}} \\ P_{load} = V_2 I \end{cases} \qquad P_{load} = V_2 I = V_2 \frac{V_1 - V_2}{R_{line}} \qquad V_2^{\,2} - V_1 V_2 + P_{load} R_{line} = 0$$

$$V_2^{\,2} - V_2 + 0.25 = 0 \qquad V_2 = 0.5 \ \ \text{(feasible)} \ \ \text{or} \ \ -0.5\,pu \ \ \text{(non-feasible)}$$

$$I = 5\,pu$$

State Estimation Problem

Given Measurements: V_m = 0.9 pu, P_m = 2.6 pu, I_m = 4.5 pu. Find V_1, V_2.

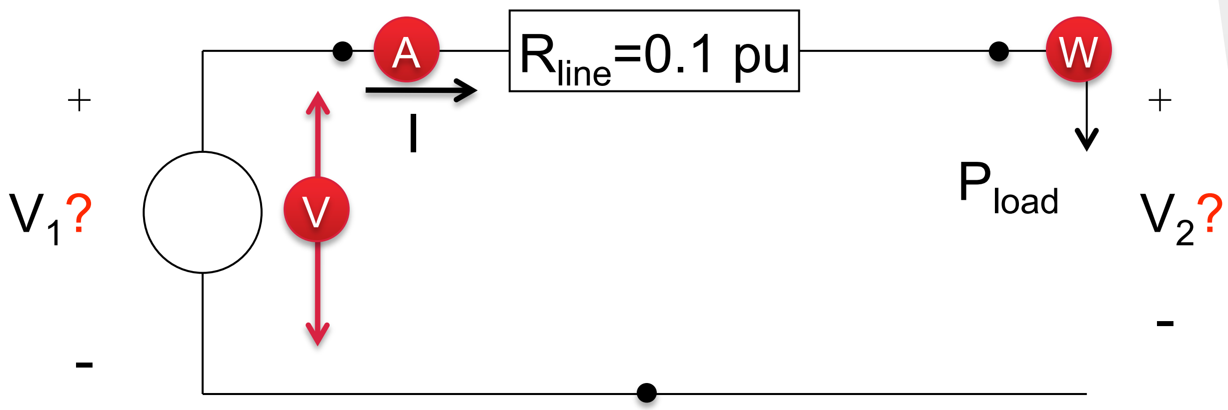


Observations:

1. Direct use of measurements results conflicting answers.
2. More measurements than necessary to solve the equations.
3. Measurements contain errors.
4. Physical laws have to be satisfied.

Measurement Equations

Given Measurements: V_m = 0.9 pu, P_m = 2.6 pu, I_m = 4.5 pu. Find V_1, V_2.



Define measurement variables and state variables:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} V_m \\ P_m \\ I_m \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Include error terms and express using state variables:

$$z = z_{true} + e = \begin{bmatrix} V_1 \\ V_2 \dfrac{V_1 - V_2}{R_{line}} \\ \dfrac{V_1 - V_2}{R_{line}} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = h(x) + e$$

Weighted Least Square Formulation

Error equations:

$$e = z - h(x)$$

Estimated errors:

$$\hat{e} = z - h(\hat{x})$$

Formulate an optimization problem using weighted least square methods:

Weights are used to indicate different levels of measurement accuracy

$$\min_{x_1, x_2} f(x_1, x_2) = \min_{V_1, V_2} f(V_1, V_2) = w_1 e_1^{\ 2} + w_2 e_2^{\ 2} + w_3 e_3^{\ 2}$$

The problem becomes solving the following two conditions:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 0, \qquad \frac{\partial f(x_1, x_2)}{\partial x_2} = 0$$

Solution Process

Expand the derivative terms:

$$\left.\frac{\partial f(x_1,x_2)}{\partial x_1}\right|_{\hat{x}} = 2\left.\left(w_1 e_1 \frac{\partial e_1}{\partial x_1} + w_2 e_2 \frac{\partial e_2}{\partial x_1} + w_3 e_3 \frac{\partial e_3}{\partial x_1}\right)\right|_{\hat{x}} = 0$$

$$\left.\frac{\partial f(x_1,x_2)}{\partial x_2}\right|_{\hat{x}} = 2\left.\left(w_1 e_1 \frac{\partial e_1}{\partial x_2} + w_2 e_2 \frac{\partial e_2}{\partial x_2} + w_3 e_3 \frac{\partial e_3}{\partial x_2}\right)\right|_{\hat{x}} = 0$$

Rewrite in matrix form:

$$\left.\begin{bmatrix} \frac{\partial e_1}{\partial x_1} & \frac{\partial e_2}{\partial x_1} & \frac{\partial e_3}{\partial x_1} \\ \frac{\partial e_1}{\partial x_2} & \frac{\partial e_2}{\partial x_2} & \frac{\partial e_3}{\partial x_2} \end{bmatrix}\right|_{\hat{x}} \begin{bmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = 0 \Rightarrow H^T W \hat{e} = 0$$

$$\Rightarrow H^T W\left[z - h(\hat{x})\right] = 0 \Rightarrow H^T W h(\hat{x}) = H^T W z$$

Solve for x hat?

Newton-Raphson Method

Linearize at an initial guess x_0 hat:

$$h(\hat{x}) = h\left(\hat{x}^{(0)}\right) + \left.\frac{\partial h(x)}{\partial x}\right|_{\hat{x}^{(0)}} \left(\hat{x}^{(1)} - \hat{x}^{(0)}\right) = h\left(\hat{x}^{(0)}\right) + H\left(\hat{x}^{(1)} - \hat{x}^{(0)}\right)$$

Solve for x hat iteratively:

$$H^T W\left[h\left(\hat{x}^{(0)}\right) + H\left(\hat{x}^{(1)} - \hat{x}^{(0)}\right)\right] = H^T W z$$

$$\boxed{\hat{x}^{(1)} = \hat{x}^{(0)} + \left(H^T W H\right)^{-1} H^T W\left[z - h\left(\hat{x}^{(0)}\right)\right]}$$

State Estimation Procedure

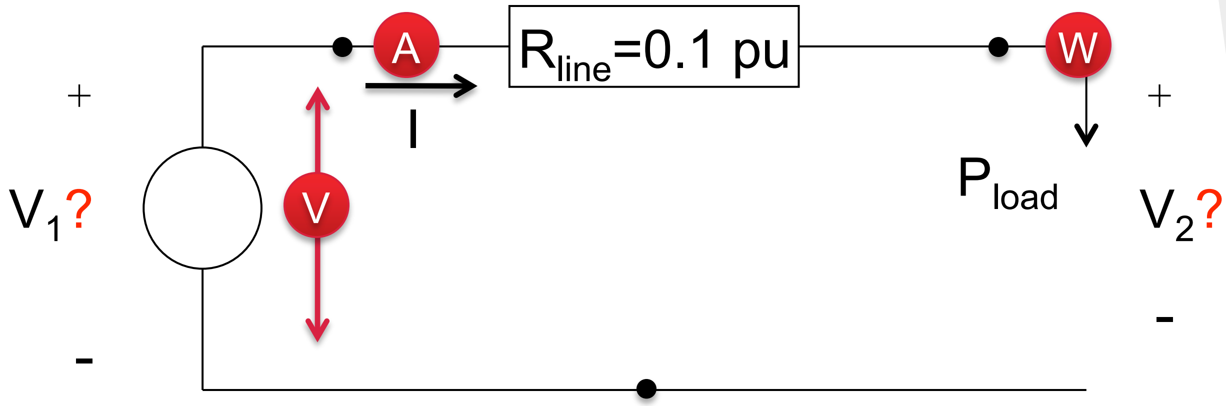
- Identify measurement variables and state variables (input and output)
  - z and x
- Formulate measurement equations
  - $z = h(x) + e$
- Derive Jacobian Matrix H: $H = \frac{\partial h(x)}{\partial x}$
- Solve for estimated states using Newton-Raphson method (*H* needs to be updated at every step)

$$\boxed{\hat{x}^{(k+1)} = \hat{x}^{(k)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(k)}\right)\right]}$$

Example – State Estimation

Problem:

Given Measurements: V_m = 0.9 pu, P_m = 2.6 pu, I_m = 4.5 pu, w = 100. Find V_1, V_2.



Solution:

Define measurement variables
and state variables:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} V_m \\ P_m \\ I_m \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Measurement equations:

$$z = h(x) + e = \begin{bmatrix} V_1 \\ V_2 \dfrac{V_1 - V_2}{R_{line}} \\ \dfrac{V_1 - V_2}{R_{line}} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 10x_1x_2 - 10x_2^{\,2} \\ 10x_1 - 10x_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Example – State Estimation *cont'd*

Jacobian matrix:

$$h(x) = \begin{bmatrix} x_1 \\ 10x_1x_2 - 10x_2{}^2 \\ 10x_1 - 10x_2 \end{bmatrix} \Rightarrow H = \frac{\partial h(x)}{\partial x} = \begin{bmatrix} 1 & 0 \\ 10x_2 & 10x_1 - 20x_2 \\ 10 & -10 \end{bmatrix}$$

Solve for x hat iteratively:

$$\hat{x}^{(0)} = \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix} \quad h\left(\hat{x}^{(0)}\right) = \begin{bmatrix} 1.0 \\ 2.5 \\ 5.0 \end{bmatrix} \quad H^{(0)} = \begin{bmatrix} 1 & 0 \\ 5 & 0 \\ 10 & -10 \end{bmatrix} \quad z = \begin{bmatrix} 0.9 \\ 2.6 \\ 4.5 \end{bmatrix} \quad W = \begin{bmatrix} 100 & & \\ & 100 & \\ & & 100 \end{bmatrix}$$

$$\hat{x}^{(1)} = \hat{x}^{(0)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(0)}\right)\right] = \begin{bmatrix} 1.015385 \\ 0.565385 \end{bmatrix}$$

$$\hat{x}^{(2)} = \hat{x}^{(1)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(1)}\right)\right] = \begin{bmatrix} 1.021688 \\ 0.571376 \end{bmatrix}$$

$$\hat{x}^{(3)} = \begin{bmatrix} 1.021685 \\ 0.571357 \end{bmatrix} pu$$

Example – State Estimation *cont'd*

Objective function *f*:

$$\min_{x_1, x_2} f(x_1, x_2) = w_1\hat{e}_1^{\,2} + w_2\hat{e}_2^{\,2} + w_3\hat{e}_3^{\,2}$$

27.000000000000000
1.642381656804740
1.554807088533027
1.554804815717737
1.554804815535279
1.554804815535258
1.554804815535259
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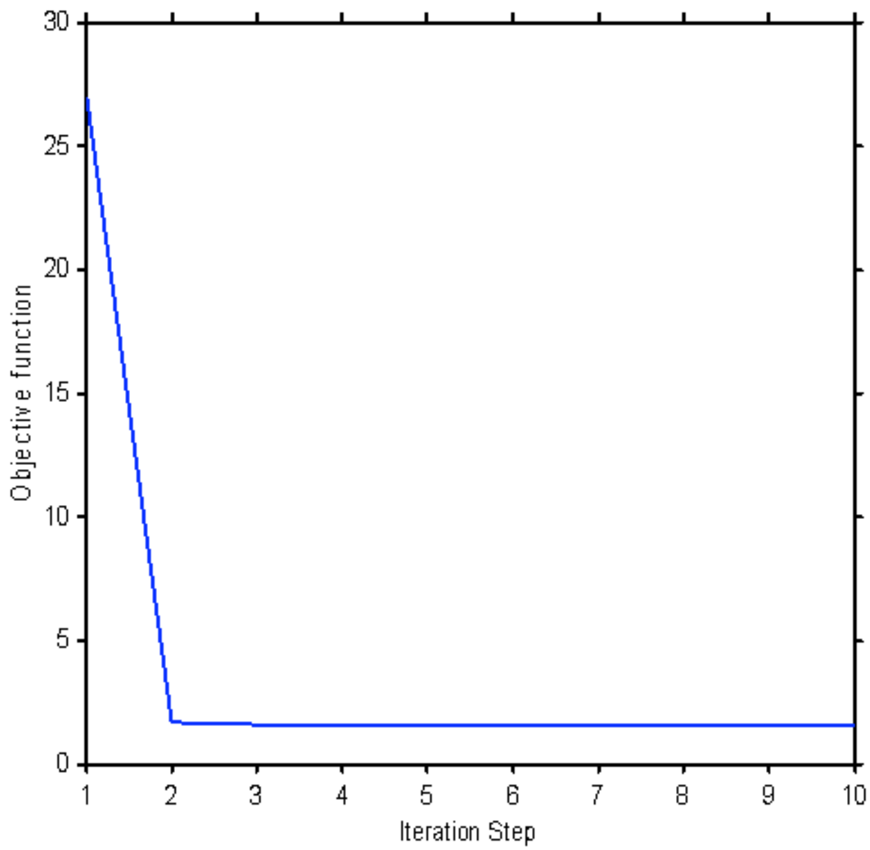
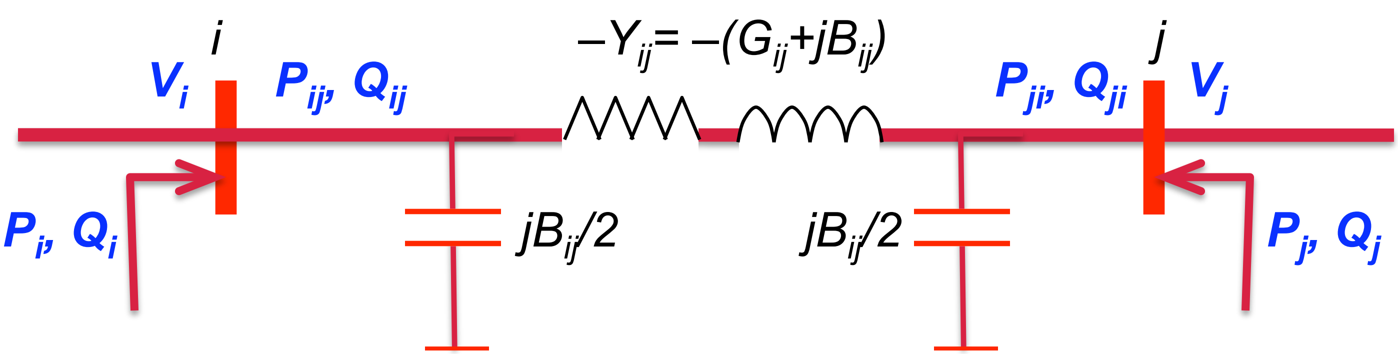


State Estimation vs. Power Flow

	Power Flow	State Estimation
Input	Given PV, PQ, Vθ	Measured $z = V_{rms}, I_{rms}, P_{line}, Q_{line}, P_{inj}, Q_{inj}$
Output	V and θ	x = V and θ
Formulation	P – P(V, θ) = 0 Q – Q(V, θ) = 0	z – h(x) = e
Objective	Drive ΔP, ΔQ towards 0.	Drive Δz towards a minimum.
Solution Method	$\begin{bmatrix} \theta^{n+1} \\ V^{n+1} \end{bmatrix} = \begin{bmatrix} \theta^{n} \\ V^{n} \end{bmatrix} - \left[J\left(x^n\right)\right]^{-1} \begin{bmatrix} \Delta P\left(x^n\right) \\ \Delta Q\left(x^n\right) \end{bmatrix}$	$\hat{x}^{(k+1)} = \hat{x}^{(k)} + \left(H^T W H\right)^{-1} H^T W \left[z - h\left(\hat{x}^{(k)}\right)\right]$
Jacobian Matrix	$J(x) = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$	$H = \left[\frac{\partial h(x)}{\partial x}\right]$ J is part of H.

Jacobian Matrix *H*

- Measurement Types
  - V_i: Voltage magnitude at bus *i*
  - P_i: Real power injection at bus *i*
  - Q_i: Reactive power injection at bus *i*
  - P_{ij}: Real power flow at bus *i* in line *ij*
  - Q_{ij}: Reactive power flow at bus *i* in line *ij*



Dimension of Jacobian Matrix

- For a *N*-bus-*B*-line power system, maximum measurements:
  - 3*N* + 4*B*.
- State variables
  - 2*N* – 1.
- Redundancy Factor
  - (3*N* + 4*B*)/(2*N* – 1)
- Jacobian Matrix
  - (3*N* + 4*B*) x (2*N* – 1)

$$z = h(x) + e = \begin{bmatrix} V_i \\ P_i \\ Q_i \\ P_{ij} \\ P_{ji} \\ Q_{ij} \\ Q_{ji} \end{bmatrix} + e \qquad H = \begin{bmatrix} \partial V_i / \partial \theta_k & \partial V_i / \partial V_k \\ \partial P_i / \partial \theta_k & \partial P_i / \partial V_k \\ \partial Q_i / \partial \theta_k & \partial Q_i / \partial V_k \\ \partial P_{ij} / \partial \theta_k & \partial P_{ij} / \partial V_k \\ \partial P_{ji} / \partial \theta_k & \partial P_{ji} / \partial V_k \\ \partial Q_{ij} / \partial \theta_k & \partial Q_{ij} / \partial V_k \\ \partial Q_{ji} / \partial \theta_k & \partial Q_{ji} / \partial V_k \end{bmatrix}$$

Jacobian Matrix – V_i Entries

Measurement equation:

$$h(x) = V_i$$

Jacobian entries:

$$\partial V_i / \partial \theta_k = 0$$

$$\partial V_i / \partial V_k = \begin{cases} 1, & \text{if } i = k \\ 0, & \text{if } i \neq k \end{cases}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \multicolumn{3}{c}{\partial P_i / \partial \theta_k} & \multicolumn{3}{c}{\partial P_i / \partial V_k} \\ \multicolumn{3}{c}{\partial Q_i / \partial \theta_k} & \multicolumn{3}{c}{\partial Q_i / \partial V_k} \\ \multicolumn{3}{c}{\partial P_{ij} / \partial \theta_k} & \multicolumn{3}{c}{\partial P_{ij} / \partial V_k} \\ \multicolumn{3}{c}{\partial P_{ji} / \partial \theta_k} & \multicolumn{3}{c}{\partial P_{ji} / \partial V_k} \\ \multicolumn{3}{c}{\partial Q_{ij} / \partial \theta_k} & \multicolumn{3}{c}{\partial Q_{ij} / \partial V_k} \\ \multicolumn{3}{c}{\partial Q_{ji} / \partial \theta_k} & \multicolumn{3}{c}{\partial Q_{ji} / \partial V_k} \end{bmatrix}$$

Jacobian Matrix – P_i, Q_i Entries

Measurement equation:

$$h(x)=\begin{cases} P_i=|V_i|\sum_{j=1}^{N}|V_j|\left(G_{ij}\cos(\theta_j-\theta_i)-B_{ij}\sin(\theta_j-\theta_i)\right) \\ Q_i=-|V_i|\sum_{j=1}^{N}|V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \end{cases}$$

Jacobian entries: (=J in the power flow problem)

$$\begin{cases} \dfrac{\partial P_i}{\partial\theta_j}=-|V_i||V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \\ \dfrac{\partial P_i}{\partial\theta_i}=|V_i|\sum_{\substack{j=1\\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \\ \dfrac{\partial Q_i}{\partial|V_j|}=-|V_i|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \\ \dfrac{\partial Q_i}{\partial|V_i|}=-\sum_{\substack{j=1\\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right)-2|V_i|B_{ii} \end{cases}$$

$$H=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \multicolumn{3}{c}{\partial P_i/\partial\theta_k} & \multicolumn{3}{c}{\partial P_i/\partial V_k} \\ \multicolumn{3}{c}{\partial Q_i/\partial\theta_k} & \multicolumn{3}{c}{\partial Q_i/\partial V_k} \\ \multicolumn{3}{c}{\partial P_{ij}/\partial\theta_k} & \multicolumn{3}{c}{\partial P_{ij}/\partial V_k} \\ \multicolumn{3}{c}{\partial P_{ji}/\partial\theta_k} & \multicolumn{3}{c}{\partial P_{ji}/\partial V_k} \\ \multicolumn{3}{c}{\partial Q_{ij}/\partial\theta_k} & \multicolumn{3}{c}{\partial Q_{ij}/\partial V_k} \\ \multicolumn{3}{c}{\partial Q_{ji}/\partial\theta_k} & \multicolumn{3}{c}{\partial Q_{ji}/\partial V_k} \end{bmatrix}$$

Jacobian Matrix – P_{ij}, Q_{ij} Entries

Measurement equation:

$$h(x)=\begin{cases} P_{ij}=-|V_i|^2G_{ij}+|V_i||V_j|\left(G_{ij}\cos(\theta_j-\theta_i)-B_{ij}\sin(\theta_j-\theta_i)\right) \\ Q_{ij}=-|V_i|^2\left(\dfrac{B_{ij}'}{2}-B_{ij}\right)-|V_i||V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right)\end{cases}$$

Jacobian entries: (only positions *ij* has value)

$$\left[\begin{matrix}0 & \dfrac{\partial P_{ij}}{\partial\theta_i} & 0 & \dfrac{\partial P_{ij}}{\partial\theta_j} & 0\end{matrix}\right|\left|\begin{matrix}0 & \dfrac{\partial P_{ij}}{\partial V_i} & 0 & \dfrac{\partial P_{ij}}{\partial V_j} & 0\end{matrix}\right]$$

$$\left[\begin{matrix}0 & \dfrac{\partial Q_{ij}}{\partial\theta_i} & 0 & \dfrac{\partial Q_{ij}}{\partial\theta_j} & 0\end{matrix}\right|\left|\begin{matrix}0 & \dfrac{\partial Q_{ij}}{\partial V_i} & 0 & \dfrac{\partial Q_{ij}}{\partial V_j} & 0\end{matrix}\right]$$

$$H=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \multicolumn{3}{c}{\partial P_i/\partial\theta_k} & \multicolumn{3}{c}{\partial P_i/\partial V_k} \\ \multicolumn{3}{c}{\partial Q_i/\partial\theta_k} & \multicolumn{3}{c}{\partial Q_i/\partial V_k} \\ \multicolumn{3}{c}{\boxed{\partial P_{ij}/\partial\theta_k}} & \multicolumn{3}{c}{\boxed{\partial P_{ij}/\partial V_k}} \\ \multicolumn{3}{c}{\boxed{\partial P_{ji}/\partial\theta_k}} & \multicolumn{3}{c}{\boxed{\partial P_{ji}/\partial V_k}} \\ \multicolumn{3}{c}{\boxed{\partial Q_{ij}/\partial\theta_k}} & \multicolumn{3}{c}{\boxed{\partial Q_{ij}/\partial V_k}} \\ \multicolumn{3}{c}{\boxed{\partial Q_{ji}/\partial\theta_k}} & \multicolumn{3}{c}{\boxed{\partial Q_{ji}/\partial V_k}} \end{bmatrix}$$

Summary of Jacobian Matrix Entries

Table 15.4
Table 15.5

It is required to understand how these elements are derived.
Feel free to let me know if you have any questions.

Assignment

Textbook Problem 15.14
Due: Sept 23.

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Questions?