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EE 521: Analysis of Power Systems

Lecture 8
Economic Dispatch

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

- Generation Cost Characteristics
- Economic Dispatch Concepts
- Economic Dispatch with Equality Constraints
- Economic Dispatch with Equality and Inequality Constraints

Generators are not born equal!

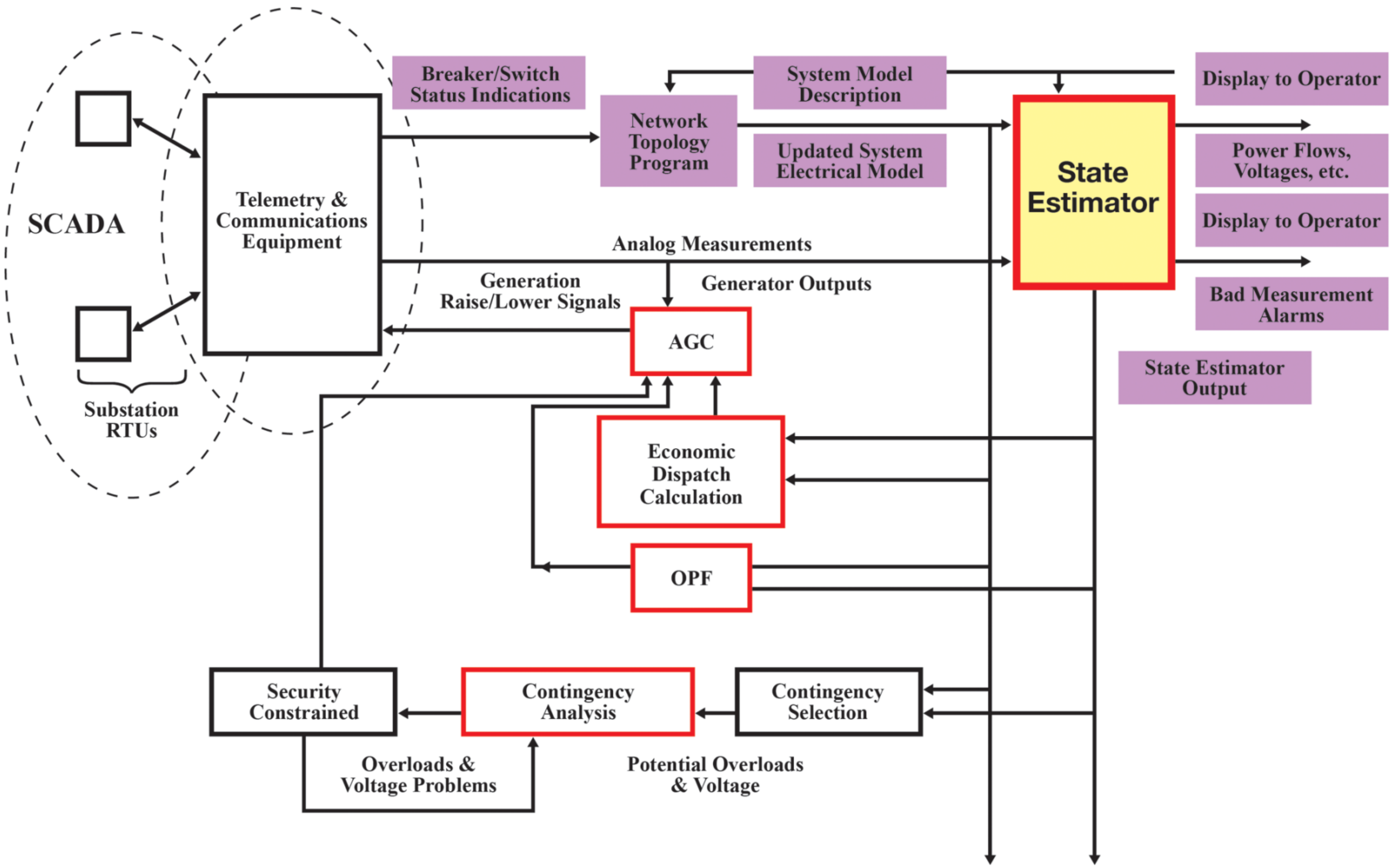
- Some cost more than others to generate the same amount of electricity.
- Some are farther away from load centers, meaning more loss to transfer electricity from those generators.
- Different generators have different lower and upper output limits.

Generation Schedule

- At any given point of time, generation and load need to be balanced.
- Generation needs to be scheduled based on forecasted load consumption.
- The goal is to meet load demand and minimize cost.

"Economic Dispatch"

Overview of Power Grid Operation



Economic Dispatch

- An optimization problem: minimize cost with load demand constraint

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1 P_{G1} + C_2 P_{G2} + C_3 P_{G3}$$

subject to

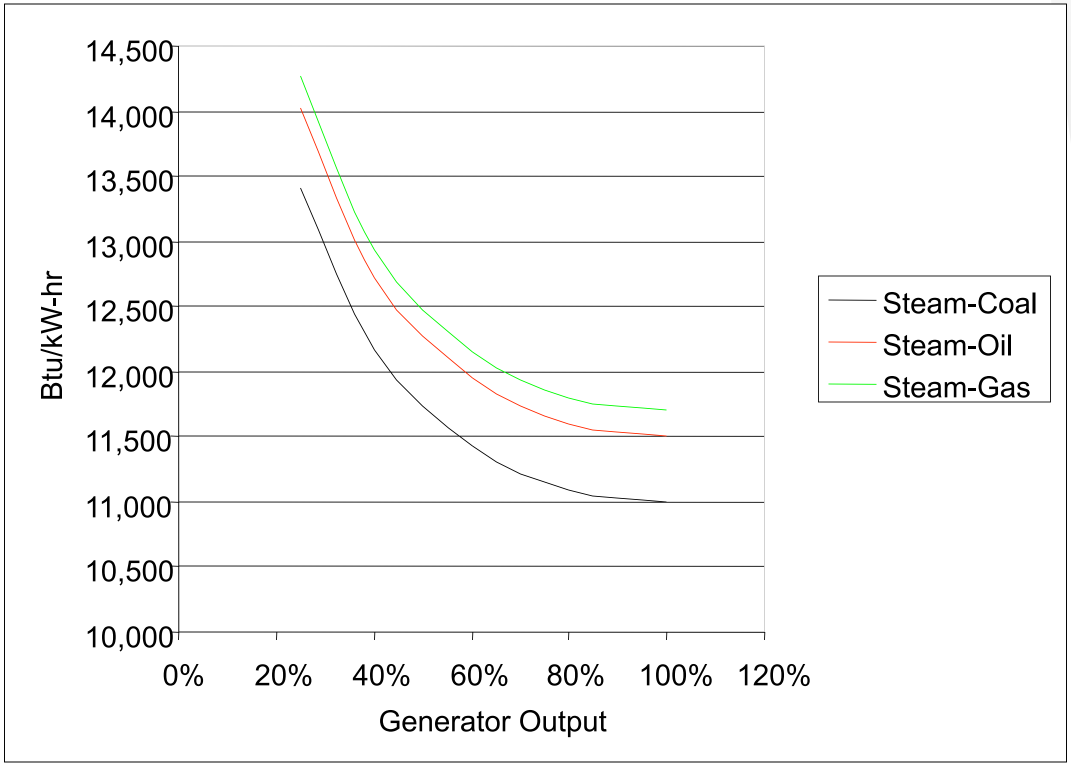
$$P_{G1} + P_{G2} + P_{G3} = P_D$$

- C_1, C_2, C_3: generator cost function

Generator Characteristics

- Heat Rate: The ratio of fuel energy input as heat per unit of work output (Btu/kWhr)
  - HHV (Higher Heating Value) – Includes the latent heat of vaporization for water
  - LHV (Lower Heating Value) – Does not included the latent heat of vaporization for water
- Heat Rate Curve
  - Usually non-linear input-output relationship!

Typical Heat Rate Curves



Heat Rates in Btu/kWhr					
	100%	80%	60%	40%	25%
Steam-Coal	11,000	11,088	11,429	12,166	13,409
Steam-Oil	11,500	11,592	11,949	12,719	14,019
Steam-Gas	11,700	11,794	12,156	12,940	14,262

Input-Output Equation

- Input-output curves give the heat rate required to obtain a given output power
  - Linear

$$H_i(P_i) = \beta_{base} + \beta_0 + \beta_1 P_i$$

  - Quadratic

$$H_i(P_i) = \beta_{base} + \beta_0 + \beta_1 P_i + \beta_2 P_i^2$$

  - Cubic

$$H_i(P_i) = \beta_{base} + \beta_0 + \beta_1 P_i + \beta_2 P_i^2 + \beta_3 P_i^3$$

  - Piecewise linear

$$H_i(P_i) = \beta_{base} + \begin{cases} \beta_0 + \beta_1 P_i & 0 \le P_i < 25 \\ \beta_3 + \beta_4 P_i & 25 \le P_i < 75 \\ \beta_5 + \beta_6 P_i & 75 \le P_i \le 100 \end{cases}$$

H: Heat input
P: Power output

Cost Curves

- Cost curves give the cost of operating a generator as a function of the output power

- The cost function is the product of the input-output function and the fuel cost (*FC*)

$$C_i(P_i) = H_i(P_i) \times FC_i$$

Economic Dispatch with Equality Constraints

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1\left(P_{G1}\right) + C_2\left(P_{G2}\right) + C_3\left(P_{G3}\right)$$

subject to

$$P_{G1} + P_{G2} + P_{G3} = P_D$$

Economic Dispatch with Equality and Inequality Constraints

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1\left(P_{G1}\right)+C_2\left(P_{G2}\right)+C_3\left(P_{G3}\right)$$

subject to

$$P_{G1}+P_{G2}+P_{G3}=P_D$$

$$P_{G1,\min} \le P_{G1} \le P_{G1,\max}$$

$$P_{G2,\min} \le P_{G2} \le P_{G2,\max}$$

$$P_{G3,\min} \le P_{G3} \le P_{G3,\max}$$

Review:
Optimization with Constraints

- Case 1: equality constraints

- Case 2: equality and inequality constraints

Review: Optimization with Equality Constraints

Problem:

$$\min_{x_1, x_2} f(x_1, x_2) = 0.25x_1^2 + x_2^2$$

Subject to: $w(x_1, x_2) = 5 - x_1 - x_2 = 0$

Solution:

Lagrange Multiplier

$$\min_{x_1, x_2, \lambda} L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda w(x_1, x_2)$$

At the optimal point:

$$\frac{\partial L}{\partial x_1} = 0 \qquad \frac{\partial L}{\partial x_2} = 0 \qquad \frac{\partial L}{\partial \lambda} = 0$$

Review: Optimization with Equality Constraints *cont'd*

$$L(x_1, x_2, \lambda) = 0.25x_1^{\,2} + x_2^{\,2} + \lambda(5 - x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 0.5x_1 - \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_2 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 5 - x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1 = 4 \qquad x_2 = 1 \qquad \lambda = 2$$

Review:
Optimization with Equality and Inequality Constraints

Problem:

$$\min_{x_1, x_2} f(x_1, x_2) = 0.25x_1^2 + x_2^2$$

Subject to:

$$w(x_1, x_2) = 5 - x_1 - x_2 = 0$$
$$g(x_1, x_2) = x_1 + 0.2x_2 - 3 \le 0$$

Solution:

Lagrange Multiplier

$$\min_{x_1, x_2, \lambda, u} L(x_1, x_2, \lambda, u) = f(x_1, x_2) + \lambda w(x_1, x_2) + u g(x_1, x_2)$$

At the optimal point:

condition 1	condition 2	condition 3	condition 4
$\frac{\partial L}{\partial x_i} = 0$	$w_i(x_1, x_2) = 0$	$g_i(x_1, x_2) \le 0$	$u_i g_i(x_1, x_2) = 0$ $u_i \ge 0$

Review:

Optimization with Equality and Inequality Constraints *cont'd*

- The 4th condition is the complementary slackness condition
- It is the condition that allows a way to handle the binding and non-binding constraints
  - When u_i=0, g_i is a non-binding constraint
  - When u_i>0, then g_i is a binding constraint and g_i = 0
- Solution Method
  - First the problem is tested with u_i=0, then with u_i>0

Review:
Optimization with Equality and Inequality Constraints *cont'd*

$$L(x_1, x_2, \lambda, u) = 0.25x_1^{\,2} + x_2^{\,2} + \lambda(5 - x_1 - x_2) + u(x_1 + 0.2x_2 - 3)$$

1) If u_i=0, then by the 1st and 2nd conditions:

$$\frac{\partial L}{\partial x_1} = 0.5x_1 - \lambda + u = 0 \qquad \frac{\partial L}{\partial x_2} = 2x_2 - \lambda + .2u = 0 \qquad \frac{\partial L}{\partial \lambda} = 5 - x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1 = 4 \quad x_2 = 1 \quad \lambda = 2$$

But this violates the 3rd condition: $g_i(x_1, x_2) = 4 + 0.2(1) - 3 = 1.2 > 0$

2) So u_i>0 and g_i=0. By the 2nd and 3rd conditions:

$$w_i(x_1, x_2) = 0 \quad g_i(x_1, x_2) \leq 0 \qquad \Rightarrow \quad x_1 = 2.5 \qquad x_2 = 2.5$$

By the 1st condition: $\lambda = 5.9375$ $\qquad u = 4.6875$

The 3rd condition is satisfied: $g_i(x_1, x_2) = 2.5 + 0.2(2.5) - 3 = 0$

Solution to Economic Dispatch with Equality Constraints

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1(P_{G1}) + C_2(P_{G2}) + C_3(P_{G3})$$

subject to

$$P_{G1} + P_{G2} + P_{G3} = P_D$$

Solution:

Lagrange Multiplier

$$\min_{x_1,x_2,\lambda} L(x_1,x_2,\lambda) = f(x_1,x_2) + \lambda w(x_1,x_2)$$

$$\min_{P_{G1},P_{G2},P_{G3}} L(P_{G1},P_{G2},P_{G3},\lambda) = C_1(P_{G1}) + C_2(P_{G2}) + C_3(P_{G3}) + \lambda(P_D - P_{G1} - P_{G2} - P_{G3})$$

Conditions: $\dfrac{\partial L}{\partial P_{Gi}} = 0, \dfrac{\partial L}{\partial \lambda} = 0$

Incremental Cost Curve

$$\min_{P_{G1},P_{G2},P_{G3}} L(P_{G1},P_{G2},P_{G3},\lambda)=C_1(P_{G1})+C_2(P_{G2})+C_3(P_{G3})+\lambda(P_D-P_{G1}-P_{G2}-P_{G3})$$

$$\frac{\partial L}{\partial P_{Gi}}=\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}}-\lambda=0 \quad \Rightarrow \quad \lambda=\frac{\partial C_1(P_{G1})}{\partial P_{G1}}=\frac{\partial C_2(P_{G2})}{\partial P_{G2}}=\frac{\partial C_3(P_{G3})}{\partial P_{G3}}$$

- The incremental cost is the cost of the next MW from a given generator
- The incremental cost is dependent on the form of the cost function

$$IC_i=\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}}$$

- At the optimal point, all generators have the same incremental cost – equal incremental cost criterion.

Example:
Economic Dispatch with Equality Constraints

Problem: Three Generators. Input-output curves are given. Fuel costs are:

FC_1 = \$1.1/MBtu, FC_2 = \$1.0/MBtu, FC_3 = \$1.0/MBtu.

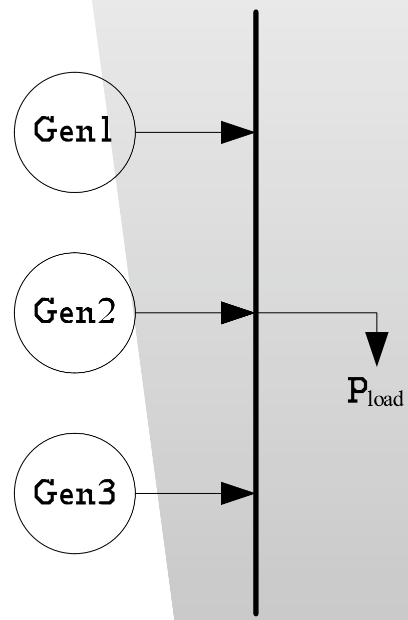
Total load P_D = 850 MW.

Determine optimal allocation.

$$H_1(P_1) = 510.0 + 7.2P_1 + 0.00142P_1^2$$

$$H_2(P_2) = 310.0 + 7.85P_2 + 0.00194P_2^2$$

$$H_3(P_3) = 78.0 + 7.97P_3 + 0.00482P_3^2$$



Example: *cont'd*
Economic Dispatch with Equality Constraints

Solution:

Determine the cost curves:

$$C_1(P_1) = H_1(P_1) \bullet FC_1 = 561 + 7.92P_1 + 0.001562P_1^2$$

$$C_2(P_2) = H_2(P_2) \bullet FC_2 = 310.0 + 7.85P_2 + 0.00194P_2^2$$

$$C_3(P_3) = H_3(P_3) \bullet FC_3 = 78.0 + 7.97P_3 + 0.00482P_3^2$$

Formulate the economic dispatch problem:

$$\min_{P_1, P_2, P_3} f_{\text{cost}}(P_1, P_2, P_3) = C_1(P_1) + C_2(P_2) + C_3(P_3)$$

subject to

$$P_1 + P_2 + P_3 = P_D = 850$$

Example: *cont'd*
Economic Dispatch with Equality Constraints

Solution:

Apply Lagrange multiplier:

$$L(P_1, P_2, P_3, \lambda) = \left\{ \begin{array}{l} \left(561 + 7.92P_1 + 0.001562P_1^2\right) + \\ \left(310.0 + 7.85P_2 + 0.00194P_2^2\right) + \\ \left(78.0 + 7.97P_3 + 0.00482P_3^2\right) + \\ \lambda\left(850 - P_1 - P_2 - P_3\right) \end{array} \right\}$$

$$\frac{\partial L}{\partial P_1} = 7.92 + 0.003124P_1 - \lambda = 0 \Rightarrow \lambda = IC_1 = 7.92 + 0.003124P_1$$

$$\frac{\partial L}{\partial P_2} = 7.85 + 0.00388P_2 - \lambda = 0 \Rightarrow \lambda = IC_2 = 7.85 + 0.00388P_2$$

$$\frac{\partial L}{\partial P_3} = 7.97 + 0.00964P_3 - \lambda = 0 \Rightarrow \lambda = IC_3 = 7.97 + 0.00964P_3$$

$$\frac{\partial L}{\partial \lambda} = 850 - P_1 - P_2 - P_3 = 0$$

Example: *cont'd*
Economic Dispatch with Equality Constraints

Solution:

Generation allocation:

$$P_1 = 393.2MW \quad P_2 = 334.6MW \quad P_3 = 122.2MW$$

Verify the equal incremental cost criterion:

$$\lambda = IC_1 = \frac{\partial C_1(P_1)}{\partial P_1} = 7.92 + 0.003124P_1 = 9.148$$

$$\lambda = IC_2 = \frac{\partial C_2(P_2)}{\partial P_2} = 7.85 + 0.00388P_2 = 9.148$$

$$\lambda = IC_3 = \frac{\partial C_3(P_3)}{\partial P_3} = 7.97 + 0.00964P_3 = 9.148$$

Example:
Economic Dispatch with Equality and Inequality Constraints

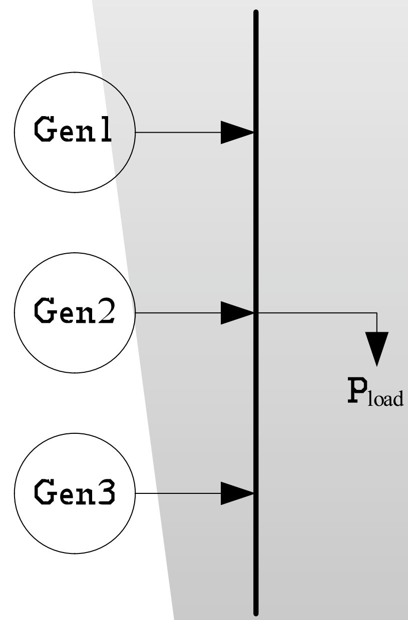
Problem: Three Generators. Input-output curves are given. Fuel costs are:

FC_1 = \$0.9/MBtu, FC_2 = \$1.0/MBtu, FC_3 = \$1.0/MBtu.

Total load P_D = 850 MW.

Determine optimal allocation.

$$H_1(P_1) = 510.0 + 7.2P_1 + 0.00142P_1^2$$
$$H_2(P_2) = 310.0 + 7.85P_2 + 0.00194P_2^2$$
$$H_3(P_3) = 78.0 + 7.97P_3 + 0.00482P_3^2$$



Added constraints:

Generator 1: Max. Output= 600 MW, Min. Output= 50 MW
Generator 2: Max. Output= 600 MW, Min. Output= 50 MW
Generator 3: Max. Output= 600 MW, Min. Output= 50 MW

Example: *cont'd*
Economic Dispatch with Equality and Inequality Constraints

Solution:

Determine the cost curves: (Generator 1's cost curve is different)

$$C_1(P_1) = H_1(P_1) \bullet FC_1 = 459 + 6.48P_1 + .00128P_1^2$$

$$C_2(P_2) = H_2(P_2) \bullet FC_2 = 310.0 + 7.85P_2 + 0.00194P_2^2$$

$$C_3(P_3) = H_3(P_3) \bullet FC_3 = 78.0 + 7.97P_3 + 0.00482P_3^2$$

Formulate the economic dispatch problem if using the same approach as the previous example:

$$\min_{P_1, P_2, P_3} f_{\text{cost}}(P_1, P_2, P_3) = C_1(P_1) + C_2(P_2) + C_3(P_3)$$

subject to

$$P_1 + P_2 + P_3 = P_D = 850$$

Example: *cont'd*

Economic Dispatch with Equality and Inequality Constraints

Solution:

Apply Lagrange multiplier:

$$L(P_1, P_2, P_3, \lambda) = \begin{Bmatrix} (459 + 6.48P_1 + .00128P_1^2) + \\ (310.0 + 7.85P_2 + 0.00194P_2^2) + \\ (78.0 + 7.97P_3 + 0.00482P_3^2) + \\ \lambda(850 - P_1 - P_2 - P_3) \end{Bmatrix}$$

$$\frac{\partial L}{\partial P_1} = 7.92 + 0.003124P_1 - \lambda = 0 \Rightarrow \lambda = IC_1 = 6.48 + 0.00256P_1$$

$$\frac{\partial L}{\partial P_2} = 7.85 + 0.00388P_2 - \lambda = 0 \Rightarrow \lambda = IC_2 = 7.85 + 0.00388P_2$$

$$\frac{\partial L}{\partial P_3} = 7.97 + 0.00964P_3 - \lambda = 0 \Rightarrow \lambda = IC_3 = 7.97 + 0.00964P_3$$

$$\frac{\partial L}{\partial \lambda} = 850 - P_1 - P_2 - P_3 = 0$$

Example: *cont'd*
Economic Dispatch with Equality and Inequality Constraints

Solution:

Generation allocation:

$$P_1 = 704.6 MW$$

$$P_2 = 111.8 MW$$

$$P_3 = 32.6 MW$$

Incremental cost:

$$\lambda = 8.284$$

Observations: $IC_1 = IC_2 = IC_3$

1) This solution meets the constraint that generation meets the demand

2) Generator 1 is above the rated maximum output power

3) Generator 3 is below the rated minimum output power

4) The next step is to assume that generator 1 is at the rated maximum output power and generator 3 is at the rated minimum output power

Example: *cont'd*

Economic Dispatch with Equality and Inequality Constraints

Solution:

Generation allocation (P_1 and P_3 are constrained by their limits):

$$P_1 = 600MW$$

$$P_2 = 200MW$$

$$P_3 = 50MW$$

Incremental cost:

$$IC_1 = \frac{\partial C_1}{\partial P_1} = 6.48 + 0.00256P_2 = 8.016$$

$$IC_2 = \frac{\partial C_2}{\partial P_2} = 7.85 + 0.00388P_2 = \lambda = 8.626$$

$$IC_3 = \frac{\partial C_3}{\partial P_3} = 7.97 + 0.00964P_2 = 8.452$$

Observations: $IC_1 < IC_2$, and $IC_3 < IC_2$

1) Generator 1: Since the incremental cost of generator 1 is less than λ, it is cheaper to operate than generator 2, but is already at maximum output ➔ Generator 1 should be left at the maximum output power

2) Generator 3: Since the incremental cost of generator 3 is less than λ, it is cheaper to operate than generator 2, but it has been forced to the minimum output power ➔ Generator 3 should not be forced to the minimum output power

Example: *cont'd*

Economic Dispatch with Equality and Inequality Constraints

Solution:

Resolve the economic dispatch problem with P_1 = 600 MW, while P_2 and P_3 are allowed to vary:

$$L(P_2, P_3, \lambda) = \begin{Bmatrix} (310.0 + 7.85P_2 + 0.00194P_2^2) + \\ (78.0 + 7.97P_3 + 0.00482P_3^2) + \\ \lambda(850 - P_1 - P_2 - P_3) \end{Bmatrix}$$

$$\frac{\partial L}{\partial P_2} = 7.85 + 0.00388P_2 - \lambda = 0 \Rightarrow \lambda = IC_2 = 7.85 + 0.00388P_2$$

$$\frac{\partial L}{\partial P_3} = 7.97 + 0.00964P_3 - \lambda = 0 \Rightarrow \lambda = IC_3 = 7.97 + 0.00964P_3$$

$$\frac{\partial L}{\partial \lambda} = 850 - P_1 - P_2 - P_3 = 0$$

Example: *cont'd*
Economic Dispatch with Equality and Inequality Constraints

Solution:

Generation allocation (P_1 is constrained by its limit):

$$P_1 = 600MW$$

$$P_2 = 187.1MW$$

$$P_3 = 62.9MW$$

Incremental cost:

$$IC_1 = \frac{\partial C_1}{\partial P_1} = 6.48 + 0.00256P_2 = 8.016$$

$$IC_2 = \frac{\partial C_2}{\partial P_2} = 7.85 + 0.00388P_2 = \lambda = 8.576$$

$$IC_3 = \frac{\partial C_3}{\partial P_3} = 7.97 + 0.00964P_2 = \lambda = 8.576$$

Observations: $IC_1 < IC_2 = IC_3$

1) Generator 1: Generator 1 is the cheapest, so it should generator as much power as possible. ➔ Generator 1 should be left at the maximum output power

2) Generator 2 and 3: their incremental costs satisfy the equal incremental cost criterion. ➔ The solution is the optimal allocation between these two generators

Homework

- Textbook problems: 13.3 and 13.6
- Due: October 12.

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Questions?