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EE 521: Analysis of Power Systems

Lecture 9
Economic Dispatch 2

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

- Economic Dispatch with Network Losses
  - System cost
  - Penalty factor
- Network Loss Equation
- Non-Fossil-Fuel Plants

Network Losses

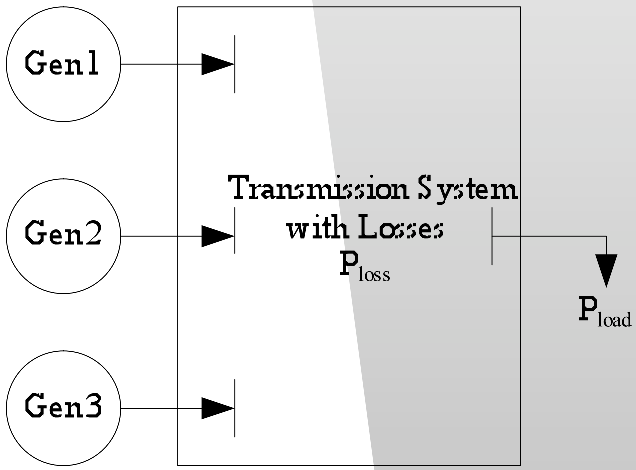
- Previously, the losses are neglected, and the balance of generation and load was treated as an equality constraint

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1(P_{G1}) + C_2(P_{G2}) + C_3(P_{G3})$$

subject to

$$P_{G1} + P_{G2} + P_{G3} = P_D$$

- In a real power system, there are network losses that are dependant on the generation dispatch



Economic Dispatch with Network Losses

- Considering network losses, the economic dispatch problem is formulated with a new equality constraint:

$$\min_{P_{G1},P_{G2},P_{G3}} f_{\text{cost}} = C_1\left(P_{G1}\right)+C_2\left(P_{G2}\right)+C_3\left(P_{G3}\right)$$

subject to

$$P_{G1}+P_{G2}+P_{G3}=P_D+P_{loss}$$

- Two basic solution approaches:
  - Incorporate the power flow equations as constraints – optimal power flow (OPF)
  - Describe the loss as a function of generator outputs

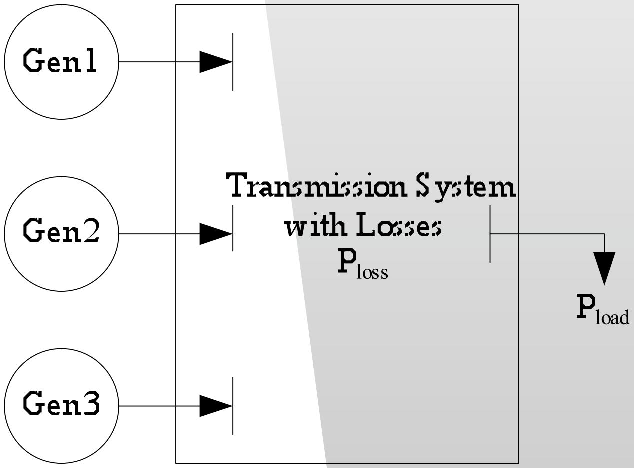
Example:
Economic Dispatch with Network Losses

Problem: Three Generators. Input-output curves are given. Fuel costs are:

FC_1 = \$1.1/MBtu, FC_2 = \$1.0/MBtu, FC_3 = \$1.0/MBtu.

Total load P_D = 850 MW.

Determine optimal allocation.



$$H_1(P_1) = 510.0 + 7.2P_1 + 0.00142P_1^2$$

$$H_2(P_2) = 310.0 + 7.85P_2 + 0.00194P_2^2$$

$$H_3(P_3) = 78.0 + 7.97P_3 + 0.00482P_3^2$$

Network Loss:

$$P_{loss} = 0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2$$

Note: This is a simplified loss expression. Most loss expressions are considerably more complex.

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

Determine the cost curves:

$$C_1(P_1) = H_1(P_1) \bullet FC_1 = 561 + 7.92P_1 + 0.001562P_1^2$$

$$C_2(P_2) = H_2(P_2) \bullet FC_2 = 310.0 + 7.85P_2 + 0.00194P_2^2$$

$$C_3(P_3) = H_3(P_3) \bullet FC_3 = 78.0 + 7.97P_3 + 0.00482P_3^2$$

Formulate the economic dispatch problem:

$$\min_{P_1, P_2, P_3} f_{\text{cost}}(P_1, P_2, P_3) = C_1(P_1) + C_2(P_2) + C_3(P_3)$$

subject to

$$P_1 + P_2 + P_3 = P_D + P_{loss} = 850 + P_{loss}$$

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

Apply Lagrange multiplier:

$$L(P_1, P_2, P_3, \lambda) = \begin{Bmatrix} (561 + 7.92P_1 + 0.001562P_1^2) + \\ (310.0 + 7.85P_2 + 0.00194P_2^2) + \\ (78.0 + 7.97P_3 + 0.00482P_3^2) + \\ \lambda(850 + P_{loss} - P_1 - P_2 - P_3) \end{Bmatrix}$$

The Lagrange function has been expanded so that the system losses are accounted for in the last term

At the optimal point:

$$\frac{\partial L}{\partial P_1} = 7.92 + 0.003124P_1 - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_1}\right) = 0$$

$$\frac{\partial L}{\partial P_2} = 7.85 + 0.00388P_2 - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_2}\right) = 0$$

$$\frac{\partial L}{\partial P_3} = 7.97 + 0.00964P_3 - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_3}\right) = 0$$

$$\frac{\partial L}{\partial \lambda} = 850 + P_{loss} - P_1 - P_2 - P_3 = 0$$

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

The following equations are yielded:

$$7.92 + 0.003124P_1 = \lambda\left(1 - 2(0.00003)P_1\right)$$

$$7.85 + 0.00388P_2 = \lambda\left(1 - 2(0.00009)P_2\right)$$

$$7.97 + 0.00964P_3 = \lambda\left(1 - 2(0.00012)P_3\right)$$

$$P_1 + P_2 + P_3 - 850 - \left(0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2\right) = 0$$

This set of equations is not linear and as such cannot be analytically solved, numeric methods are required.

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

An iterative procedure is required as with many other non-linear problems:

1. Pick a set of initial generator output powers
2. Calculate the incremental losses and the total loss
3. Calculate λ and generator outputs by solving the equations
4. Compare the calculated generator outputs with the initial output power
5. Iterate until the change in generator output powers is sufficiently small

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

1. Pick a set of initial generator output powers

$$P_1 = 400MW \qquad P_2 = 300MW \qquad P_3 = 150MW$$

2. Calculate the incremental losses and the total loss

$$\frac{\partial P_{loss}}{\partial P_1} = 2(0.00003)400 = 0.024$$

$$\frac{\partial P_{loss}}{\partial P_3} = 2(0.00012)150 = 0.036 \qquad P_{loss} = 15.6MW$$

$$\frac{\partial P_{loss}}{\partial P_2} = 2(0.00009)300 = 0.054$$

Example: *cont'd*

Economic Dispatch with Network Losses

Solution:

3. Calculate λ and generator outputs by solving the equations

$$7.92 + 0.003124P_1 = \lambda\left(1 - 2(0.00003)P_1\right) \quad \boxed{0.024}$$

$$7.85 + 0.00388P_2 = \lambda\left(1 - 2(0.00009)P_2\right) \quad \boxed{0.036}$$

$$7.97 + 0.00964P_3 = \lambda\left(1 - 2(0.00012)P_3\right) \quad \boxed{0.054}$$

$$P_1 + P_2 + P_3 - 850 - \left(0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2\right) = 0$$

$$\boxed{15.6}$$

$$P_1 = 440.68MW \qquad P_2 = 299.12MW \qquad P_3 = 125.77MW$$

$$\lambda = 9.5252$$

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

4. Compare the calculated generator outputs with the initial output power

$$\Delta P_1 = 440.68 - 400 = 40.68 MW$$

$$\Delta P_2 = 299.12 - 300 = -0.82 MW$$

$$\Delta P_3 = 125.77 - 150 = -24.23 MW$$

5. Iterate until the change in generator output powers is sufficiently small

In this case further iterations are required…

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

Second iteration:

$$\frac{\partial P_{loss}}{\partial P_1} = 2(0.00003)400.68 = 0.0264$$

$$\frac{\partial P_{loss}}{\partial P_2} = 2(0.00009)299.12 = 0.0538$$

$$\frac{\partial P_{loss}}{\partial P_3} = 2(0.00012)125.77 = 0.0301$$

$$P_{loss} = 15.78 MW$$

$$7.92 + 0.003124 P_1 = \lambda(1 - 0.0264) = \lambda(0.9736)$$

$$7.85 + 0.00388 P_2 = \lambda(1 - 0.0538) = \lambda(0.9462)$$

$$7.97 + 0.00964 P_3 = \lambda(1 - 0.0301) = \lambda(0.9699)$$

$$P_1 + P_2 + P_3 - 850 - 15.78 = 0$$

$$P_1 = 433.94 MW \quad P_2 = 300.11 MW \quad P_3 = 131.74 MW$$

$$\lambda = 9.5275$$

Example: *cont'd*
Economic Dispatch with Network Losses

Solution:

Further iterations:

	P1	P2	P3	Losses	λ
Initial	400	300	150	15.6	9.5252
1	440.68	299.12	125.77	15.78	9.5275
2	433.94	300.11	131.74	15.84	9.5285
3	435.87	299.94	130.42	15.83	9.5283
4	434.13	299.99	130.71	15.83	9.5284

Example: *cont'd*

Economic Dispatch with Network Losses

Solution:

Observations:

1. Need more generation to cover losses
2. Generation allocation is affected by the loss function
3. λ is larger: cost to deliver desired load is more expensive
4. IC's are no longer equal.

$$IC_1 = 7.92 + 0.003124P_1 = 9.2762$$

$$IC_2 = 7.85 + 0.00388P_2 = 9.0140$$

$$IC_3 = 7.97 + 0.00964P_3 = 9.2300$$

Considering losses:

$$P_1 = 434.13MW$$

$$P_2 = 299.99MW$$

$$P_3 = 130.71MW$$

$$\lambda = 9.5284$$

Not considering losses (from last lecture):

$$P_1 = 393.2MW$$

$$P_2 = 334.6MW$$

$$P_3 = 122.2MW$$

$$\lambda = 9.148$$

Revisit of the Equations

- System Incremental Cost, λ
- Penalty Factor, PF
  - Generators within a power plant have the same penalty factor
  - If network losses are negligible, the equation reduces to the earlier case shown in last class.

$$\min_{P_{Gi}} f_{\text{cost}} = \sum C_i(P_{Gi}), \quad \text{subject to} \quad \sum P_{Gi} = P_D + P_{loss}$$

$$\min_{P_{Gi},\lambda} L(P_{Gi},\lambda) = \sum C_i(P_{Gi}) + \lambda\left(P_D + P_{loss} - \sum P_{Gi}\right)$$

$$\frac{\partial L(P_{Gi},\lambda)}{\partial P_{Gi}} = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} + \lambda\left(\frac{\partial P_{loss}}{\partial P_{Gi}} - 1\right) = 0$$

$$\lambda = \left(\frac{1}{1 - \frac{\partial P_{loss}}{\partial P_{Gi}}}\right)\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = PF_i \bullet IC_i$$

$$\frac{\partial P_{loss}}{\partial P_{Gi}} \quad ?$$

Network Losses

- Network losses due to resistance – I^2R.
- Higher voltage → less current → less losses.
  - 10x higher V → 10x less I → 100x less losses
- Long distance transmission is typically done with overhead lines at voltages larger than 115 kV.
- At extremely high voltages, corona discharge losses offset the lower resistance loss in line conductors.
- Transmission and distribution losses in the US were estimated at 7.2% in 1995.

Network Loss Equation

- Network losses can be expressed as a function of generator outputs. For *K* generators:

$$P_{loss} = \sum_{i=1}^{K}\sum_{j=1}^{K} P_{gi}B_{ij}P_{Gj} + \sum_{i=1}^{K} B_{i0}P_{Gi} + B_{00}$$

- B-Matrix: loss coefficients/B-coefficients (Chapter 13.3)
  - Symmetrical
  - Power flow dependent
  - Assumed constant
  - Multiple sets of B-coefficients for large-scale power systems

$$B = \left[\begin{array}{cccc|c} B_{11} & B_{12} & \dots & & \frac{B_{10}}{2} \\ B_{21} & B_{22} & \dots & & \frac{B_{20}}{2} \\ \dots & \dots & \dots & & \dots \\ \hline \frac{B_{10}}{2} & \frac{B_{20}}{2} & \dots & & B_{00} \end{array}\right]_{K\times K}$$

Incremental Network Loss

- Change of 1 MW at a specific generator results in an incremental loss expressed as:

$$\frac{\partial P_{loss}}{\partial P_{Gi}} = 2\sum_{j=1}^{K} B_{ij} P_{gj} + B_{i0}$$

- Economic Dispatch with Network Loss Equations:

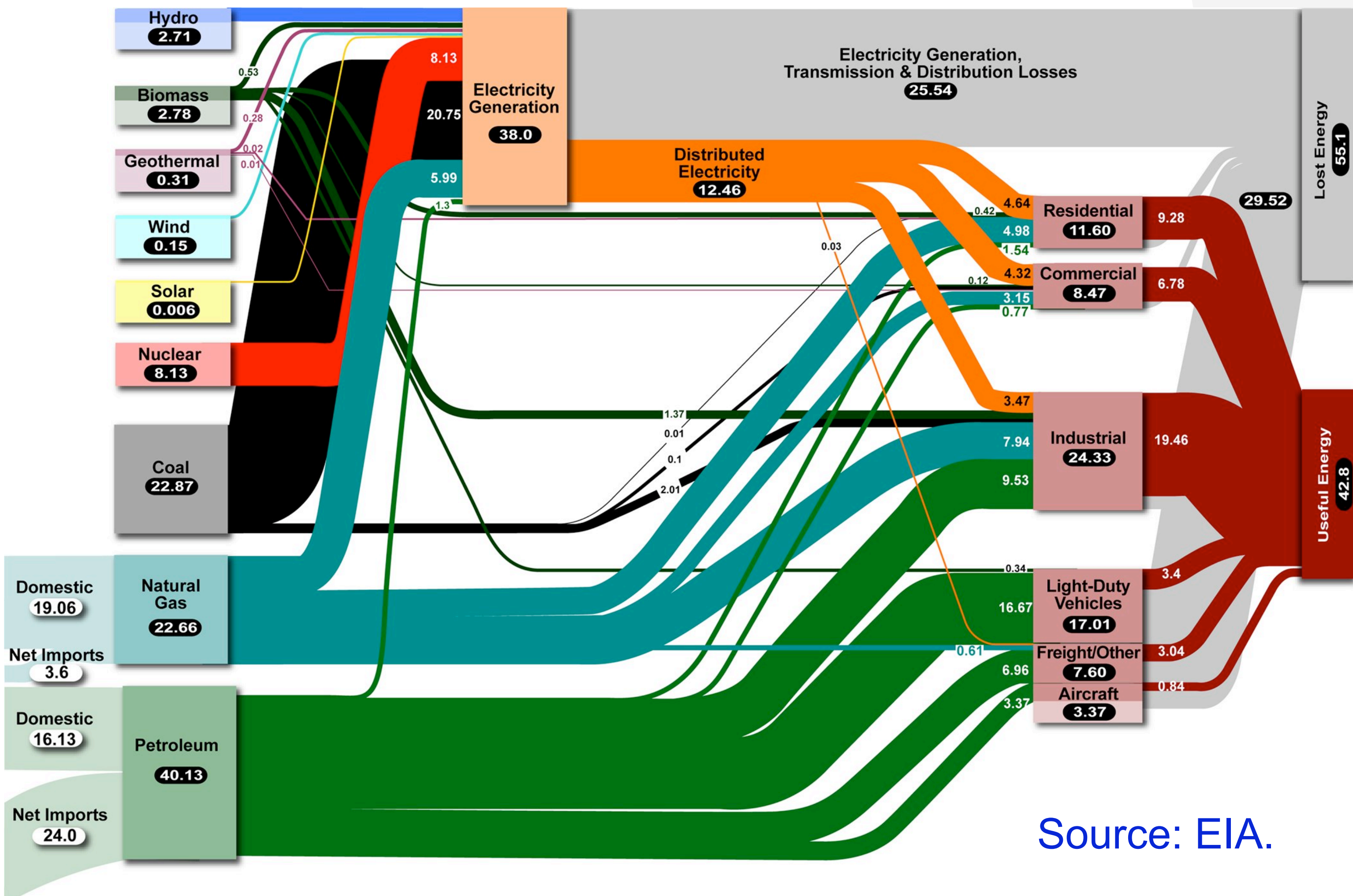
$$\frac{\partial L(P_{Gi},\lambda)}{\partial P_{Gi}} = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} + \lambda\left(\frac{\partial P_{loss}}{\partial P_{Gi}} - 1\right) = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} + \lambda\left(2\sum_{j=1}^{K} B_{ij} P_{gj} + B_{i0} - 1\right) = 0$$

$$IC_i = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = \lambda\left[1 - \left(2\sum_{j=1}^{K} B_{ij} P_{gj} + B_{i0}\right)\right]$$

Additional Methods of Solving the Economic Dispatch Problem

- As with previous non-linear problems that have been encountered in this class, there are multiple solution methods.
- For problems that involve only convex cost curves, methods such as gradient searches and Newton's method may be applied.
- For problems with non-convex cost curves, dynamic programming can be used.

Power Generation Efficiency



Source: EIA.

Practical Considerations

- Economic dispatch is primarily for fossil-fuel plants.
- Fixed cost such as installation not included.
- Non-fossil-fuel plants:
  - Nuclear: base-loaded, not participate in EC. High fixed cost, low operating cost.
  - Hydro: river optimization considering yearly water cycles, irrigation, transportation, fishing, … also wind firming …
  - Pumped hydro: peak load shaving, pumping during off-peak hours (low λ), generating during peak-hours (high λ).
  - Wind: all taken, not participate in EC. Need other forms of energy for firming.

Reading Assignments

- Unit Commitment:
  - Textbook Chapter 13.7, 13.8
  - Excerpt from “Power Generation, Operation and Control” by Wood and Wollenberg, Chapter 5 “Unit Commitment”. See handouts.
- Optimal Power Flow:
  - Excerpt from “Power Generation, Operation and Control” by Wood and Wollenberg, Chapter 13 “Optimal Power Flow”. See handouts.

Questions?

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