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EE 521: Analysis of Power Systems

Lecture 16
Transient Stability

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

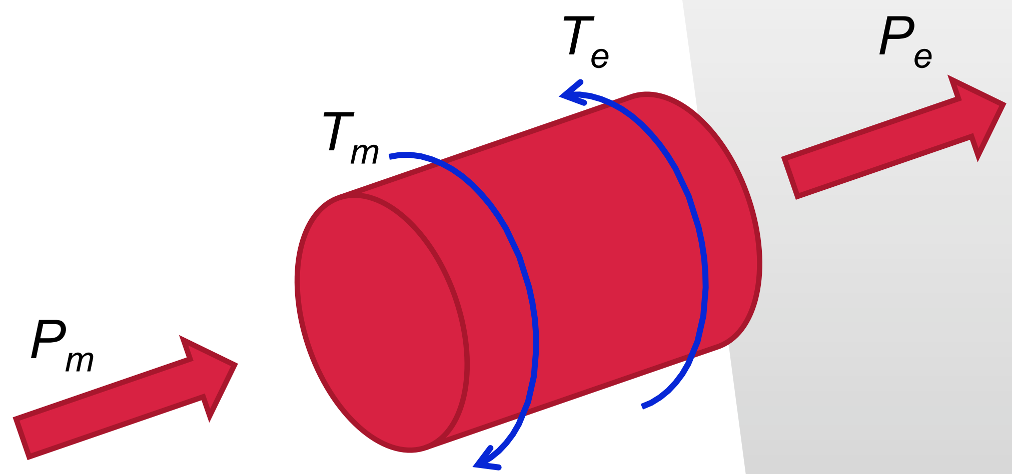
Test 216

Topics

- Machine Dynamics
- Generator Modeling
  - Generator swing equation
  - Generator grouping
  - Relative nature of stability
- Equal-Area Criterion
  - Critical clearing angle

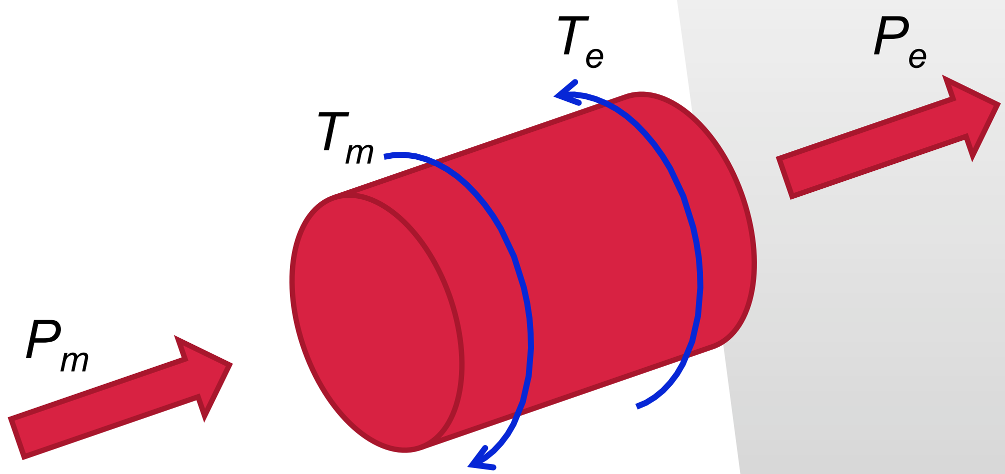
Generator Energy Conversion

- Input:
  - Mechanical power
    - Water
    - Steam
    - Wind
    - …
- Output:
  - Electrical power



Newton's Second Law

- Mathematical description of a spinning mass that is acted on by multiple torques



$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

Rotor inertia: J

Rotor angular position: $\theta_m = \omega_{sm}t + \delta_m$

Synchronous mechanical speed: ω_{sm}

Mechanical rotor angle: δ_m

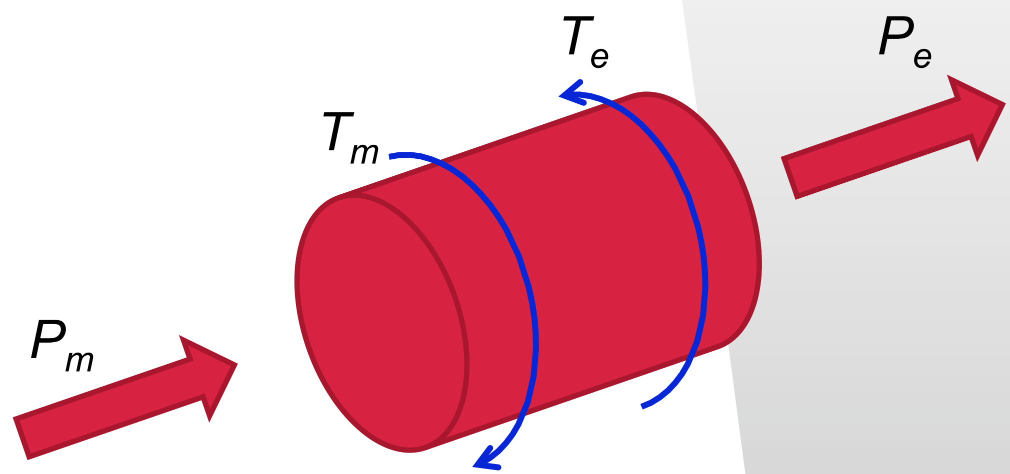
Rotor angular velocity: $\omega_m = \frac{d\theta_m}{dt}$

Representation by Rotor Angles and Power

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

$$J\frac{d^2\delta_m}{dt^2} = \frac{P_m}{\omega_m} - \frac{P_e}{\omega_m}$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = P_m - P_e$$



Generator inertia:

$$H = \frac{\frac{1}{2}J\omega_{sm}^{\;2}}{S_{rated}} \quad \text{MJ/MVA or second}$$

$$J\omega_m = \frac{2S_{rated}H}{\omega_{sm}^2}\omega_{\mathrm{m}} \approx \frac{2S_{rated}H}{\omega_{sm}}$$

$$\frac{2S_{rated}H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

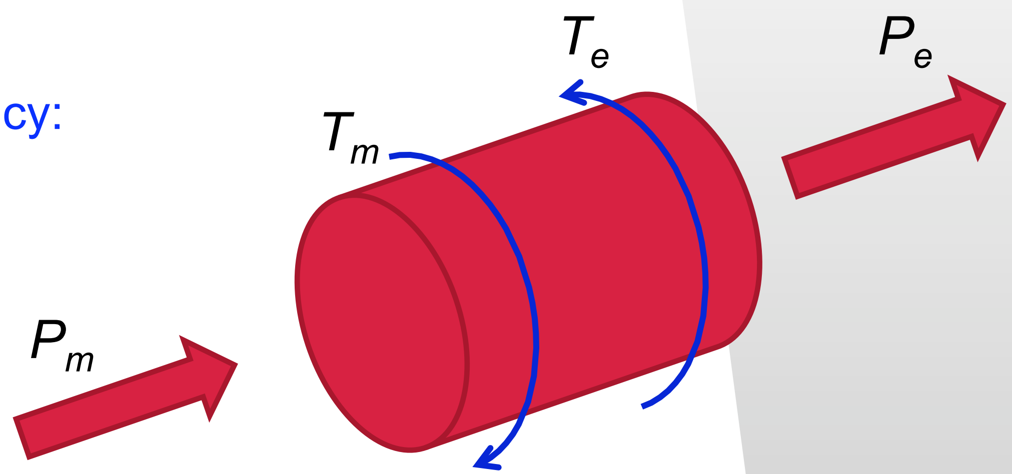
Convert Mechanical Quantities to Electrical Quantities

$$\frac{2S_{rated}H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

Electrical angle and radian frequency:

$$\delta = \frac{p}{2}\delta_m \qquad \omega = \frac{p}{2}\omega_m$$

(p: number of poles)



$$\frac{2S_{rated}H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$ In actual units

$$\boxed{\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e}$$ per unit

Generator Swing Equation

$$\begin{cases} \dfrac{2H}{\omega_s}\dfrac{d\omega}{dt} = P_m - P_e \\ \dfrac{d\delta}{dt} = \omega - \omega_s \end{cases}$$

1. The simplest model of a generator.
2. The most important equation in generator stability analysis.
3. The procedure to derive this equation:
    1) Start with Newton’s second law
    2) Covert angular positions to rotor angles
    3) Convert torque to power
    4) Convert rotor inertia *J* to generator inertia *H*
    5) Convert mechanical quantities to electrical quantities
    6) Convert actual units to per unit values

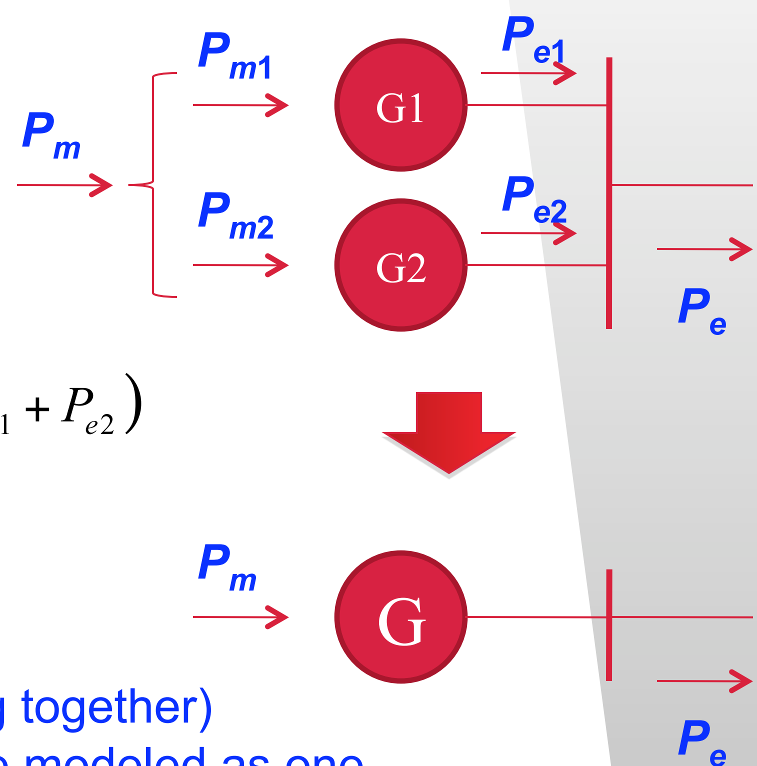
Generator Grouping

$$G1: \frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$G2: \frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

$$\Rightarrow \frac{2(H_1 + H_2)}{\omega_s}\frac{d^2\left(\dfrac{H_1\delta_1 + H_2\delta_2}{H_1 + H_2}\right)}{dt^2} = (P_{m1} + P_{m2}) - (P_{e1} + P_{e2})$$

$$\Rightarrow \frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$



1. A special case: coherent machines (swing together)
2. Multiple machines in a power plant can be modeled as one virtual machine with the total mechanical power and the total electrical power. It reduces the number of equations to be solved.

Relative Nature of Generator Stability

$$G1: \frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

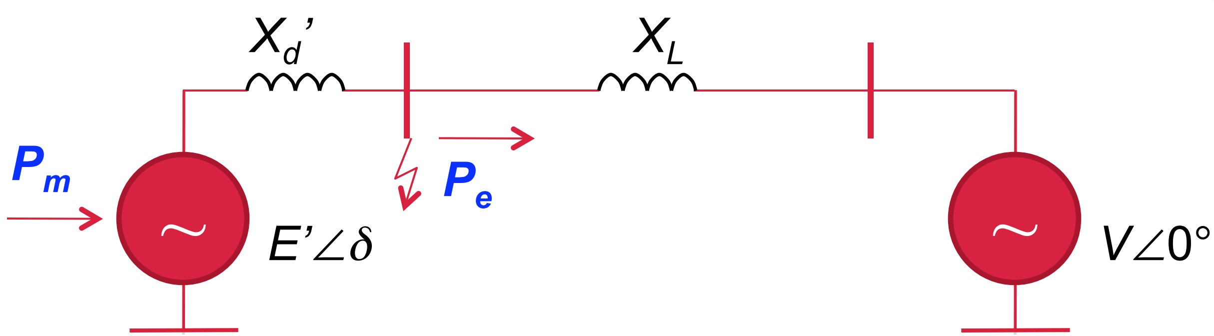
$$G2: \frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

$$\frac{2\left(\dfrac{H_1 H_2}{H_1 + H_2}\right)}{\omega_s}\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \left(\frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2}\right) - \left(\frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2}\right)$$

$$\frac{2H_{12}}{\omega_s}\frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12}$$

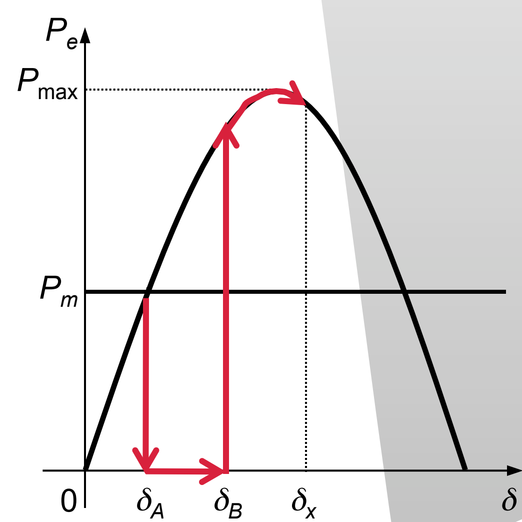
1. At the steady state: $P_{m12} - P_{e12} = 0$, so angle difference is zero. Machines are running at the synchronous speed. Or machine rotors are in synchronism.
2. Stability of a machine within a system is a relative property associated with its dynamic behavior with respect to other machines in the system.
3. A special case: G2 is an infinite bus, i.e. its inertia is very large.

Single-Machine-Infinite-Bus System



$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$P_e = \frac{E'V}{X}\sin\delta$$



1. Accelerating when $P_m > P_e$
2. Decelerating when $P_m < P_e$

Equal Area Criterion

- Machine rotor angle stops increasing when **A1 = A2**.
  - Kinetic energy built up in the rotor needs to be consumed.

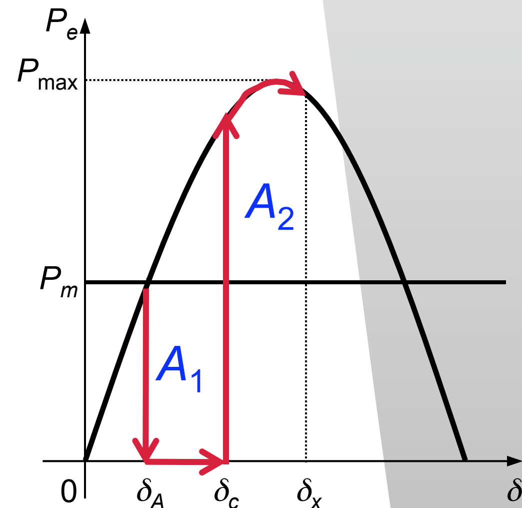
Proof:

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad \frac{d\delta}{dt} = \Delta\omega = \omega - \omega_s$$

Multiplying by $\Delta\omega$:

$$\frac{2H}{\omega_s}\Delta\omega\frac{d\Delta\omega}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

$$\frac{H}{\omega_s}\frac{d(\Delta\omega)^2}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$



Equal Area Criterion *cont'd*

Proof (cont'd):

$$\frac{H}{\omega_s}\frac{d(\Delta\omega)^2}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

Multiplying by dt and integrating between δ_A and δ_x:

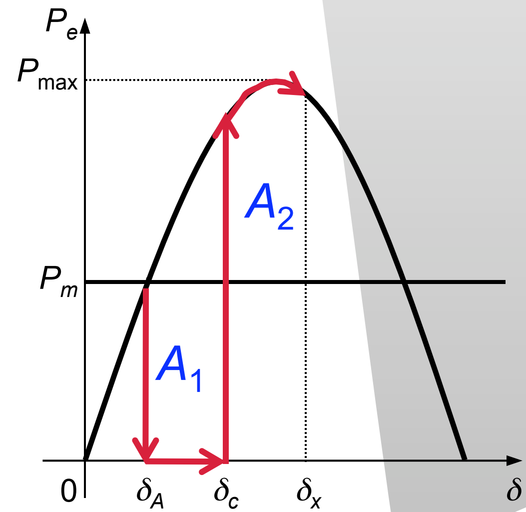
$$\int_{\Delta\omega_A}^{\Delta\omega_x}\frac{H}{\omega_s}d(\Delta\omega)^2 = \int_{\delta_A}^{\delta_x}(P_m - P_e)d\delta$$

$$0 = \frac{H}{\omega_s}\left[(\Delta\omega_x)^2 - (\Delta\omega_A)^2\right] = \int_{\delta_A}^{\delta_x}(P_m - P_e)d\delta$$

$$0 = \int_{\delta_A}^{\delta_x}(P_m - P_e)d\delta = \int_{\delta_A}^{\delta_c}(P_m - P_e)d\delta + \int_{\delta_c}^{\delta_x}(P_m - P_e)d\delta$$

$$\int_{\delta_A}^{\delta_c}(P_m - P_e)d\delta = \int_{\delta_c}^{\delta_x}(P_e - P_m)d\delta$$

$$A_1 = A_2 \quad \blacksquare$$

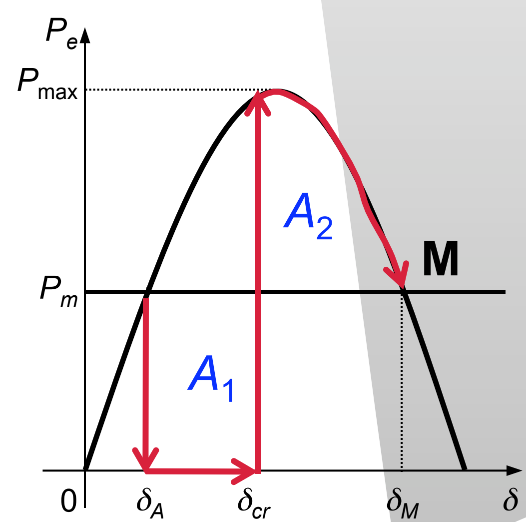


Critical Clearing Angle

- What is the maximum angle before the fault is cleared?
  - The rotor angle can not exceed point M. Otherwise the machine will continue to accelerate ➔ unstable.

$$\int_{\delta_A}^{\delta_{cr}} (P_m - P_e)\, d\delta = \int_{\delta_c}^{\delta_M} (P_e - P_m)\, d\delta$$

- δ_{cr}: Critical Clearing Angle.
- t_{cr}: Critical Clearing Time.
  - Stable if $t_c < t_{cr}$ $(\delta_c < \delta_{cr})$
  - Unstable if $t_c > t_{cr}$ $(\delta_c > \delta_{cr})$



Example: Critical Clearing Angle

- See notes.

Questions?