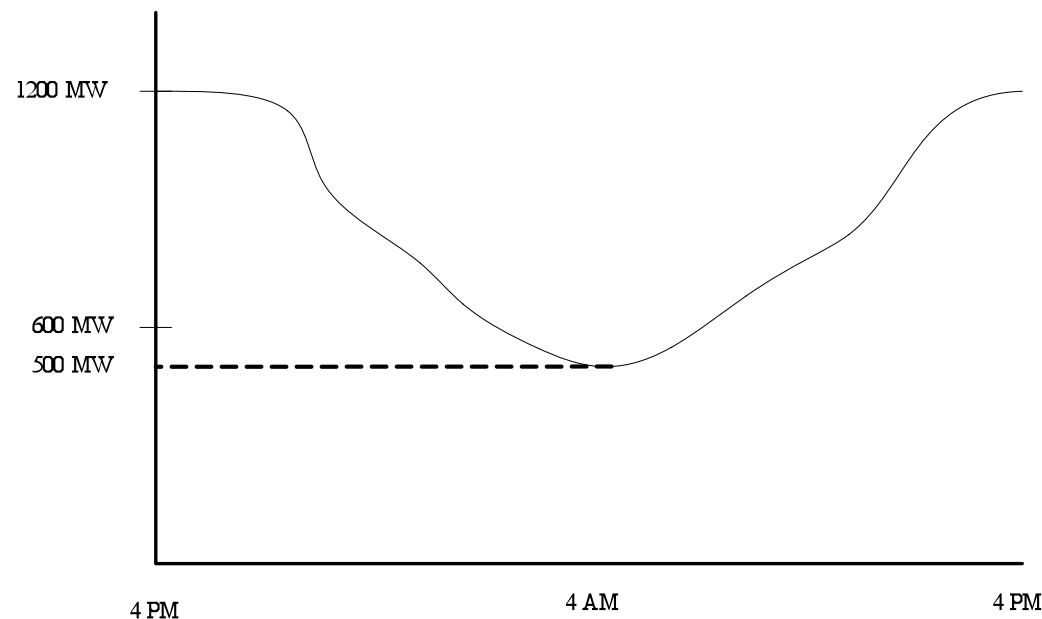


## Unit Commitment

- Economic Dispatch dealt with the problem of minimizing the cost of supplying the load with a given set of generators
- Unit Commitment deals with the more complicated problem of minimizing the cost of supplying the load when the set of generators is also variable
- Calculating a unit commitment required the calculation of numerous Economic Dispatches

## Unit Commitment

- Numerous economic dispatches are necessary because of the time varying nature of the electrical load
- Generators are generally re-dispatched every hour



## Economic Dispatch for Unit Commitment

- Because of the large number economic dispatches that must be calculated it is common to use simplified versions of the previously discussed method
- The first method that we will examine is the “priority list” method
- The second method that we will examine is the “block counting” method

## Generator Information (Wood and Wollenberg)

### ➤ Unit 1:

- Max. Output= 600 MW
- Min. Output= 150 MW
- Fuel cost=1.1 \$/MBtu

$$H_1(P_1) = 510.0 + 7.2P_1 + .00142P_1^2$$

$$C_1(P_1) = 5.61 + 7.92P_1 + .001562P_1^2$$

### ➤ Unit 2:

- Max. Output= 400 MW
- Min. Output= 100 MW
- Fuel cost=1.0 \$/MBtu

$$H_2(P_2) = 310.0 + 7.85P_2 + .00194P_2^2$$

$$C_2(P_2) = 310.0 + 7.85P_2 + .00194P_2^2$$

### ➤ Unit 3:

- Max. Output= 200 MW
- Min. Output= 50 MW
- Fuel cost=1.2 \$/MBtu

$$H_3(P_3) = 78.0 + 7.97P_3 + .00482P_3^2$$

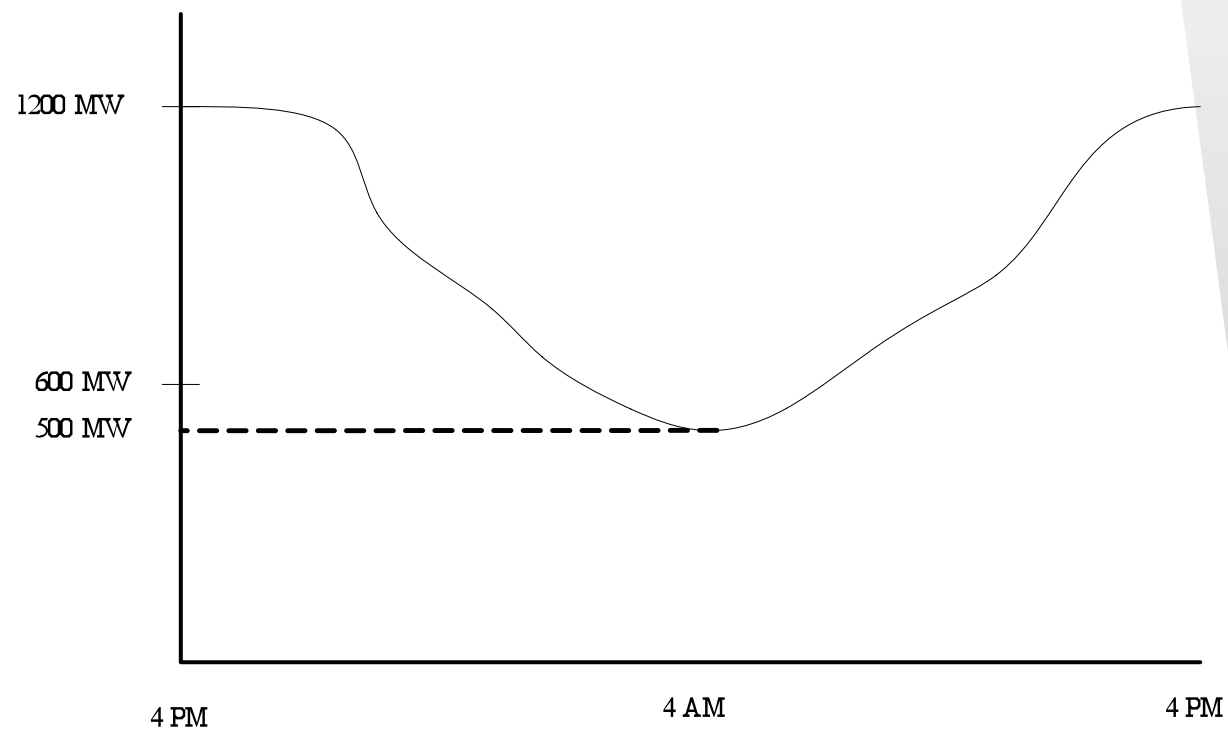
$$C_3(P_3) = 93.6 + 9.56P_3 + .00578P_3^2$$

## Example Case 1 (Wood and Wollenberg)

550 MW Load

| Unit 1 | Unit 2 | Unit 3 | Max Gen | Min Gen | P1         | P2  | P3  | C1   | C2   | C3   | Total Cost |
|--------|--------|--------|---------|---------|------------|-----|-----|------|------|------|------------|
| Off    | Off    | Off    | 0       | 0       | Infeasible |     |     |      |      |      |            |
| Off    | Off    | On     | 200     | 50      | Infeasible |     |     |      |      |      |            |
| Off    | On     | Off    | 400     | 100     | Infeasible |     |     |      |      |      |            |
| Off    | On     | On     | 600     | 150     | 0          | 400 | 150 | 0    | 3760 | 1658 | 5418       |
| On     | Off    | Off    | 600     | 150     | 550        | 0   | 0   | 5389 | 0    | 0    | 5389       |
| On     | Off    | On     | 800     | 200     | 500        | 0   | 50  | 4911 | 0    | 586  | 5497       |
| On     | On     | Off    | 1000    | 250     | 295        | 255 | 0   | 3030 | 2440 | 0    | 5471       |
| On     | On     | On     | 1200    | 300     | 267        | 267 | 50  | 2787 | 2244 | 586  | 5617       |

## Example Case 2 (Wood and Wollenberg) Cont



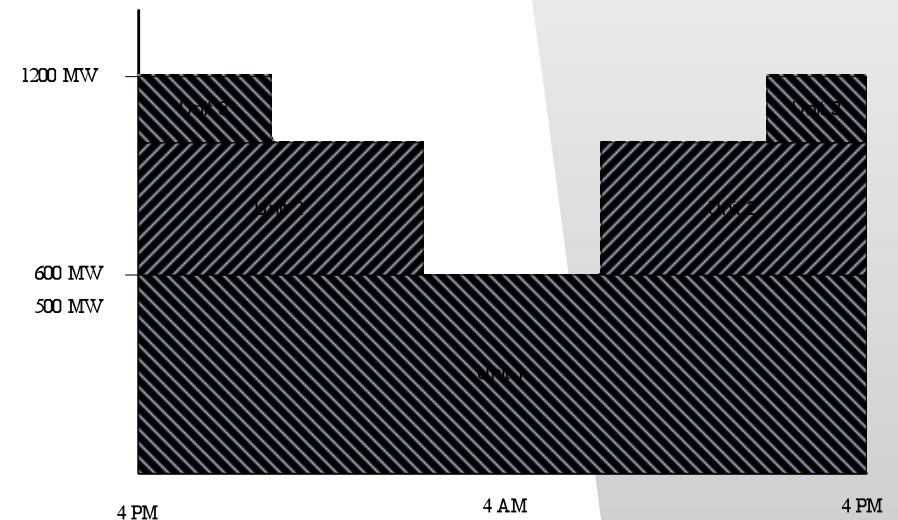
## Example Case 2 (Wood and Wollenberg) Cont

### ➤ Priority list rules

- Load < 600MW, run only unit 1
- 600MW < Load < 1,000MW, run units 1 and 2
- 1,000MW < Load, run all three units

## Example Case 2 (Wood and Wollenberg) Cont

|      | Optimal Combination |        |        |
|------|---------------------|--------|--------|
| Load | Unit 1              | Unit 2 | Unit 3 |
| 1200 | on                  | on     | on     |
| 1150 | on                  | on     | on     |
| 1100 | on                  | on     | on     |
| 1050 | on                  | on     | on     |
| 1000 | on                  | on     | off    |
| 950  | on                  | on     | off    |
| 900  | on                  | on     | off    |
| 850  | on                  | on     | off    |
| 800  | on                  | on     | off    |
| 750  | on                  | on     | off    |
| 700  | on                  | on     | off    |
| 650  | on                  | on     | off    |
| 600  | on                  | off    | off    |
| 550  | on                  | off    | off    |
| 500  | on                  | off    | off    |





## Block Counting Example

- In the previous priority list example a single set of rules was used to determine the economic dispatch
- In the block counting method each generator has its output separated into costs over ranges
  - Gen 1: \$10/MW 0-50MW  
\$12/MW 51-100MW
  - Gen 2: \$10/MW 0-25MW  
\$12/MW 26-100MW

## **Block Counting Example Cont.**

- The total cost of operation is determined by adding generation until the load is met
- The lowest cost generation is added first and thereafter in ascending order
- When the generation meets the load the total cost is calculated

## Block Counting Example Cont.

- Total Load=1000MW
  - Gen 1: \$8/MW 0-150MW  
\$10/MW 151-300MW  
\$12/MW 301-500MW
  - Gen 2: \$10/MW 0-125MW  
\$12/MW 126-250MW  
\$14/MW 251-500MW
  - Gen 3: \$9/MW 0-175MW  
\$10/MW 176-400MW  
\$12/MW 401-500MW

## Block Counting Example Cont.

- Using the table to the right the generation is added in blocks from the three generators until the generation meets the load
- The table to the lower right shows the cost per MW for the various operating regions

|          | Gen 1 | Gen 2 | Gen 3 |
|----------|-------|-------|-------|
| \$ 8.00  | 150   | 0     | 0     |
| \$ 9.00  | 0     | 0     | 175   |
| \$ 10.00 | 150   | 125   | 225   |
| \$ 11.00 | 200   | 0     | 0     |
| \$ 12.00 | 0     | 125   | 100   |
| \$ 13.00 | 0     | 250   | 0     |

| Load      | \$/MW    |
|-----------|----------|
| 0-150     | \$ 8.00  |
| 151-325   | \$ 9.00  |
| 326-825   | \$ 10.00 |
| 826-1025  | \$ 11.00 |
| 1026-1250 | \$ 12.00 |
| 1251-1500 | \$ 13.00 |

## Block Counting Example Cont.

- Total operating cost=\$9,700.00 when Load=10,000MW
  - Generator 1
    - Output: 475MW
    - Cost: \$4,625.00
  - Generator 2
    - Output: 125MW
    - Cost: \$1,250.00
  - Generator 3
    - Output: 400MW
    - Cost: \$3,825.00

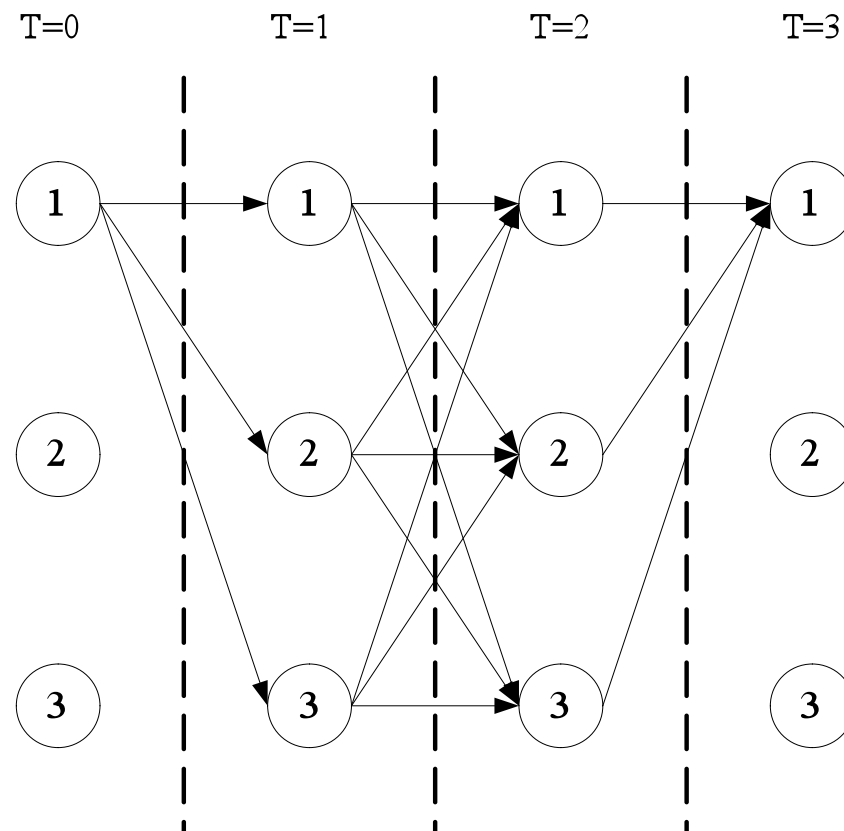
## Unit Commitment Complexity

- At each time period the number of states equals  $2^N$
- $N$  is the number of units in the system
- Even for a simple system with only 10 generators there are 1024 possible system states
- For a number of these states an economic dispatch is not necessary since the generation will not be able to meet the load

## Unit Commitment Complexity Cont.

- Once the number of possible states at each time period is determined it is necessary to examine all of the possible state transitions
- When examining the complete enumeration of the problem there is the potential to transition from any initial state to any final state
- Once again examining the 10 generator system, over a 24 hour period, yields  $1.76 \times 10^{72}$  possible state transitions

## Economic Dispatch Complexity Cont.





## Unit Commitment Constraints

- Up to this point we have examined each of the states without regard for the transition to or from the state
- In an actual system there are numerous constraints
  - Spinning Reserve
  - Minimum up time
  - Minimum down time
  - Startup/Shutdown costs

## Spinning Reserve

- Spinning reserve is defined as “the total amount of generation available from all synchronized units connected to the system, less the system load plus losses”
- Spinning reserved is used to ensure that there is sufficient generation to meet the demand under a given set of contingency conditions
- A typical contingency condition is that the spinning reserve must be sufficient to allow for the loss of the largest generator in the system

## Spinning Reserve Cont.

- Spinning reserves must also meet time requirements and can be classified according to time
  - In general only generators with high ramp rates are used
  - Some combustion turbines are used that are normally not running, but they can be brought up to speed and synchronized quickly
  
- Spinning reserves must be allocated across a system to prevent transmission congestion
  - This is especially true in areas where there are significant imports

## Example Case 5 (Wood and Wollenberg)



| Region  | Unit  | Unit Capacity | Unit Output | Regional Generation | Spinning Reserve | Regional Load | Interchange |
|---------|-------|---------------|-------------|---------------------|------------------|---------------|-------------|
| Western | 1     | 1000          | 900         | 1740                | 100              | 1900          | 160 in      |
|         | 2     | 800           | 420         |                     | 380              |               |             |
|         | 3     | 800           | 420         |                     | 380              |               |             |
| Eastern | 4     | 1200          | 1040        | 1350                | 160              | 1190          | 160 out     |
|         | 5     | 600           | 310         |                     | 290              |               |             |
| Total   | 5-Jan | 4400          | 3090        | 3090                | 1310             | 3090          |             |

## Min Up/Down Times

### ➤ Minimum up-time

- Once the unit is started it should remain running for a minimum amount of time
- In a thermal plant or combustion turbine it may be necessary to allow a generator to fully heat up for a period of time, failure to do so can result in reduction of unit life

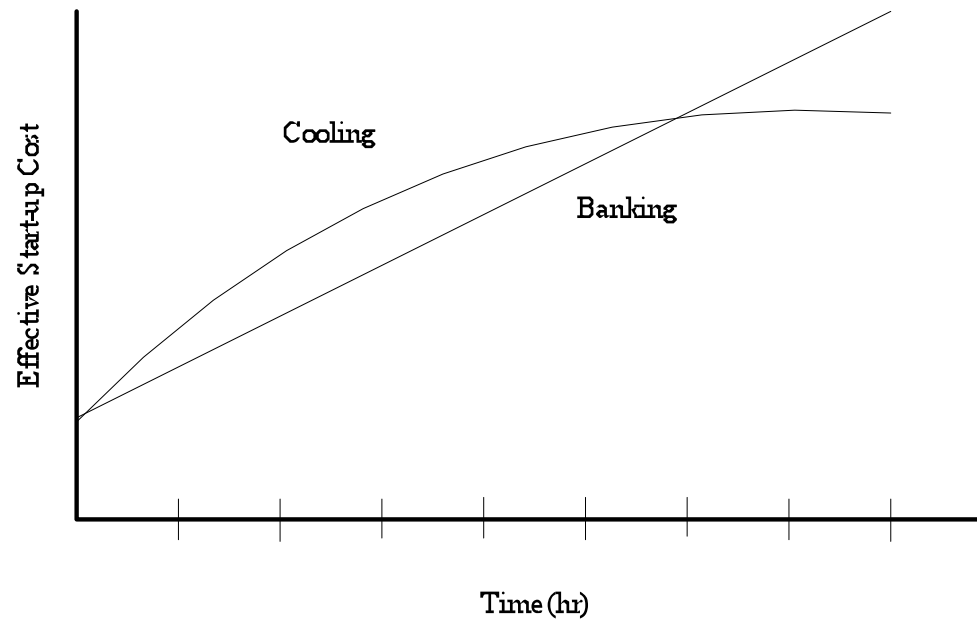
### ➤ Minimum down-time

- Once the unit is shutdown it must remain down for a minimum amount of time
- Xeon build-up in nuclear plants is an excellent example of a minimum downtime
- If a nuclear plant is at high power for a prolonged period and then shut down, it may be physically impossible to restart the reactor until the Xeon decays

## **Banking or Cooling to Cold Iron**

- When a generator is shut down it can either be allowed to cool down or it can be “banked” and kept at operating temperature
- Full cool down to “cold iron” is only allowed when the plant is expected to be shutdown for a long period of time
- A generator is “banked” when it is expected to re-dispatched in a short period of time
- For each plant there is a break even point between cooling down and banking

## Banking or Cooling to Cold Iron Cont.



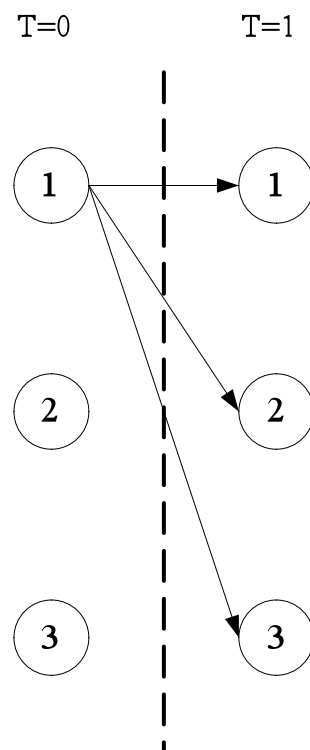
- At some point the cost of continual banking exceeds the cost of heating the plant from cold iron to normal operating temperature

## Startup/Shutdown Costs

- In general there are costs associated with starting up or shutting down a generator
- For a thermal plant heat is required to bring a generator from “cold iron” to the operating temperature, during which time no electrical power is produced
- Due to personnel requirements it may not be possible to startup or shutdown 2 generators at the same time
- These cost must be considered when transitioning from one state to the next



## Startup/Shutdown Costs Cont.



### ➤ State 1

- Gen 1:0
- Gen 2:1
- Gen 3:1

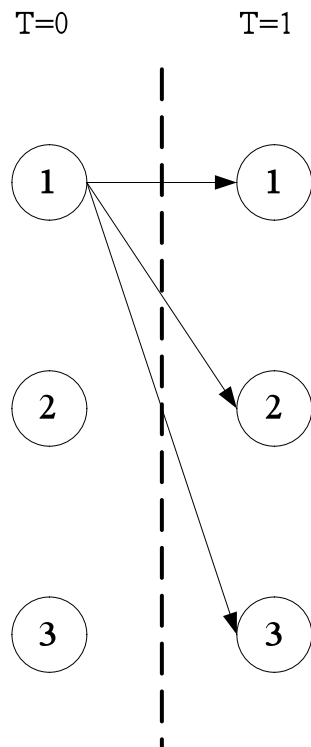
### ➤ State 2

- Gen 1:1
- Gen 2:1
- Gen 3:1

### ➤ State 3

- Gen 1:1
- Gen 2:1
- Gen 3:0

## Startup/Shutdown Costs Cont.



$$\$ = ED_1^{t=0} + (state_1^{t=0} \rightarrow state_1^{t=1}) + ED_1^{t=1} = ED_1^{t=0} + ED_1^{t=1}$$

$$\$ = ED_1^{t=0} + (state_1^{t=0} \rightarrow state_2^{t=1}) + ED_2^{t=1} = ED_1^{t=0} + SU_{gen1} + ED_2^{t=1}$$

$$\$ = ED_1^{t=0} + (state_1^{t=0} \rightarrow state_3^{t=1}) + ED_3^{t=1} = ED_1^{t=0} + (SU_{gen1} + SD_{gen3}) + ED_3^{t=1}$$

## Dynamic Programming Solution Method

- The forward method of dynamic programming has distinct advantages
- With the forward method the problem progresses forward from an initial unit dispatch
- In this way all possible and/or viable states and state transitions can be examined
- Dynamic programming allows for the inclusion of system constraints

## Dynamic Programming Solution Method Cont.

- Using the forward dynamic programming (FDP) with a strict priority list will be the examined method
- This method has the advantage of reducing the number of combinations that need to be investigated
- All states are ranked according to their total MW capacity
- All cost functions will be linear, resulting in a constant for the incremental cost (IC)

## Example Case 1 (Wood and Wollenberg)

| Unit | Max | Min | Heat Rate | No-Load Cost | IC     | Min up | Min down |
|------|-----|-----|-----------|--------------|--------|--------|----------|
|      | MW  | MW  | Btu/kWh   | \$/hr        | \$/mWh | hr     | hr       |
| 1    | 80  | 25  | 10440     | 213          | 23.54  | 4      | 2        |
| 2    | 250 | 60  | 9000      | 585.62       | 20.34  | 5      | 3        |
| 3    | 300 | 75  | 8730      | 684.74       | 19.74  | 5      | 4        |
| 4    | 60  | 20  | 11900     | 252          | 28     | 1      | 1        |

| Unit | Initial Condition | Start-up Cost |      |            |
|------|-------------------|---------------|------|------------|
|      | hrs on/off        | hot           | cold | cold start |
| 1    | -5                | 150           | 350  | 4          |
| 2    | 8                 | 170           | 400  | 5          |
| 3    | 8                 | 500           | 1100 | 5          |
| 4    | -6                | 0             | 0.02 | 0          |

## Example Case 1 (Wood and Wollenberg) Cont.

| State | Units |   |   |   | Max Capacity |
|-------|-------|---|---|---|--------------|
| 15    | 1     | 1 | 1 | 1 | 690          |
| 14    | 1     | 1 | 1 | 0 | 630          |
| 13    | 0     | 1 | 1 | 1 | 610          |
| 12    | 0     | 1 | 1 | 0 | 550          |
| 11    | 1     | 0 | 1 | 1 | 440          |
| 10    | 1     | 1 | 0 | 1 | 390          |
| 9     | 1     | 0 | 1 | 0 | 380          |
| 8     | 0     | 0 | 1 | 1 | 360          |
| 7     | 1     | 1 | 0 | 0 | 330          |
| 6     | 0     | 1 | 0 | 1 | 310          |
| 5     | 0     | 0 | 1 | 0 | 300          |
| 4     | 0     | 1 | 0 | 0 | 250          |
| 3     | 1     | 0 | 0 | 1 | 140          |
| 2     | 1     | 0 | 0 | 0 | 80           |
| 1     | 0     | 0 | 0 | 1 | 60           |
| 0     | 0     | 0 | 0 | 0 | 0            |

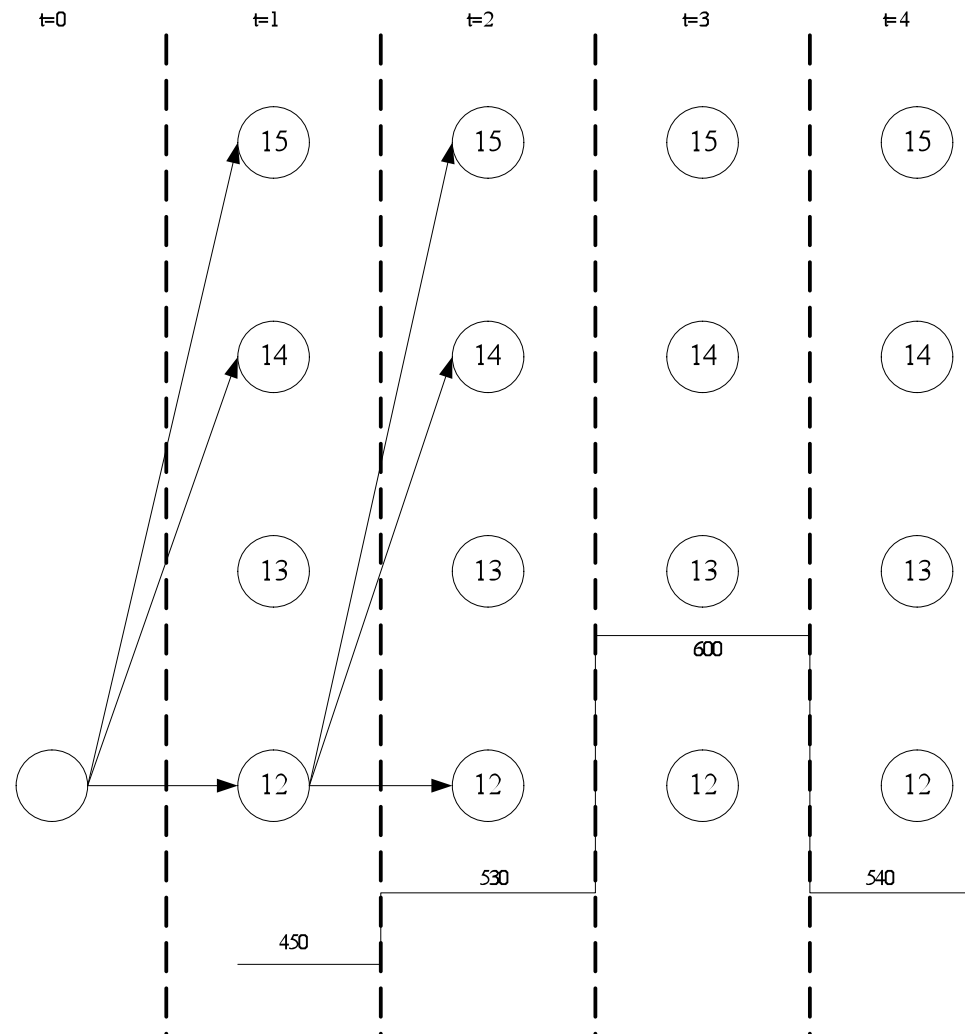
| Load Pattern |           |
|--------------|-----------|
| hr           | Load (MW) |
| 1            | 450       |
| 2            | 530       |
| 3            | 600       |
| 4            | 540       |
| 5            | 400       |
| 6            | 280       |
| 7            | 290       |
| 8            | 500       |

## Example Case 1 (Wood and Wollenberg) Cont.

- In order to make the problem more manageable only 4 of the 16 states will be considered

| State | Units |   |   |   | Max Capacity |
|-------|-------|---|---|---|--------------|
| 5     | 0     | 0 | 1 | 0 | 300          |
| 12    | 0     | 1 | 1 | 0 | 550          |
| 14    | 1     | 1 | 1 | 0 | 630          |
| 15    | 1     | 1 | 1 | 1 | 690          |

## Example Case 1 (Wood and Wollenberg) Cont.





## Example Case 1 (Wood and Wollenberg) Cont.

➤ Recursive algorithm for the FDP

$$F_{COST}(K, I) = \min [P_{COST}(K, I) + S_{COST}(K-1, L : K, I) + F_{COST}(K-1, L)]$$

$F_{COST}(K, I)$ : least total cost to arrive at state (K,I)

$P_{COST}(K, I)$ : production cost for state (K,I)

$S_{COST}(K-1, L : K, I)$ : transition cost from state (K-1,L) to (K,I)

## Example Case 1 (Wood and Wollenberg) Cont.

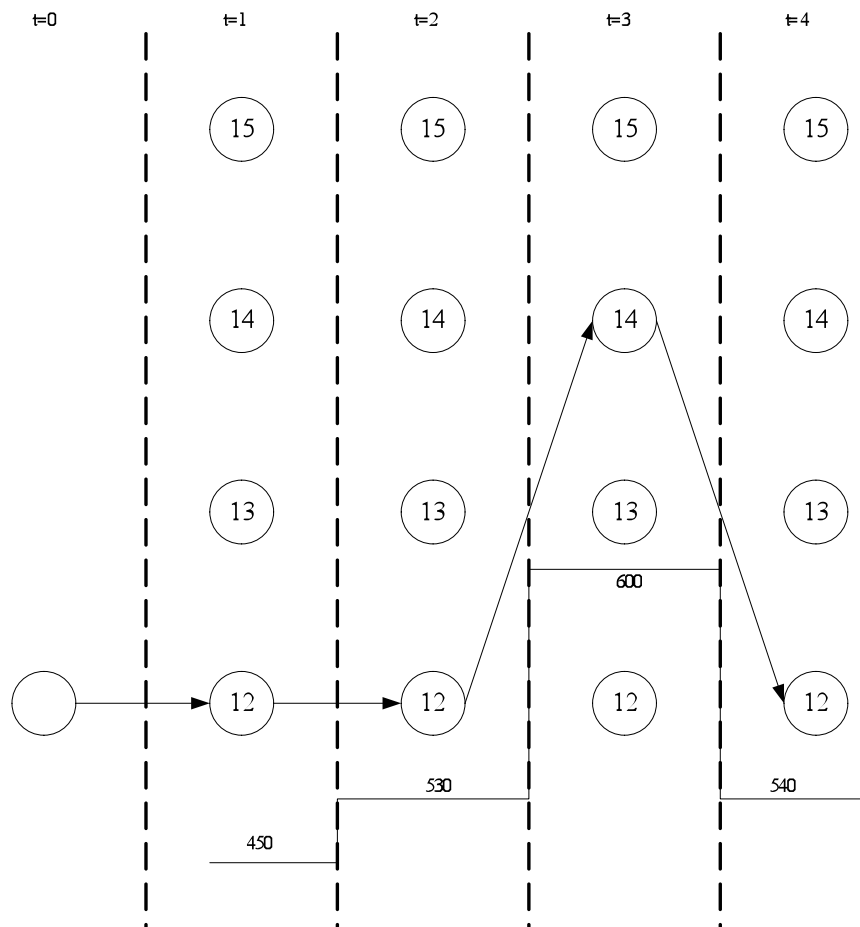
- From the initial state at  $t=0$  there are 3 possible states to transition to, state 5 is excluded for capacity reasons

$$F_{COST}(1,15) = P_{COST}(1,15) + S_{COST}(0,12:1,15) = 9861 + 350 = 10,211$$

$$F_{COST}(1,14) = P_{COST}(1,14) + S_{COST}(0,12:1,14) = 9493 + 350 = 9,843$$

$$F_{COST}(1,12) = P_{COST}(1,12) + S_{COST}(0,12:1,12) = 9208 + 0 = 9,208$$

## Example Case 1 (Wood and Wollenberg) Cont.



- For the first 4 time periods the optimal unit commitment is shown
- Minimum up and down times were not considered in this example
- A reduced set of states was used to reduce the computational requirements
- This simplification will lead to a greater than minimum operating cost

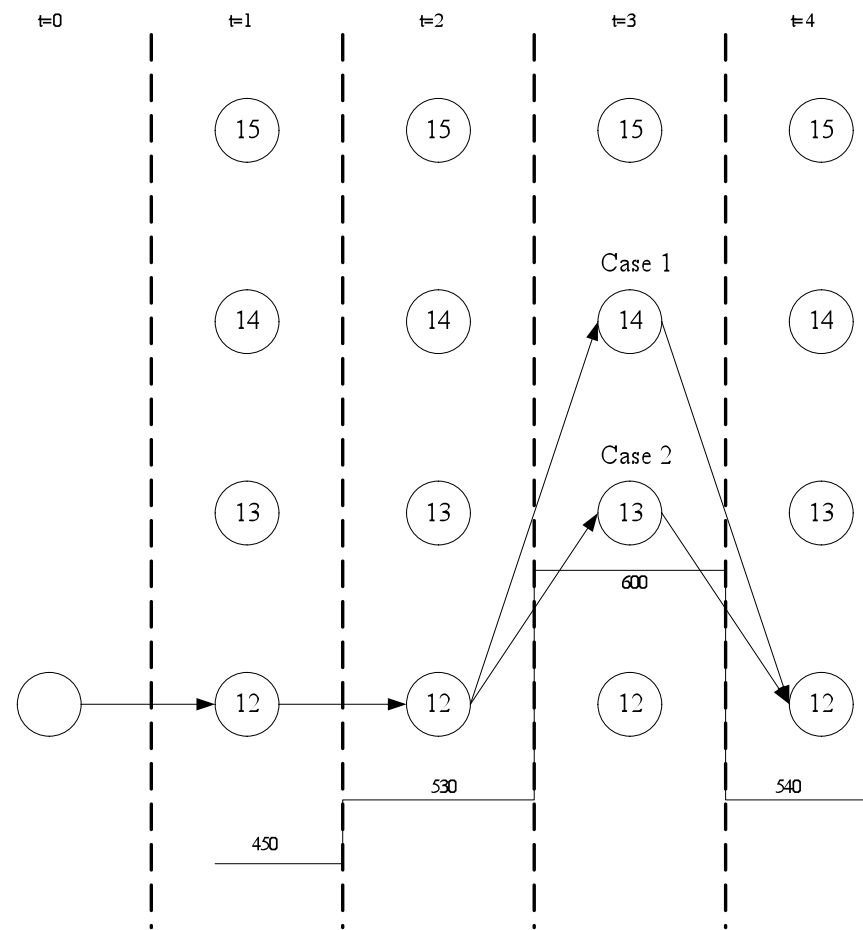
## Example Case 2 (Wood and Wollenberg)

- In the previous case the number of state was limited to 5, 12, 14, and 15
- In this example all 15 of the states will be allowed, this is the complete enumeration method
- In a practical system the complete enumeration method is not practical

## Example Case 2 (Wood and Wollenberg)

- The difference in the solution between case 1 and 2 occurs at  $t=3$
- At  $t=3$  case 1 dispatched generator 4 for 1 hour and then shut it down
- When all possible states are considered the option to start generation 1 is available
- While generator 4 is more expensive to operate than 1, it does not have an associated start up cost
- This type of unit is referred to as a “peaking” unit

## Example Case 2 (Wood and Wollenberg)



## Peaking Units

- Peaking units are generally non-thermal combustion turbines
- Peaking units are economically suited to short duration operations with relatively high ramp rates
- They are also used during high load periods when electricity is selling at higher rates
- Other rapid response generators such as hydro-plant can perform many of the functions of peaking plants

## Ramp Rates

- Generators can only change their output power at a finite rate
- For large thermal plants this is especially true
- If there is a short spike in load followed by a reduction, it may be advantageous to begin reducing the output power of low ramp rate generators before the peak
- This prevents having excessive generation when the load begins to fall



## Example Comparisons

- The total cost of operation for case 1=\$74,439
- The total cost of operation for case 2=\$73,274
- The second case yields the true optimal commitment, but at a cost of computational requirements
- In practical systems there is no guarantee that the unit commitment is the global optimum since the complete enumeration is not practical