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Instructor: Zhenyu (Henry) Huang
(509) 372-6781, zhenyu.huang@pnl.gov

EE 521: Analysis of Power Systems

Lecture 3
Power Flow

Fall 2009

Mondays & Wednesdays 5:45-7:00

August 24 – December 18

Test 216

Topics

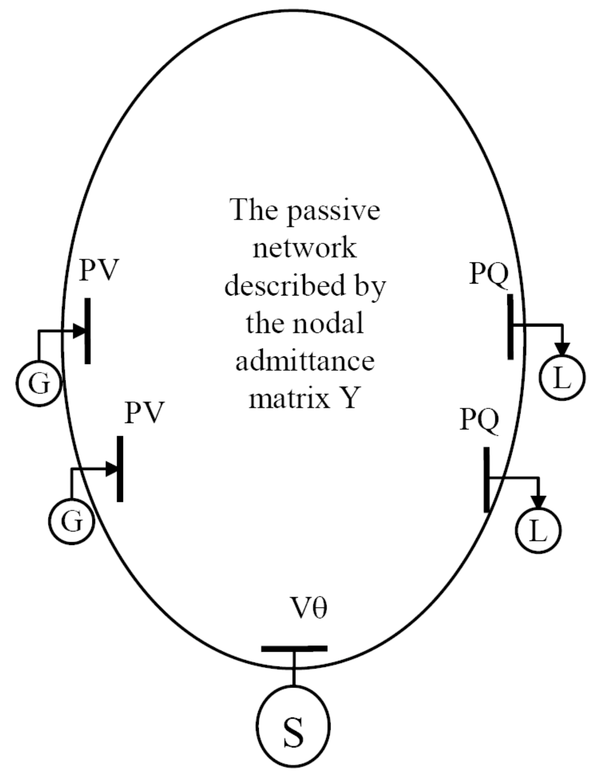
- Power Flow Basics
  - Bus Types
  - Nodal Admittance Matrix
  - Equation Derivation
- Solution Methods
  - Newton-Raphson
  - Decouple Power Flow
  - DC Power Flow

Power Flow

- One of the basic tools for examining a power system
- Used in planning studies
- Based on the concept of node injections
- A non-linear problem that generally requires an iterative solution

Power Flow Basics

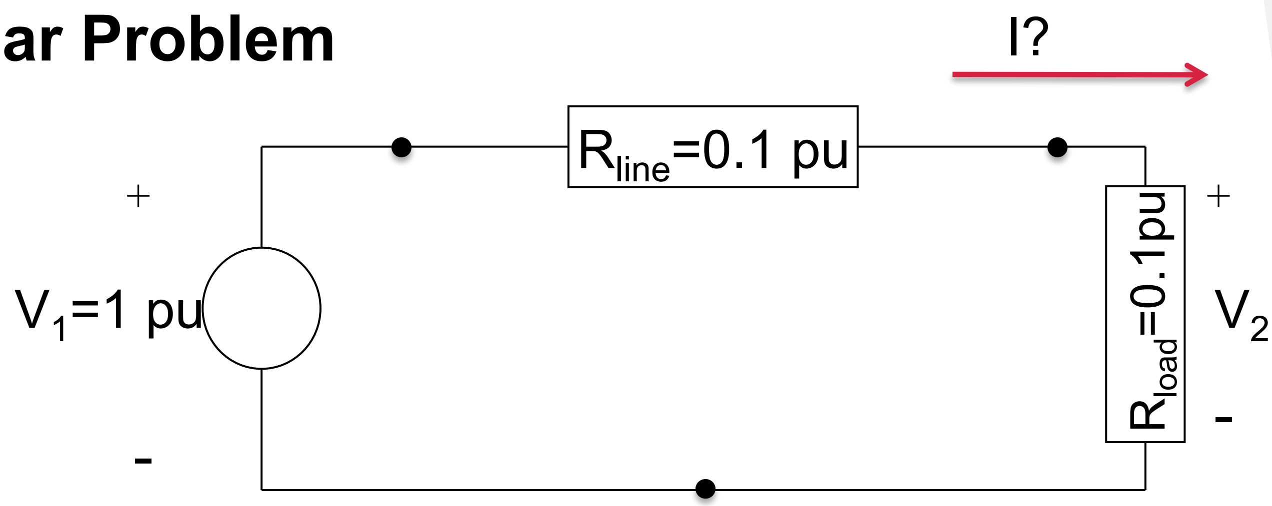
- The problem: solve electrical circuits, but at a large scale



Bus Type	Boundary Conditions				Unknowns				Total Number of Buses of the Type
	P	*Q*	*V*	*θ*	*P*	*Q*	*V*	*θ*	
PV									*r*
PQ									*N-r-1*
Vθ									*1*

A Simple Circuit

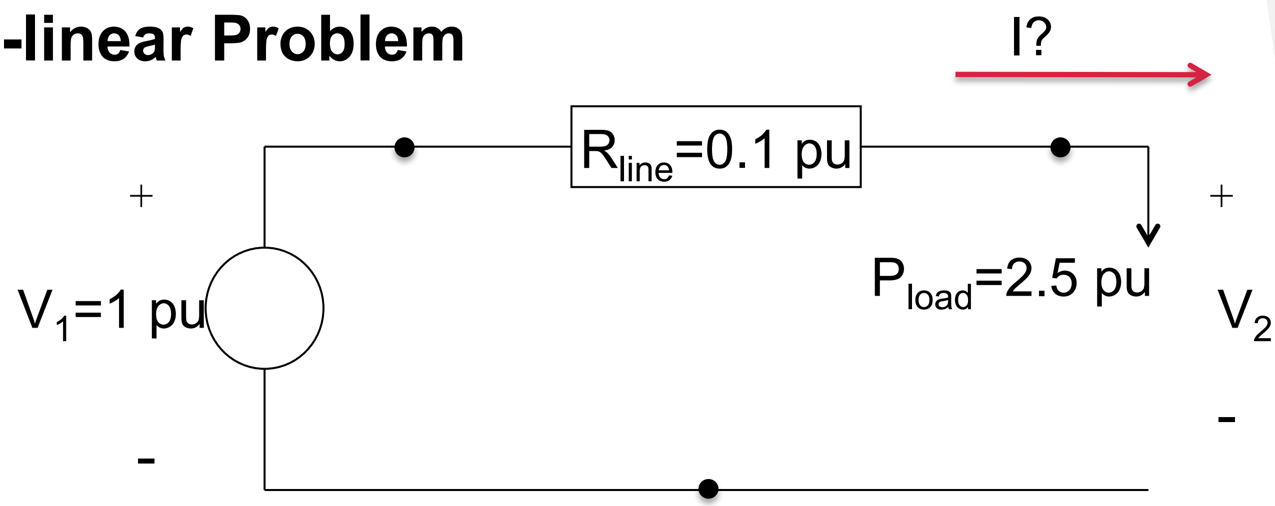
Linear Problem



$$I = YV = \frac{1}{0.1+0.1} = 5\,pu$$

Still a Simple Circuit

Non-linear Problem



$$\cancel{I = YV = \frac{1}{0.1+0.1} = 5\,pu}$$

$$\begin{cases} I = \dfrac{V_1 - V_2}{R_{line}} \\ P_{load} = V_2 I \end{cases} \qquad P_{load} = V_2 I = V_2 \frac{V_1 - V_2}{R_{line}} \qquad V_2^{\,2} - V_1 V_2 + P_{load} R_{line} = 0$$

$$V_2^{\,2} - V_2 + 0.25 = 0 \qquad V_2 = 0.5 \;\; or \;\; -0.5\,pu \qquad I = 5\,pu$$

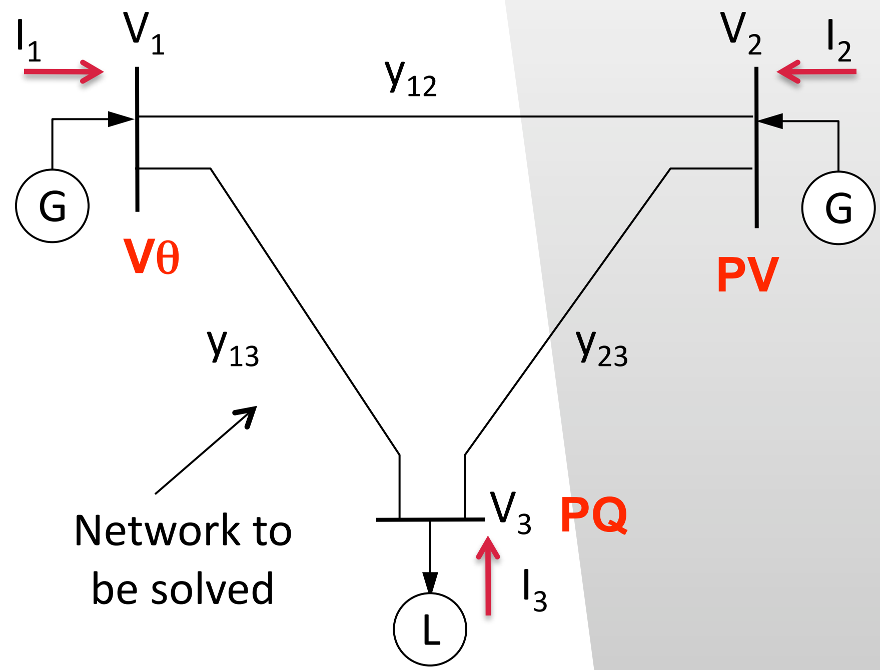
A Simple Power System

$$\boxed{I = YV}$$

$$Y = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1^* & 0 & 0 \\ 0 & V_2^* & 0 \\ 0 & 0 & V_3^* \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1^* & 0 & 0 \\ 0 & V_2^* & 0 \\ 0 & 0 & V_3^* \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$



A Simple Power System

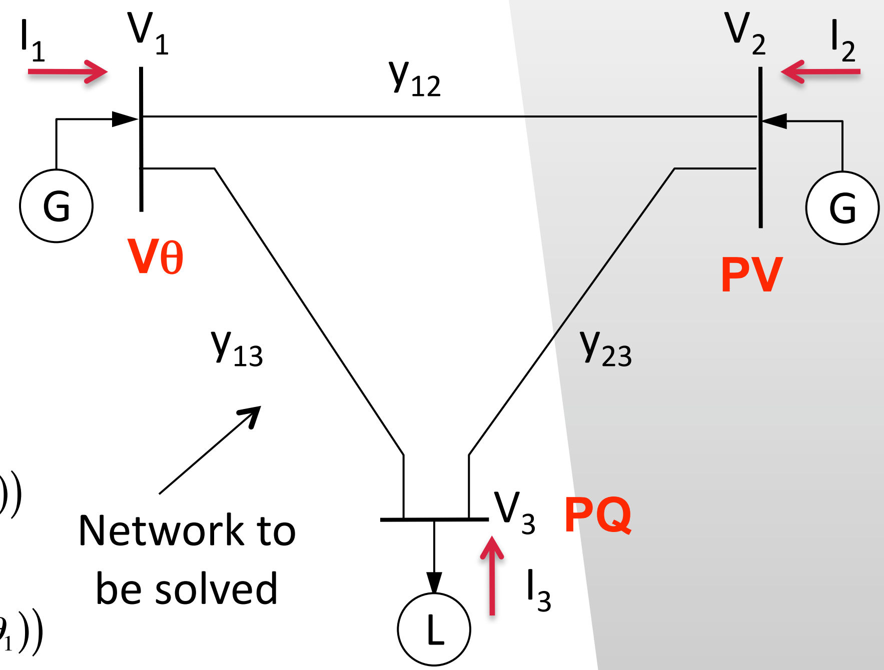
$$\boxed{E^* I = E^* Y V}$$

$$\begin{cases} P_1 - jQ_1 = |V_1|^2 Y_{11} + V_1^* V_2 Y_{12} + V_1^* V_3 Y_{13} \\ P_2 - jQ_2 = |V_2|^2 Y_{22} + V_2^* V_1 Y_{21} + V_2^* V_3 Y_{23} \\ P_3 - jQ_3 = |V_3|^2 Y_{33} + V_3^* V_1 Y_{31} + V_3^* V_2 Y_{32} \end{cases}$$

$\underline{Y = G + jB}$

$$\begin{cases} P_1 = |V_1|^2 G_{11} + |V_1||V_2|\left(G_{12}\cos(\theta_2 - \theta_1) - B_{12}\sin(\theta_2 - \theta_1)\right) \\ \qquad + |V_1||V_3|\left(G_{13}\cos(\theta_3 - \theta_1) - B_{13}\sin(\theta_3 - \theta_1)\right) \\ Q_1 = -|V_1|^2 B_{11} - |V_1||V_2|\left(G_{12}\sin(\theta_2 - \theta_1) + B_{12}\cos(\theta_2 - \theta_1)\right) \\ \qquad - |V_1||V_3|\left(G_{13}\sin(\theta_3 - \theta_1) + B_{13}\cos(\theta_3 - \theta_1)\right) \end{cases}$$

$$\begin{cases} P_1 = |V_1|\sum_{j=1}^{N}|V_j|\left(G_{1j}\cos(\theta_j - \theta_1) - B_{1j}\sin(\theta_j - \theta_1)\right) \\ Q_1 = -|V_1|\sum_{j=1}^{N}|V_j|\left(G_{1j}\sin(\theta_j - \theta_1) + B_{1j}\cos(\theta_j - \theta_1)\right) \end{cases}$$



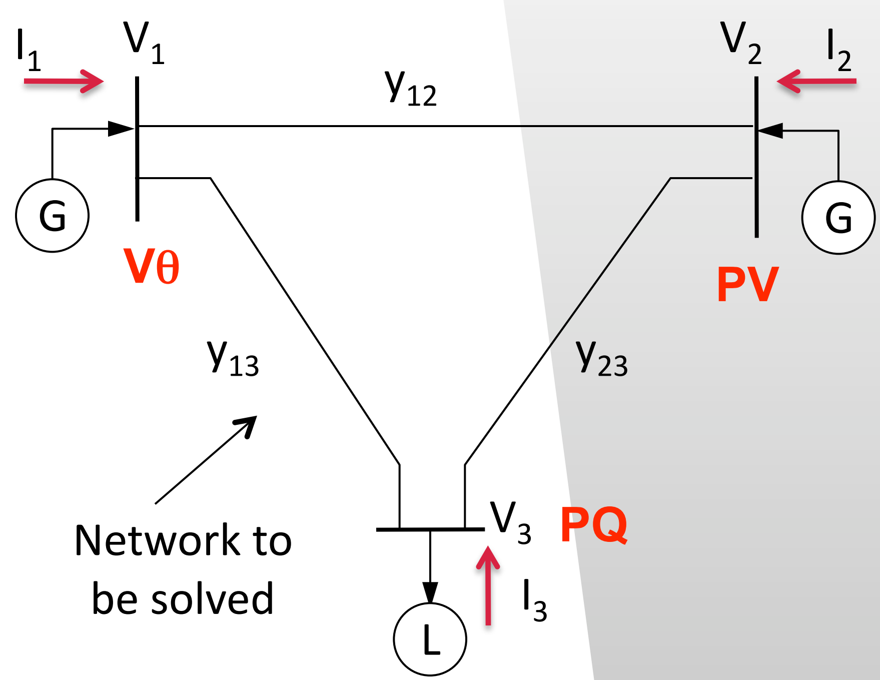
A Simple Power System

Vθ $\begin{cases} V_1 = known \\ \theta_1 = known \end{cases}$

PV $\begin{cases} P_2 = f_{P2}(|V_1|,|V_2|,|V_3|,\theta_1,\theta_2,\theta_3) \\ V_2 = known \end{cases}$

PQ $\begin{cases} P_3 = f_{P3}(|V_1|,|V_2|,|V_3|,\theta_1,\theta_2,\theta_3) \\ Q_3 = f_{Q3}(|V_1|,|V_2|,|V_3|,\theta_1,\theta_2,\theta_3) \end{cases}$

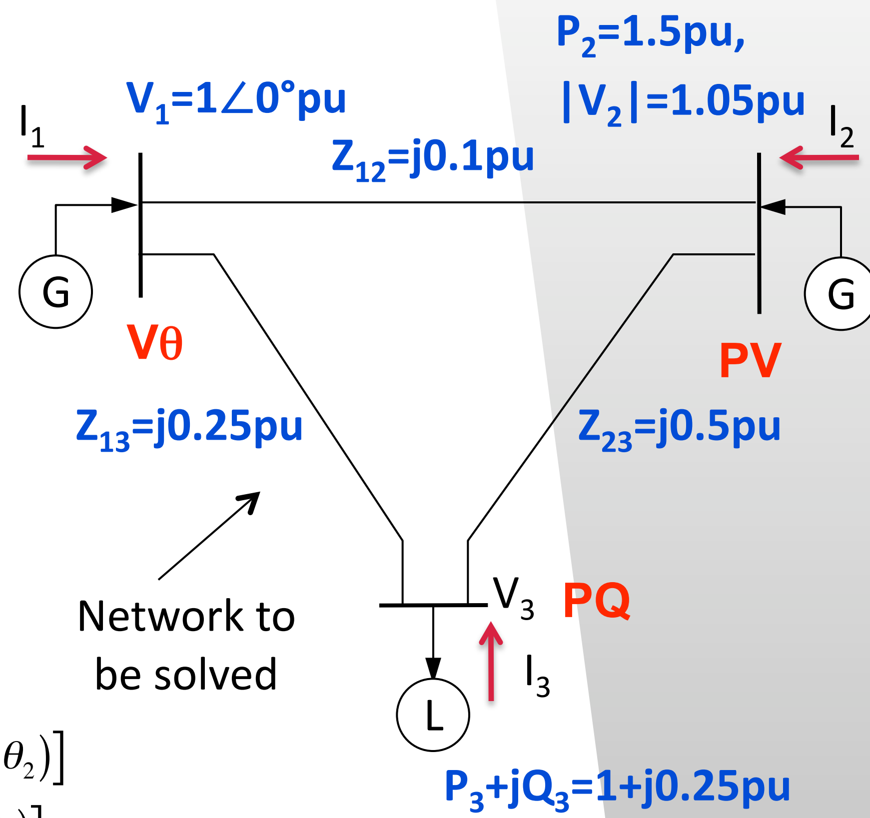
$$\begin{cases} P_2 = |V_2|\sum_{j=1}^{N}|V_j|\left(G_{2j}\cos(\theta_j-\theta_2) - B_{2j}\sin(\theta_j-\theta_2)\right) \\ P_3 = |V_3|\sum_{j=1}^{N}|V_j|\left(G_{3j}\cos(\theta_j-\theta_3) - B_{3j}\sin(\theta_j-\theta_3)\right) \\ Q_3 = -|V_3|\sum_{j=1}^{N}|V_j|\left(G_{3j}\sin(\theta_j-\theta_3) + B_{3j}\cos(\theta_j-\theta_3)\right) \end{cases}$$



A Simple Power System

$$Y = \begin{bmatrix} -j14 & j10 & j4 \\ j10 & -j12 & j2 \\ j4 & j2 & -j6 \end{bmatrix}$$

$$\begin{cases} P_2 = |V_2| \sum_{j=1}^{N} |V_j| \left(G_{2j} \cos(\theta_j - \theta_2) - B_{2j} \sin(\theta_j - \theta_2) \right) \\ P_3 = |V_3| \sum_{j=1}^{N} |V_j| \left(G_{3j} \cos(\theta_j - \theta_3) - B_{3j} \sin(\theta_j - \theta_3) \right) \\ Q_3 = -|V_3| \sum_{j=1}^{N} |V_j| \left(G_{3j} \sin(\theta_j - \theta_3) + B_{3j} \cos(\theta_j - \theta_3) \right) \end{cases}$$



$$\begin{cases} P_2 = 1.5 = 1.05 \bullet \left[-1 \bullet 10 \bullet \sin(-\theta_2) - |V_3| \bullet 2 \bullet \sin(\theta_3 - \theta_2) \right] \\ P_3 = -1 = |V_3| \bullet \left[-1 \bullet 4 \bullet \sin(-\theta_3) - 1.05 \bullet 2 \bullet \sin(\theta_2 - \theta_3) \right] \\ Q_3 = -0.25 = -|V_3| \bullet \left[1 \bullet 4 \bullet \cos(-\theta_3) + 1.05 \bullet 2 \bullet \cos(\theta_2 - \theta_3) - |V_3| \bullet 6 \right] \end{cases}$$

Power Flow Solution Methods

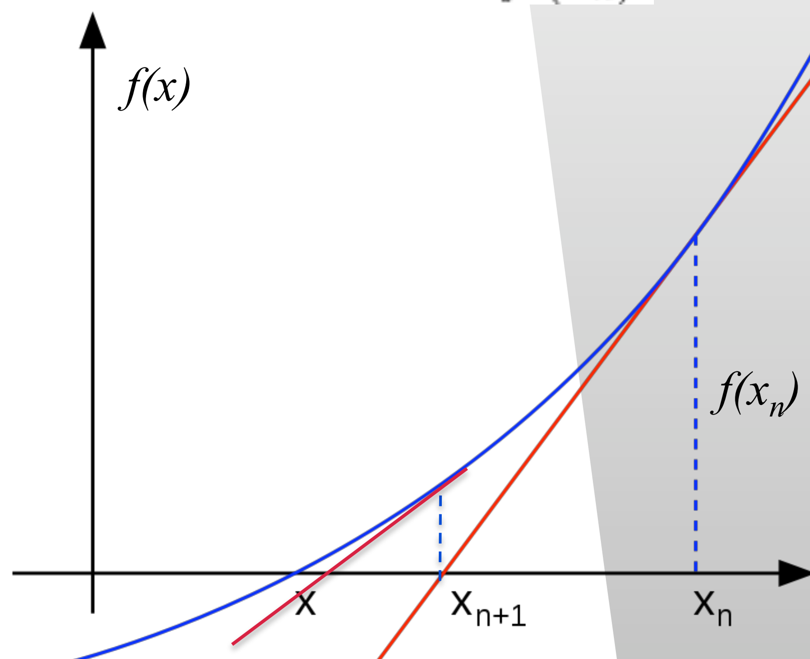
- Gauss-Seidel
- Newton-Raphson
- Decoupled
- DC

Newton-Raphson Method

- The basic principle
  - Guess the solution
  - Try and evaluate how far away from the true solution
  - If too far away, adjust your guess and try again
- The key is how to adjust the guess
  - Slope (Derivative)

$$f(x) = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$



Newton-Raphson Method for Power Flow Solution

- Jacobian Matrix

$$x^{n+1} = x^n - J\left(x^n\right)^{-1} f\left(x^n\right)$$

- Newton-Raphson Power Flow Procedure
  - First guess
    - "flat start" or a known point
    - The first guess is important
  - Evaluate power equations f(x)
  - Evaluate Jacobian matrix J(x)
  - Adjust guess point x

$$f(x) = P_i(x) - P_i = 0$$

$$J(x) = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

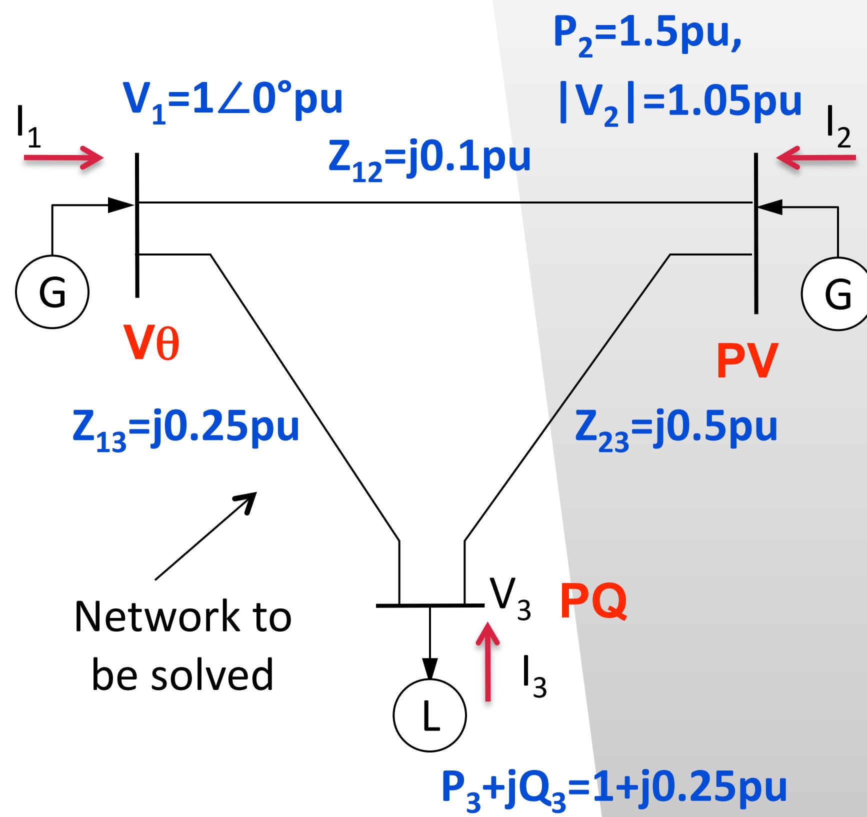
$$\left[J(x)\right]\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \theta^{n+1} \\ V^{n+1} \end{bmatrix} = \begin{bmatrix} \theta^{n} \\ V^{n} \end{bmatrix} - \left[J\left(x^n\right)\right]^{-1} \begin{bmatrix} \Delta P\left(x^n\right) \\ \Delta Q\left(x^n\right) \end{bmatrix}}$$

Newton-Raphson Power Flow Example

$$\begin{cases} P_2 = 1.5 \\ \quad = 1.05 \bullet \left[1 \bullet 10 \bullet \sin(-\theta_2) - |V_3| \bullet 2 \bullet \sin(\theta_3 - \theta_2)\right] \\ P_3 = 1 \\ \quad = |V_3| \bullet \left[1 \bullet 4 \bullet \sin(-\theta_3) - 1.05 \bullet 2 \bullet \sin(\theta_2 - \theta_3)\right] \\ Q_3 = 0.25 \\ \quad = -|V_3| \bullet \left[1 \bullet 4 \bullet \cos(-\theta_3) + 1.05 \bullet 2 \bullet \cos(\theta_2 - \theta_3) - |V_3| \bullet 6\right] \end{cases}$$

$$\begin{bmatrix} \theta^{n+1}{}_2 \\ \theta^{n+1}{}_3 \\ V^{n+1}{}_3 \end{bmatrix} = \begin{bmatrix} \theta^n{}_2 \\ \theta^n{}_3 \\ V^n{}_3 \end{bmatrix} - \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}^{-1} \begin{bmatrix} P_2(x_n) - 1.5 \\ P_3(x_n) - 1 \\ Q_3(x_n) - 0.25 \end{bmatrix}$$



Newton-Raphson Power Flow Example

t

	Initial	1	2	3	4
Theta 2	0	5.546556	5.593182	5.593524	5.593524
Theta 3	0	-7.43451	-7.88595	-7.89849	-7.89849
Voltage 3	1	0.9745	0.95735	0.956977	0.956977

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Decoupled Power Flow

- The objective
  - Reduce computation in evaluating Jacobian matrix
- The basic principle
  - Δθ primarily affects P
  - ΔV primarily affects Q

$$J(x)=\begin{bmatrix}\frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V}\\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V}\end{bmatrix}\approx\begin{bmatrix}\frac{\partial P}{\partial \theta} & 0\\ 0 & \frac{\partial Q}{\partial V}\end{bmatrix}$$

$$\begin{bmatrix}\theta^{n+1}\\ V^{n+1}\end{bmatrix}=\begin{bmatrix}\theta^{n}\\ V^{n}\end{bmatrix}-\left[J\left(x^n\right)\right]^{-1}\begin{bmatrix}\Delta P\left(x^n\right)\\ \Delta Q\left(x^n\right)\end{bmatrix}$$

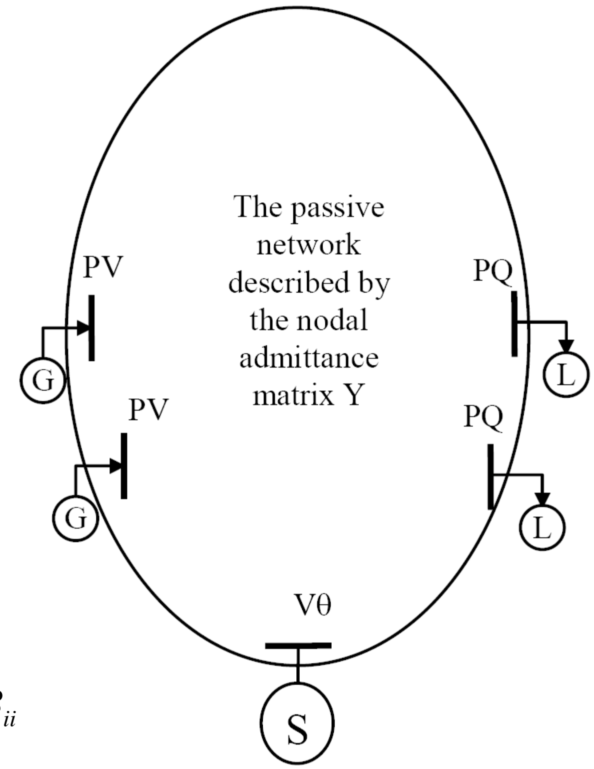
$$\begin{cases}\left[\theta^{n+1}\right]=\left[\theta^{n}\right]-\left[\frac{\partial P}{\partial \theta}\right]^{-1}\left[\Delta P\left(\theta^n\right)\right]\\ \left[V^{n+1}\right]=\left[V^{n}\right]-\left[\frac{\partial Q}{\partial V}\right]^{-1}\left[\Delta Q\left(V^n\right)\right]\end{cases}$$

Re-examination of Power Flow Equations

$$\boxed{E^{*}I = E^{*}YV}$$

$$\begin{cases} P_i = |V_i|\sum_{j=1}^{N}|V_j|\left(G_{ij}\cos\left(\theta_j-\theta_i\right)-B_{ij}\sin\left(\theta_j-\theta_i\right)\right) \\ Q_i = -|V_i|\sum_{j=1}^{N}|V_j|\left(G_{ij}\sin\left(\theta_j-\theta_i\right)+B_{ij}\cos\left(\theta_j-\theta_i\right)\right) \end{cases}$$

$$\begin{cases} \dfrac{\partial P_i}{\partial \theta_j} = -|V_i||V_j|\left(G_{ij}\sin\left(\theta_j-\theta_i\right)+B_{ij}\cos\left(\theta_j-\theta_i\right)\right) \\ |V_j|\dfrac{\partial Q_i}{\partial |V_j|} = -|V_i||V_j|\left(G_{ij}\sin\left(\theta_j-\theta_i\right)+B_{ij}\cos\left(\theta_j-\theta_i\right)\right) \\ \dfrac{\partial P_i}{\partial \theta_i} = |V_i|\sum_{\substack{j=1 \\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin\left(\theta_j-\theta_i\right)+B_{ij}\cos\left(\theta_j-\theta_i\right)\right) = -Q_i-|V_i|^2 B_{ii} \\ |V_i|\dfrac{\partial Q_i}{\partial |V_i|} = -|V_i|\sum_{\substack{j=1 \\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin\left(\theta_j-\theta_i\right)+B_{ij}\cos\left(\theta_j-\theta_i\right)\right)-2|V_i|^2 B_{ii} = Q_i-|V_i|^2 B_{ii} \end{cases}$$



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Re-examination of Power Flow Equations

$$\begin{cases} \dfrac{\partial P_i}{\partial \theta_j} = -|V_i||V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \\ |V_j|\dfrac{\partial Q_i}{\partial |V_j|} = -|V_i||V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) \\ \dfrac{\partial P_i}{\partial \theta_i} = |V_i|\sum_{\substack{j=1\\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) = -Q_i-|V_i|^2 B_{ii} \\ |V_i|\dfrac{\partial Q_i}{\partial |V_i|} = -|V_i|\sum_{\substack{j=1\\ j\neq i}}^{N}|V_j|\left(G_{ij}\sin(\theta_j-\theta_i)+B_{ij}\cos(\theta_j-\theta_i)\right) - 2|V_i|^2 B_{ii} = Q_i-|V_i|^2 B_{ii} \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{\partial P_i}{\partial \theta_j} = -|V_i||V_j|B_{ij} \\ |V_j|\dfrac{\partial Q_i}{\partial |V_j|} = -|V_i||V_j|B_{ij} \\ \dfrac{\partial P_i}{\partial \theta_i} = -|V_i|^2 B_{ii} \\ |V_i|\dfrac{\partial Q_i}{\partial |V_i|} = -|V_i|^2 B_{ii} \end{cases}$$

$$\left.\begin{array}{l} \cos(\theta_j-\theta_i)\approx 1 \\ \sin(\theta_j-\theta_i)\approx \theta_j-\theta_i \\ B_{ij} >> G_{ij} \end{array}\right\} \Rightarrow G_{ij}\sin(\theta_j-\theta_i) << B_{ij}\cos(\theta_j-\theta_i)$$

$$Q_i << |V_i|^2 B_{ii}$$

Further Decoupled Power Flow

$$\begin{cases} \left[\theta^{n+1}\right] = \left[\theta^{n}\right] - \left[\dfrac{\partial P}{\partial \theta}\right]^{-1} \left[\Delta P\left(\theta^{n}\right)\right] \\ \left[V^{n+1}\right] = \left[V^{n}\right] - \left[\dfrac{\partial Q}{\partial V}\right]^{-1} \left[\Delta Q\left(V^{n}\right)\right] \end{cases}$$

?

$$\begin{cases} \left[\theta^{n+1}\right] = \left[\theta^{n}\right] - \left[-B\right]^{-1} \left[\dfrac{\Delta P\left(\theta^{n}\right)}{|V|}\right] \\ \left[V^{n+1}\right] = \left[V^{n}\right] - \left[-B\right]^{-1} \left[\dfrac{\Delta Q\left(V^{n}\right)}{|V|}\right] \end{cases}$$

Decoupled Power Flow Procedure

- Make a guess
- Calculate ΔP/V
- Update θ
- Use new θ to calculate ΔQ/V
- Update V
- Repeat until ΔP & ΔQ within tolerance
- Assignment (due: Sept 9):
  - 1. Derive the decoupled power flow equations
  - 2. Solve the 3-bus example using the decoupled power flow procedure

$$\begin{cases} \left[\theta^{n+1}\right] = \left[\theta^{n}\right] - \left[-B\right]^{-1}\left[\dfrac{\Delta P\left(\theta^{n}\right)}{|V|}\right] \\ \left[V^{n+1}\right] = \left[V^{n}\right] - \left[-B\right]^{-1}\left[\dfrac{\Delta Q\left(V^{n}\right)}{|V|}\right] \end{cases}$$

DC Power Flow

- Assumptions
  - All transfer conductance values are set to zero i.e. $G_{ij}=0$
  - Small angle assumption
  - All voltages are set to 1 pu
- Properties
  - Not an accurate power flow solution – only calculate real power flow
  - Becomes a linear problem
  - Used for quick system assessment

$$\cos(\theta_j - \theta_i) \approx 1$$

$$\sin(\theta_j - \theta_i) \approx \theta_j - \theta_i$$

$$\begin{cases} P_i = |V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} \cos(\theta_j - \theta_i) - B_{ij} \sin(\theta_j - \theta_i) \right) \\ Q_i = -|V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} \sin(\theta_j - \theta_i) + B_{ij} \cos(\theta_j - \theta_i) \right) \end{cases}$$

$$\boxed{[-B][\theta] = [P]}$$

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Questions?

Gauss-Seidel

- First effective method to be implemented on digital computers
- In general has a linear convergence rate
- Can generally accept a “flat start”
- Has some issues with reactance reactance branches
- Each bus is treated individually

Newton-Raphson

- Based on the concept of driving an error function to zero
- Generally has a quadratic convergence rate
- Some issues exist with "flat start" values
- Each iteration requires more computations than for a Gauss-Seidel, but quadratic convergence results in fewer iterations
- Very robust, but the values of the Jacobian must be updated at each iteration

Decoupled Power Flow

- Exploits the decoupled nature of voltage and angle, thus reducing the number of non-zero entries in the Jacobian
- Assumptions are made that greatly improve the convergence time
- When the assumptions are not true, convergence issues may arise

DC Power Flow

- Linear/non-iterative solution to the power flow problem
- Will only calculate the MW flows in the system, not MVAR
- Effective for situations requiring an approximate solution, e.g. contingency analysis