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Optimal Power Flow (OPF)

- The OPF optimizes a given objective function while ensuring that the power flow equations are satisfied
- The controllable values of power system are adjusted in order to minimize the objective function
- Controllable values include:
  - Generator voltage
  - Generator power P&Q
  - Phase shifters
  - HVDC & FACTS

Optimal Power Flow (OPF) cont.

- One solution method for the economic dispatch problem is the OPF method, where the total cost is minimized subject to the constraints of the power flow equations

- In this instance the controllable system values are the real power injections of the generators

- This method will account for the incremental system losses since the losses are accounted for if the power flow equations are satisfied

Optimal Power Flow (OPF) cont.

- In addition to minimizing the system losses/minimizing cost the OPF can be solved with many other objective functions

- Reactive Power
- Voltage
- Line Flow AC&DC
- Control Actions

- When coupled with contingency analysis the normal OPF becomes a security constrained OPF

Optimal Power Flow (OPF) cont.

- Lambda iteration method
- Gradient method
- Newton’s method
- Linear programming method
- Interior point method

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OPF Gradient and Newton Methods

- We will examine both the Gradient and Newton OPF methods
- In both instances the total cost of operation will be used as the objective function, constrained by the power flow equations
- In both cases the calculations are extensive and only increase as the size of the system is increased
- For a full size power system sparse matrix techniques would be implemented

Optimal Power Flow (OPF) cont.

$$L(x,u,p) = f(x,u) + \lambda^T g(x,u,p)$$

- x= vector of state variables
- u= vector of control variables
- p= vector of fixed parameters
- λ= vector of Lagrange multipliers
- s= set of equality constraints
- f= the objective function

Optimal Power Flow (OPF) cont.

- The output of the OPF at each iteration is the updated values for the control parameters

- In this way the total cost of operation is minimized by adjusting the system controls

- While minimizing the total cost the system, constraints ensure the load plus losses are supplied by the generators

OPF-Gradient Method

- The gradient method is based on the well established principles of following the gradient toward the minimum

- This method will effectively handle equality constraints, but inequality constrains are much more difficult to account for

- This method only requires the calculation of first order partial differential equations, in the form of a Jacobian matrix

OPF-Gradient Method Cont.

- For this example the objective function will be the total cost function that was examined in the economic dispatch problem

$$\sum_{i=1}^{n} F_i(P_i) = \sum_{i=1}^{n} H_i(P_i) \times FC_i$$

- The objective function does not have to be related to total cost, instead it could be related to minimizing transmission line congestion or ensure that all voltages are above a certain level

OPF-Gradient Method Cont.

$$x = \begin{bmatrix} \theta_i \\ |V_i| \\ \theta_i \end{bmatrix} \begin{array}{l} \text{On each PQ bus} \\ \text{On each PQ bus} \\ \text{On each PV bus} \end{array}$$

- Unlike the economic dispatch the unknown state variables are not the generator outputs
- Similar to state estimation the generator powers will form the equations for obtaining the state variables
- The state variables are the same as for the power flow problem

OPF-Gradient Method Cont.

- The next vector to be defined is the y vector
- The y vector is made of up all the parameters that must be specified
- Some of these parameters are fixed but some are adjustable
- The y vector is separated into 2 parts, u and p

$$y = \begin{bmatrix} \theta_i \\ |V_i| \\ P_i \\ Q_i \\ P_i \\ |V_i|^{sch} \end{bmatrix} \begin{array}{l} \text{On the swing bus} \\ \text{On the swing bus} \\ \text{On each PQ bus} \\ \text{On each PQ bus} \\ \text{On each PV bus} \\ \text{On each PV bus} \end{array}$$

$$y = \begin{bmatrix} u \\ p \end{bmatrix} \begin{array}{l} \text{Controllable values} \\ \text{Non-controllable values} \end{array}$$

OPF-Gradient Method Cont.

$$g(x,y)=\begin{bmatrix} P_i(x)-P_i \\ Q_i(x)-Q_i \\ P_i(x)-P_i \end{bmatrix} \begin{matrix} \text{On each PQ bus} \\ \text{On each PQ bus} \\ \text{On each PV bus} \end{matrix}$$

- In the Lagrangian the objective function is minimized subject to a set of constraints
- In the OPF the constraints are the power flow equations
- It is these constraints that ensure that the minimized objective function includes the system losses

Optimal Power Flow (OPF) cont.

- Prior to the solution of the power flow problem it is not possible to know the output power of the swing bus generator

- For this reason the output power of the swing bus generator cannot be considered an independent

- For this reason the cost function is separated, just as the y vector was, into controllable and uncontrollable values

$$COST = C_{ref}\left(P_{ref}\right) + \sum_{i=1}^{n} C_i\left(P_i\right)$$

OPF-Gradient Method Cont.

- Using the previously defined matrices the Lagrangian becomes:

$$L(x,u,p) = C_{ref}\left(P_{ref}\right) + \sum_{i=1}^{n}\left(C_i\left(P_i\right)\right) + \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_m \end{bmatrix} \begin{bmatrix} P_i(x) - P_i \\ Q_i(x) - Q_i \\ P_i(x) - P_i \\ \vdots \end{bmatrix}$$

- The Lagrangian is used to solve:

$$u^1 = u^0 - \alpha \nabla L_u$$

OPF-Gradient Method Cont.

- As with the previous examples the optimum solution is found when the gradient of the Lagrangian is zero

$$\nabla L = 0$$

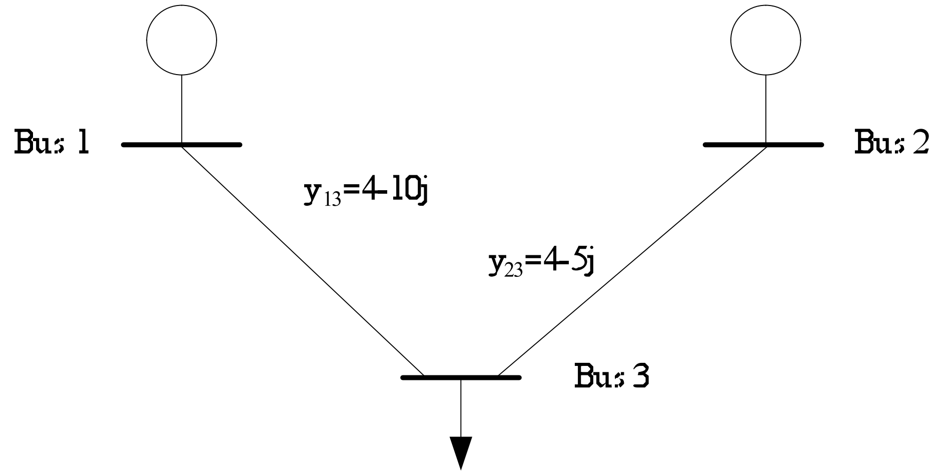
$$\nabla L_x = \frac{\partial L}{\partial x} = \frac{\partial f}{\partial x} + \left[\frac{\partial g}{\partial x}\right]^T \lambda$$

$$\nabla L_u = \frac{\partial L}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u}\right]^T \lambda$$

$$\nabla L_\lambda = \frac{\partial L}{\partial \lambda} = g(x,u,p)$$

Example 13B (Wood & Wollenberg)

- Given the system shown below, with fixed real power injections, determine the voltage regulator setting that minimize the real power losses



Example 13B (Wood & Wollenberg) Cont.

- Since the generator at bus 2 is fixed, the only remaining control variables will be the voltage magnitudes at buses 1 and 2

$$u = \begin{bmatrix} |E_1| \\ |E_2| \end{bmatrix}$$

- The state variables are the same as they would be in the traditional power flow

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |E_3| \end{bmatrix}$$

- The remaining variables form the vector of fixed, non-controllable, parameters

$$p = \begin{bmatrix} \theta_1 \\ P_2 \\ P_3 \\ Q_3 \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- The constraints are the power flow equations

- The constraints are differentiated with respect to the state variables and the control variables

$$g(x,u)=\begin{bmatrix} P_2(x)-P_2 \\ P_3(x)-P_3 \\ Q_3(x)-Q_3 \end{bmatrix} \qquad \frac{\partial g}{\partial x}=\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial |E_3|} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial |E_3|} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial |E_3|} \end{bmatrix} \qquad \frac{\partial g}{\partial u}=\begin{bmatrix} \frac{\partial P_2}{\partial |E_1|} & \frac{\partial P_2}{\partial |E_2|} \\ \frac{\partial P_3}{\partial |E_1|} & \frac{\partial P_3}{\partial |E_2|} \\ \frac{\partial Q_3}{\partial |E_1|} & \frac{\partial Q_3}{\partial |E_2|} \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- In this case there is more than 1 possible way to express the objective function
- The first method is express the losses as a summation of the individual line losses
- The second method recognizes that the generation at node 2 and 3 is fixed, thus any changes in losses is directly reflected in the generation at bus 1

Example 13B (Wood & Wollenberg) Cont.

$$f = P_1$$

$$\frac{\partial P_{losses}}{\partial x} = \frac{\partial P_1}{\partial x}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \dfrac{\partial P_1}{\partial \theta_2} \\ \dfrac{\partial P_1}{\partial \theta_3} \\ \dfrac{\partial P_1}{\partial |E_3|} \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- Initially a set of values for the control variables is selected and a power flow run

- At each subsequent iteration of the OPF a power flow solution is necessary

- This contributes to the computational complexity of the OPF problem

Example 13B (Wood & Wollenberg) Cont.

- From the initial power flow, for a given real power dispatch, the initial control vector and Jacobian are obtained:

$$\begin{bmatrix} |E_1| \\ |E_2| \end{bmatrix}^0 = \begin{bmatrix} 1.1 \\ .9 \end{bmatrix} \qquad \frac{\partial g}{\partial x} = \begin{bmatrix} 8.14 & 8.14 & 1.54 \\ 6.96 & 12.0 & 3.85 \\ -4.5 & -7.85 & 10.0 \end{bmatrix}$$

- Once the state variables are known it is possible to determine the partial derivative of the objective function with respect to the state variables

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 \\ 4.36 \\ 4.14 \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- With these known values we return to the gradient of the Lagrangian in the direction of the state variables

$$\nabla L_x = \frac{\partial L}{\partial x} = \frac{\partial f}{\partial x} + \left[\frac{\partial g}{\partial x}\right]^T \lambda$$

- Rearranging the terms in order to solve for the vector of λ values:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = -\left[\frac{\partial g}{\partial x}\right]^{T^{-1}} \frac{\partial f}{\partial x} = \begin{bmatrix} .743 \\ -.98 \\ -.154 \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- Solve for the partial differential of the objective function with respect to the control variables:

$$\frac{\partial f}{\partial u} = \begin{bmatrix} \dfrac{\partial P_1}{\partial V_1} \\ \dfrac{\partial P_1}{\partial V_2} \end{bmatrix} = \begin{bmatrix} 5.533 \\ 0 \end{bmatrix}$$

- Solve for the partial differential of the equality constraints with respect to the control variables:

$$\left[\frac{\partial g}{\partial u}\right]^T = \begin{bmatrix} 0 & 3.354 & 5.0 \\ 4.94 & 4.5 & 6.96 \end{bmatrix}$$

Example 13B (Wood & Wollenberg) Cont.

- With these known values we return to the gradient of the Lagrangian in the direction of the control variables

$$\nabla L_u = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u}\right]^T \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 2.25 \\ -1.78 \end{bmatrix}$$

- Using the values that have been calculated, the updated control values can be determined

$$\begin{bmatrix} |E_1| \\ |E_2| \end{bmatrix}^1 = \begin{bmatrix} |E_1| \\ |E_2| \end{bmatrix}^0 - \alpha \nabla L_u \quad \rightarrow \quad \begin{bmatrix} |E_1| \\ |E_2| \end{bmatrix}^1 = \begin{bmatrix} 1.1 \\ .9 \end{bmatrix} - .3 \begin{bmatrix} 2.25 \\ -1.78 \end{bmatrix} = \begin{bmatrix} .95 \\ 1.03 \end{bmatrix}$$

OPF-Newton Method

- The Newton method is based on the well established principles displayed in the NR power flow algorithm

- This method will effectively handle equality constraints, but inequality constrains are much more difficult to account for

- This method requires the calculation of second order partial differential equations, in the form of a Hessian matrix

OPF-Newton Method Cont.

$$\begin{bmatrix} x \\ u \\ \lambda \end{bmatrix}^1 = \begin{bmatrix} x \\ u \\ \lambda \end{bmatrix}^0 - \begin{bmatrix} \frac{\partial}{\partial x}\nabla L_x & \frac{\partial}{\partial u}\nabla L_x & \frac{\partial}{\partial \lambda}\nabla L_x \\ \frac{\partial}{\partial x}\nabla L_u & \frac{\partial}{\partial u}\nabla L_u & \frac{\partial}{\partial \lambda}\nabla L_u \\ \frac{\partial}{\partial x}\nabla L_\lambda & \frac{\partial}{\partial u}\nabla L_\lambda & \frac{\partial}{\partial \lambda}\nabla L_\lambda \end{bmatrix}^{-1} \begin{bmatrix} \nabla L_x \\ \nabla L_u \\ \nabla L_\lambda \end{bmatrix}$$

OPF-Newton Method Cont.

- The Hessian matrix:

$$
\begin{bmatrix}
\dfrac{\partial^2 f(x)}{\partial x_1^2} & \dfrac{\partial^2 f(x)}{\partial x_1 \partial x_2} & \cdots & \cdots \\
\dfrac{\partial^2 f(x)}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f(x)}{\partial x_2^2} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial^2 f(x)}{\partial x_n \partial x_1} & \dfrac{\partial^2 f(x)}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2 f(x)}{\partial x_n^2}
\end{bmatrix}
$$

Inequality Constraints

- Without inequality constraints the solution of the OPF could potentially be invalid

- Example constraints

  - Generator voltage setting
  - Real output power of a generator
  - Reactive output power of a generator
  - Inter-area power flows

Inequality Constraints Cont.

- A common method for ensuring that inequality constraints are meet is to apply quadratic penalty functions

- These function significantly increase the value of the objective function if the inequality constraints are not meet

- These penalty functions can be included in the existing cost functions in order to prevent another iterative loop