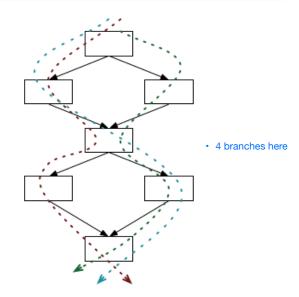
## STRUCTURAL TESTING & TEST ADEQUACY - PART 2

How good are your tests?

Path-based criteria, cyclomatic complexity, data-flow testing

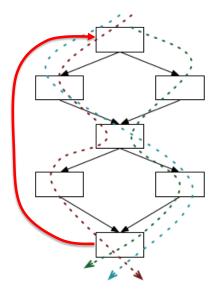
#### Paths? (beyond individual branches)

- Should we explore sequences of branches (paths) in the control flow?
- Many more paths than branches
  - Must combine all possible outcomes of all decisions
  - A pragmatic compromise will be needed



#### Paths? (beyond individual branches)

- Should we explore sequences of branches (paths) in the control flow?
- If program has loops => too many paths

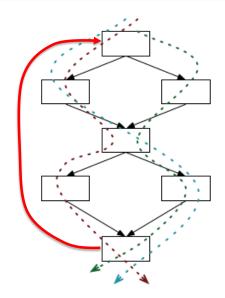


#### Optimized versions of path coverage exist

## Example: loop boundary adequacy

Each loop is executed

- 0,
- 1, and
- multiple times



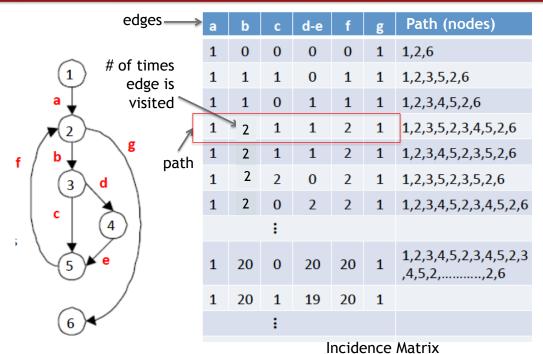
# Cyclomatic Complexity and Cyclomatic Adequacy (McCabe's Basis Path Coverage)

A compromise between full path coverage and simple branch coverage

# cyclomatic complexity is a common measure of code complexity related to the vector representation of paths

higher ←→ program's logical (branching) structure is more complex

## To understand cyclomatic complexity fully, we need to represent paths through the CFG as vectors



#### Mathematically: cyclomatic complexity is the max number of linearly independent paths that we can have in a CFG

- P<sub>1</sub>, ..., P<sub>n</sub>: paths represented as vectors
- $P_1,..., P_n$  are *linearly independent* if none of the  $P_i$  can be expressed as a linear combination of the others
  - If  $P_1$  (or any other path) can be expressed as a linear combination of the others, e.g.,

$$P_1 = a_2 P_2 + ... + a_n P_n$$

... then  $P_1,..., P_n$  are linearly dependent

#### Determining cyclomatic complexity, V(G), is easy!

- Consider a directed graph G with
- e = # of edges, n = # of nodes,
   c = # of connected regions of a graph, then...

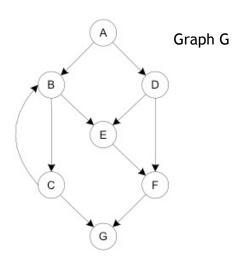


- V(G) = e n + 2c for an arbitrary graph
- It can be proven that: V(G) = e n + 2 if G is connected (c = 1 for CFG of a contiguous piece of code)

A graph theoretic result!

### Cyclomatic complexity of a CFG

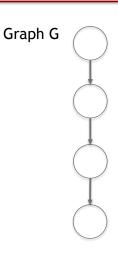




Cyclomatic complexity = V(G) = e - n + 2

#### Cyclomatic complexity of a CFG



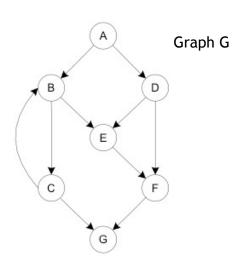


Minimum cyc. complexity: 1
Corresponds to a program fragment with no control structures

Cyclomatic complexity = V(G) = 4 - 3 + 2 = 1

#### Cyclomatic complexity of a CFG

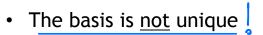


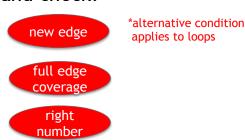


Cyclomatic complexity = V(G) = e - n + 2 = 10 - 7 + 2 = 5

#### Linear independence = Basis paths

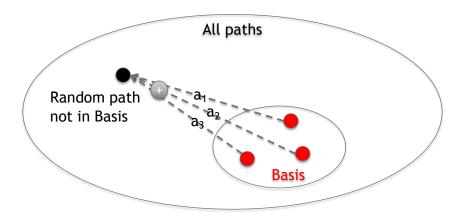
- Any set of linearly independent paths of size V(G) is called a basis of G
- The basis covers all paths in G, it does not just cover G
- Each path in the basis is called a basis path
- How to determine whether a set of paths forms a basis?
   Order basis paths from shortest to longest, and check:
  - each basis path introduces at least one new edge that is not already included in any other basis path\*
  - every edge of G is included in a basis path
  - there are V(G) paths in the set





## The basis covers "all paths" in G, <u>not</u> just G, in a special mathematical sense

## Any path through the CFG can be represented as a linear combination of the paths in the basis



#### Cyclomatic adequacy

- Do the executed paths include a basis?
- In other terms: Is a basis covered?
   YES => full cyclomatic coverage

Attractive because it answers the question:

- To what extent does the test suite compensate for program's complexity?
  - Closely related to branch coverage, but stronger

how may paths are we short?

Coverage measure: V(G) - (min # extra paths needed to make a basis)

V(G)

#### Example



A Java method with V(G) = 5

- Test suite covers 10 paths
- 1 more path needed to make a basis
- Cyclomatic coverage = ?



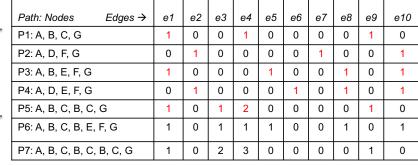
#### Example



A Java method with V(G) = 5

- Test suite covers 10 paths
- 1 more path needed to make a basis
- Cyclomatic coverage = 4/5 = 80%

#### Cyclomatic adequacy = Linear independence



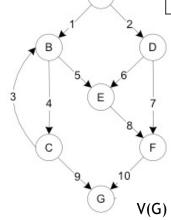


Each row is a vector that represents a path

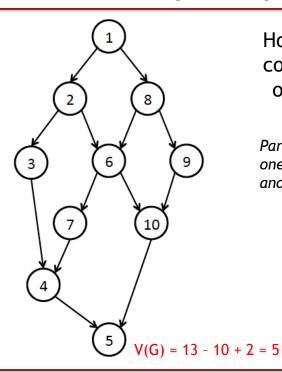
Sample paths as linear combinations of basis paths:

• 
$$P6 = P5 + P3 - P1$$

• 
$$P7 = 2 \cdot P5 - P1$$



basis

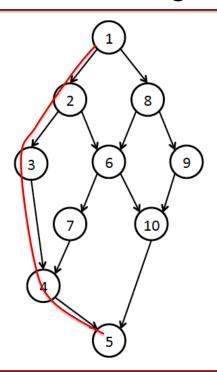


How do I exercise all feasible combinations of all outcomes of all decisions without full path coverage?

Part of a combination can be covered in one path and the remaining part in another path

- -1,2,3,4,5
- 1,2,6,7,4,5
- -1,2,6,10,5
- 1,8,6,7,4,5
- 1,8,6,10,5
- 1,8,9,10,5

Take advantage of overlapping test cases like in All-Pairs or MC/DC



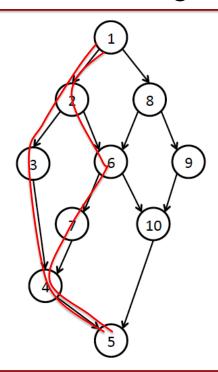
Paths

**Decisions** 

<del>- 1,2,3,4,5</del>

Combine 1 and 2

- 1,2,6,7,4,51,2,6,10,5
- 1,8,6,7,4,5
- 1,8,6,10,5
- -1,8,9,10,5



Paths

**Decisions** 

<del>- 1,2,3,4,5</del>

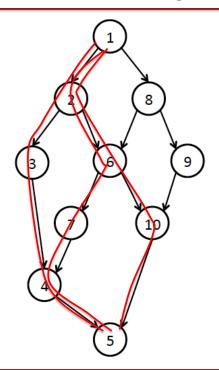
<del>1,2,6,7,4,5</del> Combine 1 and !2 and 6

-1,2,6,10,5

- 1,8,6,7,4,5

- 1,8,6,10,5

-1,8,9,10,5



Paths **Decisions** 

1,2,3,4,5

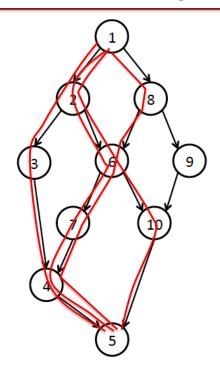
1,2,6,7,4,5

1,2,6,10,5 Combine 1 and !2 and !6

-1,8,6,7,4,5

-1,8,6,10,5

-1,8,9,10,5



Paths Decisions

<del>- 1,2,3,4,5</del>

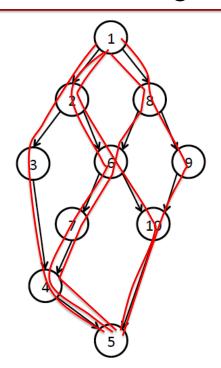
<del>- 1,2,6,7,4,5</del>

<del>- 1,2,6,10,5</del>

- 1,8,6,7,4,5 Combine !1 and 8 and 6

- 1,8,6,10,5

- 1,8,9,10,5



Paths Decisions

<del>- 1,2,3,4,5</del>

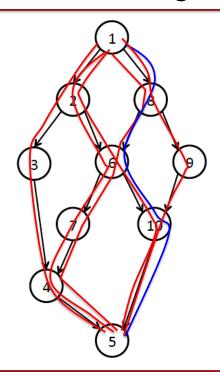
<del>- 1,2,6,7,4,5</del>

<del>- 1,2,6,10,5</del>

<del>- 1,8,6,7,4,5</del>

- 1,8,6,10,5

- 1,8,9,10,5 Combine !1 and !8



Paths Decisions

<del>- 1,2,3,4,5</del>

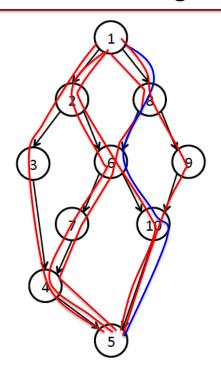
<del>- 1,2,6,7,4,5</del>

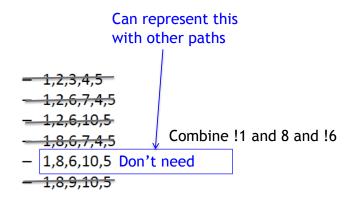
-1,2,6,10,5

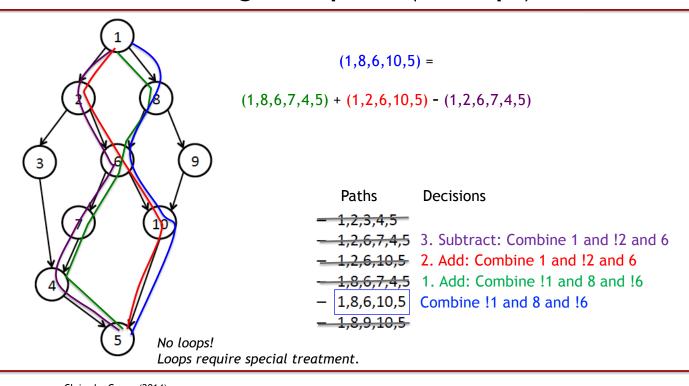
<del>- 1,8,6,7,4,5</del>

- 1,8,6,10,5 ?

- 1,8,9,10,5



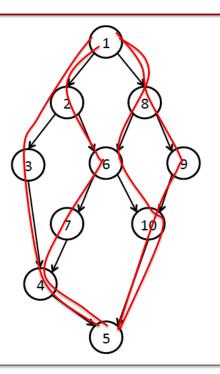




Claire Le Goues (2014)

#### Not enough paths?



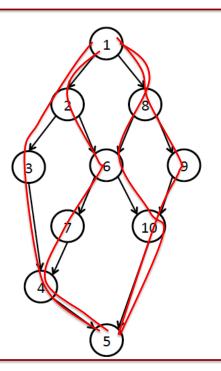


What about these 4 paths? They cover G, but do they form a basis?

- 1,2,3,4,5
- 1,2,6,7,4,5
- -1,8,6,10,5
- -1,8,9,10,5

#### Not enough paths?





What about these 4 paths?

They cover G, but do they form a basis?

No, because you can't reproduce all other paths with a linear combination of them.

E.g., for 1,2,6,10,5:

You need:

- 1,2,6,7,4,5 (because of edge 2-6)
- 1,8,6,10,5 (because of edge 6-10)

But you have to subtract edge 6-7, but there are no other paths that contain that edge!

- -1,2,3,4,5
- <del>1,2,6,7,4,5</del>
- <del>1,8,6,10,5</del>
- -1,8,9,10,5

#### Heuristic for cyclomatic testing: generate a basis

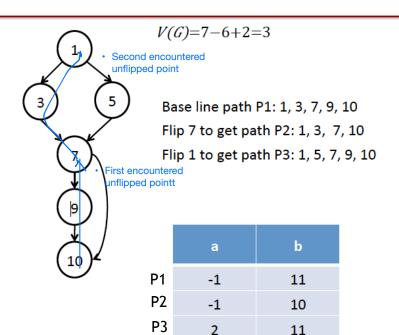
- 1. Produce the CFG G of program
- 2. Generate all basis paths
  - a. Pick a baseline path that represents a nominal behavior, not an error outcome
     avoid entering loops
  - b. Retrace the baseline path and "flip" the first un-flipped decision encountered
    - loop exit decisions require special care: when you encounter a new loop, choose exit decision before entry decision so skipping loop is for sure covered
  - c. Repeat this until all decisions have been flipped/covered or when you reach  $V(\mathsf{G})$
- 3. Analyze the conditions that force the execution along each basis path
- 4. Write test cases that will generate the conditions defined in 3

#### Note:

- the basis is not unique
- there might be infeasible basis paths (impossible to exercise)

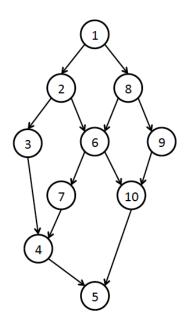
#### Generating test cases from a basis (no loops)

F

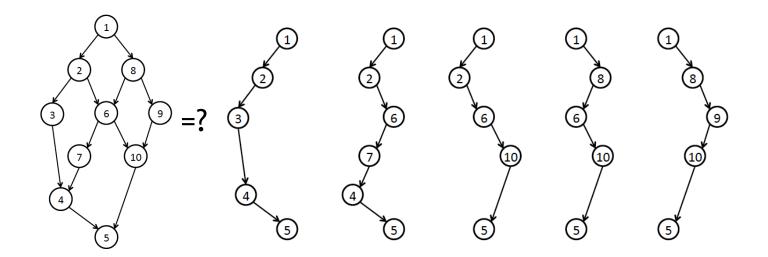




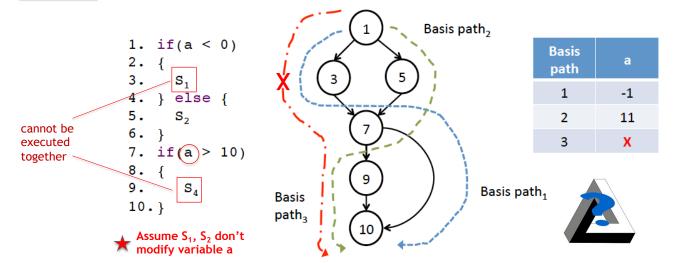
### Find a basis for this CFG using the algorithm described!



#### Is the RHS a basis for the CFG on the LHS?



#### Infeasibility in cyclomatic testing

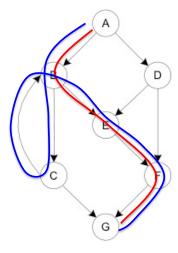


Some paths that can be derived from the topology of the CFG are semantically impossible due to unreachable code. This usually happens when a variable appears in more than one condition along the same path.

→ Might indicate a faulty logic in the program or bad coding practice.

#### Basis with loops

- Loops must be both skipped and executed!
- Need both red and blue paths even though red is included in blue;
- Choose red (exit) path before blue (entry) path, and apply the same heuristic



Cyclomatic complexity = 
$$V(G)$$
 =  $e - n + 2 = 10 - 7 + 2 = 5$ 

You can verify that without red path, it is impossible to construct a basis with V(G) paths!

## Algorith for cyclomatic testing: generate a basis with loops treated specially

- 1. Produce the CFG G of program
- 2. Generate all basis paths
  - a. Pick a baseline path that represents a nominal behavior, not an error outcome
     avoid entering loops
  - b. Retrace the baseline path and "flip" the first un-flipped decision encountered
    - loop exit decisions require special care: when you encounter a new loop, choose the exit decision before the entry decision so skipping loop is for sure covered
  - c. Repeat this until all decisions have been flipped/covered or when you reach  $V(\mathsf{G})$
- 3. Analyze the conditions that force the execution along each basis path
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#### Note:

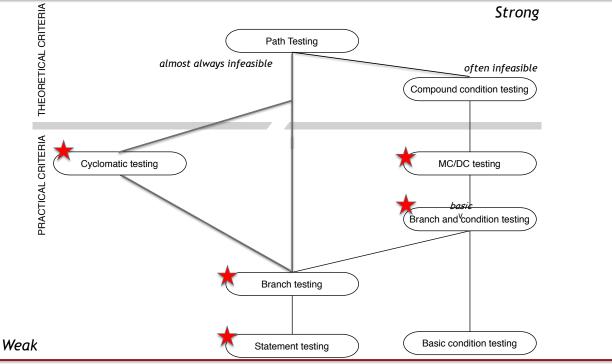
- the basis is not unique
- there might be infeasible basis paths (impossible to exercise)

## Cyclomatic adequacy

#### Advantages

- Simple, heuristic to compute basis paths (linearly independent paths) from CFG
- Size of test suite linear in #edges
- More complex program require proportionally more test cases
- Reduced number of test cases
- Subsumes branch (decision) coverage
- Disadvantages
  - No guarantee that basis paths are feasible (can we force execution by selecting right inputs?)
    - Manually too laborious, error prone: Need automated tools to generate the paths and path conditions for oracles
    - Full coverage may be impossible (but this is also a disadvantage with branch coverage) -- we'll simply assume infeasible paths don't exist

## Subsumption relation for CFG coverage



#### Path-based testing process

- Select the coverage criterion
- Execute spec-based tests
- Generate a CFG
- Identify paths in the CFG satisfying the criterion, but not covered in specbased tests
- Is the spec, or spec-based test design missing something?
- Build test cases determine the oracles
  - derive path condition expressions from the selected paths and solve those expressions to derive test inputs (involves constraint solving - SAT problem)
- Execute test cases

This may be a lot of work... some of it can be automated (tools exist to generate test cases)

# Structural testing: key messages

- We defined a number of adequacy criteria
  - NOT test design techniques per se, but lead to test generation/supplementation techniques!
- Different criteria address different classes of errors
- Full coverage for strong criteria may be unattainable (too many test cases)
  - ...and when attainable, creating actual test cases may be hard
  - How do I find program inputs (oracle problem) allowing to cover something buried deeply in the CFG?
  - Oracle problem reduces to constraint satisfaction (SAT)
  - Automated support (e.g., symbolic execution) may be necessary
  - Some paths may be infeasible that's ok, we'll have to ignore them
- Therefore, rather than requiring full adequacy, the "degree of adequacy" of a test suite is estimated and reported by coverage measures
  - Adequacy drives test suite and test design improvement

#### Sources

Pezze & Young: Software Analysis & Testing; Ch 12, 13

Jorgensen: Software Testing: A Craftsman's Approach

Claire Le Goues and Eduardo Miranda, CMU-CS & CMU-ISR

#### Midterm Review Head Start

All-Pairs Coverage Example

#### 4 characteristics

- A: 4 blocks {a1,a2, a3, a4 }
- B: 3 blocks {b1, b2, b3 }
- C: 2 blocks {c1, c2 }
- D: 2 blocks {d1, d2 }

Possible ways of choosing characteristic pairs:

$$C(4, 2) = 4!/2!(4 - 2!) = (4 \times 3 \times 2)/(2 \times 2) = 6$$

# Pairwise combinations that need to be covered

Characteristic Pair	Possible Pairwise Combinations
(A, B)	4 x 3 = 12
(A, C)	4 x 2 = 8
(A, D)	4 x 2 = 8
(B, C)	3 x 2 = 6
(B, D)	3 x 2 = 6
(C, D)	2 x 2 = 4
Total: 6 Pairs	44 Pairwise Combinations

- If each case covers 6 entirely new pairs...
  - Min # of test cases = Ceiling(44/6) = 8, but...
  - to cover largest attributes (A, B), must have at least  $4 \times 3 = 12$  test cases Max(8,12) = 12
- If each case covers just 1 new pair (no overlaps):
  - Max # of test cases = 44

# Calculating all pairs coverage

pair examples: no duplicates



ı	Case #	A	В	С	D	# new	Cum. new
	1	<b>a</b> 1	b1	c1	d1	6	6
	2	a1	b2	c1	d2	5	11
	3	a1	b3	c2	d2	5	16
	4	a2	b1	c1	d1	3	19
	5	a2	<u>b2</u>	c1	<u>d2</u>	2	21
	6	a2	b3	c2	d2	2	23
	7	a3	b1	<b>c</b> 1	d1	3	26
	8	<b>a</b> 3	<u>b2</u>	c1	d2	2	28
	9	a3	b3	c2	d2	2	30
	10	a4	b1	c1	d1	3	33
	11	a4	b2	c1	d2	2	35
	12	a4	b3	c2	d2	2	37

given test cases

Total pairwise combinations to be covered

= 44

Covered pairwise combinations = 37

All-Pairs Coverage

= 37/44 = 84%

# APPENDIX - OTHER PATH COVERAGE CRITERIA

(not included in exam)

## Path testing

- Decision (branch) and condition adequacy criteria consider individual program decisions
- Path testing considers combinations of decisions along paths through the CFG
- Adequacy criterion: each path must be executed at least once
- Coverage measure
   # executed paths
   # total paths

Loops?

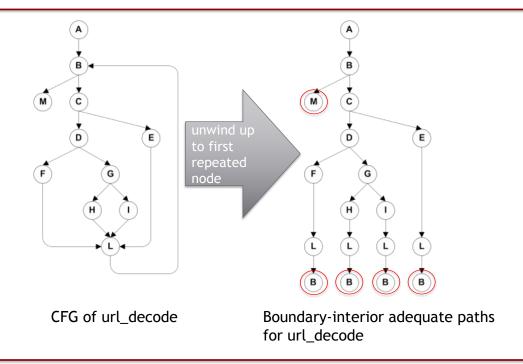
#### Practical path coverage criteria

- The number of paths in a program with loops is unbounded
  - the simple criterion is usually impossible to satisfy
- For a feasible criterion: partition infinite set of paths into a finite number of blocks
- Useful criteria can be obtained by:
  - limiting number of traversals of loops
  - limiting length of the paths to be traversed (depth limit)
  - exploiting subpath relationships

## Boundary-interior path testing

- Group together paths that differ only in the sub-path they follow when repeating the body of a loop
  - finite path X with sub-path Y repeated n times ≡ finite path X' where X differs from X' only in that sub-path Y is repeated n+1 times
  - unwind CFG to follow each path in the control flow graph up to the first repeated node
  - the set of paths from the root of the tree to each leaf is the required set of sub-paths for boundary-interior coverage

## Boundary interior adequacy for url-decode



## Limitations of boundary-interior adequacy

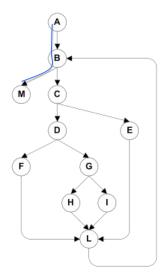
The number of paths can still grow exponentially

```
if (a) {
   S1;
if (b) {
   S2;
if (c) {
   S3;
if (x) {
   Sn;
```

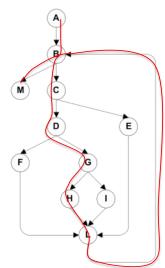
- The subpaths through this control flow can include or exclude each of the statements Si, so that in total N branches result in 2<sup>N</sup> paths that must be traversed
- Choosing input data to force execution of one particular path may be very difficult, or even impossible if the conditions are dependent

## Loop-boundary adequacy

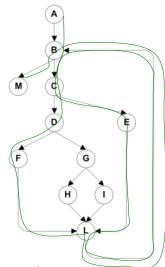
- Variant of the boundary-interior criterion that treats loop boundaries similarly but is less stringent with respect to other differences among paths
- Criterion: A test suite satisfies the loop boundary adequacy criterion iff for every loop:
  - in at least one test case, the loop body is iterated zero times
  - in at least one test case, the loop body is iterated once
  - in at least one test case, the loop body is iterated more than once
- Emulates "proof by induction"



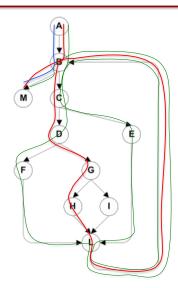
1st case: No loop execution



2<sup>nd</sup> case: Loop executed once



3<sup>rd</sup> case: Loop executed twice



- Weak criterion: covered the loop according to loop-boundary adequacy, but some branches within the loop are not executed
- · Ok if combined with branch coverage

#### LCSAJ adequacy

- Linear Code Sequence And Jump: sequential subpath in the CFG starting and ending with a jump
  - TER<sub>1</sub> = statement coverage
  - TER<sub>2</sub> = (almost) branch coverage
  - $-TER_{n+2}$  = coverage of *n* consecutive LCSAJs

# A QUICK LOOK AT

#### **DATA FLOW TESTING**

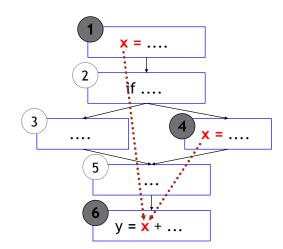
... another kind of structural testing ...

#### Motivation for data flow testing

- Middle ground in structural testing
  - Statement and branch coverage don't cover fine-grained interactions among program elements
  - Path-based criteria require impractical number of test cases
    - and only a few paths uncover additional faults, anyway
  - Need to distinguish "important" paths
- Intuition: statements interact through data flow
  - Value computed (defined) in one statement, used in another
  - Bad value computation revealed only when it is used

## Data flow concept

- Value of x at 6 could be computed at 1 or at 4
- Bad computation at 1 or 4 could be revealed only if they are used at 6
- Nodes (1,6) and (4,6) are def-use (DU) pairs
  - defined at 1,4
  - used at 6



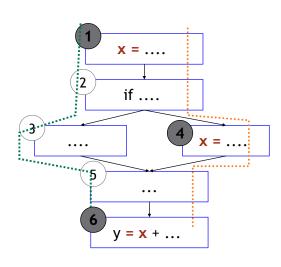
#### **Terms**

• **DU pair**: a pair of *definition* and *use* for some variable, such that at least one DU path exists from the definition to the use

```
x = ... is a definition of x
... = ... x ... is a use of x
```

- **DU path**: a *definition-clear* path in the CFG from a definition to a use of the same variable
  - **Definition-clear**: value is not updated on path
  - Loops may create infinite DU paths between a definition and a use

## **Definition-clear path**



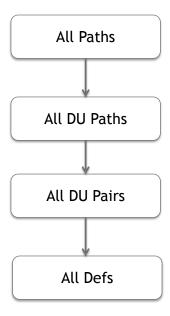
- 1,2,3,5,6 is a definition-clear path from 1 to 6
  - x is not updated between 1 and6
- 1,2,4,5,6 is *not* a definitionclear path from 1 to 6
  - the value of x is killed (updated) at node 4
  - (1,6) is a DU pair because
     1,2,3,5,6 is a definition-clear
     path

## Adequacy criteria

- All DU Pairs: each DU pair is exercised by at least one test case
- All DU Paths: each simple (non looping) DU path (def-clear) is exercised by at least one test case
- All Defs: for each definition, there is at least one test case which exercises a DU pair containing it
  - every computed value is used somewhere

Corresponding coverage measures (fractions) can also be defined

# Data flow testing subsumption



#### Difficult cases

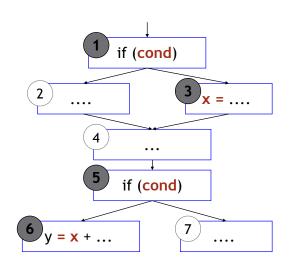
```
• x[i] = ...; ...; y = x[j]
                                      use?
  - DU pair (only) if i==i
                                                        use
• x = ...; p = &x; ...; *p = 99; ...; q = *p
  - *p is an alias of x
• m.putFoo(...); ...; def
  y = n.qetFoo(...);
                             use?
  – Are m and n the same object?
  – Do m and n share a foo field?
```

Alias problem: which references are (always or sometimes) the same?

#### Data flow coverage with complex structures

- Arrays and pointers are critical for data flow testing
  - Under-estimation of aliases may fail to include some DU pairs
  - Over-estimation, on the other hand, may introduce unfeasible test obligations
- For testing, it may be preferable to accept under-estimation of alias set rather than over-estimation or expensive analysis
  - Controversial: in other applications (e.g., compilers), a conservative over-estimation of aliases is usually required
  - Alias analysis may rely on external guidance or other global analysis to calculate good estimates
  - Undisciplined use of dynamic storage, pointer arithmetic, etc. may make the whole analysis infeasible

#### Infeasibility in data flow analysis



- Suppose cond has not changed between 1 and 5
  - or the conditions could be different, but the first implies the second
- Then (3,6) is not a (feasible) DU pair
  - but it is difficult or impossible to determine which pairs are infeasible
- Infeasible test obligations are a problem
  - no test case can cover them

#### Coverage in data flow analysis

- In practice, reasonable coverage is (often, not always) achievable
  - Number of DU pairs is exponential in worst case, but often linear
  - All DU Paths is more often impractical

## Data flow testing process

#### Similar to general structural testing...

- Draw an annotated CFG for the program
- Select the DF coverage criterion
- Identify paths in the CFG not covered according to the criterion
- Build test cases determine the oracles
  - derive path condition expressions from the selected paths and solve those expressions to derive test inputs (involves constraint solving)
- Execute test cases

This may be a lot of work...

# Data flow testing: key messages

- Data flow testing attempts to distinguish "important" paths: interactions between statements
  - intermediate between simple statement and branch coverage and more expensive path coverage
- Cover Def-Use (DU) pairs: from computation of value to its use
  - intuition: bad computed value is revealed only when it is used
  - Levels: All DU Pairs, All DU Paths, All Defs, ...
- Limits: aliases, infeasible paths
  - Worst case is bad (undecidable properties, exponential blowup of paths), so pragmatic compromises are required