MATH3013 Final Project

DC126238 OUYANG LAM CHEONG

May 20, 2024

Table of Contents

Introduction

- Water environments
- Control water temperature in these environments is by adding hot water

Figure: Bathtub

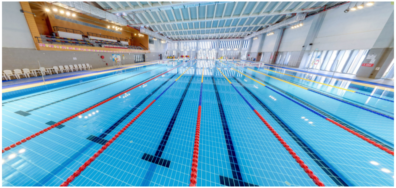

Figure: Swimming Pool in University of Macau

Problem Background

- How to maintain the temperature of water in a bathtub?
- How does the temperature of bathtub water change?
- What impact do different parameters have on the temperature change?
- What is the spatial temperature distribution?

Assumptions

- All the physical properties of air, water, and bathtub are stable.
- Any addition of water results in an immediate drainage of the same volume.
- The user is considered as an extension of the bathtub water.
- All radiation effects are neglected.

Geometric consideration

- The bathtub has dimensions of 1.7m $\times$ 1.0m $\times$ 0.7m.
- The faucet is placed at $x = 0$.
- Assume that the flow and heat transfer of the water occur only in the x-direction.

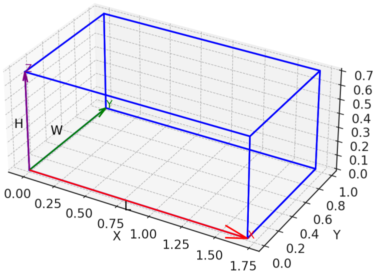

Figure: Perspective view

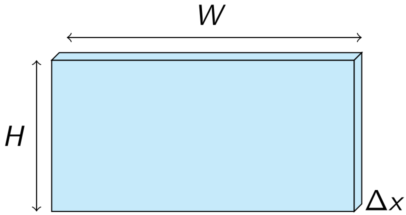

Figure: Cross section of the bathtub

Model I

- Cooling:

$$\frac{dT}{dt} = -\frac{hS}{cm_{tub}}(T - T_\infty)$$

- Heating:

$$\frac{dT}{dt} = \frac{dm}{dt}\frac{(T_{in} - T)}{m_{tub}}$$

- Complete Model:

$$\frac{dT}{dt} = \left(\frac{-hS - \dot{m}c}{cm_{tub}}\right)T + \left(\frac{hST_\infty + \dot{m}cT_{in}}{cm_{tub}}\right)$$

Results

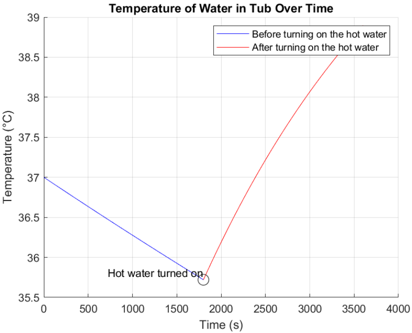

Figure: Temperature change within 1 hour

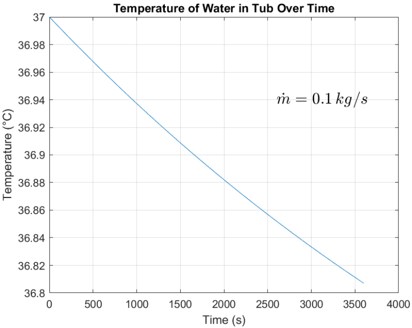

Figure: Temperature change with trickling hot water

Results

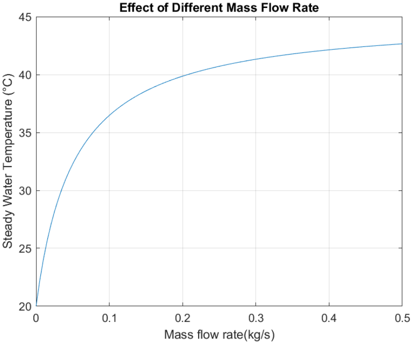

Figure: Sensitivity of flow rate

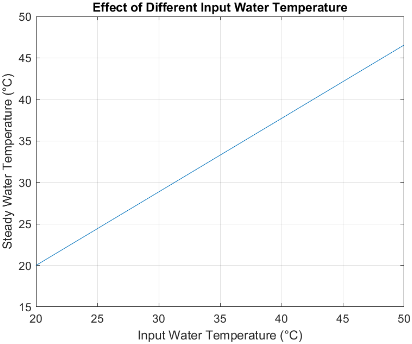

Figure: Sensitivity of inlet temperature

Model II

General one dimension heat equation:

$$
\begin{aligned}
\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \tilde{Q}(x,t), \quad 0 < x < L, \quad t > 0, \quad \alpha \in \mathbb{R}, \\
u_x(x=0,t) = a(t), \quad t > 0, \\
u_x(x=L,t) = b(t), \quad t > 0, \\
u(x,t=0) = \phi(x), \quad 0 < x < L.
\end{aligned}
$$

Model II

Cooling Process:

$$
\begin{aligned}
\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \tilde{Q}(x,t), \quad 0 < x < L, \quad t > 0, \quad \alpha \in \mathbb{R}, \\
u_x(x=0,t) = 0, \quad t > 0, \\
u_x(x=L,t) = 0, \quad t > 0, \\
u(x,t=0) = 37^\circ\mathrm{C}, \quad 0 < x < L, \\
\tilde{Q}(x,t) = \tilde{h}(u_\infty - u(x,t)), \quad 0 < x < L, \quad t > 0.
\end{aligned}
$$

Numerical Scheme

Complete scheme:

- $x_i = i\Delta x, \quad i = 0, 1, \ldots, N, \quad \Delta x = \frac{L}{N}$
- $t_n = n\Delta t, \quad n = 0, 1, 2, \ldots$
- $u_i^0 = 37^\circ\mathrm{C}, \quad i = 0, 1, \ldots, N$
- $u_1^n = u_0^n, \quad u_N^n = u_{N-1}^n, \quad n \geq 0$
- $$u_i^{n+1} = u_i^n + \Delta t \left(\alpha \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2} + \tilde{h}(u_\infty - u_i^n) \right)$$
  $$i = 1, 2, \ldots, N-1, \quad n \geq 0$$

Stability condition:

- $\Delta t \leq \min \left(\frac{2}{\tilde{h}}, \frac{2}{\frac{4\alpha}{(\Delta x)^2} + \tilde{h}} \right)$

Results

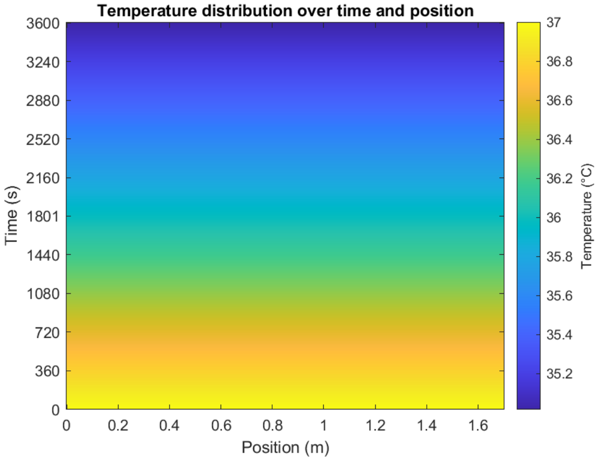

Figure: Temperature change within 60 minutes in PDE model

Model II

Cooling and Heating Process:

$$
\begin{aligned}
\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \tilde{Q}(x,t), &\quad 0 < x < L, \quad t > 0, \quad \alpha \in \mathbb{R}, \\
u_x(x=0,t) = \hat{h}(u_{in} - u(0,t)), &\quad t > 0, \\
u_x(x=L,t) = 0, &\quad t > 0, \\
u(x,t=0) = 37^\circ\mathrm{C}, &\quad 0 < x < L, \\
\tilde{Q}(x,t) = \tilde{h}(u_\infty - u(x,t)), &\quad 0 < x < L, \quad t > 0.
\end{aligned}
$$

Results

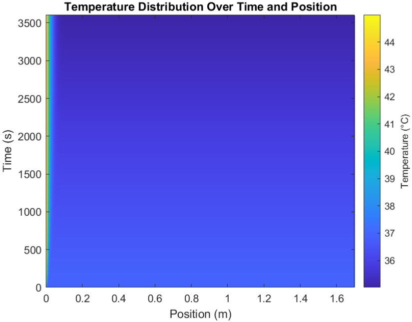

Figure: Temperature change with input hot water

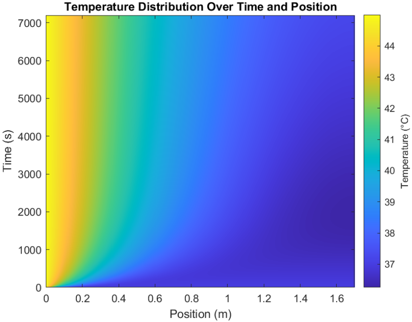

Figure: Temperature change with increased thermal diffusivity

Thermal diffusivity

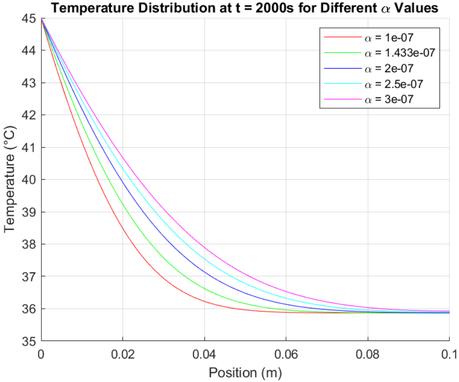

Figure: Sensitivity of temperature to thermal diffusivity

Remark of Model I

Strengths:

- Straightforward simulation
- Helps in understanding the influence of each parameter

Weaknesses:

- Unrealistic assumption of uniform temperature distribution
- Limit applicability to complex systems

Remark of Model II

Strengths:

- Presentation of spatial temperature change
- Comprehensive view of the thermal dynamics

Weaknesses:

- Increase of mathematical and computational complexity
- Accuracy is highly dependent on the precise values of parameters

Model Improvement and Extension

Consider the advection-diffusion equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} - \epsilon v \frac{\partial u}{\partial x} + \hat{Q}(x, t)$$

Consider the presence of user in the bathtub

Possibility to extend the application of model to other scenarios

References

B. Barnes and G.R. Fulford.
Mathematical Modelling with Case Studies Using Maple and MATLAB, Third Edition.
Textbooks in Mathematics. CRC Press, 2014.

R.L. Burden and J.D. Faires.
Numerical Analysis.
Cengage Learning, 2010.

Y.A. cCengel, J.M. Cimbala, M. Kanouglu, and R.H. Turner.
Fundamentals of Thermal-fluid Sciences.
McGraw-Hill Education, 2016.

W.A. Strauss.
Partial Differential Equations An Introduction.
Wiley, 2007.

The End

Thank you for your attention!

Any questions?