

Prob 140 Fall 2019 Final Exam Reference Sheet

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name and parameters	values	mass function or density	cdf F or survival function	expectation	variance	mgf $M(t)$
Uniform	$m \leq k \leq n$	$1/(n-m+1)$		$(m+n)/2$	$((n-m+1)^2-1)/12$	
Bernoulli (p)	$0, 1$	$p_1 = p, p_0 = q$		p	pq	$q + pe^t$
Binomial (n, p)	$0 \leq k \leq n$	$\binom{n}{k} p^k q^{n-k}$		np	npq	$(q + pe^t)^n$
Poisson (μ)	$k \geq 0$	$e^{-\mu} \mu^k / k!$		μ	μ	$\exp(\mu(e^t - 1))$
Geometric (p)	$k \geq 1$	$q^{k-1} p$	$P(X > k) = q^k$	$1/p$	q/p^2	
"Negative binomial" (r, p)	$k \geq r$	$\binom{k-1}{r-1} p^{r-1} q^{k-r} p$		r/p	rq/p^2	
Geometric (p)	$k \geq 0$	$q^k p$	$P(X > k) = q^{k+1}$	q/p	q/p^2	
Negative binomial (r, p)	$k \geq 0$	$\binom{k+r-1}{r-1} p^{r-1} q^k p$		rq/p	rq/p^2	
Hypergeometric (N, G, n)	$0 \leq g \leq n$	$\binom{g}{g} \binom{N-g}{b-g} / \binom{N}{b}$		$n \frac{G}{N}$	$n \frac{G}{N} \cdot \frac{B}{N} \cdot \frac{N-n}{N-1}$	
Uniform	$x \in (a, b)$	$1/(b-a)$	$F(x) = (x-a)/(b-a)$	$(a+b)/2$	$(b-a)^2/12$	
Beta (r, s)	$x \in (0, 1)$	$\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1} (1-x)^{s-1}$	by uniform order statistics if r, s integers	$r/(r+s)$	$rs/((r+s)^2(r+s+1))$	
Exponential (λ) = Gamma ($1, \lambda$)	$x \geq 0$	$\lambda e^{-\lambda x}$	$F(x) = 1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$	
Gamma (r, λ)	$x \geq 0$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	for integer r : $F(x) = 1 - \sum_{k=0}^{r-1} e^{-\lambda x} \frac{(\lambda x)^k}{k!}$	r/λ	r/λ^2	$(\lambda/(\lambda-t))^r, t < \lambda$
Chi-square (n)	$x \geq 0$	same as gamma ($n/2, 1/2$)		n	$2n$	
Normal ($0, 1$)	$x \in \mathbb{R}$	$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	cdf: $\Phi(x)$	0	1	$\exp(t^2/2)$
Normal (μ, σ^2)	$x \in \mathbb{R}$	$\frac{1}{\sigma} \phi((x-\mu)/\sigma)$	cdf: $\Phi((x-\mu)/\sigma)$	μ	σ^2	
Rayleigh	$x \geq 0$	$xe^{-\frac{1}{2}x^2}$	$F(x) = 1 - e^{-\frac{1}{2}x^2}$	$\sqrt{\pi}/2$	$(4-\pi)/2$	
Cauchy	$x \in \mathbb{R}$	$1/\pi(1+x^2)$	$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan(x)$			

• If X_1, X_2, \dots, X_n are i.i.d. with variance σ^2 , then $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator of σ^2 but $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ is not.

• For $r > 0$, the integral $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx$ satisfies $\Gamma(r+1) = r\Gamma(r)$. So $\Gamma(r) = (r-1)!$ if r is an integer. Also, $\Gamma(1/2) = \sqrt{\pi}$.

• If Z_1 and Z_2 are i.i.d. standard normal then $\sqrt{Z_1^2 + Z_2^2}$ is Rayleigh. • If Z is standard normal then $E(|Z|) = \sqrt{2/\pi}$

• The k th order statistic $U_{(k)}$ is k th smallest of U_1, U_2, \dots, U_n i.i.d. uniform $(0, 1)$, so $U_{(1)}$ is min and $U_{(n)}$ is max. Density of $U_{(k)}$ is beta $(k, n-k+1)$.

• If S_n is the number of heads in n tosses of a coin whose probability of heads was chosen according to the beta (r, s) distribution, then the distribution of S_n is beta-binomial (r, s, n) with $P(S_n = k) = \binom{n}{k} C(r, s) / C(r+k, s+n-k)$ where $C(r, s) = \Gamma(r+s) / (\Gamma(r)\Gamma(s))$ is the constant in the beta (r, s) density.

• If \mathbf{X} has mean vector μ and covariance matrix Σ then $\mathbf{A}\mathbf{X} + \mathbf{b}$ has mean vector $\mathbf{A}\mu + \mathbf{b}$ and covariance matrix $\mathbf{A}\Sigma\mathbf{A}^T$.

• If \mathbf{X} has the multivariate normal distribution with mean vector μ and covariance matrix Σ , then \mathbf{X} has density $f(\mathbf{x}) = \frac{1}{(\sqrt{2\pi})^n \sqrt{\det(\Sigma)}} \exp(-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu))$

• The least squares linear predictor of Y based on the $p \times 1$ vector \mathbf{X} is $\hat{Y} = \mathbf{b}^T (\mathbf{X} - \mu_{\mathbf{X}}) + \mu_Y$ where $\mathbf{b} = \Sigma_{\mathbf{X}}^{-1} \Sigma_{\mathbf{X}, Y}$. Here the i th element of the $p \times 1$ vector $\Sigma_{\mathbf{X}, Y}$ is $\text{Cov}(X_i, Y)$. In the case $p = 1$ this is the equation of the regression line, with slope $\text{Cov}(X, Y) / \text{Var}(X) = rSD(Y) / SD(X)$ and intercept $E(Y) - \text{slope}E(X)$.

• If $W = Y - \hat{Y}$ is the error in the least squares linear prediction, then $E(W) = 0$ and $\text{Var}(W) = \text{Var}(Y) - \Sigma_Y \mathbf{X} \Sigma_{\mathbf{X}}^{-1} \Sigma_{\mathbf{X}, Y}$. In the case $p = 1$, $\text{Var}(W) = (1-r^2)\text{Var}(Y)$.

• If Y and \mathbf{X} are multivariate normal then the formulas in the above two bullet points are the conditional expectation and conditional variance of Y given \mathbf{X} .

• If Y and X are standard bivariate normal with correlation r , then $Y = rX + \sqrt{1-r^2}Z$ for some standard normal Z independent of X .

• Under the multiple regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, the least squares estimate of β is $\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$.