

# Introduction to Symbolic AI

## Coursework 2: Adversarial Search on Connect Four

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February 6, 2022

The times measured in this report does not correspond to any real world unit of time, because it was measured by `time.perf_counter()`.

### 1 Question 3

See Tables 1, 2, 3 and Figures 1, 2, 3, 4.

There is a significant increase in selection time for actions as each of  $m, n, k$  increases. Out of the three, increase in  $k$  brought the most dramatic increase (refer to the Tables and Figures). In Figure 1 and 2, each line looked almost linear (except for  $k = 1$ ), suggesting that the running time is similar to exponential in both  $m$  and  $n$ .

Figure 3 looked very similar to Figure 4 for  $k = 2$  and  $k = 3$ , suggesting that the running time of minimax and the number of states visited are highly positively correlated. However, they looked very different for  $k = 1$ .

### 2 Question 4

See Tables 4, 5, 6, and Figures 5, 6, 7 and 8.

Compared to the unpruned plots, the gradients of the lines in both the  $\text{Log}(\text{Time})$  and  $\text{Log}(\text{Count})$  plots are smaller (see Figures 7 and 8), suggesting that the change in running time with respect to  $m, n$  and  $k$  is much smaller than the unpruned game. Also, the lines aren't straight, but rather bend downwards, revealing the time is not truly exponential in  $m$  and  $n$ , but rather a little bit less than exponential.

Figure 5 looked very similar to Figure 6 for  $k = 1, 2, 3$ , suggesting that the running time of minimax and the number of states it visits are highly positively correlated.

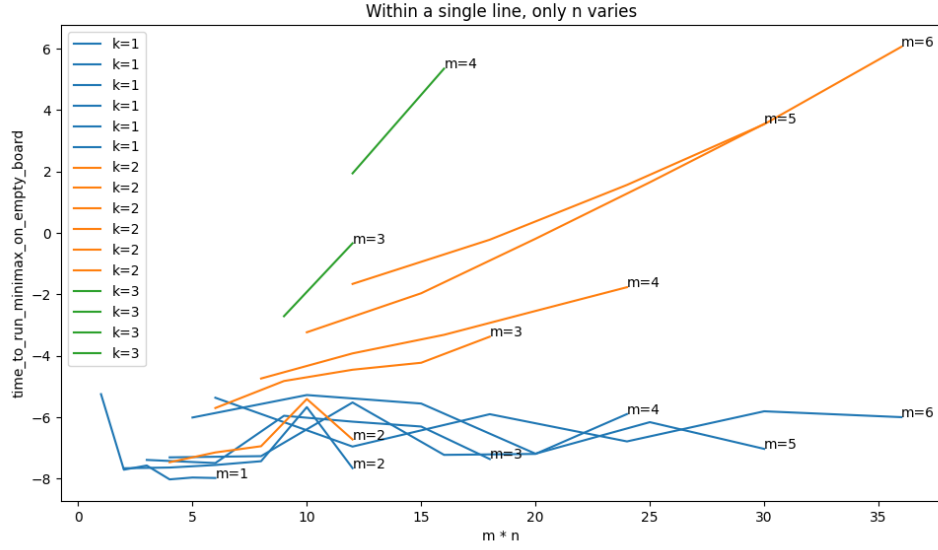


Figure 1:  $\text{Log}(\text{time})$ ; Unpruned. Curves are parametrized with respect to  $n$ .

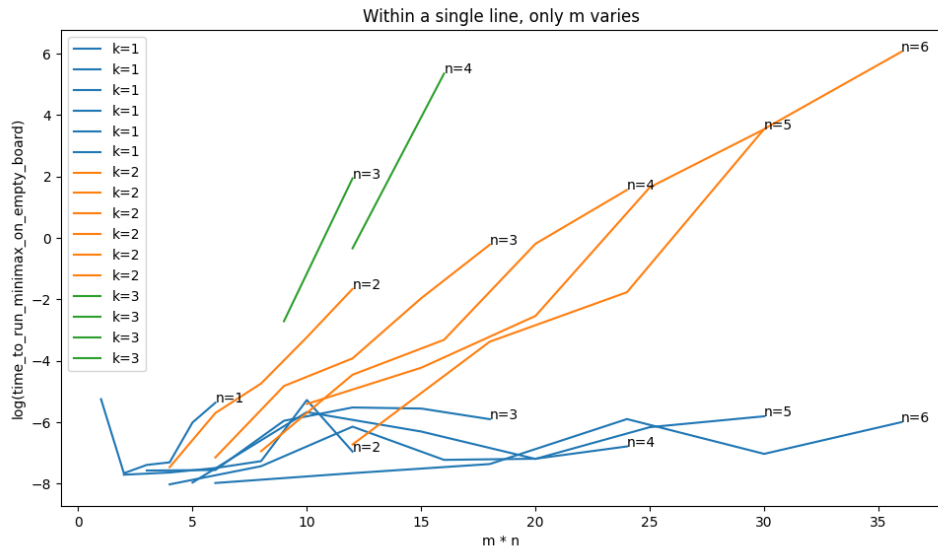


Figure 2:  $\text{Log}(\text{time})$ ; Unpruned. Curves parametrized with respect to  $m$ .

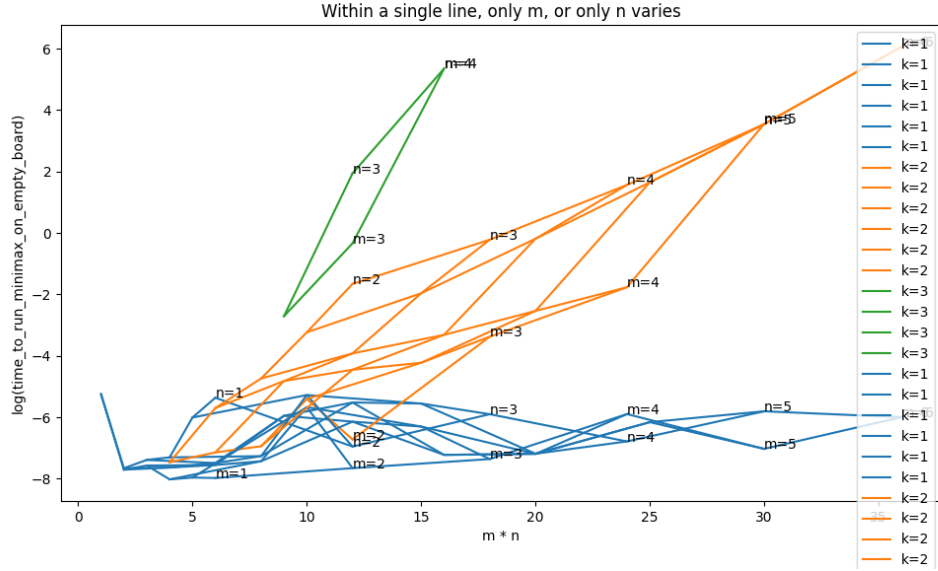


Figure 3:  $\text{Log}(\text{time})$ ; Unpruned. (Figure 1 and 2 combined.)

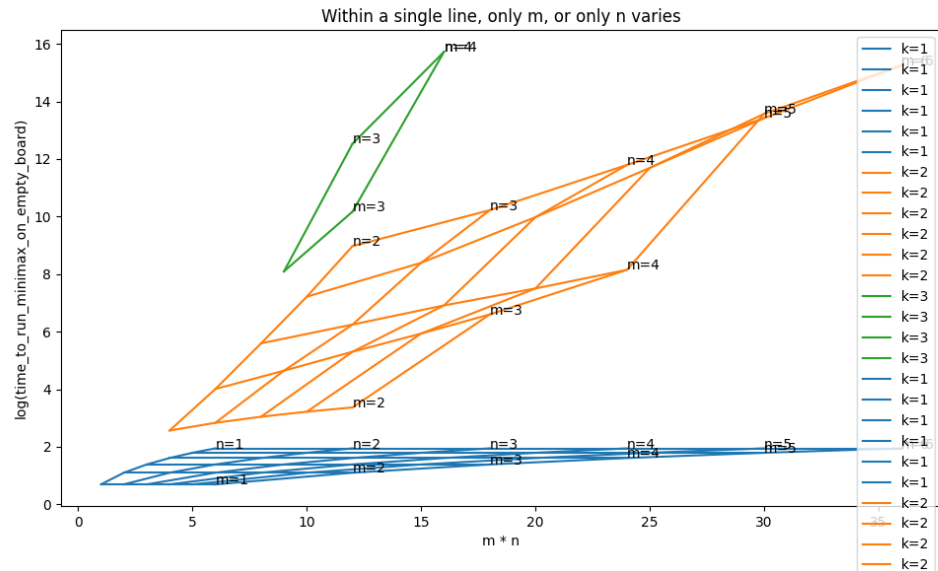


Figure 4:  $\text{Log}(\text{count})$ ; Unpruned. (the y-axis is mislabelled)



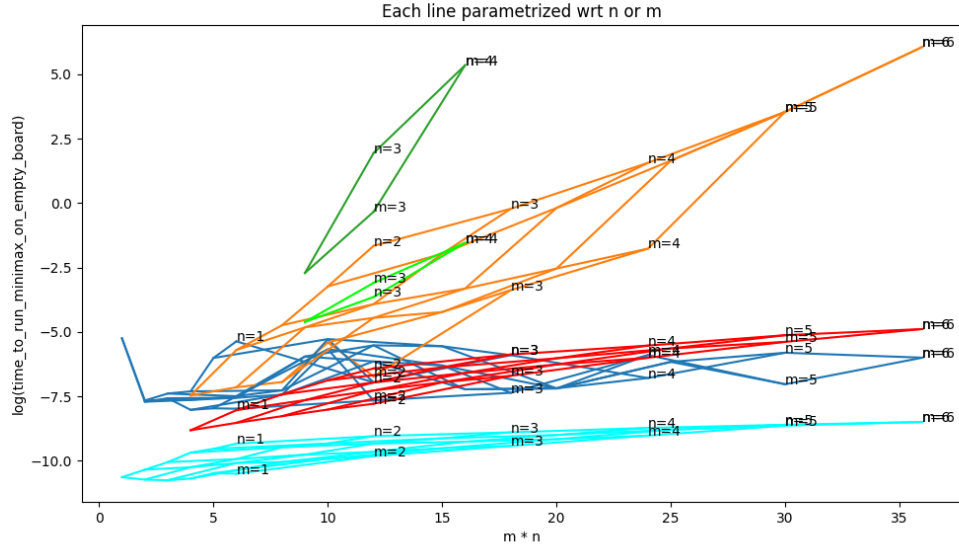


Figure 7: Log(time) plots combined (Figure 3 and 5). The brighter colors indicate the pruned version.

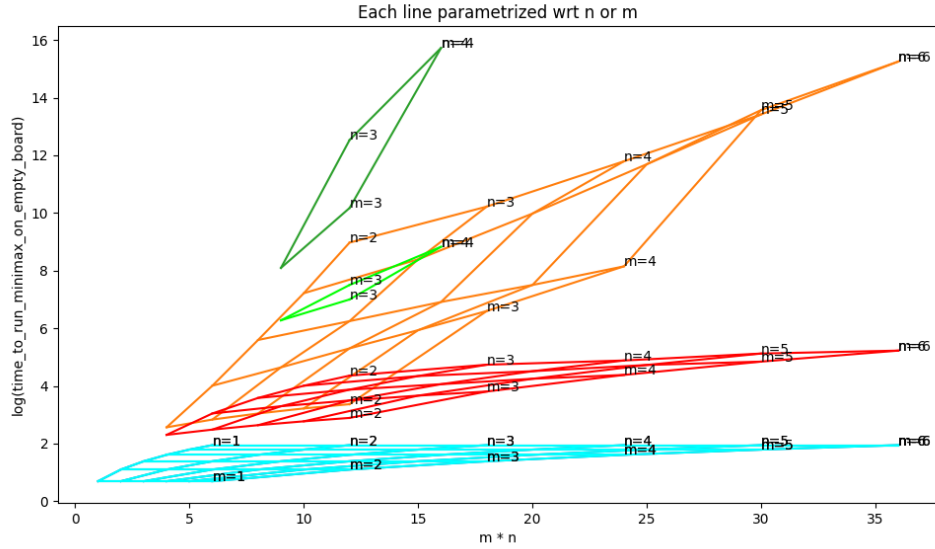


Figure 8: Two Log(count)(The y-axis is mislabelled) plots combined (Figure 4 and 6). The brighter colors indicate the pruned version. The cyan actually completely obscures the unpruned version's darker blue lines here.

n\m	1	2	3	4	5	6
1	$4.050 \times 10^{-3}$	$4.261 \times 10^{-4}$	$3.748 \times 10^{-4}$	$4.171 \times 10^{-4}$	$2.464 \times 10^{-3}$	$4.690 \times 10^{-3}$
2	$3.509 \times 10^{-4}$	$4.377 \times 10^{-4}$	$3.317 \times 10^{-4}$	$4.555 \times 10^{-3}$	$5.121 \times 10^{-3}$	$9.571 \times 10^{-4}$
3	$3.034 \times 10^{-4}$	$4.438 \times 10^{-4}$	$3.795 \times 10^{-4}$	$3.185 \times 10^{-4}$	$3.882 \times 10^{-3}$	$2.745 \times 10^{-3}$
4	$2.780 \times 10^{-4}$	$3.692 \times 10^{-4}$	$1.474 \times 10^{-3}$	$3.549 \times 10^{-4}$	$7.584 \times 10^{-4}$	$1.131 \times 10^{-3}$
5	$3.493 \times 10^{-4}$	$3.455 \times 10^{-3}$	$1.838 \times 10^{-3}$	$7.534 \times 10^{-4}$	$2.118 \times 10^{-3}$	$3.017 \times 10^{-3}$
6	$3.439 \times 10^{-4}$	$4.741 \times 10^{-4}$	$6.375 \times 10^{-4}$	$2.755 \times 10^{-3}$	$8.881 \times 10^{-4}$	$2.492 \times 10^{-3}$

Table 1: Unpruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 1$ .

n\m	2	3	4	5	6
2	$5.748 \times 10^{-4}$	$3.354 \times 10^{-3}$	$8.772 \times 10^{-3}$	$3.950 \times 10^{-2}$	$1.906 \times 10^{-1}$
3	$7.903 \times 10^{-4}$	$8.082 \times 10^{-3}$	$1.986 \times 10^{-2}$	$1.406 \times 10^{-1}$	$8.042 \times 10^{-1}$
4	$9.677 \times 10^{-4}$	$1.167 \times 10^{-2}$	$3.635 \times 10^{-2}$	$8.293 \times 10^{-1}$	$4.783 \times 10^{-0}$
5	$4.500 \times 10^{-3}$	$1.464 \times 10^{-2}$	$7.908 \times 10^{-2}$	$5.211 \times 10^{-0}$	$3.449 \times 10^{-1}$
6	$1.205 \times 10^{-3}$	$3.422 \times 10^{-2}$	$1.712 \times 10^{-1}$	$3.480 \times 10^{-1}$	$4.290 \times 10^{-2}$

Table 2: Unpruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 2$ .

n\m	3	4
3	$6.664 \times 10^{-2}$	$6.974 \times 10^0$
4	$7.152 \times 10^{-1}$	$2.106 \times 10^2$

Table 3: Unpruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 3$ .

n\m	1	2	3	4
1	2.58864375e-05	2.95562500e-05	4.20523750e-05	5.36285000e-05
2	2.15188750e-05	3.24377500e-05	4.95180625e-05	6.64923750e-05
3	2.17856875e-05	3.66650625e-05	5.59633125e-05	8.79735625e-05
4	2.19276875e-05	4.20060625e-05	6.41628125e-05	8.88194375e-05

Table 4: Pruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 1$ .

n\m	2	3	4
2	0.00014108	0.00032122	0.00059654
3	0.00019683	0.00049385	0.00094588
4	0.00026159	0.00069692	0.00137228

Table 5: Pruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 2$ .

$n \backslash m$	3	4
3	0.00994167	0.02623095
4	0.04536541	0.21473945

Table 6: Pruned game: the running times for minimax for different values of  $m$  and  $n$ , when  $k = 3$ .