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PANTHÉON SORBONNE

Time Series Analysis : Early Detection of Crisis Signals in Financial Data, Cryptocurrencies, and Climate Data

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Abstract

In the field of ecology, as well as more generally in complex systems, the question of detecting tipping points, bifurcations, or critical transitions arises, these points corresponding to the shift from one state to another. Regulators are therefore interested in implementing advanced indicators, known as "early warning signals." A set of indicators is based on a system's lack of resilience approaching a bifurcation, connected to the theory of Critical Slowing Down (CSD). This thesis proposes to calculate these indicators using financial data and to evaluate their relevance in the approach of a financial crisis, such as the one in 2007. We will also examine the Bitcoin price, as well as self-generated meteorological data, to extend our analysis.

Keywords: Early Warnings Signals, Critical Slowing Down, Bifurcations, Critical transitions, Advanced indicators, Financial crisis, Bitcoin price, Meteorological
SAS tools: Proc IML

Contents

1 Introduction

The stability of complex systems, whether economic, financial, or meteorological, relies on a delicate balance. This equilibrium can be disrupted by unforeseen events or sudden changes. These critical transitions are crucial for understanding and managing these systems, particularly in the financial sector, where stock market crashes can have long-lasting impacts on the economy. Therefore, predicting financial crises has become a major concern for regulators, investors, and financial institutions in recent years.

In response to this concern, the theory of Critical Slowing Down (CSD) emerges as a promising tool for detecting early warning signs of significant changes in a complex system. The CSD theory suggests an increasing trend in the time series of CSD indicators near catastrophic events (a critical transition). These indicators are often based on early warning signals that can alert to changes in variability, correlation, or data distribution.

The CSD theory, formalized by Kenneth Wilson, Nobel Prize laureate in physics in 1982, is based on the renormalization method he introduced in his 1971 paper "Renormalization Group and Critical Phenomena"[8], published in the journal Physical Review B. This powerful method allows for the analysis of critical systems and phase transitions, playing a crucial role in the development of CSD.

In this thesis, we explore the application of the CSD theory to financial data to predict crises, such as the 2007 financial crisis. We also extend our analysis to other domains such as the Bitcoin market and meteorological data to understand the prediction of critical transitions in different contexts: economic, technological, or environmental. Our multidisciplinary approach will allow us to evaluate the relevance and reliability of early warning indicators based on the Critical Slowing Down theory.

Thus, through this thesis, we will attempt to answer the question: "Is it possible to detect tipping points in financial time series using early warning signals derived from the Critical Slowing Down theory?" To answer this question, our report is organized as follows: In Section 2, we present the Critical Slowing Down theory and early warning signals, detailing their functioning and various application domains. Section 3 is dedicated to the methodology used to collect and process our data and calculate the early warning indicators. Then, in Section 4, we conduct an in-depth analysis of the main results, highlighting the importance of early warning indicators. Finally, we conclude our report with a thorough discussion of our results, conclusions and an opening.

2 Theoretical Approach

2.1 Theory of Critical Slowing Down

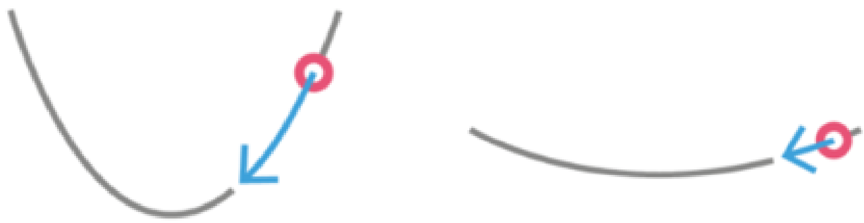
2.1.1 The Theory

In complex dynamic systems, transitions from one state to another are commonplace, whether they result from major external shocks or minor disturbances. The theory of critical slowdown comes from statistical physics. This provides a general framework for interpreting such transitions. Critical slowdowns are the result of processes near critical points in the phase transitions of systems. Basically, these critical points are the limits beyond which the system switches from one state to another in a meaningful way.

It manifests itself by the slowdown in the recovery of the system towards its initial state following a disturbance. Such a slowdown can also be observed from the variance and autocorrelation coefficients of the data. In other words, critical slowdown indicators make it possible to detect a loss of stability in the system, therefore an increased tendency to transition to a new state.
This slowdown in recovery time toward an event manifests itself in the critical slowdown theoretical framework as an increasing trend in critical slowdown indicators in time series as catastrophic events approach. Indeed, before the system reaches a critical point and undergoes a significant transition, its dynamic characteristics change. That is, it becomes more variable and correlated, which creates warning signals for state change.

The objective of these indicators is to capture the characteristics of changes in time series that can signify the change of state of a dynamic system. Identifying these early signals helps anticipate impending transitions and take steps to mitigate their likely effects. Therefore, critical slowdown theory provides a powerful conceptual framework for understanding and predicting transitions in complex dynamic systems.

We can illustrate this theory as the “ball in the well” analogy to compare a system that is (on the left) far from tipping and (on the right) close to tipping



(a) stable System (b) System Close to Tilting

When the system is more stable (represented by the steeper-sided well), recovery from a given disturbance is faster (the ball returns faster). A system closer to tipping (represented by a shallower well) has a slower recovery after the same disturbance (the ball takes longer to return).

Indeed, the system that is furthest from tipping recovers more quickly from disturbances, with the steeper sides of the well describing the strongest restoration feedbacks to the system. Near the tilt, the sides of the well are shallower, so the system will take time to return from the same disturbance because the restoration feedbacks are weaker.

2.1.2 Fields of application : Climate, Financial crises and Cryptocurrency markets

This theory has become a generic early warning signal applied in most heterogeneous fields. It finds special applications in climate research, where it is used to forecast critical changes in climate and their possible effects. In finance, critical downturn theory finds its applications in the identification of warning signs of approaching financial crises.
In addition, it is applied in the cryptocurrency markets, such as bitcoin, in the detection of critical transitions and periods of increased volatility. This goes to demonstrate how strong and relevant critical slowdown theory is as a technique of analysis in the prediction and management of complex dynamic systems.

- **The Climate :**

Climate[3], as a complex dynamic system, has tipping points where it can change dynamic regimes. When the climate approaches critical tipping points, it can undergo rapid and irreversible changes, such as the accelerated melting of ice sheets or the intensification of extreme weather events. CSD indicators in this context can reveal significant trends in time series of temperature, precipitation or other climate variables, signaling critical transition phases. As the climate approaches a tipping point, an increasing trend is observed in the time series of CSD indicators, such as increasing variance and spectral density. By using critical slowdown theory, researchers can better understand climate change and anticipate extreme weather events.

- **Financial crises :**

Economic systems[4] can be viewed as complex dynamic systems with tipping points. When the economy is near a critical point, it may be susceptible to external disturbances, such as financial shocks or banking crises, which can trigger rapid transitions to alternative states. CSD indicators in this context could exhibit significant changes in the time series, signaling increased volatility or unusual correlation patterns between financial markets. Thus, critical downturn theory could be useful in identifying early warning signals of impending financial crises, thereby providing potentially valuable tools for economic risk management.

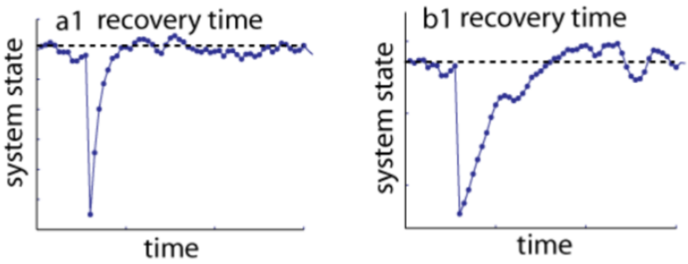
- **Cryptocurrency markets :**

Cryptocurrency markets are highly volatile and subject to significant fluctuations. As the Bitcoin price approaches a critical point, it may become more sensitive to external factors such as regulatory announcements, changes in investor sentiment, or the activities of large cryptocurrency holders. These disruptions can lead to abrupt changes in price dynamics, perhaps signaling periods of increased volatility or intense speculation. CSD indicators in this context could reveal unusual temporal patterns in Bitcoin price data, highlighting times when the market is near a critical point. Thus, the application of critical slowdown theory offers valuable tools for risk analysis and early detection of significant trends in cryptocurrency markets.

2.2 Early Warnings Signals

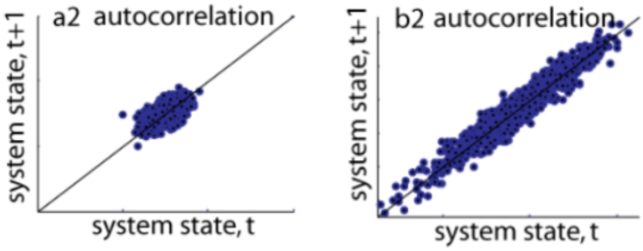
Early warning signals[7] are indicators that exhibit distinctive changes before a critical transition.
Some early warning signals may arise directly from critical slowing down, resemblance to its past, and increasing variance :

Slow recovery after disturbances : As the system approaches bifurcation, the recovery rate after small disturbances decreases.



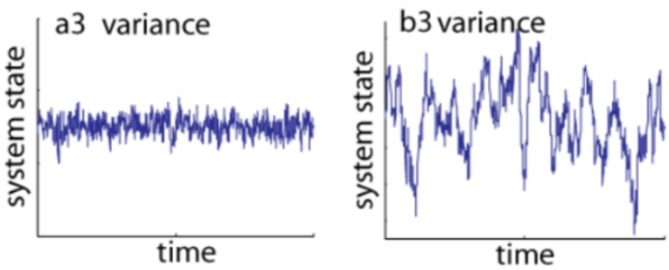
(a) Far from the critical transition (b) Close to critical transition

Increasing autocorrelation : As the system approaches the transition, its state shows increasing resemblance to its past state.



(a) Far from the critical transition (b) Close to critical transition

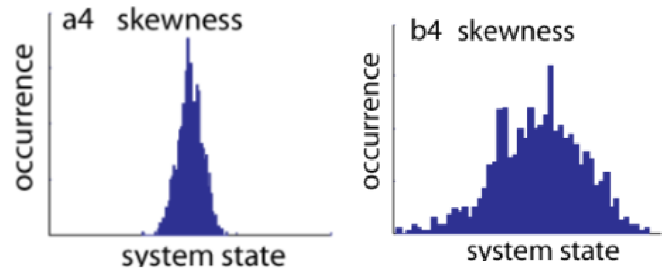
Increasing variance : Before the transition, the cumulative impact of non-decreasing shocks increases the variance [6] of the state variable.



(a) Far from the critical transition (b) Close to critical transition

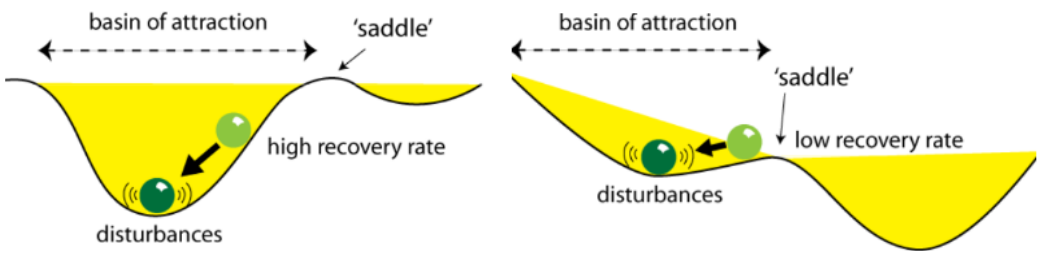
Other early warning signals may be related to asymmetries in the stability landscape or jumps between alternative basins of attraction:

Increasing skewness : Near saddle points, rates of change are low, resulting in increasing skewness [1] in the stability landscape. The system spends more time near the saddle, leading to a highly skewness distribution of the state variable.



(a) Far from the critical transition (b) Close to critical transition

Flicker : The probability of a stochastic disturbance temporarily moving the system between alternative basins of attraction is higher near a bifurcation. As a result, the variance and skewness of the state variable's frequency distribution increase.



(a) Far from the critical transition (b) Close to critical transition

3 Methodology

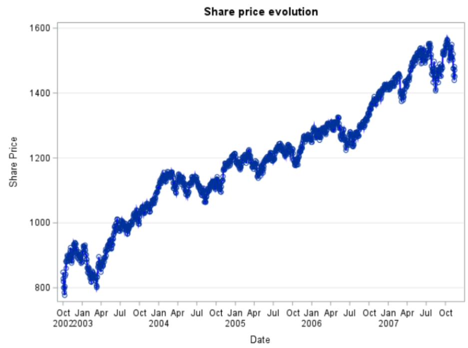
3.1 Data Exploitation

In this section, we will detail the methods used to conduct our research into early signals of financial crises, drawing on Critical Slowing Down (CSD) theory and other leading indicators. The rigorous methodology we have chosen aims to guarantee the validity and reliability of our results, while allowing in-depth and multidimensional analysis of the data.

- **Financial data :**

We selected the S&P 500 stock index between October 2007 and October 2009 as financial data to anticipate the crisis of 2008, also known as the "Great Recession", one of the most significant crises since the Great Depression of the 1930s.

(a) Share price evolution (1994-2017)



(b) Share price evolution (2002-2007)

This crisis began in the United States with the collapse of the subprime mortgage market and then spread globally, seriously impacting the economy and the financial system [2]. The causes of this crisis are diverse and complex: risky mortgage loans, the securitization of these loans into complex financial products such as Collateralized Debt Obligations (CDOs), the insufficient regulatory framework, the excessive leverage of financial institutions, errors rating agencies, among others. The consequences have been devastating, with major bank failures, credit crunches, massive job losses, stock market collapses and recessions in many countries.

Analyzing this crisis through the theory of Critical Slowing Down (CSD) offers an opportunity to evaluate the effectiveness of early warning signals in the context of a real financial crisis. According to CSD theory, as a critical transition approaches, a complex system shows signs of loss of resilience, detectable through statistical indicators such as first-order autocorrelation, variance, and asymmetry.

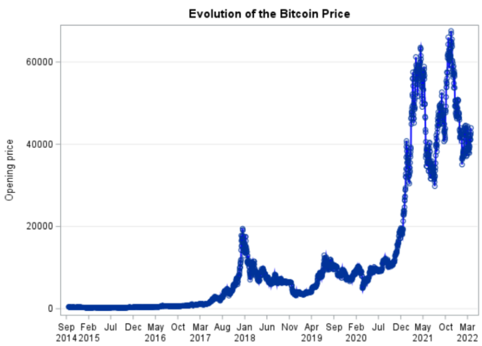
In summary, using S&P 500 stock index data complements our analysis by providing valuable insights into the dynamics of complex financial systems, their resilience, and their potential for change.

- **Bitcoin Data :**

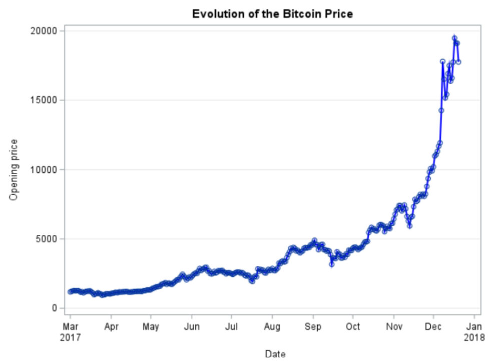
To enrich our analysis, we integrated data on bitcoin from March 2017 to December 2018 in order to identify possible critical transitions in a complex system subject to non-linear dynamics and multiple interactions.

During this period, Bitcoin experienced a meteoric rise, attracting increasing media attention and generating massive enthusiasm among investors, before suffering a brutal fall, thus marking one of the most tumultuous phases in the history of cryptocurrencies. currencies.

This rapid decline in the price of Bitcoin has been influenced by several factors, including increased regulation in certain countries, growing concerns about the security of cryptocurrency exchanges, as well as fears of speculative bubbles. One-off events, such as exchange hacks and negative statements from influential figures in the financial field, also contributed to worsening the price drop. This period of extreme volatility and rapid upheaval provides a favorable context for applying the concepts of Critical Slowing Down (CSD) theory.



(a) Bitcoin price (2014-2022)



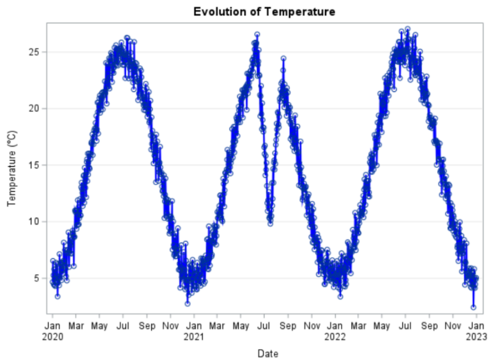
(b) Bitcoin price (2017-2018)

By closely analyzing the evolution of variance, skewness and autocorrelation before and during the Bitcoin price crash, we can assess whether these indicators may have provided warning signals of the impending crisis. This in-depth analysis will allow us to better understand the mechanisms underlying these drastic fluctuations and identify the key factors that contributed to the transition from rapid growth to the dramatic fall of Bitcoin.

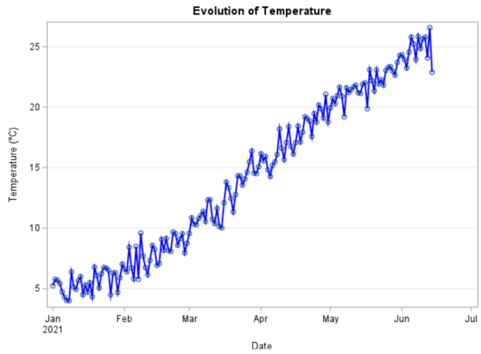
This in-depth understanding of cryptocurrency market dynamics can provide valuable insights for investors and regulators to assess the stability and resilience of these markets. By integrating these insights into the development of risk management strategies and regulatory policies, we will be able to better anticipate and mitigate the impacts of future crises in this emerging and constantly evolving area.

- **Meteorological Data :**

In order to complete our analysis, we considered the integration of meteorological data into our study to be crucial. By specifically incorporating weather data generated for our purposes, we were able to explore the potential impacts of different weather scenarios on other variables or systems.



(a) Evolution of temperature (2020-2023)



(b) Evolution of temperature (Jan 2021 - Jul 2021)

This approach allowed us to assess the direct impact of weather conditions on various aspects of our environment, while understanding how these effects could diffuse through complex networks and influence areas such as the economy, health public and the environment.

To do this, we used data generation code that relies on a sinusoidal function to reproduce seasonal variations in temperature, combined with a normal distribution to introduce noise and simulate random fluctuations. In addition, we intentionally introduced a significant drop in temperature between June 15, the peak heat period, and July 15, in order to simulate an unusual weather event. This steep drop resulted in a decrease of 17 degrees Celsius in just a few days, illustrating an extreme scenario of weather change.

3.2 Detrending Techniques

In time series analysis, a crucial step is to eliminate long-term trends to focus on short-term fluctuations, a process known as detrending[9]. This process is essential in our memory, as it helps reveal the underlying dynamics and early signals of critical transitions in various complex systems, such as financial markets, cryptocurrencies and climate data. By applying rigorous detrending methods, we can isolate meaningful variations that might otherwise be masked by long-term trends. This approach improves our ability to detect anomalies, predict potential crises and understand the interactions between different phenomena.

- **Financial Data : S&P 500**

The S&P 500 index, comprising the 500 largest companies listed in the United States, is a key barometer of the health of the American stock market. To prepare this data for our analysis, we follow several methodological steps: Calculation of daily returns: Daily returns are calculated using the logarithm of the ratio between the current day's closing price and that of the previous day. This logarithmic transformation stabilizes the variance of the data, which is crucial for time series analysis:

$$Dailyreturn = \log\left(\frac{Closingprice_t}{Closingprice_{t-1}}\right)$$

Data normalization: Daily yields are then normalized for easier comparisons and analysis. Normalization involves subtracting the average from daily returns and dividing by the standard deviation, placing all values on a common scale:

$$Normalizeddata = \frac{Dailyyield - Mean}{Standarddeviation}$$

Elimination of long-term trends (Detrending): To isolate short-term fluctuations and eliminate long-term trends, we apply a moving average smoothing method (MA) with a bandwidth of $\sigma = 15$. This method smoothes the data and eliminates long-term components:

$$MA_t = \frac{\sum_{r=1}^{T} G(r-t) z_r}{\sum_{r=1}^{T} G(r-t)}$$

where G is a Gaussian smoothing function. The detrended series is then obtained by subtracting the moving average from the original series:

$$y_t = z_t - MA_t$$

This transformation allows us to focus our analysis on significant short-term variations and identify early signals of potential financial crises.

- **Bitcoin Data :**

Bitcoins, as a digital currency and emerging financial asset, offer a unique perspective on the dynamics of modern financial markets. To prepare this data, we follow a similar detrending process:

Calculation of daily returns: Bitcoin daily returns are calculated in the same way as for traditional financial data, using the logarithm of the ratio between the current day's closing price and the previous day's closing price:

$$Dailyreturn = \log\left(\frac{Closingprice_t}{Closingprice_{t-1}}\right)$$

Data normalization: Daily bitcoin returns are then normalized by subtracting the mean and dividing by the standard deviation, allowing them to be directly compared with other time series:

$$Normalizeddata = \frac{Dailyyield - Mean}{Standarddeviation}$$

Elimination of Long-Term Trends (Detrending): To isolate short-term fluctuations, we also use a moving average smoothing method, adjusting the bandwidth appropriately for the bitcoin data:

$$MA_t = \frac{\sum_{r=1}^{T} G(r-t) z_r}{\sum_{r=1}^{T} G(r-t)}$$

The detrended series is obtained by subtracting the moving average from the original series:

$$y_t = z_t - MA_t$$

By processing bitcoin data in this way, we can identify early signals of volatility and critical transitions in this relatively new and often volatile financial market.

- **Meteorological Data :**

Weather data, particularly global temperature anomalies, are crucial for understanding the impacts of climate change and their potential interactions with financial systems. To prepare this data, we also apply a rigorous detrending process:

Calculation of temperature anomalies: Temperature anomalies are calculated using the logarithm of the ratio between the observed temperature and the average temperature of a reference period (for example, 1951-1980). This logarithmic transformation stabilizes the variance of the data:

$$Temperatureanomaly = \log\left(\frac{Observedtemperature}{Averagereferencetemperature}\right)$$

Data normalization: Temperature anomalies are normalized by subtracting the mean and dividing by the standard deviation, allowing these anomalies to be compared across different periods and regions:

$$Normalizeddata = \frac{Temperatureanomaly - Mean}{Standarddeviation}$$

Detrending: We apply a moving average smoothing method to eliminate long-term trends, using a Gaussian function to smooth the data and remove long-term components:

$$MA_t = \frac{\sum_{r=1}^{T} G(r-t) z_r}{\sum_{r=1}^{T} G(r-t)}$$

The detrended series is obtained as follows:

$$y_t = z_t - MA_t$$

This approach allows us to detect early signals of abrupt and potentially critical climate changes, which can have major implications for ecological and financial systems.

By applying these transformation and detrending methods to our financial, bitcoin and weather data, we ensure consistent and rigorous preparation of our time series. This allows us to consistently and accurately analyze early signals of critical transitions in different complex systems, thereby contributing to our understanding of underlying dynamics and prediction of financial and ecological crises.

3.3 Calculation of Leading Indicators

To anticipate financial crises and critical transitions in complex systems, the calculation of specific leading indicators based on the theory of Critical Slowing Down (CSD) is crucial. In our study, we focus on three main statistical indicators: first-order autocorrelation (AR(1)), variance and skewness. These indicators are applied to financial data, bitcoin data and weather data.

3.3.1 Autocorrelation of order 1 (AR(1))

First-order autocorrelation is calculated by estimating a first-order autoregressive model without a constant term. The AR(1) indicator is obtained by ordinary least squares (OLS) estimation. According to CSD theory, as a critical transition approaches, the first-order autocorrelation should increase, indicating that the system takes longer to return to its equilibrium state after a disturbance.

$$AR(1) : z_t = \phi z_{t-1} + \epsilon_t$$

where ϕ is the estimated autocorrelation coefficient and ϵ_t is the error term.

3.3.2 Variance

Variance, expressing the dispersion of data relative to the mean, provides insight into the stability of a given system. As a bifurcation approaches, an increase in variance suggests increasing instability. Indeed, a high variance means that the values are widely dispersed around the mean, thus indicating high variability or instability. Conversely, low variance indicates that values are more concentrated around the mean, reflecting greater stability and predictability in the system. Therefore, analysis of variance in the context of time series allows one to evaluate fluctuations and possible changes in behavior over time. For each sliding window, the variance is calculated according to the following formula:

$$Variance = \frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z})^2$$

where z_i are the values of the time series in the window, $\bar{z}$ is the average of these values, and n is the number of observations in the window.

3.3.3 Skewness

Skewness, which assesses the symmetry of the data around the mean, is crucial for detecting changes in the distribution of values, especially when approaching a critical transition. When a distribution is symmetrical, it has a balanced shape where the values are equally distributed on either side of the mean. On the other hand, an asymmetric distribution presents an imbalance in the distribution of values around the mean.

As a critical transition approaches, a decrease in asymmetry may indicate a change in the distribution of values. For example, a decrease in skewness can result in a broadening of the tail of the distribution, meaning that extreme or rare values become more frequent.

This phenomenon is often observed in complex systems as they move toward a critical or unstable state. The formula used to calculate the asymmetry is:

$$Skewness = \frac{\frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z})^2\right)^{3/2}}$$

where the terms are defined as before.

4 Analysis of Results

We will now analyze the significance of advanced indicators such as variance, skewness, and first-order autocorrelation in predicting various crises. [5]

4.1 Variance Analysis

- **Financial Data :**

To analyze the financial data, we observed the S&P 500 index over the period from April 2005 to December 2007. In particular, we focus on the evolution of variance, which is one of the key indicators of Critical Slowing Down (CSD) theory. Here is the detailed interpretation of the variance observed over this period.

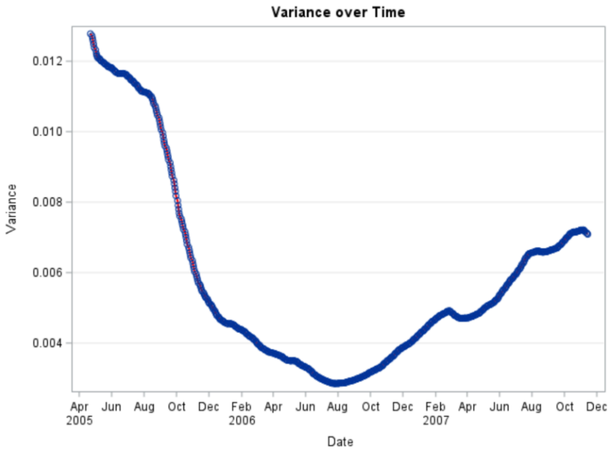


Figure 10: Variance over time

Initially, the variance shows a decreasing trend, from 0.012 in May 2005 to almost 0 in August 2006. This decrease in variance can be interpreted as a period of relative stability in market returns. This low variance phase indicates that fluctuations in returns have become extremely small, which can be seen as a period of market stabilization after previous turbulence.

However, after this period of low variance, we observe an increase in variance, reaching approximately 0.007 in December 2007. This recovery in variance is particularly significant because it can be seen as an early sign of increasing market instability. A marked increase in variance at this stage indicates that swings in returns are becoming larger, reflecting increasing uncertainty among investors and greater sensitivity to economic and financial news. This phenomenon is crucial because it often precedes critical transitions in the market, potentially heralding the arrival of a crisis.

- **Bitcoin Data :**

We chose to examine the variance on bitcoin data from March 2017 to January 2018. The variance graph reveals several key points when examined closely. During the period from April 2017 to December 2017, the variance oscillates between 0 and 0.1, signaling relative

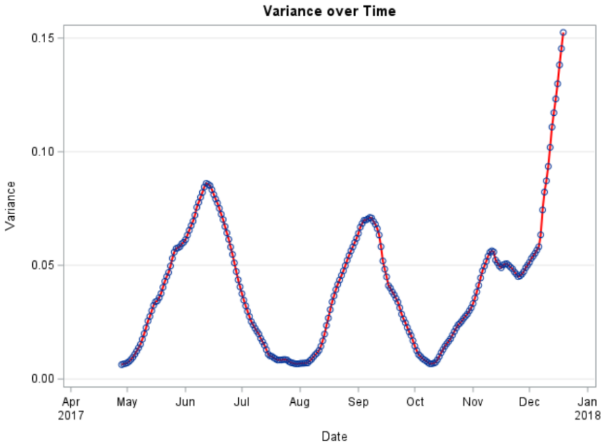


Figure 11: Variance over time

stability in Bitcoin prices. Initially, from April to mid-June 2017, the variance increases, then decreases from mid-June to August 2017.

This trend corresponds to an increase in Bitcoin prices until mid-June, followed by a slight decrease until to mid-July, then a slight increase from mid-July to September, and finally, a further decline until almost October.

However, in December 2017, a significant increase in variance is observed, reaching critical levels. It goes from 0.05 at the start of the month to 0.15 at the end, thus tripling in just one month. This rapid increase is a major signal of imminent change in the market.

Thus, the fluctuation of the variance between 0 and 0.1 during this period suggests relative stability with moderate volatility. This indicates that despite some variations, the Bitcoin market has not been significantly disrupted. Low and stable variance means that Bitcoin prices were generally predictable, and the market was relatively resilient to small fluctuations.

However, the sharp increase in variance in December 2017, reaching 0.15 by the end of the month, indicates a much wider dispersion in prices, signaling increasing instability. This variation suggests a loss of system resilience and an approach to a critical transition. Increased instability is typical of phases where the market becomes very sensitive to external and internal disruptions.

- **Meteorological Data :**

In our analysis of weather data, we observe a decline in variance.

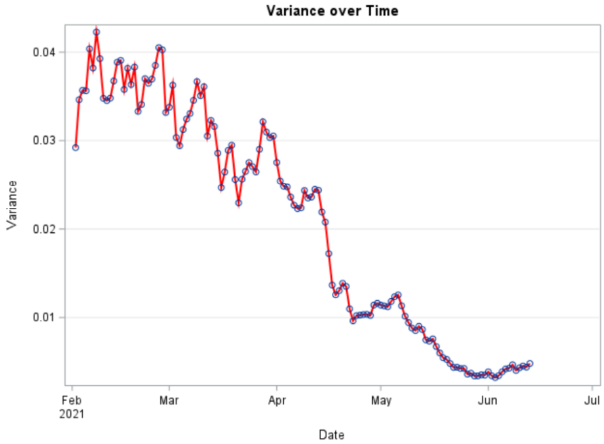


Figure 12: Variance over time

Initially fluctuating, the variance gradually decreases. In April 2021, it drops significantly, from 0.03 to around 0.01. A second notable decrease is observed from the beginning of May until June 1st. This decline may indicate relative stability in weather conditions, with fewer fluctuations and less sensitivity to disturbances.

However, a decrease in variance can also foreshadow an impending fall. According to Critical Slowing Down (CSD) theory, this may signal an increasing sensitivity of the system to disturbances, potentially leading to extreme weather events or significant climate change.

Towards the end of the observed period, after June 1, the variance begins to increase slightly. This recovery may be a harbinger of further instability in the weather system, with increased fluctuations and greater sensitivity to climate change, pointing to a possible weather crisis.

In summary, the initial drop in variance can be interpreted as a moment of relative calm but also as a warning of upcoming instability. This highlights the importance of carefully monitoring these signals to anticipate and manage risks associated with sudden fluctuations in weather conditions.

4.2 Skewness Analysis

- **Financial Data :**

To analyze early warning signals in the evolution of S&P 500 index returns, we observed the asymmetry (skewness) of the data over the period from November 2002 to October 2007.

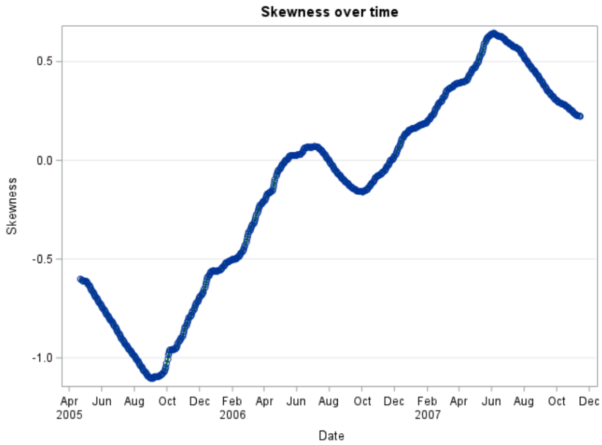


Figure 13: Skewness over time

Between April 2005 and September 2005, skewness decreases from nearly 0.6 to around -1.1, signaling an increase in negative tail returns, reflecting increased investor perceptions of risk. This move towards a more asymmetric distribution of returns suggests increasing market instability. After this sharp decline, the skewness begins to gradually increase, reaching around 0.2 in December 2007. This trend toward a positive or near-zero value can be interpreted as a sign of an impending critical market transition, indicating a loss of resilience according to the CSD theory.

Analyzing asymmetry during this period reveals crucial insights into the underlying dynamics of the financial market. Negative skewness, as observed between April and September 2005, suggests an increase in negative tail returns, reflecting increased perception of downside risk by investors. This period of strong negative asymmetry highlights increased instability and sensitivity to economic and financial events.

The subsequent rise in skewness to a positive value or close to zero by December 2007 is particularly concerning, as it may indicate a growing loss of market resilience, making it more vulnerable to potential crises. This fluctuation towards a more positive asymmetry suggests a market restructuring, a precursor to significant fluctuations and major disruptions.

According to Critical Slowing Down (CSD) theory, changes in asymmetry can be leading indicators of critical transitions in complex systems, such as financial markets. Frequent swings and increasing skew toward more positive values may signal a loss of market resilience as a crisis approaches. In particular, the evolution of the S&P 500 skewness from

2002 to 2007 shows a period of great instability, culminating with a significant increase in skewness in 2007, foreshadowing the 2008 financial crisis.
These observations highlight the importance of monitoring asymmetry as an indicator of market dynamics. However, its high variability may make this measure less reliable than variance in predicting critical transitions. Variance, more consistent across phases of stability and instability, could be a more robust indicator of market resilience and approaching critical transitions.

- **Bitcoin Data :**

The chart below reflects the asymmetry of bitcoin price data. During the month of May

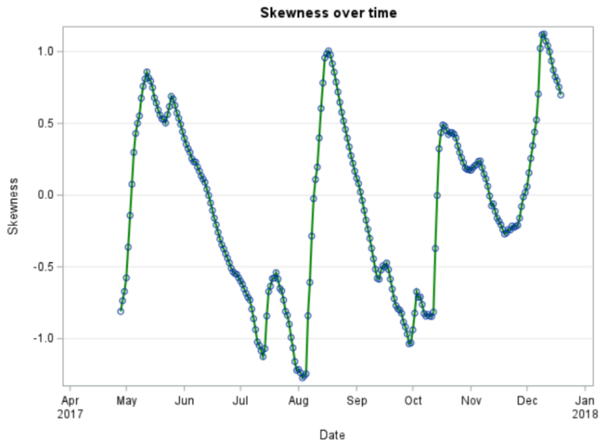


Figure 14: Skewness over time

2017, the skewness of the Bitcoin price changed, going from -0.8 to 0.8. This increase suggests a shift in yields towards positive values, signaling an increase in price increases relative to declines. Between mid-May and August 2017, the skewness decreased, reaching almost -1.5. This decline indicates greater fairness in returns, with more balanced price rises and falls. This period corresponds to a market correction, where Bitcoin prices gradually stabilize.

In August 2017, the skewness increased again, reaching 1. This rise indicates a resumption of significant price increases relative to declines, likely due to specific events or positive developments in the cryptocurrency sector. However, from mid-August to October 2017, the skewness dropped again to -1. During this period, the market was likely marked by uncertainty or trend reversals.

Between October and the end of December 2017, skewness overall increased from -1 to around 0.7, although it experienced temporary declines. It is important to note that before the fall in January 2018, skewness increased significantly in early to mid-December 2017, only to decrease before the fall.

Fluctuations in skewness between positive and negative values reflect changes in the perception of risk in the market. These variations are important to regulators and financial analysts because they can provide insights into potential critical transitions and periods of increased market vulnerability.

Compared to variance, skewness appears to be a less stable and perhaps less reliable indicator of Bitcoin market dynamics. While variance shows a more consistent trend across periods of stability and instability, skewness varies more frequently and less predictably. This suggests that variance may be a better leading indicator of market resilience and the approach of critical transitions according to Critical Slowing Down (CSD) theory.

- **Meteorological Data :**

By analyzing the temperature skewness graph before a major event, we see a fluctuation between positive and negative values.

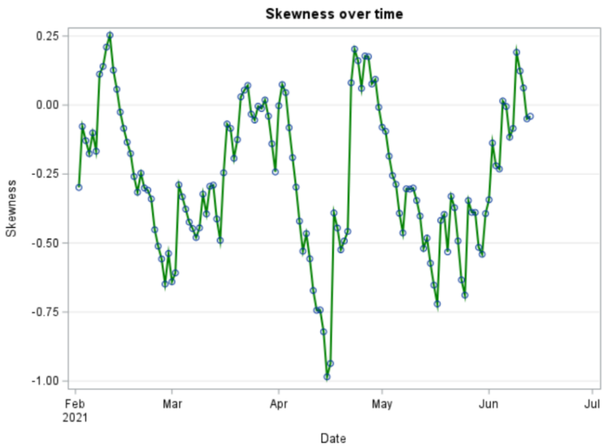


Figure 15: Skewness over time

The predominance of negative values suggests an asymmetric distribution of temperatures, with a tendency towards lower values. This observation corresponds to the cyclical nature of temperatures, often characterized by significant drops.

Negative skewness can signal a change in system behavior, potentially indicating increasing vulnerability to shocks. This implies that the system spends more time in negative extreme states, which may foreshadow increased instability.

Monitoring temperature skewness can therefore be crucial for early detection of signs of an imminent shock. An increase in negative asymmetry could warn of an upcoming critical event. If the skewness becomes progressively more negative, this could be a sign that the system is approaching a critical state, where a shock is likely.

4.3 Analysis of first-order autocorrelation :

- **Financial Data :**

First-order autocorrelation (AR(1)), measured for the S&P 500 Index over the period October 2002 to October 2007, provides crucial information about the persistence of market movements over time. Changes in autocorrelation can reveal trends and changes in financial market dynamics, and are particularly important within the framework of Critical Slowing Down (CSD) theory.

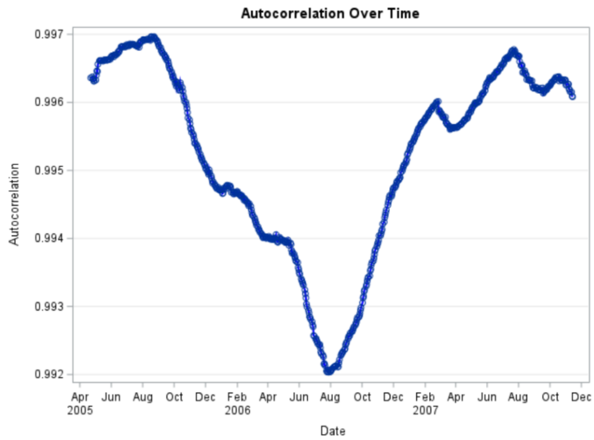


Figure 16: Autocorrelation over time

Between April 2005 and August 2006, the autocorrelation decreases notably, from 0.997 to 0.992. This decline indicates a decrease in the persistence of market trends, suggesting a period where investors react less consistently to economic and financial events. This may reflect a transitional phase where the market is less stable and more sensitive to minor disruptions.

However, after this period of decline, the autocorrelation begins to rise, reaching 0.997 again in December 2007. This increase in autocorrelation is particularly significant. According to CSD theory, an increase in autocorrelation can be an early sign of a critical transition. It says the system is showing an increasing resemblance to its past state, meaning the market is becoming slower to respond to disruptions, signaling a loss of resilience and increased vulnerability to an impending crisis.

Analyzing autocorrelation over this period reveals critical insights into the underlying dynamics of the financial market. High autocorrelation indicates that market movements are heavily influenced by past states, which can be interpreted as apparent but potentially misleading stability. The initial decline in autocorrelation between April 2005 and August 2006 suggests a phase of increased volatility and uncertainty, where market trends are less predictable.

The rise in autocorrelation towards the end of the period studied, peaking at 0.997 in December 2007, is particularly worrying. This increase may indicate that the market is approaching a critical point. In this context, high autocorrelation reflects a dynamic where the market becomes slower to react to new information, thereby increasing the risk of abrupt transitions and major crises.

- **Bitcoin Data :**

We will now look at the first-order autocorrelation graph of the bitcoin price.

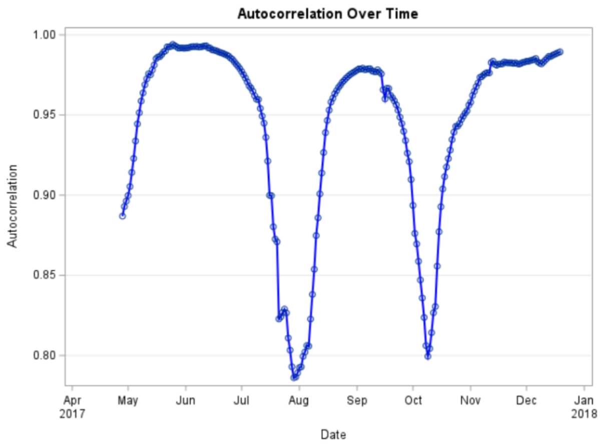


Figure 17: Autocorrelation over time

Now let's look at the first-order autocorrelation graph. We observe fluctuations between approximately 0.78 and 1, with alternating periods of growth and decay. As with variance, autocorrelation increases when Bitcoin prices rise and decreases when Bitcoin prices fall. For example, we see a growth in autocorrelation from April to mid-June, reaching its maximum at this time, followed by a decay until August. This pattern repeats from August to October, with growth followed by a decline in autocorrelation, corresponding to fluctuations in Bitcoin prices.

We particularly look at the period of growth in autocorrelation before the fall, from October 2017 to January 2018. This phase is interesting because it could indicate persistence in price fluctuations. Indeed, past data shows that when the price of Bitcoin experiences a slight decline, the autocorrelation also immediately drops, going from a peak to a trough. Thus, the strong growth in Bitcoin prices between October 2017 and January 2018, accompanied by an increase in autocorrelation from 0.8 to around 0.98, suggests an imminent fall.

Overall, the fluctuation of autocorrelation between 0.78 and 1 during this period reveals a marked persistence of Bitcoin price movements. An increase in autocorrelation indicates greater price predictability and a possible loss of market resilience, typical signals of a critical transition, such as a major price drop.

Changes in autocorrelation can be linked to specific market events, such as sudden price increases or regulatory announcements. For example, the rise in autocorrelation at the end of 2017 could be associated with a series of speculative events and positive news around Bitcoin. However, this rapid increase and the strong autocorrelation signaled underlying instability, anticipating the fall in early 2018.

In conclusion, increases in autocorrelation can be early warning signals for major price declines. In the case of Bitcoin, high autocorrelation before the significant drop in 2018 confirms this hypothesis. Monitoring autocorrelation, combined with other indicators like variance, can be a valuable tool for anticipating crises and significant fluctuations in the Bitcoin market. Increasing autocorrelation can indicate increased price predictability, ironically signaling a loss of resilience and increasing market vulnerability.

- **Meteorological Data :**

In our study of weather data, autocorrelation, represented by AR(1), reveals essential information about the persistence of climate trends over time.

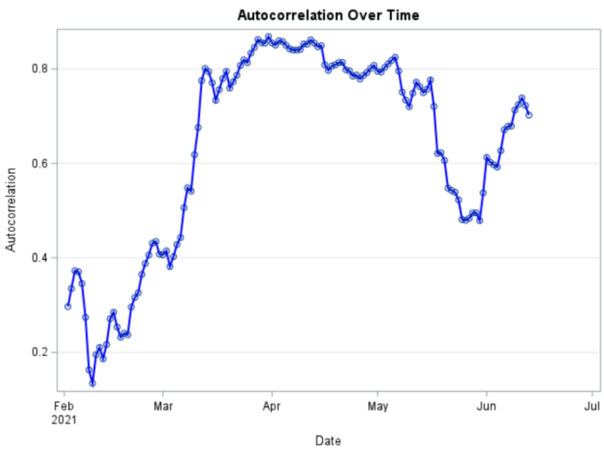


Figure 18: Autocorrelation over time

Initially, in February, we observe a decline in autocorrelation from around 0.4 to almost 0 in mid-February, suggesting a decrease in consistency in weather trends. However, towards the end of February through the beginning of March, the autocorrelation rises sharply to around 0.8, signaling a return to some form of persistence in climate trends. This sudden increase can be attributed to specific meteorological phenomena reinforcing the consistency of the observed conditions.

Between March and mid-May, the autocorrelation remains relatively high, oscillating around 0.8, indicating constant persistence in the observed climate trends. However, from mid-May until June, we observe a gradual decrease in autocorrelation, marking a decrease in the persistence of climate trends, perhaps due to a period of transition to more volatile conditions. Finally, around mid-June, the autocorrelation rises slightly, reaching around 0.7, suggesting a reappearance of persistence in climate trends.

The analysis reveals significant fluctuations in autocorrelation over the study period, highlighting the complexity of climate dynamics. A high autocorrelation indicates persistence in weather patterns, while a decrease may signal a decrease in consistency in trends, associated with increased volatility and extreme events.

Close monitoring of autocorrelation is crucial for anticipating future climate changes and understanding system resilience to environmental changes. This analysis can guide adaptation and mitigation measures in the face of extreme weather events, highlighting the importance of AR(1) as a valuable tool in climate risk management.

5 Conclusion

In conclusion, this dissertation aimed to evaluate the relevance of various indicators for predicting crises using the theory of Critical Slowing Down (CSD). This theory, derived primarily from physics and biology, proposes that complex systems exhibit warning signs of critical transitions, including a decrease in their resilience to disruptions. Building on this theory, we sought to determine whether specific indicators could signal impending crises, by studying their behavior in financial and weather contexts.

To do this, we selected three key indicators: variance, skewness and first-order autocorrelation (AR(1)). These indicators were chosen because of their ability to capture different aspects of complex system dynamics.

Variance has proven to be a good leading indicator for preventing crises in financial and cryptocurrency markets, where marked increases in variance preceded significant falls. However, in the context of weather data, the variance did not provide sufficiently clear early warning signals, highlighting the need to adjust parameters and perhaps use additional indicators for better crisis forecasting in this domain. We noted that the growth in variance turned out to be too insignificant to claim that we could identify critical transitions in the climate data. A decrease in variance may precede increasing instability, while an increase indicates increased vulnerability. However, the variance increased very little as the climate crisis approached.

The skewness indicator showed variable usefulness depending on the types of data analyzed. For financial data from the S&P 500 and the Bitcoin market, skewness provided insights into critical transitions and risk perceptions. However, its high variability relative to variance makes it a less stable and potentially less reliable indicator for predicting seizures. For weather data, skewness provides information about the asymmetry of the temperature distribution, signaling a predominance of large drops. Persistent negative skewness may indicate changing system behavior and increased sensitivity to shocks. In sum, although skewness can provide useful insights into system dynamics, variance remains a more robust and reliable leading indicator for anticipating critical transitions.

Autocorrelation of order 1 (AR(1)) has shown varying usefulness depending on the types of data analyzed. For financial data from the S&P 500 and the Bitcoin market, autocorrelation provided valuable insights into critical transitions and resilience losses, confirming its potential as a leading indicator. In contrast, for weather data, although autocorrelation can offer insights into climate dynamics, it appears to be a less reliable and less predictive indicator for critical transitions due to the complexity of climate systems and the multiple factors at play. However, among the three indicators, autocorrelation was found to be the most reliable for predicting critical points in weather data. In short, although these warning signals are not always 100% reliable, they can be very useful in anticipating critical transitions in various contexts.

6 Discussion

The analysis of different statistical indicators in this report highlights several crucial aspects of detecting critical transitions in complex systems. First, it is important to recognize that the effectiveness of indicators can vary depending on the type of data and the context studied. In the case of financial markets, variance, asymmetry and autocorrelation have proven to be valuable tools for anticipating crises and critical transitions, providing complementary perspectives on market dynamics. However, for weather data, the performance of these indicators was more mixed, highlighting the unique challenges of forecasting crises in complex natural systems.

An interesting aspect to consider is the combination of indicators for better anticipation of critical transitions. While each indicator offers valuable insight into system dynamics, using them in conjunction can make forecasts more reliable. For example, although variance may be less sensitive in a meteorological context, it can still provide useful insights when interpreted in combination with other measures such as skewness and autocorrelation. This holistic approach could be further explored in future studies to improve the prediction ability of critical transitions.

Another important consideration is the need to adapt analysis methods according to the specific characteristics of the data and the system studied. For example, in the case of meteorological data, it may be necessary to develop more sophisticated models taking into account the spatial and temporal variability of climatic phenomena. Similarly, for financial markets, incorporating additional data such as trading volumes or economic events could enrich the analysis and improve crisis prediction ability.

In conclusion, this study highlights the importance of developing robust and adaptive analysis methods to detect critical transitions in various complex systems. By combining deep theoretical understanding with advanced statistical techniques, it is possible to improve crisis forecasting and strengthen the resilience of systems to future shocks. This work provides valuable insight into the challenges and opportunities in this evolving research area, paving the way for further advances in risk prediction and management in contexts as diverse as financial markets and climate phenomena.

7 Opening

In a context where the interactions between financial markets, cryptocurrencies and climate change are becoming increasingly complex, early detection of crisis signals is of paramount importance. This section explores avenues of research that could enrich our understanding of critical transitions and open new perspectives in this evolving field.

- **Development of New Leading Indicators :**

Future research should focus on developing new leading indicators based on Critical Slowing Down (CSD) theory. These indicators could be designed to capture specific aspects of financial and non-financial system dynamics that are not fully explored by existing indicators. For example, metrics based on spatio-temporal correlation between different financial assets or fluctuations in volatility could provide valuable insights into critical phases of markets.

- **Multiple Data Integration :**

Integrating data from multiple sources, such as social media, investor sentiment data, and macroeconomic data, represents an important opportunity to improve the understanding of critical transitions in complex systems. Future research should explore advanced data integration methods, such as transfer learning and deep learning, to extract hidden patterns and nonlinear relationships between different data sources.

- **Modeling of Complex Interactions:**

Given the interconnected nature of financial and non-financial systems, it is imperative to develop more sophisticated models that can capture the complex interactions between variables. Approaches based on complex networks, machine learning and artificial intelligence offer promising possibilities to more realistically model emerging dynamics and transition phenomena. For example, recurrent neural networks can be used to model temporal interactions between different financial time series.

- **Studies on Exogenous Factors :**

Future research should pay particular attention to identifying and modeling exogenous factors that may influence financial crises. Analyzing geopolitical events, regulatory changes, and technological innovations could provide valuable insights into potential crisis triggers and ways to anticipate them. For example, longitudinal studies could be conducted to assess the impact of monetary and fiscal policies on long-term financial stability.

- **Real-Time Validation and Application :**

It is crucial to validate leading indicators in real-time environments to assess their effectiveness in detecting emerging financial crises. Future research should strive to develop tools and methodologies to monitor and prevent crises in real time, thereby strengthening the capacity of regulators and policymakers to take preventive action. For example, systemic risk monitoring platforms could be developed to detect early warning signals and inform decision-makers in real time of potential threats to financial stability.

The avenues for future research identified in this section pave the way for significant advances in the detection and prevention of financial crises. By investing in these areas, researchers could help strengthen the resilience of financial markets and mitigate systemic risks, thereby providing tangible benefits to the global economy and society as a whole. A collaborative and interdisciplinary approach will be essential to address these complex challenges and meet the challenges of the modern financial world.

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