EEE5062计算方法 作业九

习题P275：1、3(1)

计算实习题P277：1（代码、输出、备注等，清晰截图）

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Q1

1. 利用格什戈林圆盘定理估计下面矩阵特征值的界：

(1) $\begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 1 \\ -1 & -1 & 2 \end{bmatrix}$； (2) $\begin{bmatrix} 4 & -1 & & & \\ -1 & 4 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 4 & -1 \\ & & & -1 & 4 \end{bmatrix}$.

解：（1） 根据格什戈林圆盘定理，矩阵的特征值λ_i分别位于圆盘$|\lambda_1-(-1)|\le 2, |\lambda_2-0|\le 1, |\lambda_3-2|\le 1$之内

即矩阵特征值的界为 $-3\le\lambda_1\le 1, -1\le\lambda_2\le 1, 1\le\lambda_3\le 3$

（2）根据格什戈林圆盘定理，矩阵的特征值λ_i位于圆盘$|\lambda_i-4|\le 2, i=1,2,\ldots,n$之内

即矩阵特征值的界为 $2\le\lambda_i\le 6, i=1,2,\ldots,n$

Q3（1）

3. 用幂法计算下列矩阵的主特征值及对应的特征向量：

(1) $\mathbf{A}_1=\begin{bmatrix} 7 & 3 & -2 \\ 3 & 4 & -1 \\ -2 & -1 & 3 \end{bmatrix}$；

当特征值有 3 位小数稳定时迭代终止.

解：使用幂法公式，$u_0\ne 0, v_k=Au_{k-1}, u_k=\dfrac{v_k}{max(v_k)}, k=1,2,\ldots$

取$u_0=(1,1,1)^T\ne 0$，根据给定的A_1，代入得：

k	u_k^T	$max(v_k)$
1	[1. , 0.75, 0.]	8
2	[1. , 0.64864865, -0.2972973]	9.25
3	[1. , 0.61756374, -0.37110482]	9.54054054054054
4	[1. , 0.60879835, -0.38883968]	9.594900849858357
5	[1. , 0.60641274, -0.39309539]	9.604074402125775
6	[1. , 0.60577683, -0.39412075]	9.605429001813766
7	[1. , 0.60560975, -0.39436892]	9.60557200236834

故A_1的主特征值$\lambda_1 = 9.60557200236834$, 特征向量$x_1 = [1, 0.60560975, -0.39436892]^T$

计算实习题Q1

1. 已知矩阵

$$A = \begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 & 4 & 5 & 6 \\ 4 & 4 & 5 & 6 & 7 \\ 0 & 3 & 6 & 7 & 8 \\ 0 & 0 & 2 & 8 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_6 = \begin{bmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{7} \\ \vdots & \vdots & & \vdots \\ \frac{1}{6} & \frac{1}{7} & \cdots & \frac{1}{11} \end{bmatrix}.$$

(1) 用 MATLAB 函数“eig”求矩阵全部特征值.

(2) 用基本 QR 算法求全部特征值(可用 MATLAB 函数“qr”实现矩阵的 QR 分解).

(1) Using python, and numpy.linalg.eig() provides us with the same function as eig in matlab.

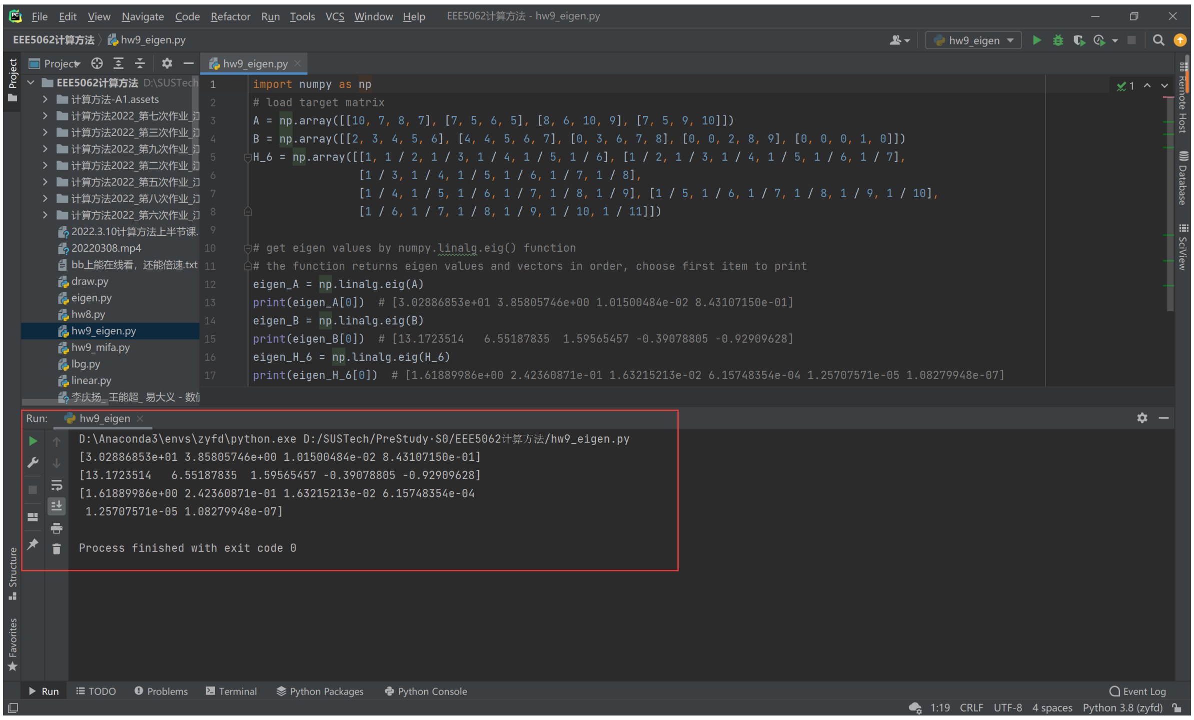
```
import numpy as np

# load target matrix
A = np.array([[10, 7, 8, 7], [7, 5, 6, 5], [8, 6, 10, 9], [7, 5, 9, 10]])
B = np.array([[2, 3, 4, 5, 6], [4, 4, 5, 6, 7], [0, 3, 6, 7, 8], [0, 0, 2, 8,
9], [0, 0, 0, 1, 0]])
H_6 = np.array([[1, 1 / 2, 1 / 3, 1 / 4, 1 / 5, 1 / 6], [1 / 2, 1 / 3, 1 / 4, 1
/ 5, 1 / 6, 1 / 7],
                [1 / 3, 1 / 4, 1 / 5, 1 / 6, 1 / 7, 1 / 8],
                [1 / 4, 1 / 5, 1 / 6, 1 / 7, 1 / 8, 1 / 9], [1 / 5, 1 / 6, 1 /
7, 1 / 8, 1 / 9, 1 / 10],
                [1 / 6, 1 / 7, 1 / 8, 1 / 9, 1 / 10, 1 / 11]])

# get eigen values by numpy.linalg.eig() function
# the function returns eigen values and vectors in order, choose first item to
print
eigen_A = np.linalg.eig(A)
print(eigen_A[0])  # [3.02886853e+01 3.85805746e+00 1.01500484e-02 8.43107150e-
01]
eigen_B = np.linalg.eig(B)
```

```
print(eigen_B[0])  # [13.1723514   6.55187835  1.59565457 -0.39078805
-0.92909628]
eigen_H_6 = np.linalg.eig(H_6)
print(eigen_H_6[0])  # [1.61889986e+00 2.42360871e-01 1.63215213e-02
6.15748354e-04 1.25707571e-05 1.08279948e-07]
```

Output screenshot:



(2) Using python, and numpy.linalg.qr() provides us with the same function as qr in matlab. Then iterations start until reaching limitations.

```
import numpy as np
# load target matrix
A = np.array([[10, 7, 8, 7], [7, 5, 6, 5], [8, 6, 10, 9], [7, 5, 9, 10]])
B = np.array([[2, 3, 4, 5, 6], [4, 4, 5, 6, 7], [0, 3, 6, 7, 8], [0, 0, 2, 8,
9], [0, 0, 0, 1, 0]])
H_6 = np.array([[1, 1 / 2, 1 / 3, 1 / 4, 1 / 5, 1 / 6], [1 / 2, 1 / 3, 1 / 4, 1
/ 5, 1 / 6, 1 / 7],
                [1 / 3, 1 / 4, 1 / 5, 1 / 6, 1 / 7, 1 / 8],
                [1 / 4, 1 / 5, 1 / 6, 1 / 7, 1 / 8, 1 / 9], [1 / 5, 1 / 6, 1 /
7, 1 / 8, 1 / 9, 1 / 10],
                [1 / 6, 1 / 7, 1 / 8, 1 / 9, 1 / 10, 1 / 11]])

# QR method
def eigen_qr(matrix, eps=1e-7):
    # define iteration and limitation
    matrix_k = matrix
    while True:
        matrix_k_1 = matrix_k
        q_k, r_k = np.linalg.qr(matrix_k)  # using np.linalg.qr() for qr
decomposition
        matrix_k = np.dot(r_k, q_k)
        if np.sum(np.diag(np.abs(matrix_k - matrix_k_1))) < eps:
            # when sum of elements in diagonal is less than limit, stop
iteration
            break
    return np.diag(matrix_k)
```

```
print(eigen_qr(A))
print(eigen_qr(B))
print(eigen_qr(H_6))
```

Output screenshot: