EEE5015: Machine Learning & Artificial Intelligence

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Lecture 05: Regression

- Regression
 - Features
 - > Training examples
 - ➤ Model
 - > Loss function
 - Optimization

o What should I watch this Friday?



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o Goal: Predict movie rating automatically!



o Goal: How many followers will I get?



. Goal: Predict the price of the house





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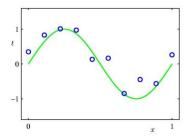
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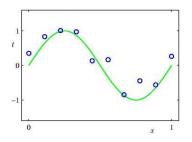
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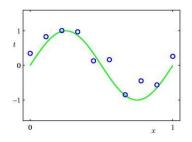
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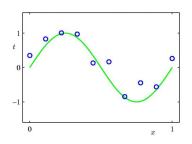


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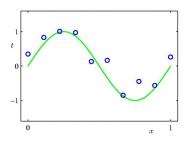
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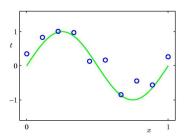
- In green is the "true" curve that we don't know
- Goal: We want to fit a curve to these points



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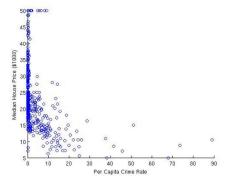


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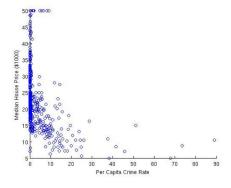
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- ► How do we optimize fit to unseen test data (generalization)?

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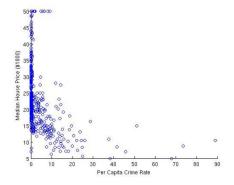


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- Look at first possible attribute (feature): per capita crime rate



- Use this to predict house prices in other neighborhoods
- Is this a good input (attribute) to predict house prices?

- Data is described as pairs $\mathcal{D} = \{(x^{(1)}, t^{(1)}), \cdots, (x^{(N)}, t^{(N)})\}$
 - $x \in \mathbb{R}$ is the input feature (per capita crime rate)
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- Divide the dataset into training and testing examples
 - ► Use the training examples to construct hypothesis, or function approximator, that maps *x* to predicted *y*
 - Evaluate hypothesis on test set

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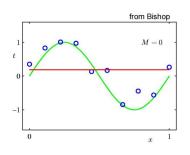
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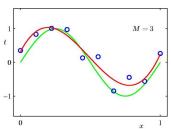
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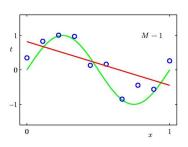


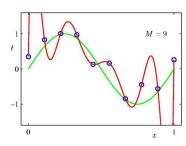
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 - Additional attributes not taken into account by data attributes, affect target values (latent variables). In the example, what else could affect house prices?
 - Model may be too simple to account for data targets

Which fit is best?









```
import torch
    import torch.nn as nn
    import numpy as np
    import matplotlib.pvplot as plt
    # Hyper-parameters
   input size = 1
   output size = 1
10 num epochs = 60
    learning rate = 0.001
13 i iter = np.array([])
    loss iter = np.array([])
    # Toy dataset
    x train = np.array([[3.3], [4.4], [5.5], [6.71], [6.93], [4.168],
18
                        [9.779], [6.182], [7.59], [2.167], [7.042],
                        [10.791], [5.313], [7.997], [3.1]], dtype=np.float32)
    y_train = np.array([[1.7], [2.76], [2.09], [3.19], [1.694], [1.573],
                        [3.366], [2.596], [2.53], [1.221], [2.827],
                        [3.465], [1.65], [2.904], [1.3]], dtype=np.float32)
    # Linear regression model
    model = nn.Linear(input size, output size)
```

```
# Loss and optimizer
    criterion = nn.MSELoss()
    optimizer = torch.optim.SGD(model.parameters(), lr=learning rate)
    # Train the model
    for epoch in range(num epochs):
        # Convert numpy arrays to torch tensors
        inputs = torch.from numpy(x train)
        targets = torch.from numpy(v train)
        # Forward pass
        outputs = model(inputs)
        loss = criterion(outputs, targets)
        # Backward and optimize
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
        i iter = np.append(i iter, epoch)
47
        loss iter = np.append(loss iter, loss.item())
        if (epoch+1) % 5 == 0:
            print ('Epoch [{}/{}], Loss: {:.4f}'.format(epoch+1, num epochs, loss.item()))
```

```
# Plot the graph
    predicted = model(torch.from_numpy(x_train)).detach().numpy()
    plt.figure()
    plt.plot(x_train, y_train, 'ro', label='Original data')
    plt.plot(x train, predicted, label='Fitted line')
    plt.legend()
    plt.figure()
    plt.plot(i iter,loss iter)
    plt.show()
64
    # Save the model checkpoint
```

torch.save(model.state dict(), 'model.ckpt')

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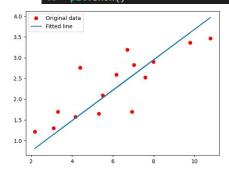
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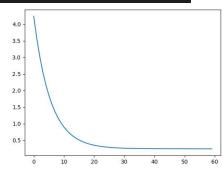
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plt.plot(i_iter,loss_iter)

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```





Summary of Regression

Regression: to predict continuous outputs *t*

- Consider proper features (inputs): x (or x if vectors)
- Training examples, many x(i) for which t(i) is known (labeled)
- A model, a function that represents the relationship between x and t

$$y = f(x, w)$$

- A loss or a cost or an objective function, which tells us how well our model approximates the training examples
- Optimization, a way of finding the parameters w of our model that minimizes the loss function