Modelling a Cold Atom using a Driven Gyroscope

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Abstract

A gyroscope is proposed to model a cold atom using the Bloch sphere concept. The inner disk of a gyroscope will be driven by a motor and a torque in the vertical axis will be applied using permanent magnets in a magnetic field on the central gimbles of the gyroscope causing it to process. The angular velocity of the spinning inner disk and procession of the gyroscope will be quantified, and an analysis carried out to establish the accuracy of the gyroscope as a model of the Bloch sphere and therefore the cold atom.

Introduction

The Faraday disk (or homopolar generator) is a direct current generator that works due to electromagnetic induction. It was proposed to build a Faraday disk that would be driven in the Earth's magnetic by passing a current along the gyroscope's inner disk radius and measuring the angular velocity to determine the strength of the Earth's magnetic field. This was discovered to be unfeasible, so it is now proposed to build on a pervious project carried out in the second-year labs to use the gyroscope as a model of the cold atom. A cold atom is a quantum system, it can be represented geometrically as a Bloch sphere and a gyroscope can be used to model this due to similarities in their equations of motion. The spin of the inner disk of the gyroscope represents the angular momentum or intrinsic spin of the atom and the procession caused by an applied torque is very like the procession of a cold atom when it's magnetic moment interacts with an external magnetic field [7].

The gyroscope is proposed to be driven first by passing a current along the radius of the inner disk as a Faraday disk then secondly through the permanent magnets placed on the disk gimbles as the magnets have a stronger magnetic field so would provide a greater torque. The effect of electromagnetic induction is the same and driving force produced is equivalent for both methods following Faraday and Lenz's laws of electromagnetic induction.

The presence of friction is unavoidable in almost all mechanical systems including the Faraday disk. The motion of the disk can be modelled as an angular oscillator: $I\alpha + b\omega = \tau$ (1) where the first part is the torque of the driving force (I is the moment of inertia and α the angular acceleration) and the second part is the torque due to friction (b is the friction constant and ω is the angular velocity). A solution to this equation is: $\omega = \omega_0 e^{-\frac{b}{I}t}$ (2) where ω is angular velocity, ω_0 is initial angular velocity, b is the friction constant, I is the moment of inertia and t is the time passed. The force on a current in a magnetic field is given by $\boldsymbol{F} = I\boldsymbol{l} \times \boldsymbol{B}$ **[3]** (3), where $\boldsymbol{F}$ is the vector force, I is the current, $\boldsymbol{l}$ is the length travelled by the current and $\boldsymbol{B}$ is the magnetic field. Torque is given by $\tau = \boldsymbol{F} \times \boldsymbol{r}$ (4) where $\boldsymbol{F}$ is the force and $\boldsymbol{r}$ the radius of the object, therefore the torque exerted by a current in a magnetic field is: $\tau = BIlr$ (5) when the current is perpendicular to the magnetic field. Equating the torque given by friction (in equation 2) and the torque on a current in a magnetic field (equation 5), angular velocity can be related to the friction and current by: $\omega = \frac{BIlR}{b}$ (6) where ω is angular velocity, B is magnetic field strength, I is the moment of inertia, R is the radius of the disk and b is the friction constant. Driving the gyroscope using electromagnetic induction was discovered to be unfeasible using both methods, the gyroscope will instead be driven by a motor which sadly is a less elegant method but is fit for purpose.

A torque will be applied to the gyroscope using permanent magnets on the gimbles of the inner disk, this will cause it to process. The torque is given by: $\boldsymbol{\tau} = \boldsymbol{B}_{ext} \wedge \boldsymbol{\mu}$ (7) where $\boldsymbol{\tau}$ is the torque, $\boldsymbol{B}_{ext}$ is the strength of the external magnetic field and $\boldsymbol{\mu}$ is the magnetic moment of the permanent magnets. The magnetic moment of the permanent magnets is given by: $\boldsymbol{\mu} = \frac{2\boldsymbol{B}_{mag}\wedge\boldsymbol{R}}{\mu_0} \wedge \boldsymbol{A}$ (8) where $\boldsymbol{\mu}$ is the magnetic moment of the permanent magnets, $\boldsymbol{B}_{mag}$ is the field strength of the permanent magnets, $\boldsymbol{R}$ is the radius of the permanent magnets and $\boldsymbol{A}$ is their cross-sectional area. The procession rate of a gyroscope is given by $\Omega = \frac{\tau}{I\omega}$ (9) where Ω is the procession rate, τ is the torque and I is the moment of inertia and ω is the angular velocity of the inner disk of the gyroscope [1] & [8]. Providing this torque using magnets is preferable to a torque from gravity by placing a mass on one side as it will not cause a nutation in the gyroscope that would need to be quantified along with the many other ranges of motion.

The inner disk of the gyroscope must be driven to keep it at an appropriate angular velocity for a reasonable procession rate to be produced and this must be done for an extended period. The decay of velocity and dependence of the inner disk's angular velocity on the procession rate was one of the main limiting factors of the previous experiment carried out on this gyroscope [1]. By driving the inner disk, it can be kept at a chosen angular velocity for a long period of time allowing consistent measurements to be made. Another limiting factor on the pervious experiment was the inconsistency of the friction given by the bearings [1], by driving the gyroscope this effect is eliminated.

Method

The experiment will be carried out using a gyroscope built for a previous project and will further develop their experiment. The gyroscope will be driven mechanically using a battery powered motor that will be designed in week 4 and time has been allowed during weeks 4-6 for it to be acquired. Care will be taken to balance out the weight of this motor on the other side of the gyroscope such that no unwanted torque is produced. The driven gyroscope will have magnets symmetrically along the arms providing a torque in the vertical axis causing the gyroscope the process. The magnets and motor will be fixed in week 7 such that measurements can be taken in week 8.

The angular velocity of both the gyroscope's inner disk spin and procession is to be measured using a tachometer RM1501[6]. The tachometer needs a clear mark on the gyroscope surface to read once in each cycle, this is achieved by the disk surface being covered with black card with one white line painted onto it and is proving to be effective. The tachometer will be connected to the computer to read out the data directly, therefore, increasing the accuracy of measurement by eliminating potential human inaccuracy and minimising workload. The data is outputted from the tachometer in an odd format as hexadecimals in a pattern of bytes. The instruction manual is used to decode the format and a process followed to obtain a readable result for the angular velocity of the gyroscope. A python programme will be written enabling the data to be automatically interpreted and made into a legible format for analysis. This will be carried out in weeks 5-7.

The effectiveness of driving the procession of the driven gyroscope with a magnetic field will be analysed and compared to the equations of motion of a Bloch sphere. From this comparison it can be concluded how accurate the gyroscope is as a model of the Bloch sphere and therefore of a cold atom.

Feasibility Study

A feasibility study has been carried out through research into the theory and previous experiments carried out within this area and preliminary testing has taken place. The moment of inertia and friction constant were estimated for the gyroscope such that an order of magnitude value for the driving current needed could be obtained to see if the Faraday experiment would be possible.

The estimate for the required current to drive the gyroscope was obtained by manually spinning the gyroscope and measuring its angular velocity as it decayed at five spaced intervals. The angular velocity was read off the screen of the tachometer and the time from a stopwatch simultaneously to obtain these values. The moment of inertia of the gyroscope's inner disk was calculated by measuring the dimensions, taking a known value for the density of brass and using known equations for the moment of inertia of a disk. Equation (2), stated above, was then used to calculate the friction constant for these values of time and angular velocity. The average value of the friction constant is $(3 \pm 2) \times 10^{-5} kgm^3s^{-1}$ but it is dependent on the angular velocity hence the large error in the result. However, as an order of magnitude estimate is required this result is useful.

Using equation (6) and a rough value of the Earth's magnetic field an approximate value of 37000A was obtained for the driving current required. This is unreasonably large by four orders of magnitude making the experiment unfeasible as this high value of current cannot be produced and is far too dangerous to use in the laboratory. As previously proposed, the design of the new gyroscope cannot sufficiently reduce the current needed through altered dimensions making that plan unfeasible. Therefore, a Faraday disk cannot be built for this project and so the Earth's magnetic field cannot be measured in this way. It is assumed that Faraday achieved his experimental aims with his disk as he was driving it manually and measuring the small electromotive force produced.

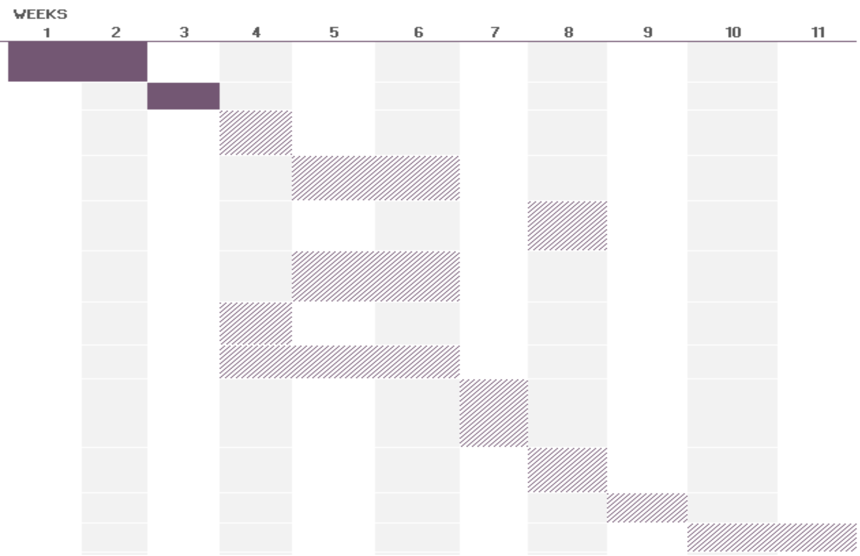
The original plan of driving the gyroscope then applying a torque onto it as a Bloch sphere is returned to and a feasibility study carried out on this plan. With the magnets placed along the gimbles of the gyroscope, as alternatively suggested, the current required to drive the gyroscope (obtained in the same way as above) is roughly 80A. This is still unfeasibly large, so the gyroscope cannot be driven in this manner either and so cannot be driven by electromagnetism at all.

A feasibility study is also done for the processional torque provided by the permanent magnets placed on the gyroscope arms. Using equations (7), (8) and (9) and the angular velocity of the disk rotation as 10rpm (the minimum range of the tachometer to give the maximum procession rate) the procession rate to an order of magnitude is estimated as 0.005 ± 0.001 rads^{-1}. This gives a rough period 1090s per procession which is too long a period for the tachometer to register. This will be overcome by marking more than one line, evenly spaced on the black card for the tachometer to read then dividing the angular velocity given by that number of lines. Using this method, the procession can still be measured using the tachometer, so the experiment is feasible.

If the plan stated above cannot be carried out for any reason and experimental results become impossible to obtain, a study of the undriven motion of the original gyroscope will be carried out.

Schedule

ACTIVITY	PLAN START	PLAN DURATION	ACTUAL START	ACTUAL DURATION	PERCENT COMPLETE
Creating concept	1	2	1	2	100%
Proposal	3	1	3	1	90%
Presentation creation	4	1			0%
Presentation giving	5	2			0%
Project demonstration	8	1			0%
Tachometer calibration	5	2			0%
Motor calculation	4	1			0%
Finding motor	4	3			0%
Motor and magnet attachment	7	1			0%
Measurement taking	8	1			0%
Analysis	9	1			0%
Report writing	10	2			0%



Conclusion

Unfortunately, the gyroscope cannot be driven as a Faraday disk by the Earth's magnetic field due to numerous factors including predominantly that the Earth's magnetic field is too weak, meaning an unfeasibly large current of 37000A would be needed which cannot be produced in the lab. The current needed is also too high (at 80A) to drive the rotation of the gyroscope with magnets placed on its gimbles meaning that the inner disk of the gyroscope cannot be driven using electromagnetic induction. The gyroscope will instead be driven mechanically using a fixed motor and magnets will be placed on its gimbles. The magnets will provide a torque on the gyroscope causing it to process due to the driven rotation of the inner disk. The procession of the gyroscope will be quantified using the tachometer and used to analyse the accuracy of the gyroscope as a model of the cold atom with the Bloch sphere model.

References

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