Laboratory Proposal

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Modelling a Cold Atom using a Driven Gyroscope

Introduction

The project was started by looking into the principles of electromagnetic induction with the Faraday disk. The Faraday disk, also known as the homopolar generator, was used by Faraday to demonstrate the ability of a conducting rotating disk to generate a current when in motion (from the axis to the rim of the conductor). This was the first example of an electromagnetic generator demonstrating his principle of electromagnetic induction. Here (in Maxwell's differential form) he proposed that:

$$\boldsymbol{\nabla} \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Equation 1: Differential form of Faraday's law

Here with Stokes's theorem Faraday's law can also be written as:

$$\oint_{\Gamma} \boldsymbol{E} \cdot d\boldsymbol{S} = -\frac{d}{dt}\int_{S} \boldsymbol{B} \cdot d\boldsymbol{S}$$

Equation 2: Stokes's theorem with Faraday's law

Thus, showing when the flux of the magnetic field changes, the electrons experience an electromotive force causing a current. [1] The use of such a device has enabled stored rotational kinetic energy to be converted into electrical energy. The device is a low voltage, high current device which can store a large amount of energy continuously from a low power source. The rotor that is spun in order to achieve this, is supported by bearings which are supported by a further 'yoke' like structure. A type of electrical brush is used to transfer current to and from the rotor, with a dc power sourced used to drive an external magnetic field. There are two main electrical configurations for homopolar generators called disk and drum. The disc machines use a magnetic field applied axially in the rotor and a current flowing radially to produce the stopping torque (via the Lorentz force) during the discharge of the machine. Meanwhile, drum machines use exactly the opposite of this. [2]
Our apparatus (see Figure 1) has similarities to this important experiment [3].

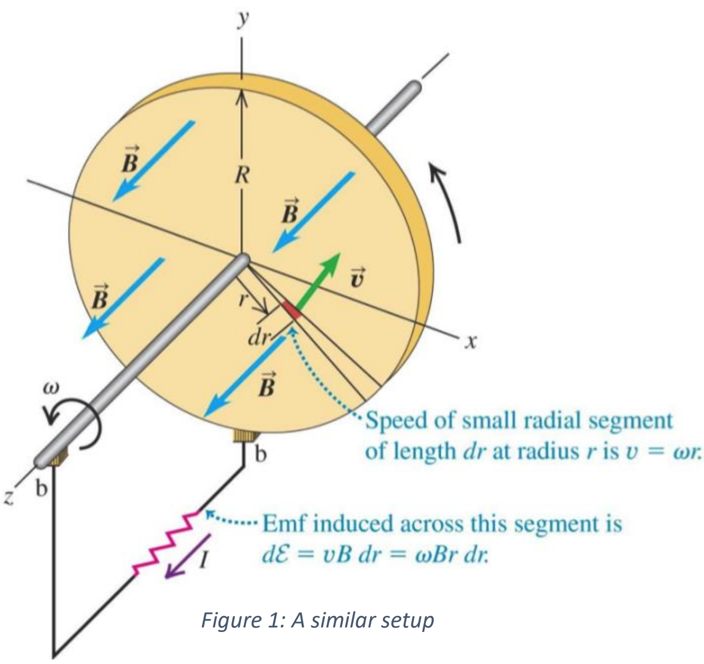


Figure 1: A similar setup

A cold atom can be represented geometrically as a Bloch sphere and the gyroscope can easily show many properties associated with its motion. The spinning of the rotor within a gyroscope is a good model of the intrinsic spin associated with the angular momentum of such objects. The system itself mirrors the two-level quantum mechanical system Bloch developed [4]. Here the general superposition state can be written as:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|\downarrow\rangle$$

Equation 3: The general superposition state

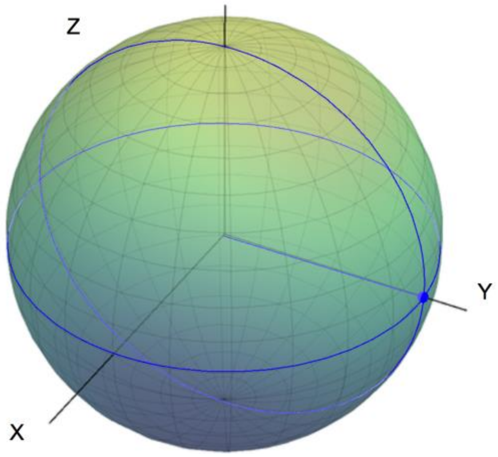


Figure 2: A Bloch sphere for a spin-1/2 particle

Where the angles θ and ϕ can be seen to be the azimuthal and polar angles [5].

Objective

The main objective of our experiment is to continue from a previous experiment in demonstrating the precession of a gyroscope using permanent magnets and a driving motor. Factoring into this aerodynamic frictional forces and how optical methods could enable rotational measurement. With this equipment, we aim to demonstrate that the torque due to an external field from an internal magnetic moment can cause a precession. Our secondary objective is then to measure the angular speed and acceleration of the disk alongside a fixed motor running at a constant slow angular speed. Here obtaining the direct hexadecimal readings from this device and deciphering and reformatting the results in a meaningful way such that a set of graphs can be made. Using the graphs, we aim to obtain a value of the magnetic field of the earth from the relationships between these quantities. Further, assessing the validity of some of the theorised relations of the variables.

Frictional considerations

Within our experiment we realised that we needed to incorporate the fact that dry friction and fluid drag would be present. With the frictional torque and angular speed of the form:

$$\tau_{frictional} = A + B\omega$$

Equation 4: Frictional torque

$$\omega = \left(\omega_0 + \frac{A}{B}\right)e^{-\frac{Bt}{I}} - \frac{A}{B}$$

Equation 5: Angular velocity with frictional parameters

(Here A and B are constants with I the moment of inertia of the disk taken to be $\frac{1}{2}mr^2$ for an ideal disk) [6].

Experimental procedure

A torque is created from the magnetic moment of the permanent magnets and the external magnetic field of the earth. This torque has a magnitude:

$$\tau = B_{ext} \wedge \mu$$

Equation 6: Torque from magnetic moments

Here the magnetic moment of the permanent magnets can be calculated using

$$\mu = \frac{2B_{by\ magnet} \wedge R}{\mu_0} A$$

Equation 7: Magnetic moment of permanent magnet

and the dimensions of the permanent magnet, incorporating the values of A and R. The precession rate as a result is

$$\Omega = \frac{\tau}{|L|} = \frac{\tau}{I\omega}$$

Equation 8: Rate of precession

Here I is the moment of inertia, L is the angular momentum and ω is the rate of rotation of the spinning disk (who's minimum value achievable, we know to be $10\ rpm$ using the tachometer device RM1501 [7]).

Feasibility studies

The motion of the rotor spinning was seen to obey the simple harmonic oscillator equation:

$$I'\ddot{\theta} + b\dot{\theta} = \Gamma$$

Equation 9: Torque with a simple harmonic oscillator

Here $I'\ddot{\theta}$ is the applied torque with I' the moment of inertia, Γ is the resultant torque I is the moment of inertia and b is a constant representing the damping effect of friction. When the rotor is spinning the motion of the rotor obeys the equation:

$$\dot{\theta} = \dot{\theta}_0 e^{-\frac{b}{I}t}$$

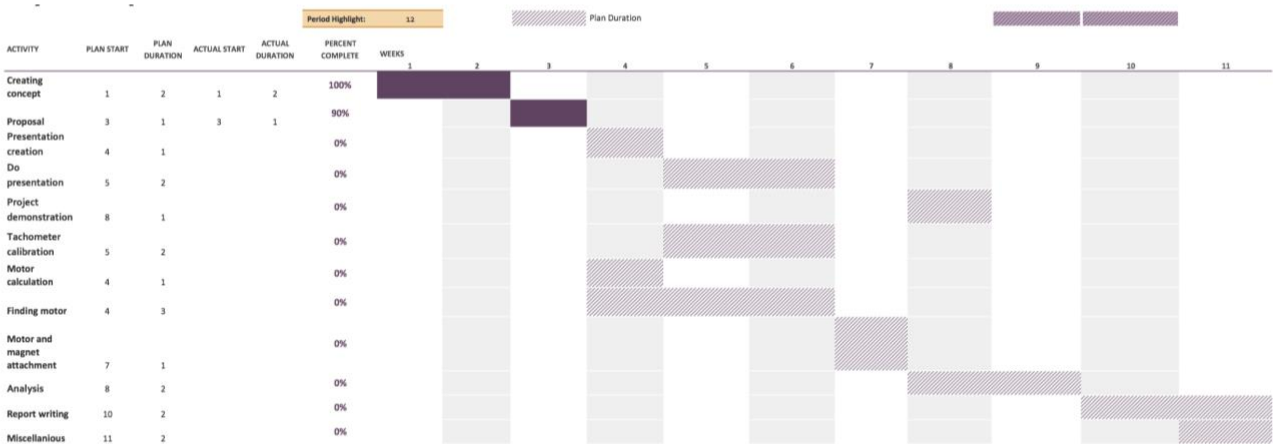
Equation 10: Angular velocity of the rotor

The value for b was calculated for the gyroscope to be on the order of $\approx 10^{-15} kgm^3 s^{-1}$using video recording prior to the tachometer setup.
The value of I was calculated to be $\approx 10^{-3} kgm^3$ taking into account the imperfections of the architecture of the rotor in comparison to the idealised disk.
From previous feasibility studies, we realised that the current needed to directly cause the disk to spin would need to be far higher than achievable ($\approx 3000A$) within a realistic

laboratory (when equating $\tau = BIL \wedge R = b\dot{\theta}$). With small conducting permanent magnets, put on the axis of rotation on either side of the rotor, a similar, high value of current was calculated $(\approx 80A)$. Thus, the only feasible way in which to drive the gyroscope was determined to be by using a motor.

A feasibility study on the rate of achievable precession using equations (6),(7) and (8) resulted in a precession rate $\approx 10^{-3} rads^{-1}$ being predicted. This equates to a time of 18 minutes per revolution. Here the tachometer measuring system would need modification in applying equally spaced strips of reflective Tipp-Ex as opposed to the single strip that we had applied during preliminary results.

Project Timeline



Thus far we have already completed feasibility studies to rule out possible driving processes, considering factors such as the mass, moment of inertia, aerodynamics and conductivity of the material. Progress has also been made in taking readings from the optical tachometer where hexadecimal values have been taken, corresponding to the rate of rotation of the rotor within the gyroscope. In the near future, we aim to have interpreted the hexadecimal results and created a python coding system with which meaningful graphs of angular velocity can be obtained.

Bibliography

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