$$training = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1D} \\ a_{21} & a_{22} & \cdots & a_{2D} \\ \vdots & \vdots & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{ND} \end{bmatrix}_{N_{train}} \quad \begin{matrix} \leftarrow train[1] \;/\; \vec{a}_1 \\ \leftarrow train[2] \;/\; \vec{a}_2 \\ \vdots \\ \leftarrow train[N] \;/\; \vec{a}_N \end{matrix}$$

D.

$$test = \begin{bmatrix} b_{11} & b_{12} & b_{13} \cdots & b_{1D} \\ b_{21} & b_{22} & \cdots & b_{2D} \\ \vdots & \vdots & & \vdots \\ b_{M1} & b_{M2} & \cdots & b_{MD} \end{bmatrix}_{M_{test}} \quad \begin{matrix} \leftarrow test[1] & \vec{b}_1 \\ \leftarrow test[2] & \vec{b}_2 \\ \vdots & \\ \leftarrow test[M] & \vec{b}_M \end{matrix}$$

D.

$$Distance = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1N} \\ D_{21} & D_{22} & \cdots & D_{2N} \\ \vdots & \vdots & & \vdots \\ D_{M1} & D_{M2} & \cdots & D_{MN} \end{bmatrix}_{\#\text{ of test } M.}$$

$\#$ of train. N.

ith test point & jth train point.

$$Dists[i,j] = \sqrt{(train[j,1]-test[i,1])^2 + (train[j,2]-test[i,2])^2 + (train[j,3]-test[i,3])^2 + \cdots + (train[j,D]-test[i,D])^2}$$

$$= \sqrt{\sum (train[j,:]-test[i,:])^2}$$

$\sum$: add all the numbers. $-$: elementwise. ^{2}: elementwise.

To simplify,

$D_{ij} = \Sigma(\vec{a}_j - \vec{b}_i)^2$ all the operations are element wise.

$$\text{Distance} = \begin{bmatrix} \Sigma(\vec{a}_1 - \vec{b}_1)^2 & \Sigma(\vec{a}_2 - \vec{b}_1)^2 & \cdots & \Sigma(\vec{a}_N - \vec{b}_1)^2 \\ \Sigma(\vec{a}_1 - \vec{b}_2)^2 & \Sigma(\vec{a}_2 - \vec{b}_2)^2 & \cdots & \Sigma(\vec{a}_N - \vec{b}_2)^2 \\ \Sigma(\vec{a}_1 - \vec{b}_3)^2 & \Sigma(\vec{a}_2 - \vec{b}_3)^2 & \cdots & \Sigma(\vec{a}_N - \vec{b}_3)^2 \\ \vdots & \vdots & & \vdots \\ \Sigma(\vec{a}_1 - \vec{b}_M)^2 & \Sigma(\vec{a}_2 - \vec{b}_M)^2 & \cdots & \Sigma(\vec{a}_N - \vec{b}_M)^2 \end{bmatrix}$$

$$= \begin{bmatrix} \Sigma(\vec{a}_1^2 + \vec{b}_1^2 - 2\vec{a}_1 \cdot \vec{b}_1) & \Sigma(\vec{a}_2^2 + \vec{b}_1^2 - 2\vec{a}_2 \cdot \vec{b}_1) & \cdots & \Sigma(\vec{a}_N^2 + \vec{b}_1^2 - 2\vec{a}_N \cdot \vec{b}_1) \\ \Sigma(\vec{a}_1^2 + \vec{b}_2^2 - 2\vec{a}_1 \cdot \vec{b}_2) & \Sigma(\vec{a}_2^2 + \vec{b}_2^2 - 2\vec{a}_2 \cdot \vec{b}_2) & \cdots & \Sigma(\vec{a}_N^2 + \vec{b}_2^2 - 2\vec{a}_N \cdot \vec{b}_2) \\ \vdots & \vdots & & \vdots \\ \Sigma(\vec{a}_1^2 + \vec{b}_M^2 - 2\vec{a}_1 \cdot \vec{b}_M) & \Sigma(\vec{a}_2^2 + \vec{b}_M^2 - 2\vec{a}_2 \cdot \vec{b}_M) & \cdots & \Sigma(\vec{a}_N^2 + \vec{b}_M^2 - 2\vec{a}_N \cdot \vec{b}_M) \end{bmatrix}$$

$$= \sum \begin{bmatrix} \vec{a}_1^2 & \cdots & \vec{a}_N^2 \\ \vec{a}_1^2 & \cdots & \vec{a}_N^2 \\ \vdots & & \vdots \\ \vec{a}_1^2 & \cdots & \vec{a}_N^2 \end{bmatrix} + \sum \begin{bmatrix} \vec{b}_1^2 & \cdots & \vec{b}_1^2 \\ \vdots & & \vdots \\ \vec{b}_M^2 & \cdots & \vec{b}_M^2 \end{bmatrix} - 2 \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_N \\ \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_N \\ \vdots & \vdots & & \vdots \\ \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_N \end{bmatrix} \cdot \begin{bmatrix} \vec{b}_1 & \cdots & \vec{b}_1 \\ \vdots & & \vdots \\ \vec{b}_M & \cdots & \vec{b}_M \end{bmatrix}$$

element wise. meaning the sum of every number

element wise. meaning we multiplies one here

$$= \sum \left[\vec{a}_1^{\,2}\ \vec{a}_2^{\,2} \cdots \vec{a}_N^{\,2}\right] + \sum \begin{bmatrix} \vec{b}_1^{\,2} \\ \vec{b}_2^{\,2} \\ \vdots \\ \vec{b}_M^{\,2} \end{bmatrix} - 2\sum \left[\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_N\right] \cdot \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vdots \\ \vec{b}_M \end{bmatrix}$$

$\sum$ means sum up the numbers inside each vector.

gives a $[\ ]_N$ matrix

$$= \text{sum}\left((\text{training}^T)^2\right) + \text{sum}\left(\text{testing}^2\right) - 2\,\text{sum}\left(\text{train}^T \cdot \text{test}\right)$$

axis = 1 axis = 1

all the operations are elementwise.