Introduction to Deep Learning

Embeddings, Word2wec, fastText, GloVe

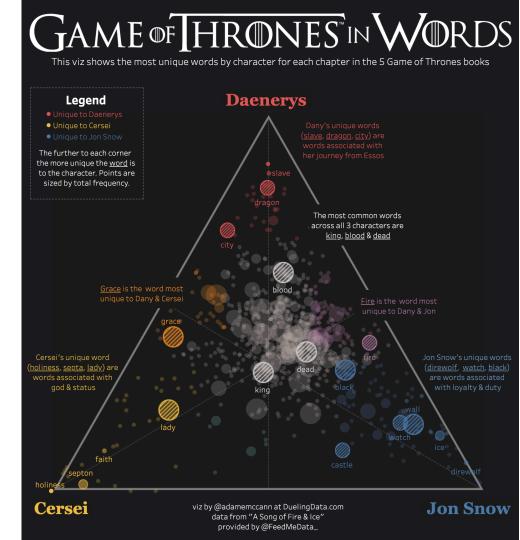
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8:30-9:00	Continental Breakfast
9:00-9:45	Introduction and Setup
9:45-10:30	Neural Networks 101
10:30-10:45	Break
10:45-11:15	Machine Learning Basics
11:15-11:45	Context-free Representations for Language
11:45-12:15	Convolutional Neural Networks
12:15-13:15	Lunch Break
13:15-14:00	Recurrent Neural Networks
14:00-14:45	Attention Mechanism and Transformer
14:45-15:00	Coffee Break
15:00-16:15	Contextual Representations for Language
16:15-17:00	Language Generation



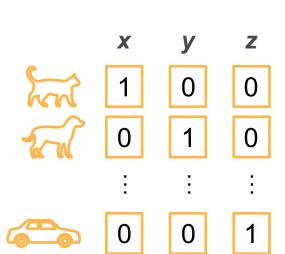
Word2Vec



Motivation

- One-hot vectors map objects/ words into fixed-length vectors
- These vectors only contain the identity information, not semantic meaning, e.g.

$$\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{z}, \mathbf{y} \rangle = 0$$



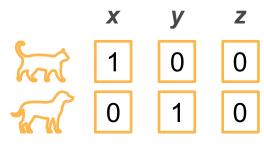


Word2vec

- Learn an embedding vector for each word
- Use $\langle x, y \rangle$ to measure the similarity

$$\langle \mathbf{x}, \mathbf{y} \rangle > \langle \mathbf{z}, \mathbf{y} \rangle$$

- Build a probability model
- Maximize the likelihood function to learn the model

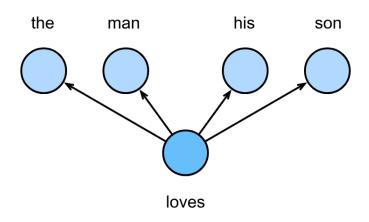






The Skip-Gram Model

- A word can be used to generate the words surround it
- Given the center word, the context words are generated independently



```
ℙ("the", "man", "his", "son" | "loves")
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= \mathbb{P}(\text{"the"} \mid \text{"loves"}) \cdot \mathbb{P}(\text{"man"} \mid \text{"loves"})
```

$$\cdot \, \mathbb{P}(\text{"his"} \mid \text{"loves"}) \cdot \mathbb{P}(\text{"son"} \mid \text{"loves"})$$



Likelihood Function

Summing over all words is too expensive

• Given length *T* sequence, context window *m*, the likelihood function:

$$\prod_{t=1}^{T} \prod_{-m \le j \le m, \ j \ne 0} \mathbb{P}(w^{(t+j)} \mid w^{(t)})$$



Negative Sampling

 Treat a center word and a context word appear in the same context window as an event

$$\mathbb{P}\left(D = 1 \mid w_c, w_o\right) = \sigma\left(\mathbf{u}_c^T \mathbf{v}_o\right) \qquad \sigma(x) = \frac{1}{1 + \exp(-x)}$$

• Change the likelihood function from $\prod_{t=1}^{T} \prod_{-m \le j \le m, j \ne 0} \mathbb{P}(w^{(t+j)} \mid w^{(t)})$ to

$$\prod_{t=1}^{I} \prod_{-m \le j \le m, \ j \ne 0} \mathbb{P}(D=1 \mid w^{(t)}, w^{(t+j)})$$



Naive solution: infinity

Negative Sampling

• Sample noise word w_n that doesn't appear in the window

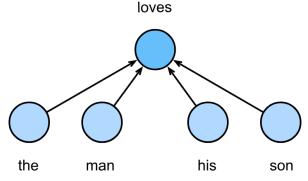
$$\mathbb{P}\left(D=0 \mid w_c, w_n\right) = 1 - \sigma\left(\mathbf{u}_n^T \mathbf{v}_c\right)$$

- Add into the likelihood function as well
- Maximizing the likelihood equals to solve a binary classification problem with a binary logistic regression loss



Continuous Bag Of Words (CBOW)

The center word is generated based on the context words



P("loves" | "the", "man", "his", "son")



Likelihood Function

Compute the probability

$$\mathbb{P}(w_c \mid w_{o_1}, \dots, w_{o_{2m}}) = \frac{\exp\left(\frac{1}{2m}\mathbf{u}_c^{\mathsf{T}}(\mathbf{v}_{o_1} + \dots + \mathbf{v}_{o_{2m}})\right)}{\sum_{i \in \mathcal{V}} \exp\left(\frac{1}{2m}\mathbf{u}_i^{\mathsf{T}}(\mathbf{v}_{o_1} + \dots + \mathbf{v}_{o_{2m}})\right)}$$

Likelihood

$$\prod_{t=1}^{I} \mathbb{P}(w^{(t)} \mid w^{(t-m)}, ..., w^{(t-1)}, w^{(t+1)}, ..., w^{(t+m)})$$



FastText

- English words usually have internal structures and formation methods
 - dog, dogs, dogcatcher



- Each center word is represented as a set of subwords
 - "where" -> "<where>" -> n-gram
 - n=3: "<wh", "whe", "her", "ere", "re>"
- Useful for long but infrequent words
 - e.g. pneumonoultramicroscopicsilicovolcanoconiosis



FastText

- For word w, \mathcal{G}_w is the union of subwords with length from 3 to 6
- The center vector is then

$$\mathbf{u}_{w} = \sum_{g \in \mathcal{G}_{w}} \mathbf{u}_{g}$$

The rest model is same as skip-gram



Word Embedding with Global Vectors (GloVe)

• Denote by
$$q_{ij} = \frac{\exp(\mathbf{u}_j^{\mathsf{T}} \mathbf{v}_i)}{\sum_{k \in \mathcal{K}} \exp(\mathbf{u}_k^{\mathsf{T}} \mathbf{v}_i)}$$

Rewrite the negative log-likelihood function of skip-gram

$$-\log \left[\prod_{t=1}^{T} \prod_{-m \le j \le m, \ j \ne 0} \mathbb{P}(w^{(t+j)} \mid w^{(t)}) \right]$$

• as $-\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} x_{ij} \log q_{ij}$ with proper counts x_{ij}



Glove

Further rewrite

$$-\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}x_{ij}\log\ q_{ij}=-\sum_{i\in\mathcal{V}}x_i\sum_{j\in\mathcal{V}}p_{ij}\log\ q_{ij}$$
 with $x_i=\sum x_{ij},\ p_{ij}=x_i/x_{ij}$ Cross entropy



Glove

Replace the cross entropy with a log square loss

$$\sum_{j \in \mathcal{V}} p_{ij} \log q_{ij} \to \sum_{j \in \mathcal{V}} (\log p_{ij} - \log q'_{ij})^2$$

with an easy to compute $q'_{ij} = \exp(\mathbf{u}_j^{\mathsf{T}} \mathbf{v}_i)$

- Add bias term for center and context words
- Replace the weights x_i with a monotone increasing function in [0,1]

$$\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} h(x_{ij}) \left(\mathbf{u}_j^{\mathsf{T}} \mathbf{v}_i + b_i + c_j - \log x_{ij} \right)^2$$

