

Vision and Perception

Motion estimation and Optical flow



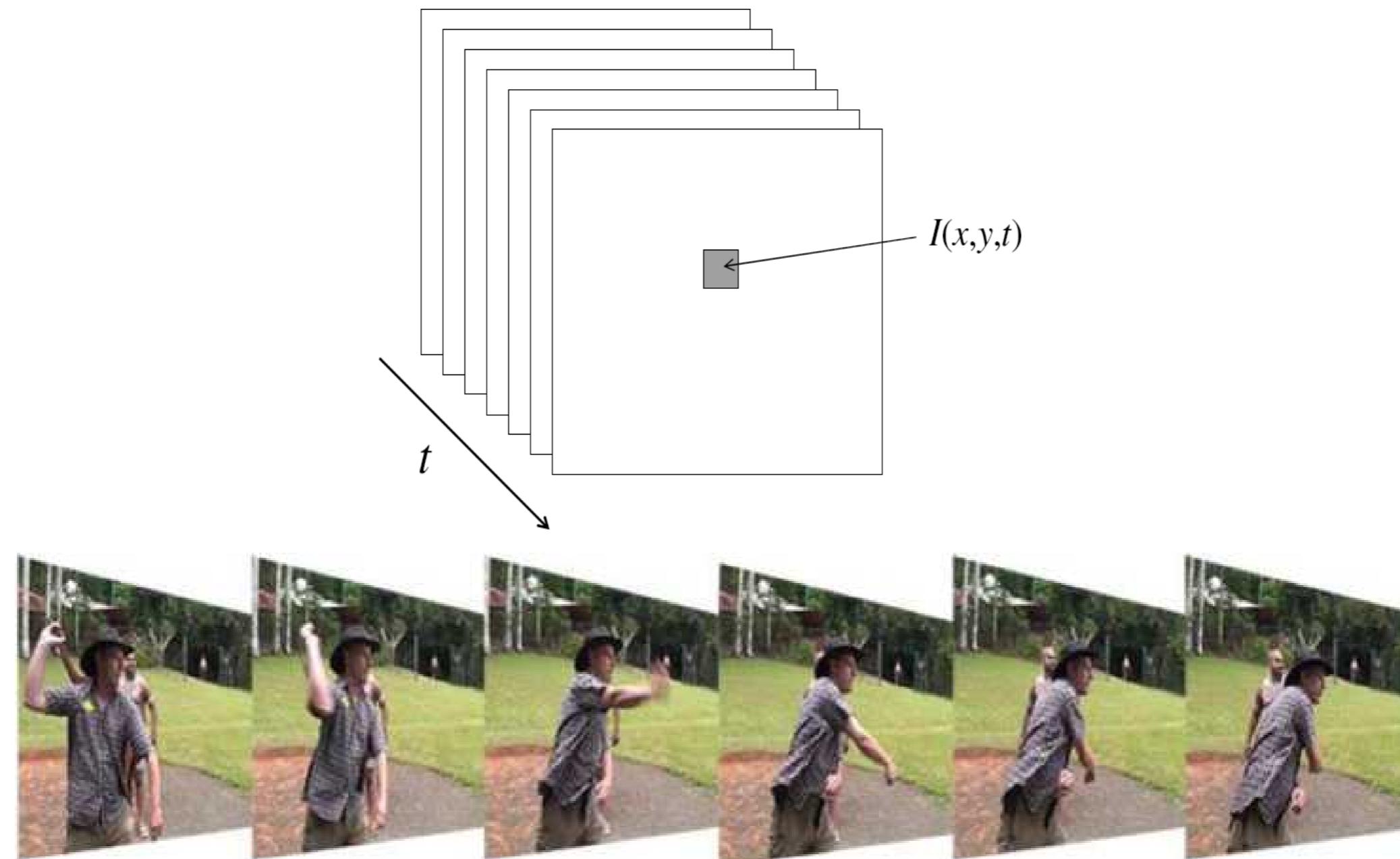
SAPIENZA
UNIVERSITÀ DI ROMA

References

- Basic reading: Szeliski, Chapter 9.3-9.4

From images to videos

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Uses of motion

The estimation of every pixel in a sequence is a problem with many applications in computer vision

- Improving video quality
 - Motion stabilization
 - Super resolution
- Segmenting objects based on motion cues
- Tracking objects
- Recognizing events and activities

Super-resolution

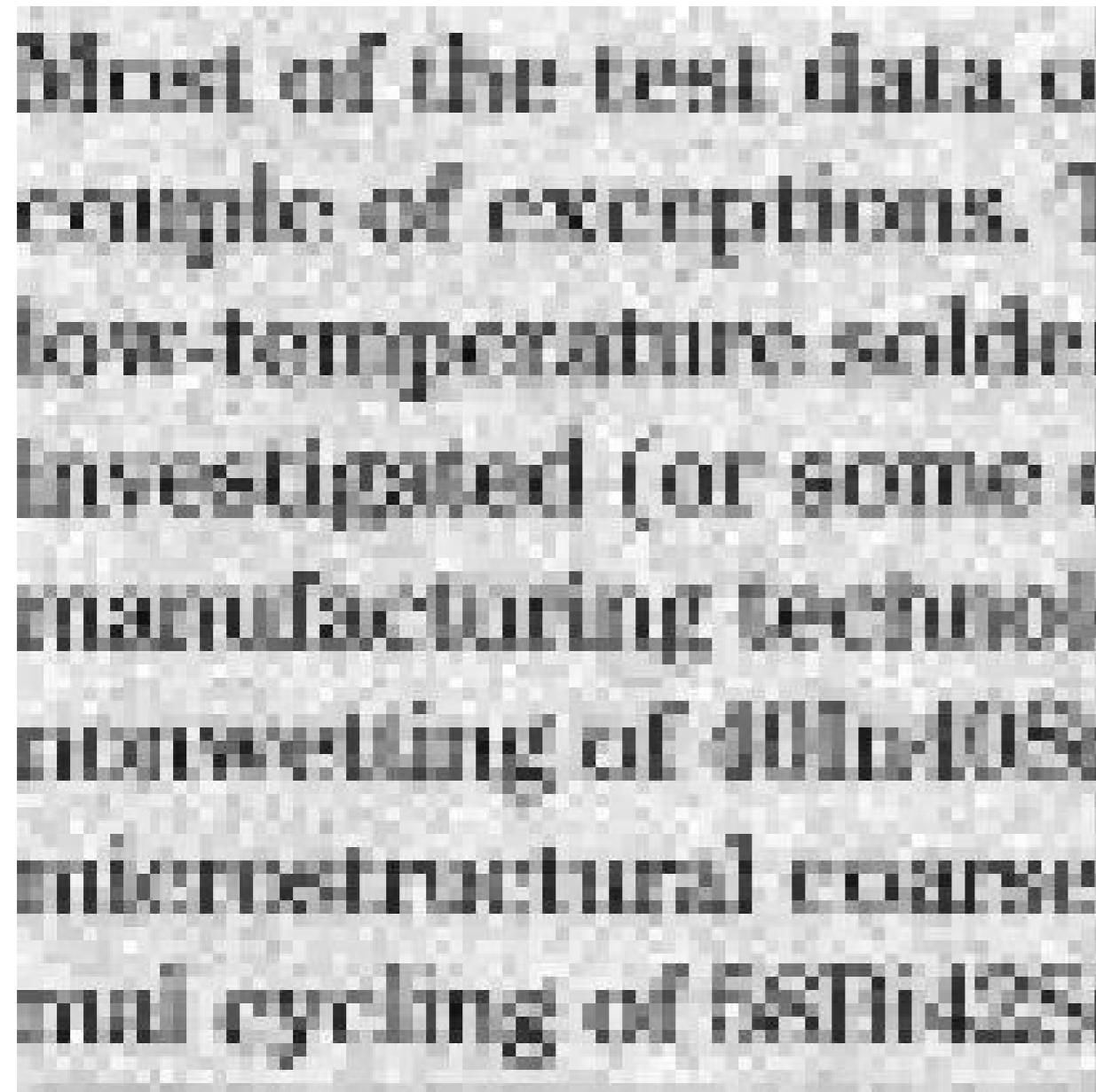
Example: a set of low quality images

Most of the test data o couple of exceptions. I low-temperature solder investigated (or some c manufacturing technol nonwetting of 40In40S microstructural coarse mal cycling of 58Bi42S	Most of the test data o couple of exceptions. I low-temperature solder investigated (or some c manufacturing technol nonwetting of 40In40S microstructural coarse mal cycling of 58Bi42S	Most of the test data o couple of exceptions. I low-temperature solder investigated (or some c manufacturing technol nonwetting of 40In40S microstructural coarse mal cycling of 58Bi42S
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- Irani, M.; Peleg, S. (June 1990). "Super Resolution From Image Sequences". International Conference on Pattern Recognition
- Fast and Robust Multiframe Super Resolution, Sina Farsiu, M. Dirk Robinson, Michael Elad, and Peyman Milanfar, EEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 13, NO. 10, OCTOBER 2004

Super-resolution

Each of these images looks like this:

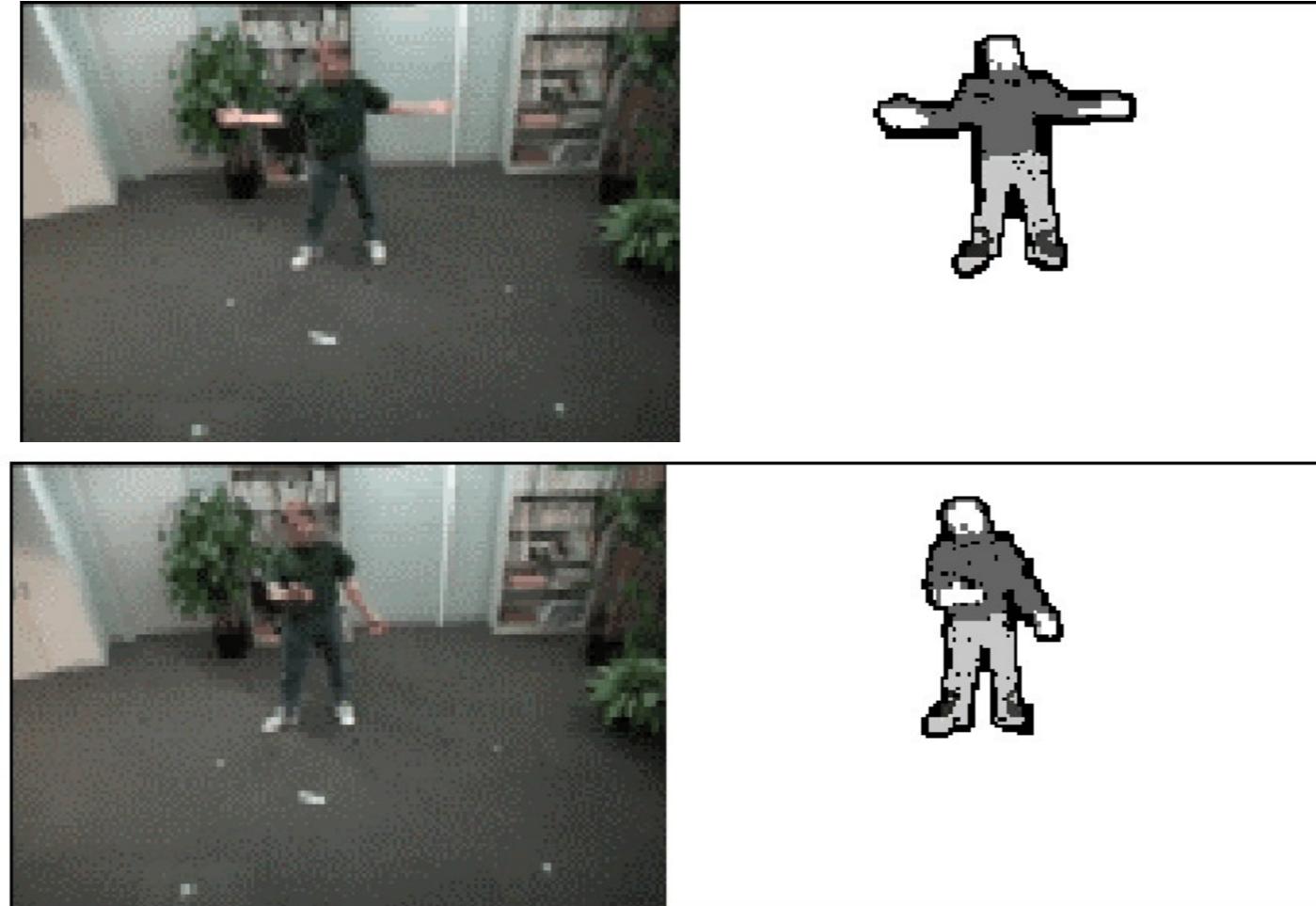


The recovery result:

Most of the test data o
couple of exceptions. T
low-temperature solder
investigated (or some o
manufacturing technolo
nonwetting of 40In40Sr
microstructural coarse
mal cycling of 58Bi42Si

Segmenting objects based on motion cues

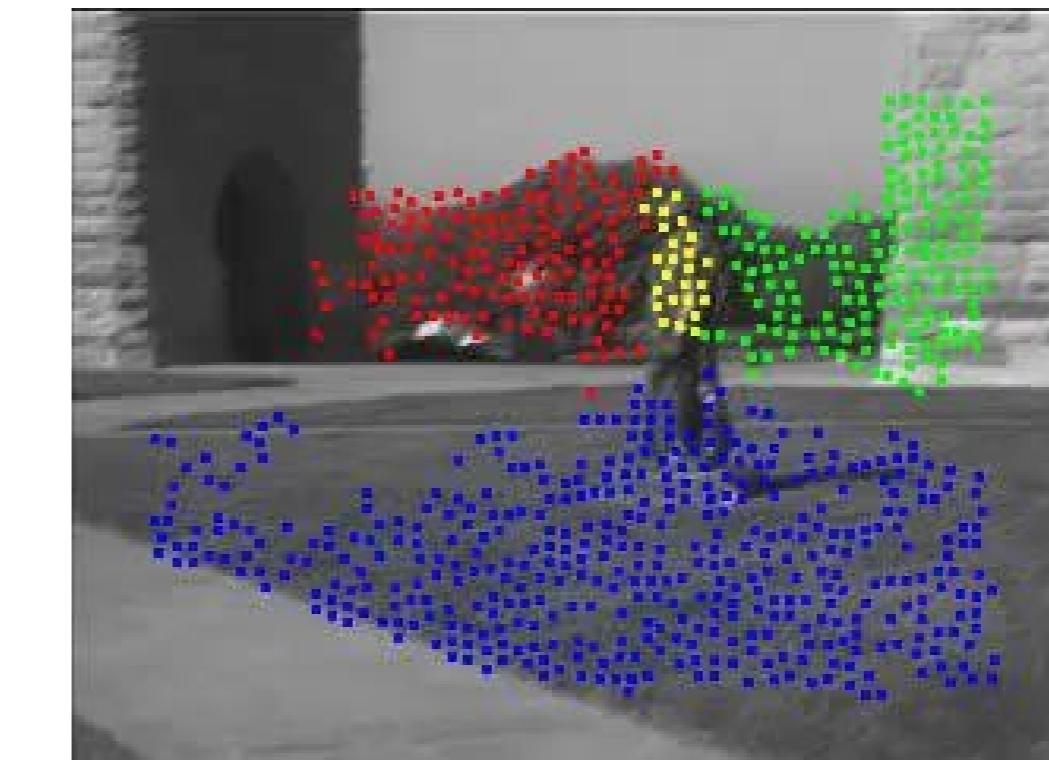
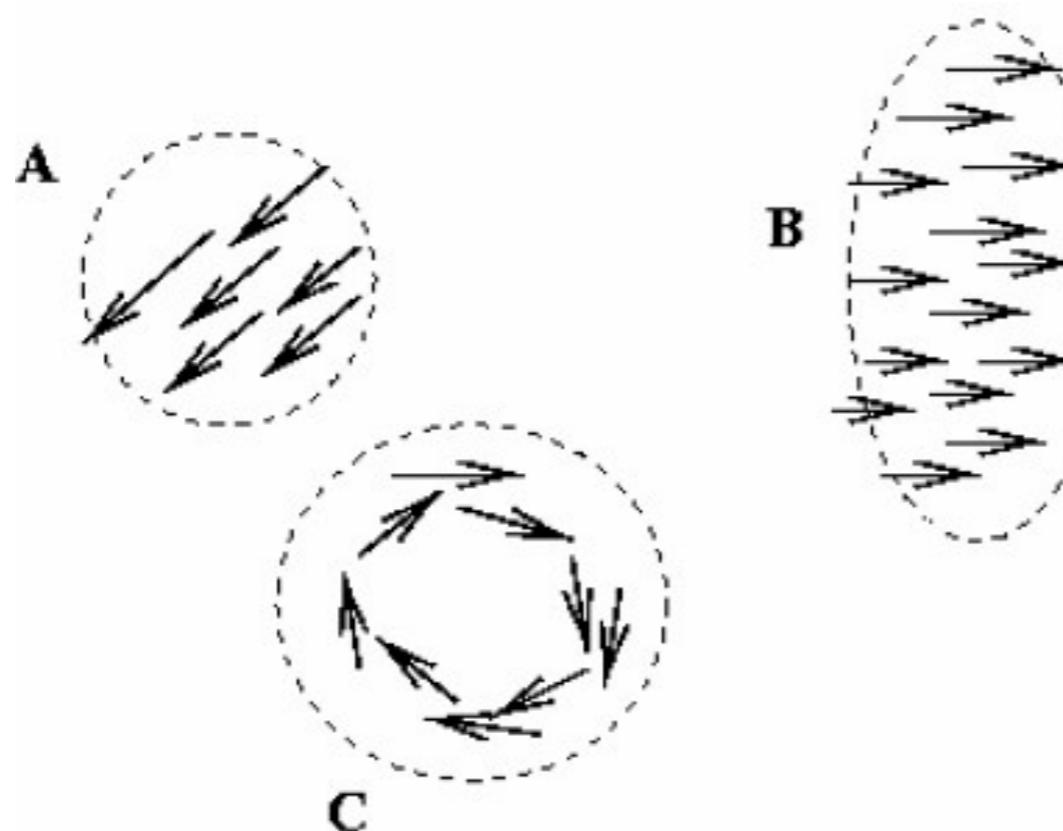
- Background subtraction
 - A static camera is observing a scene
 - Goal: separate the static *background* from the moving *foreground*



<https://www.youtube.com/watch?v=YAszeOaInUM>

Segmenting objects based on motion cues

- Motion segmentation
 - Segment the video into multiple *coherently* moving objects



S. J. Pundlik and S. T. Birchfield, Motion Segmentation at Any Speed,
Proceedings of the British Machine Vision Conference 2006

Tracking objects

- Facial tracking on openCV

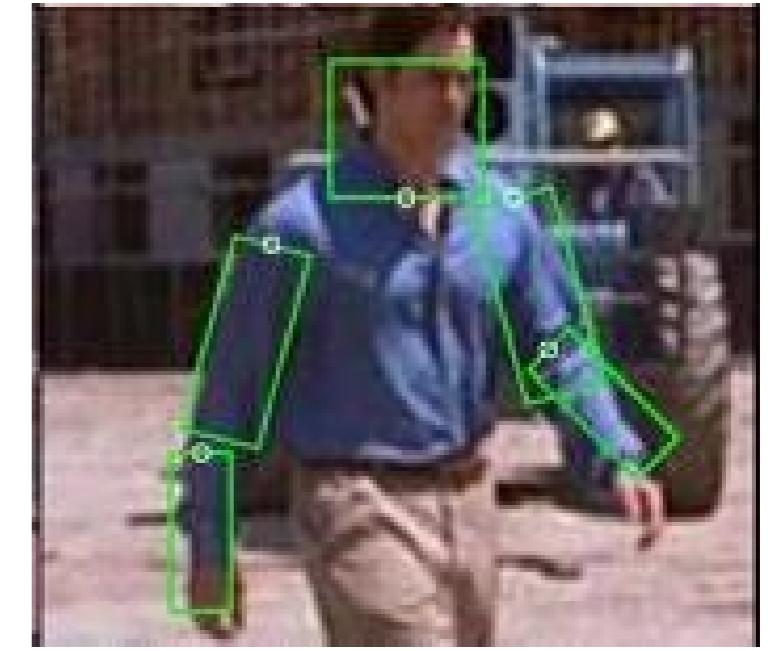
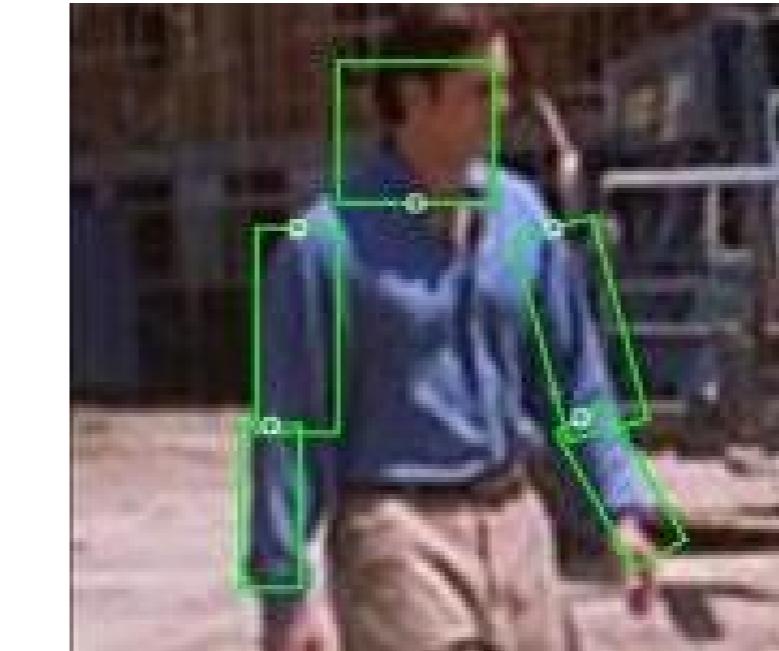
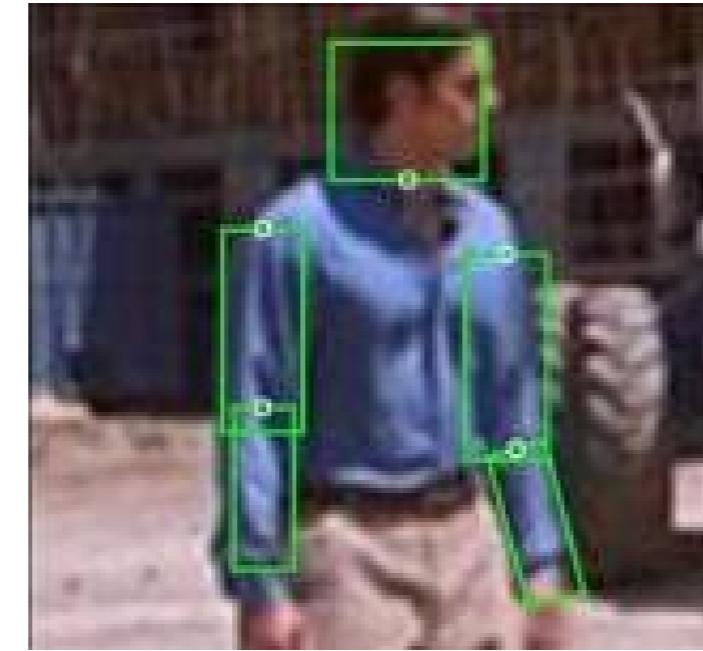
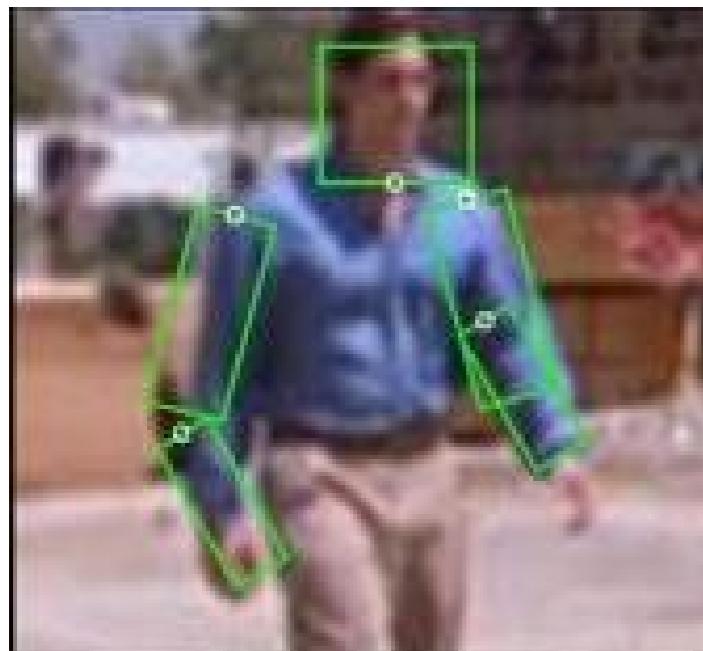


OpenCV's face tracker uses an algorithm called Camshift (based on the meanshift algorithm)

http://www.youtube.com/watch?v=HTk_UwAYzVk

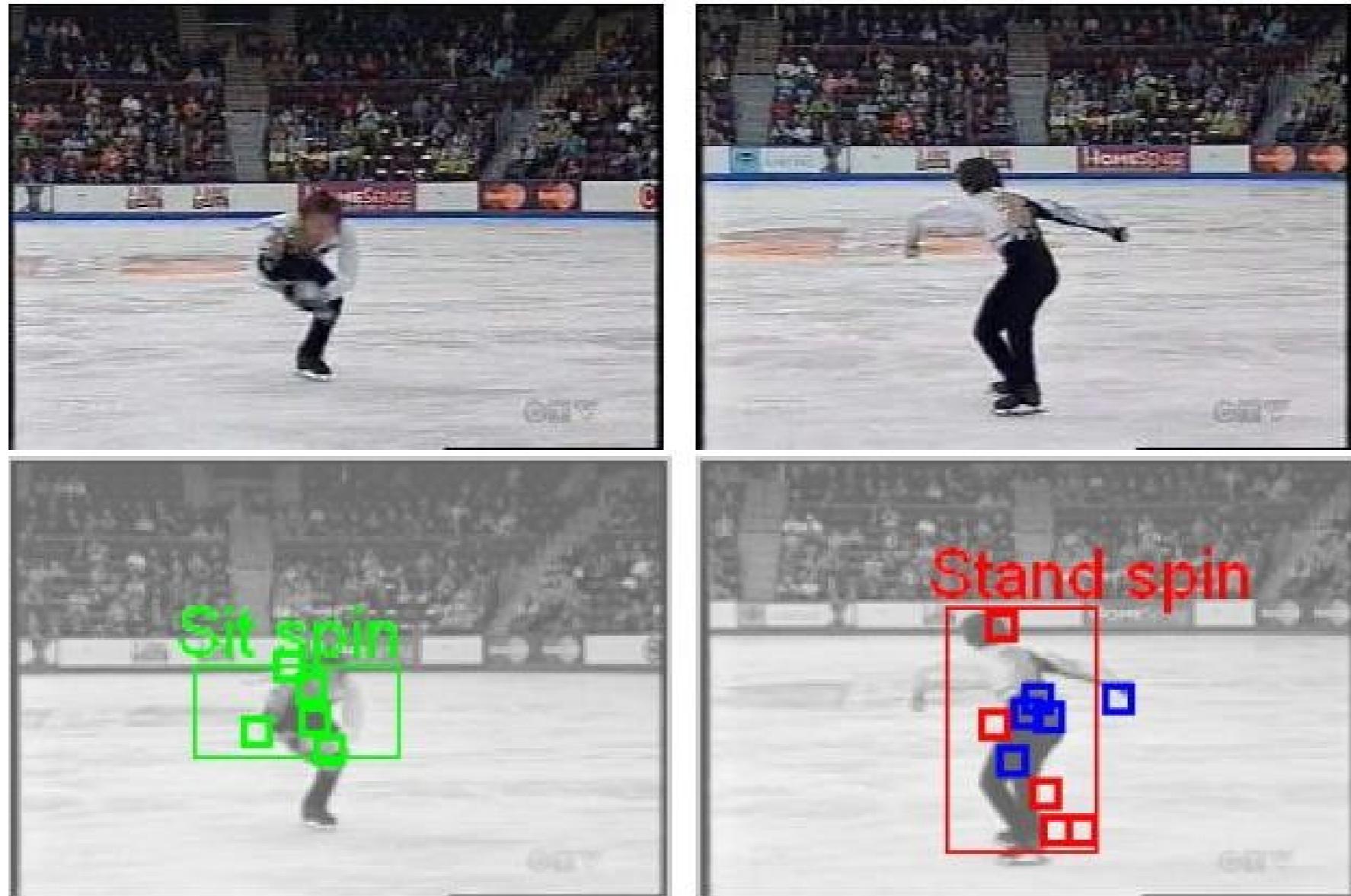
<http://learnopencv.com/wp-content/uploads/2017/02/real-time-face-tracking.gif>

Tracking body parts



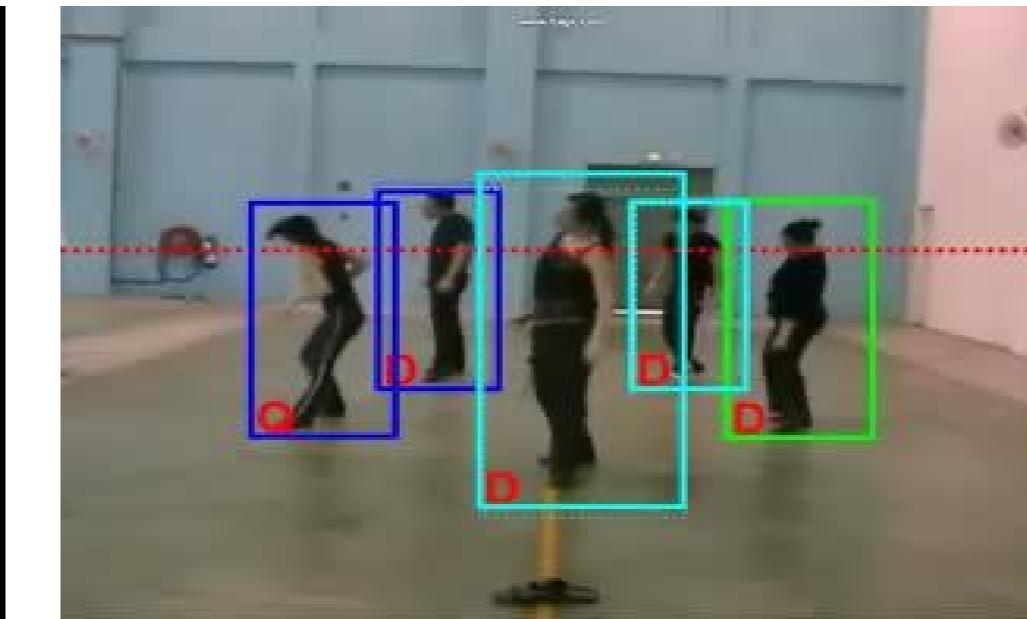
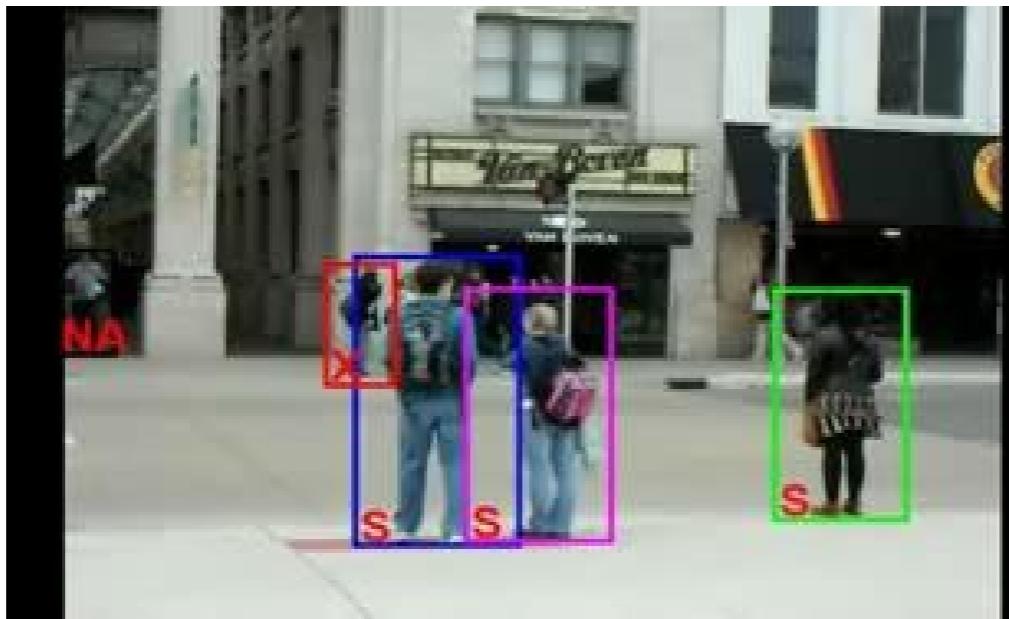
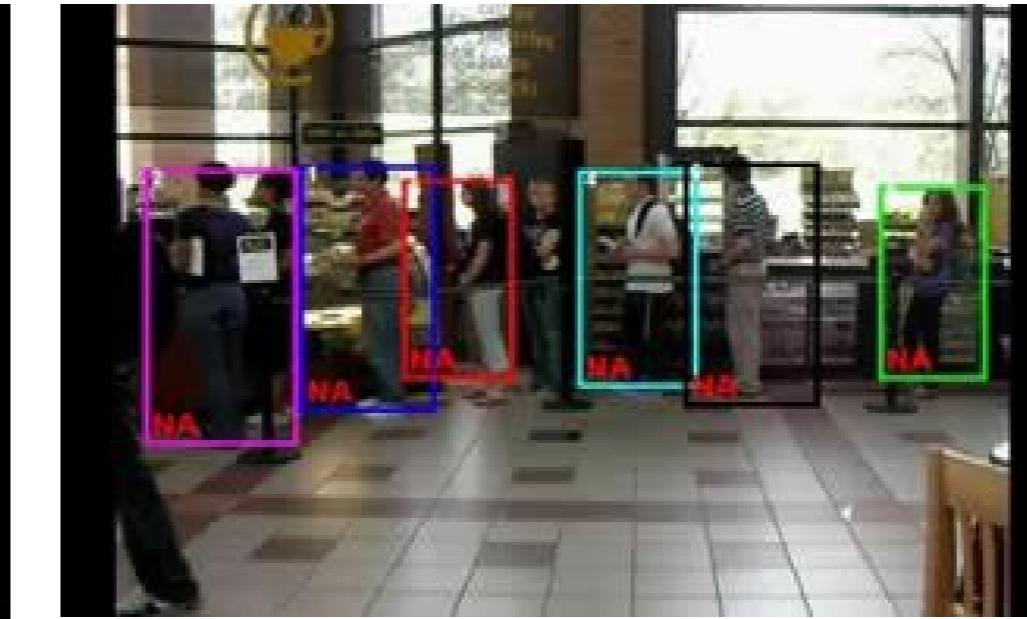
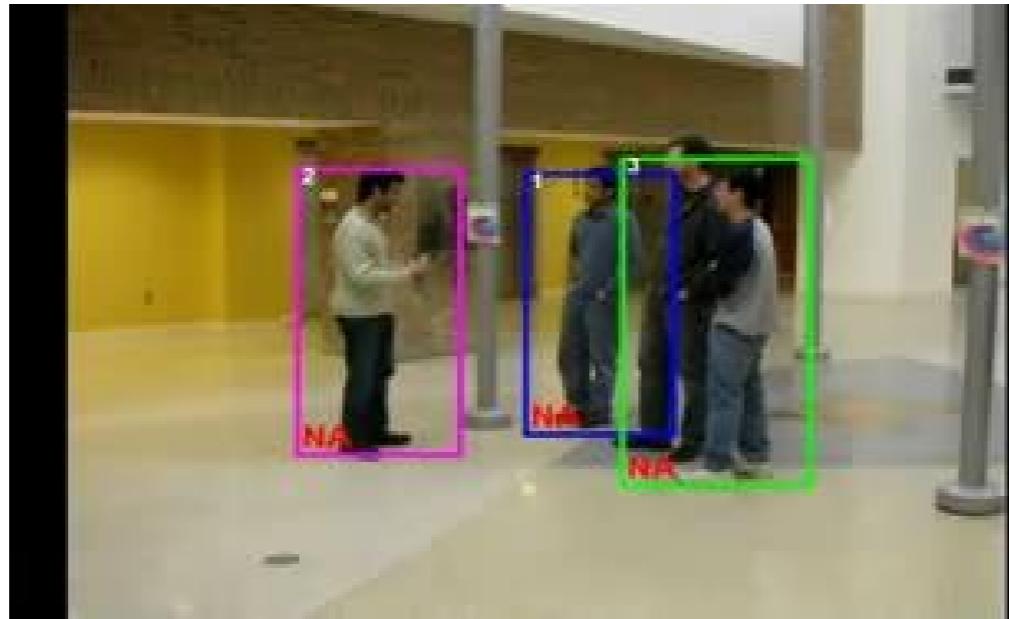
Courtesy of Benjamin Sapp

Recognizing events and activities



Recognizing group activities

Crossing – Talking – Queuing – Dancing – jogging



X: Crossing, S: Waiting, Q: Queuing,
W: Walking, T: Talking, D: Dancing

Motion estimation techniques

Optical flow

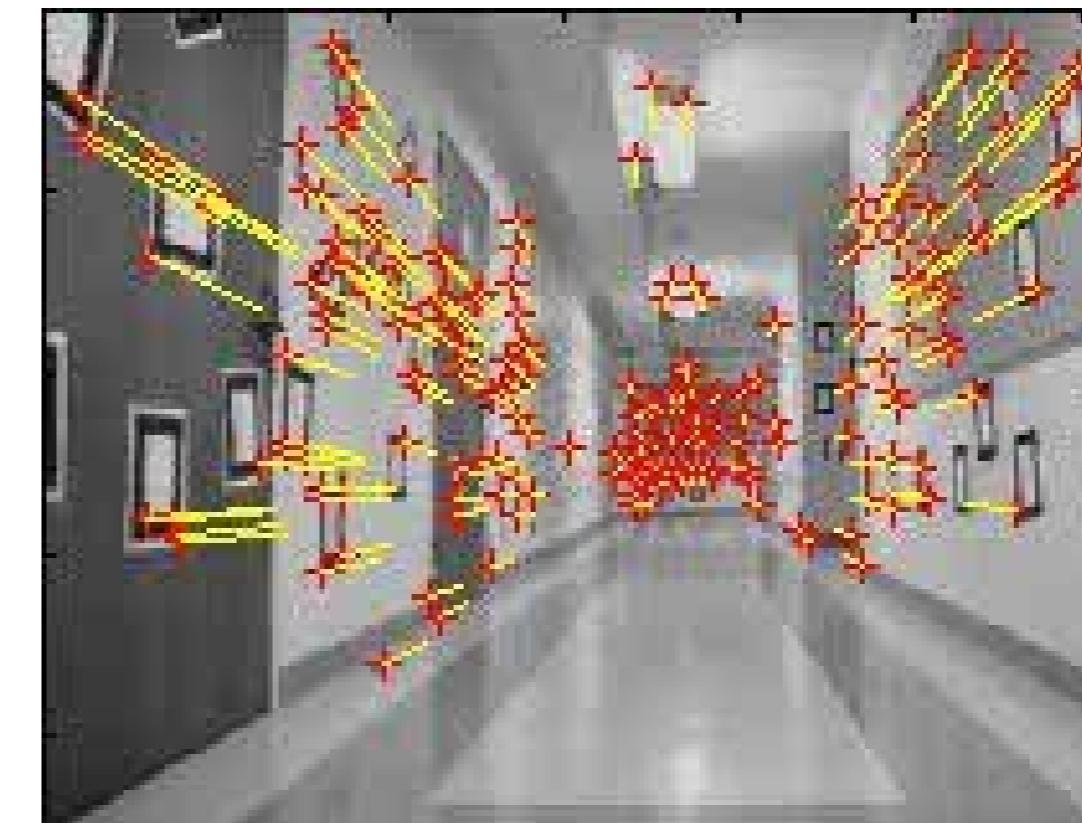
- Recover image motion at each pixel from spatio-temporal image brightness variations

Feature-tracking

- Extract visual features (corners, textured areas) and “track” them over multiple frames

Tracking features

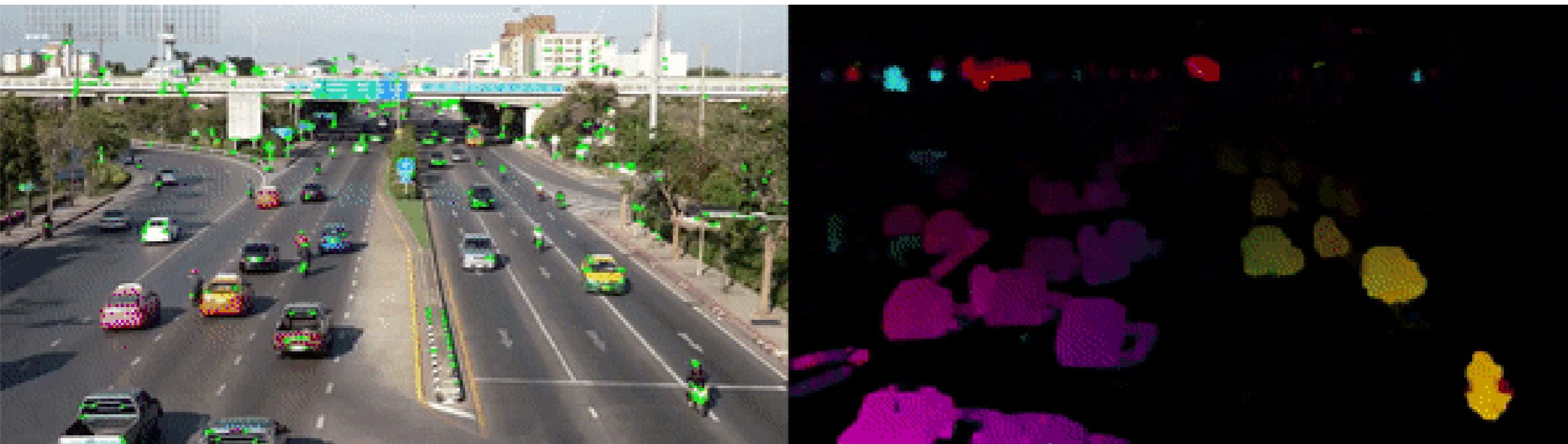
Tracking object regions frame to frame



Courtesy of Jean-Yves Bouguet – Vision Lab, California Institute of Technology

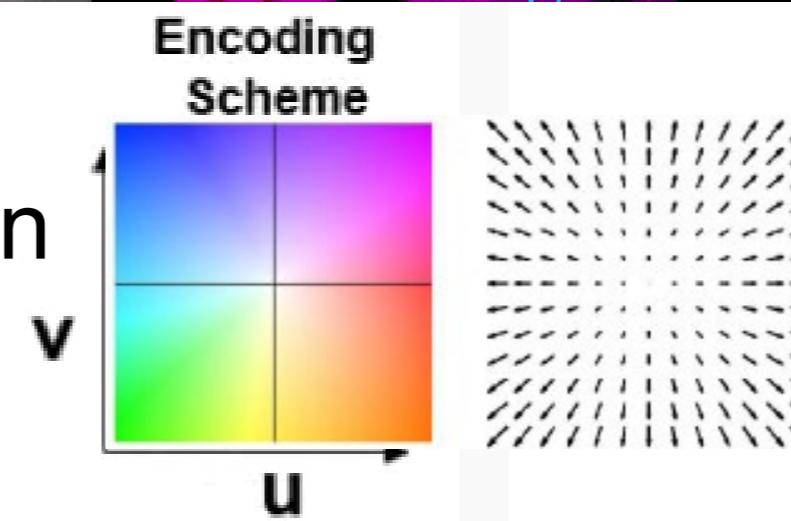
Optical flow

Optical flow is used to see how every point is moving frame to frame in a video sequence



Colour code for visualisation

HSV encoding scheme: hue, saturation, value
Vector field: displacement, velocity



Optical flow

1. **Sparse Optical Flow:** this method processes the flow vectors of only a few of the most interesting pixels from the entire image, within a frame.
2. **Dense Optical Flow:** the flow vectors of all pixels in the entire frame are processed which, in turn, makes this technique slower but more accurate.

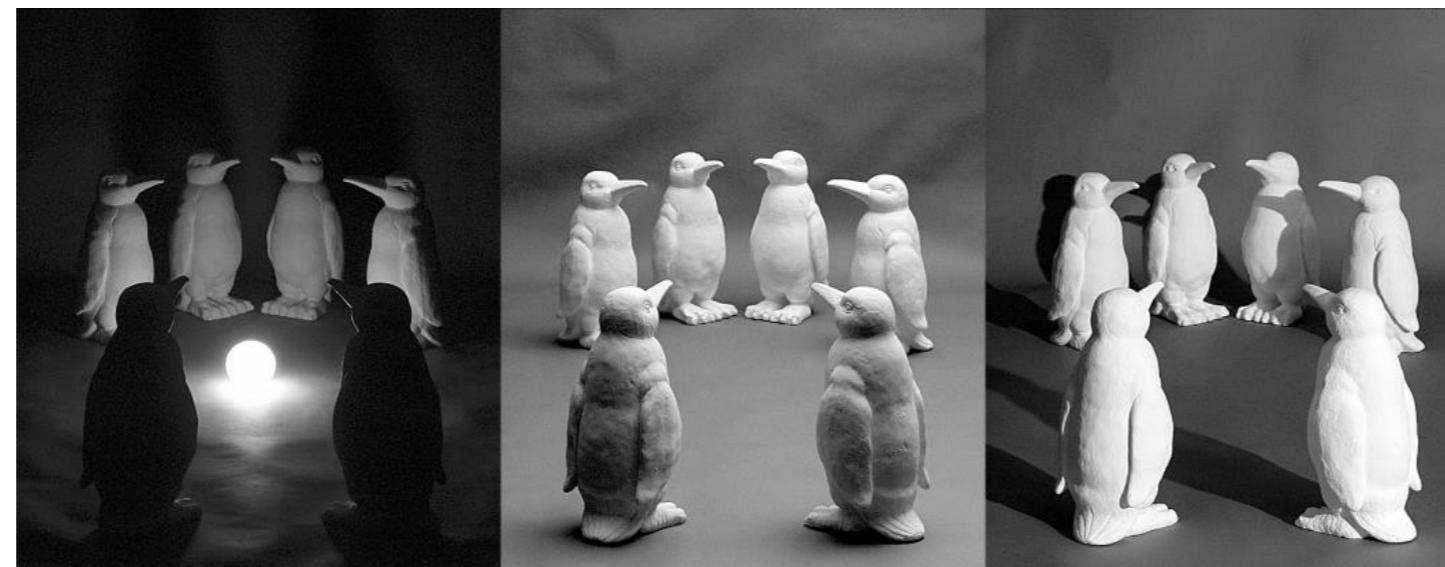
Optical flow



Definition: optical flow is the *apparent* motion of brightness patterns in the image

GOAL: Recover image motion at each pixel by optical flow. Pattern of motion of pixels between two consecutive frames. The motion can be caused either by the movement of a scene or by the movement of the camera.

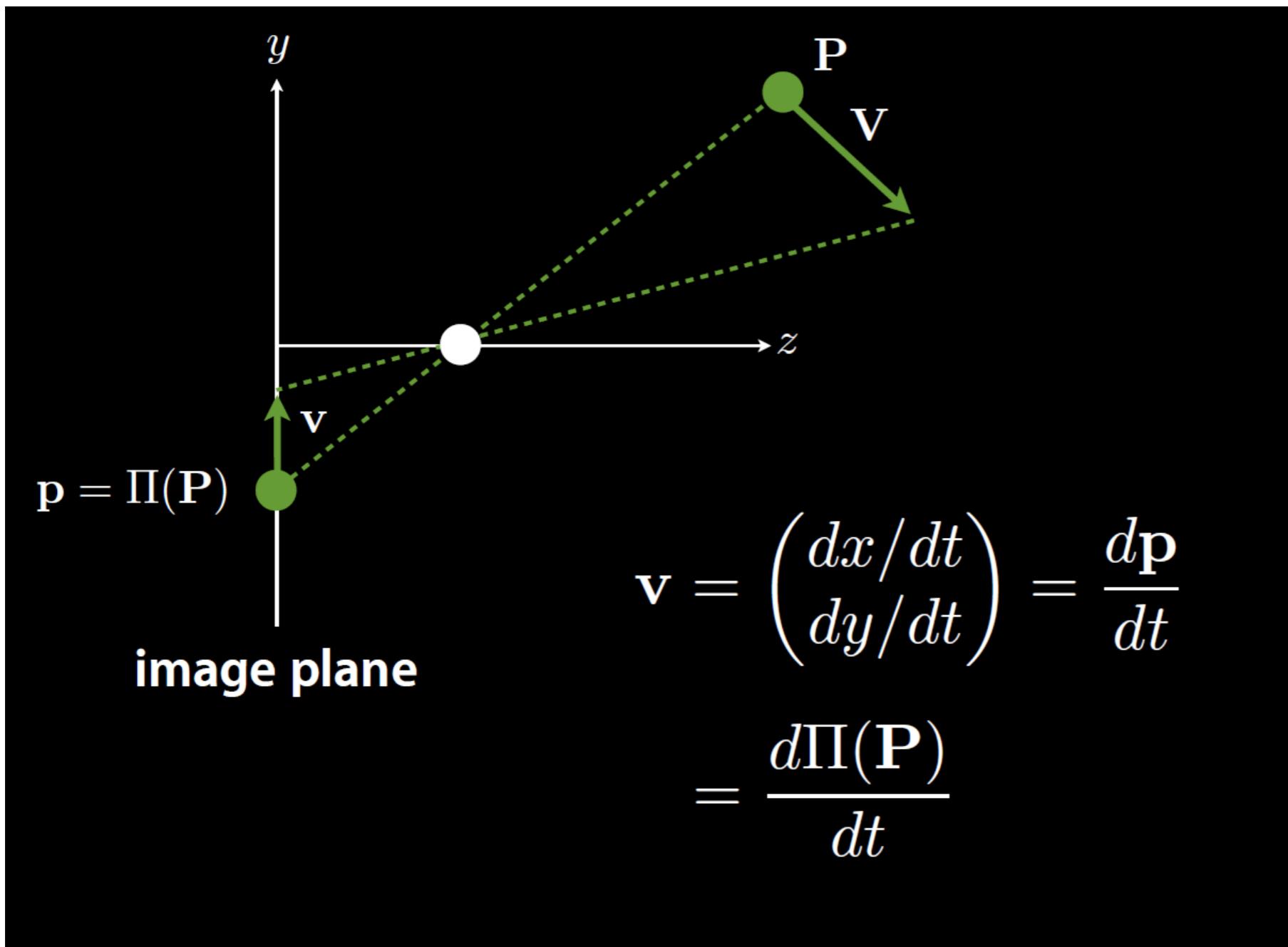
Note: apparent motion can be caused by lighting changes without any actual motion
(optical flow is different from the concept of motion field)



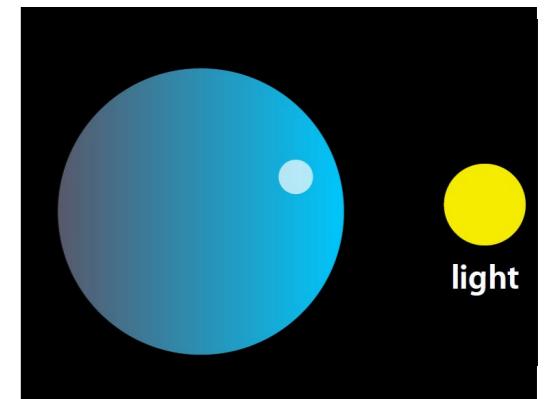
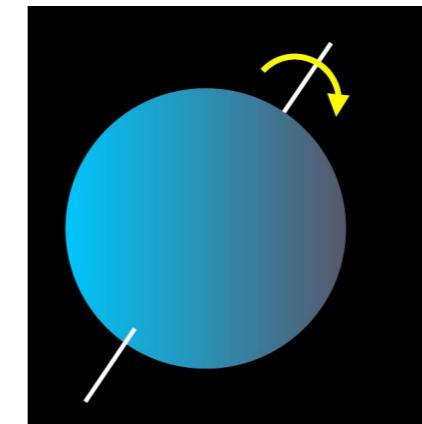
Motion field



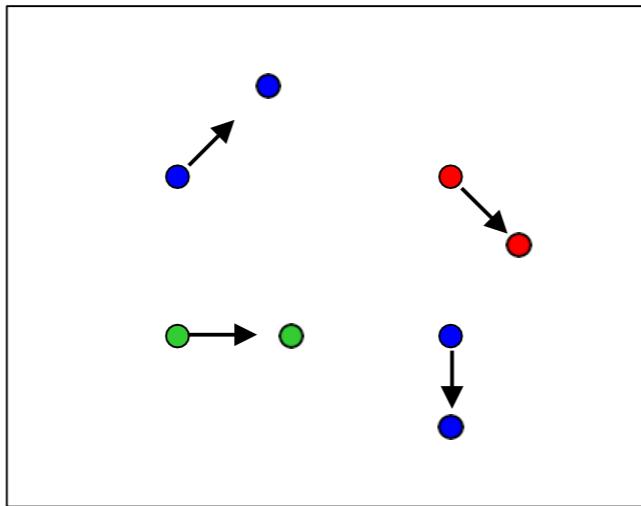
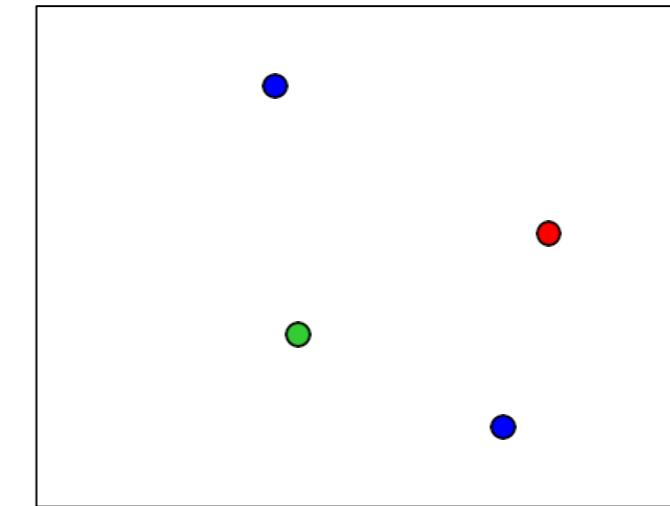
Projection of the 3D scene velocities onto the image plane



It is a geometric concept
The optical flow is a photometric concept
Ideally OF e MF are equivalent



Estimating optical flow

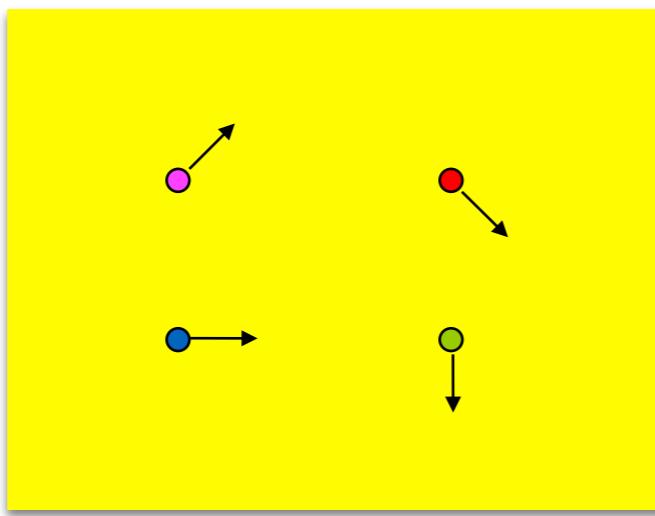
 $I(x,y,t)$  $I(x,y,t')$

Given two subsequent frames, estimate the apparent motion field $u(x,y), v(x,y)$ between them.

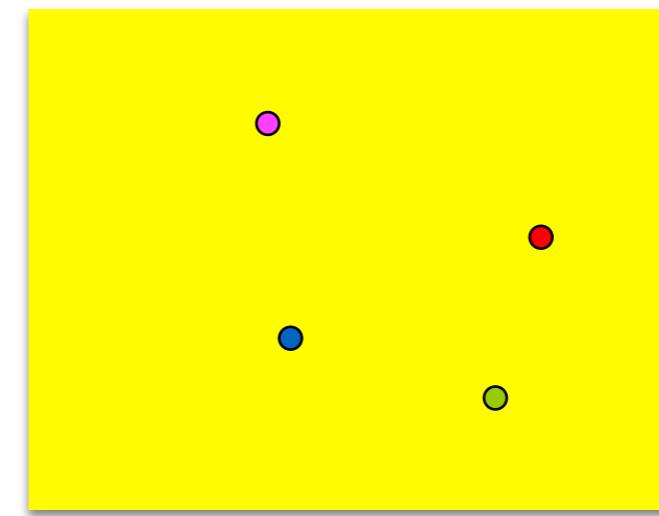
Key assumptions:

- **Brightness constancy:** projection of the same point looks the same in every frame
- **Small motion:** points do not move very far
- **Spatial coherence:** points move like their neighbors

The approach



$I(x, y, t)$



$I(x, y, t')$

Look for *nearby pixels with the same color*

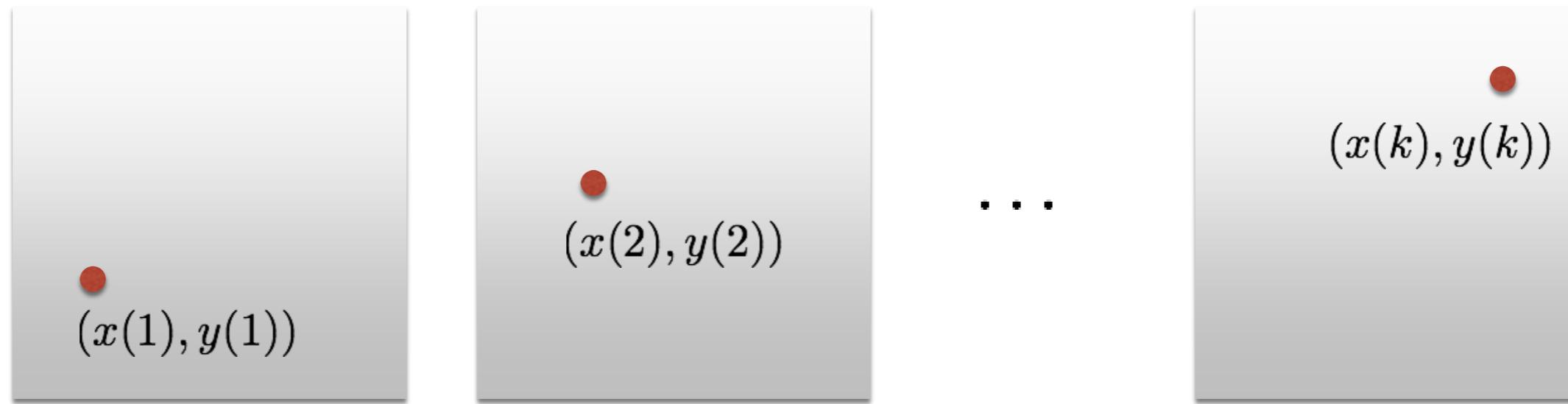
(small motion)

(color constancy)

Assumption 1

Brightness constancy

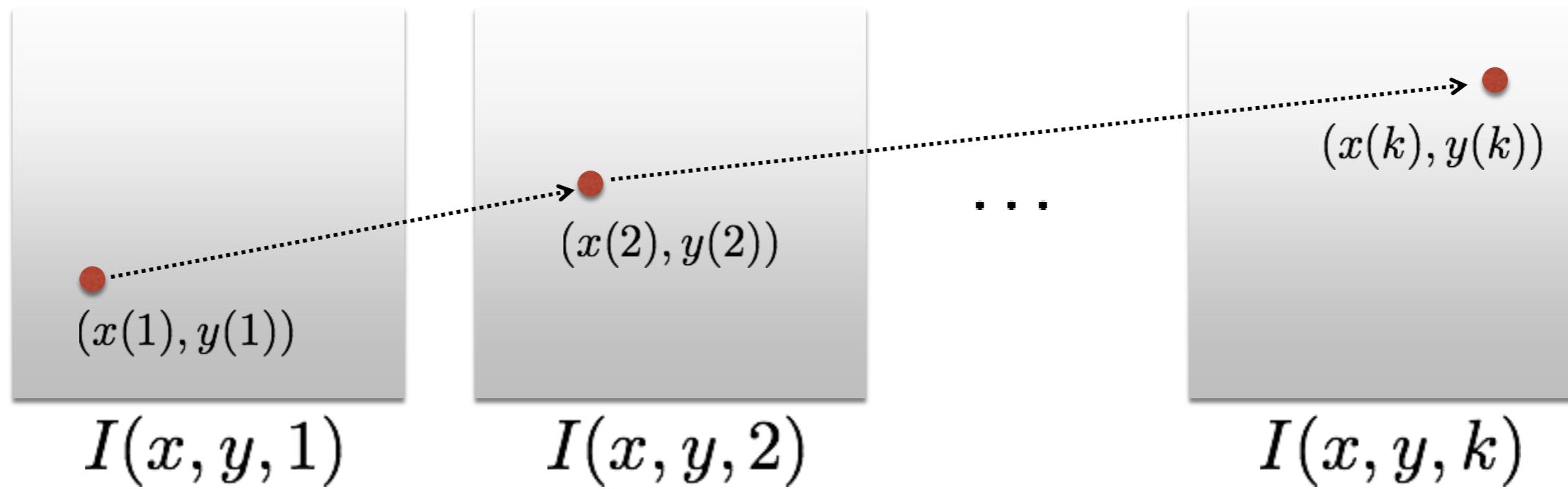
Scene point moving through image sequence



Assumption 1

Brightness constancy

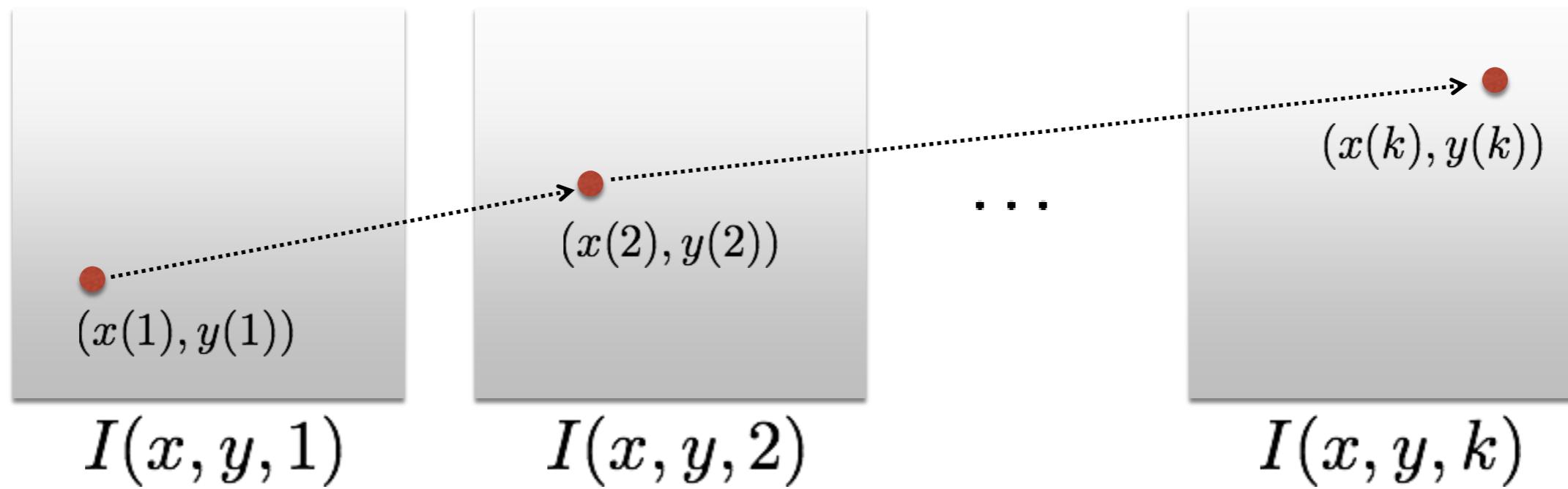
Scene point moving through image sequence



Assumption 1

Brightness constancy

Scene point moving through image sequence

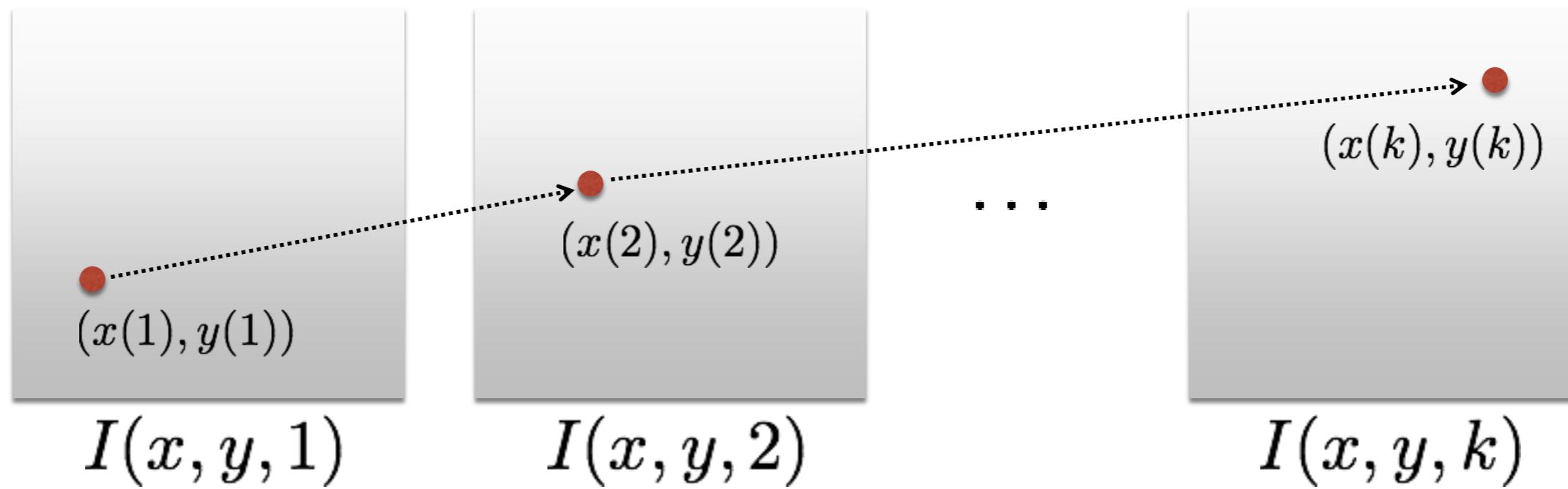


Assumption: brightness of the point will remain the same

Assumption 1

Brightness constancy

Scene point moving through image sequence



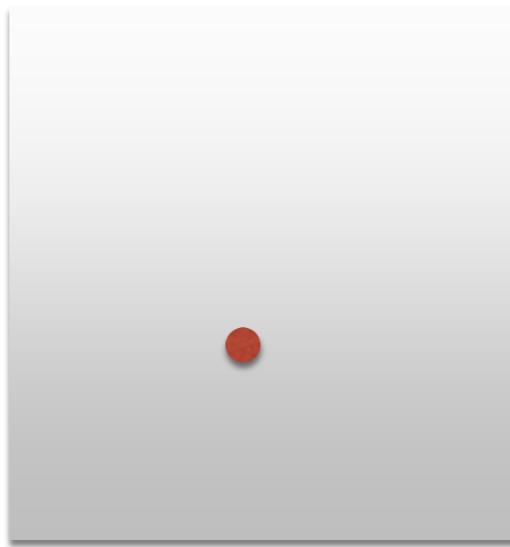
Assumption: Brightness of the point will remain the same

$$I(x(t), y(t), t) = C$$

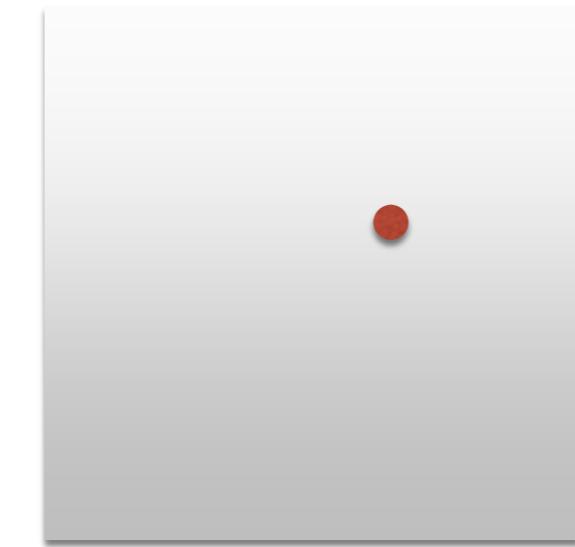
constant

Assumption 2

Small motion



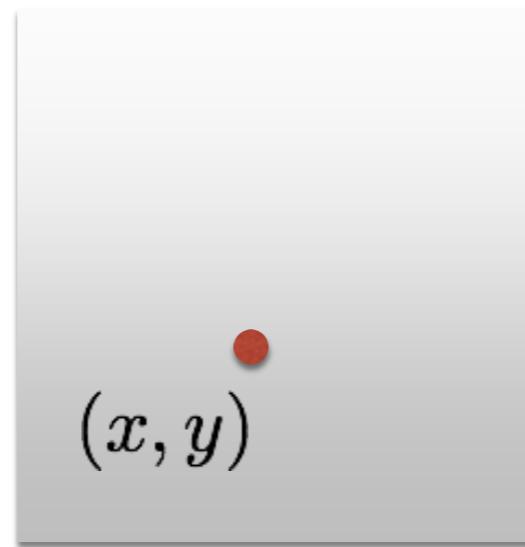
$I(x, y, t)$



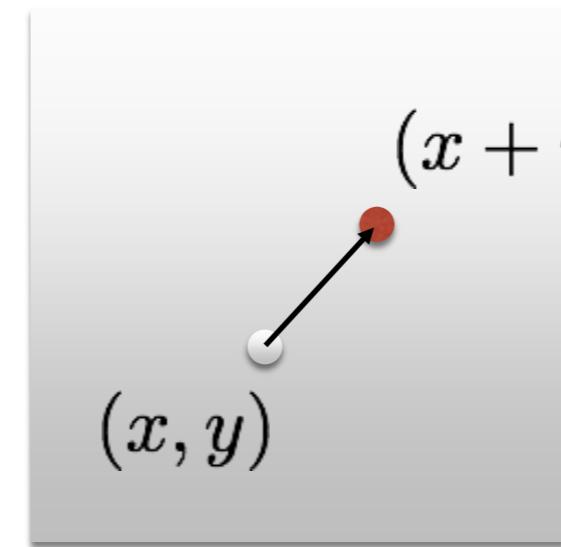
$I(x, y, t + \delta t)$

Assumption 2

Small motion



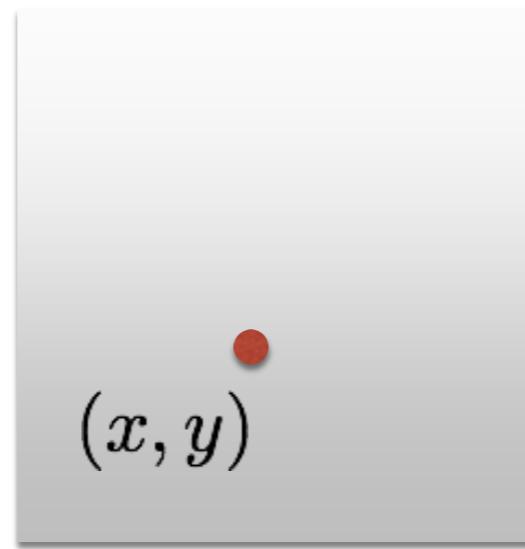
$$I(x, y, t)$$



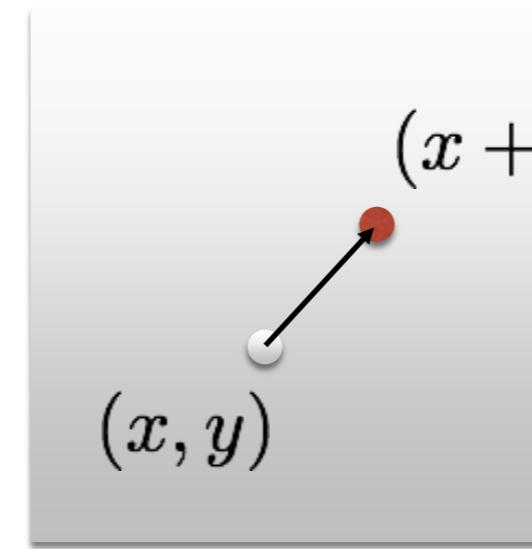
$$I(x, y, t + \delta t)$$

Assumption 2

Small motion



$$I(x, y, t)$$



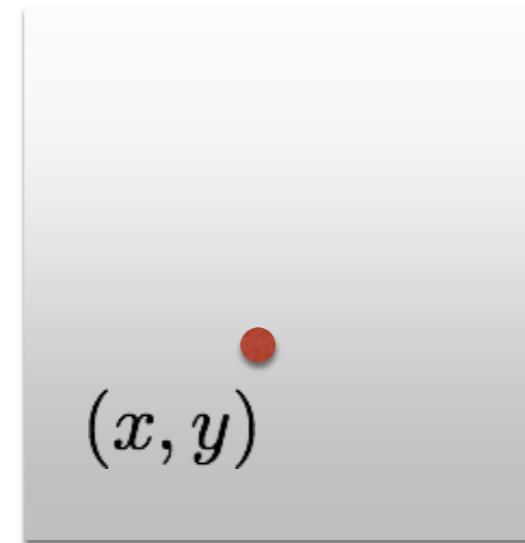
$$I(x, y, t + \delta t)$$

Optical flow (velocities): (u, v)

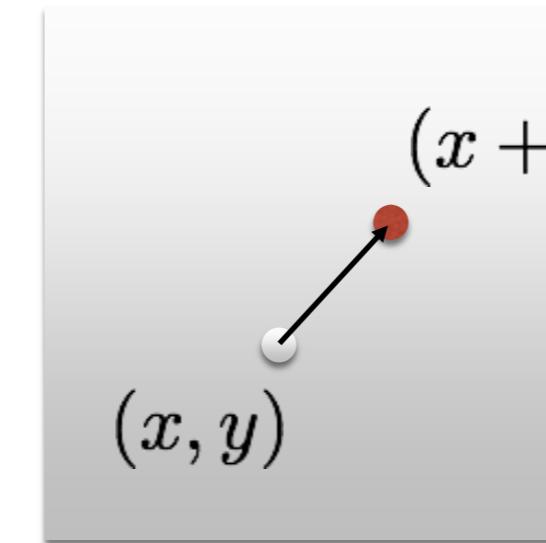
Displacement: $(\delta x, \delta y) = (u\delta t, v\delta t)$

Assumption 2

Small motion



$$I(x, y, t)$$



$$I(x, y, t + \delta t)$$

Optical flow (velocities): (u, v)

Displacement: $(\delta x, \delta y) = (u\delta t, v\delta t)$

For a small space-time step

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

the brightness between two consecutive image frames is the same

Taylor series expansion

For small space-time step, brightness of a point is the same

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$
$$\frac{\delta x}{\delta y}$$

Insight:

If the time step is really small, we can *linearize* the intensity function with first order approximation of the Taylor series expansion

Expand a function as an infinite sum of its derivatives

For one variable

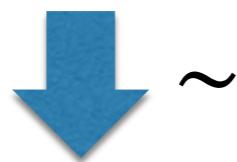
$$f(x + \delta x) = f(x) + \frac{\partial f}{\partial x} \delta x + \frac{\partial^2 f}{\partial x^2} \frac{\delta x^2}{2!} + \cdots + \frac{\partial^n f}{\partial x^n} \frac{\delta x^n}{n!}$$

If δx is small:

$$f(x + \delta x) = f(x) + \frac{\partial f}{\partial x} \delta x + \boxed{O(\delta x^2)} \rightarrow \text{Almost Zero}$$

Taylor series expansion

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$



~

$$I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = I(x, y, t)$$

assuming small motion

fixed point

Partial derivative

$$\frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0$$

cancel terms

$$\frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0$$

divide by δt

take limit $\delta t \rightarrow 0$

$$\frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Brightness Constancy
Equation

Brightness constancy equation

$$\frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

**Brightness
Constancy Equation**

$$I_x u + I_y v + I_t = 0$$

(x-flow)

(y-flow)

shorthand notation

$$\nabla I^\top \mathbf{v} + I_t = 0$$

(1 x 2) (2 x 1)

vector form

Brightness constancy equation

What do the term of the brightness constancy equation represent?

$$I_x u + I_y v + I_t = 0$$

flow velocities

Image gradients
(at a point p)

temporal gradient

The diagram illustrates the brightness constancy equation $I_x u + I_y v + I_t = 0$. It features three terms: $I_x u$, $I_y v$, and I_t . The term $I_x u$ is associated with 'flow velocities' (blue arrows pointing downwards) and 'Image gradients (at a point p)' (green arrows pointing upwards). The term $I_y v$ is also associated with 'flow velocities' (blue arrows pointing downwards) and 'Image gradients (at a point p)' (green arrows pointing upwards). The term I_t is associated with a 'temporal gradient' (purple arrow pointing upwards).

Brightness constancy equation

$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Forward difference

Sobel filter

Derivative-of-Gaussian filter

...

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

Frame differencing

Frame differencing

Example of a forward difference

$$I_t = \frac{\partial I}{\partial t}$$

t $t + 1$

-

1	1	1	1	1
1	1	1	1	1
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10

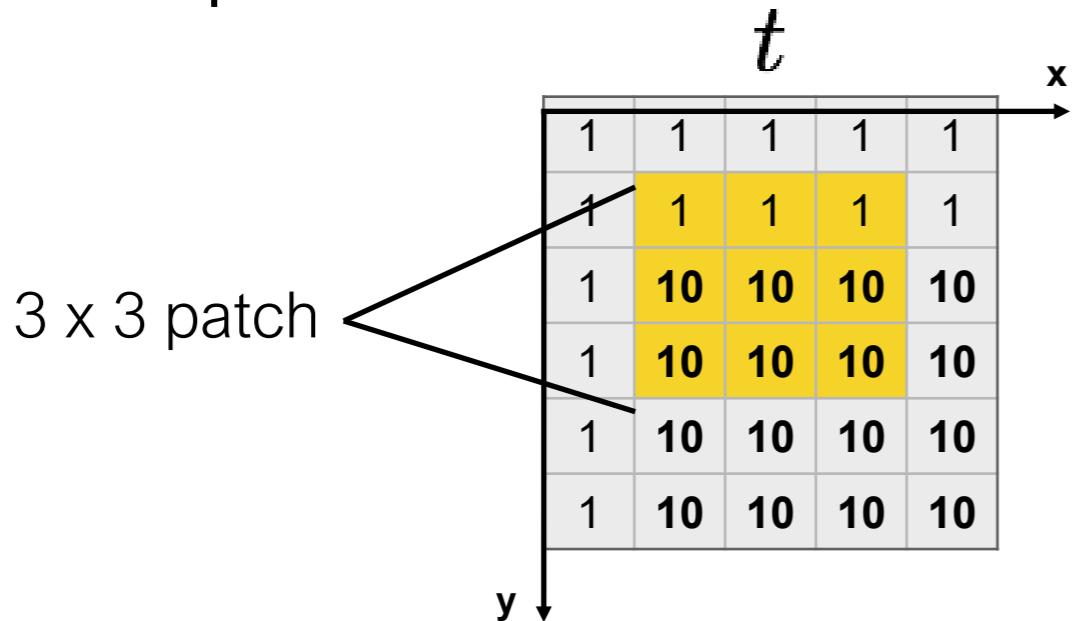
=

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	10	10	10
1	1	10	10	10
1	1	10	10	10

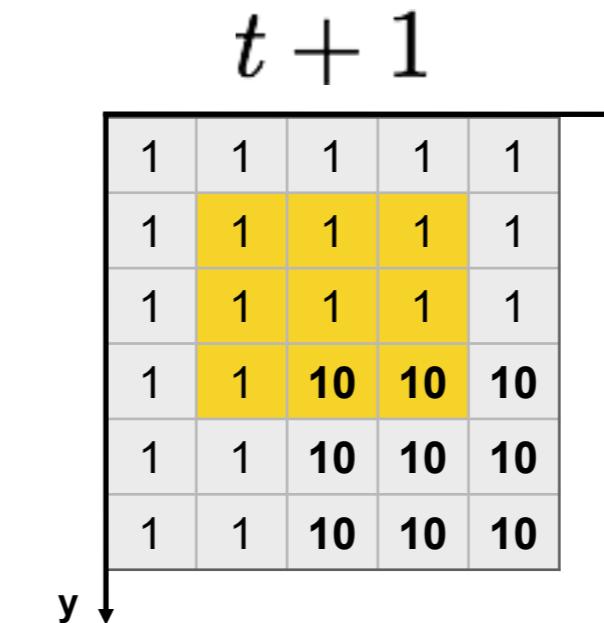
=

0	0	0	0	0
0	0	0	0	0
0	9	9	9	9
0	9	0	0	0
0	9	0	0	0
0	9	0	0	0

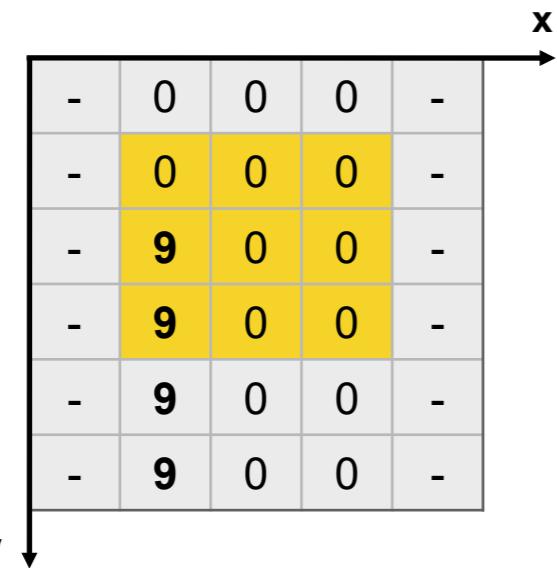
Example:



3 x 3 patch

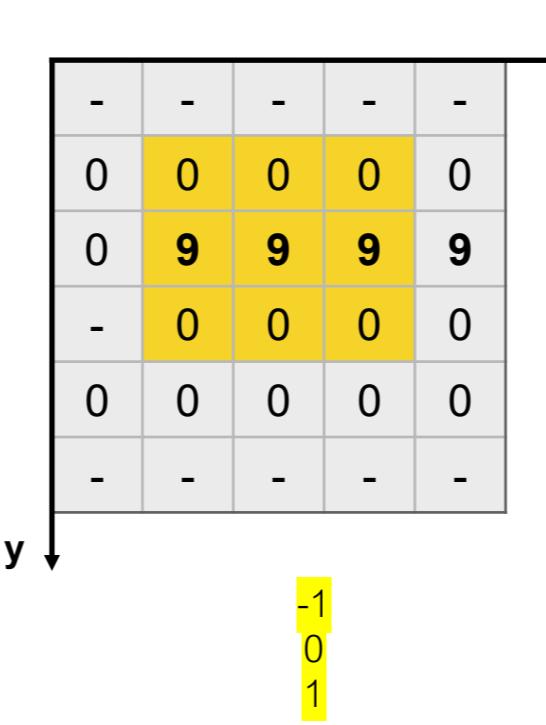


$$I_x = \frac{\partial I}{\partial x}$$

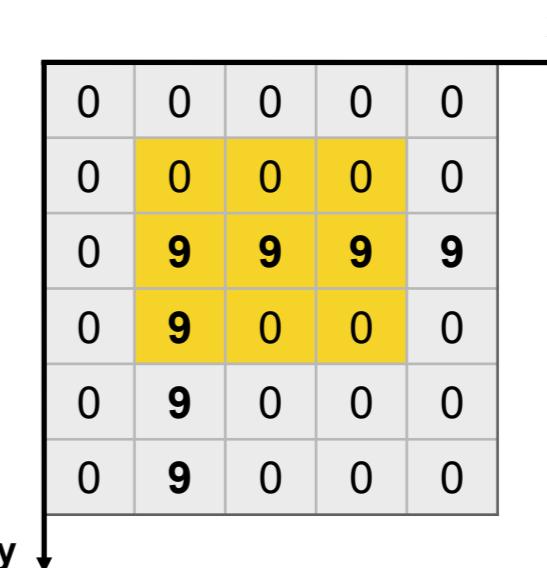


-1 0 1

$$I_y = \frac{\partial I}{\partial y}$$



$$I_t = \frac{\partial I}{\partial t}$$



Brightness constancy equation

$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Forward difference

Sobel filter

Derivative-of-Gaussian filter

...

$$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$$

optical flow

unknown

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

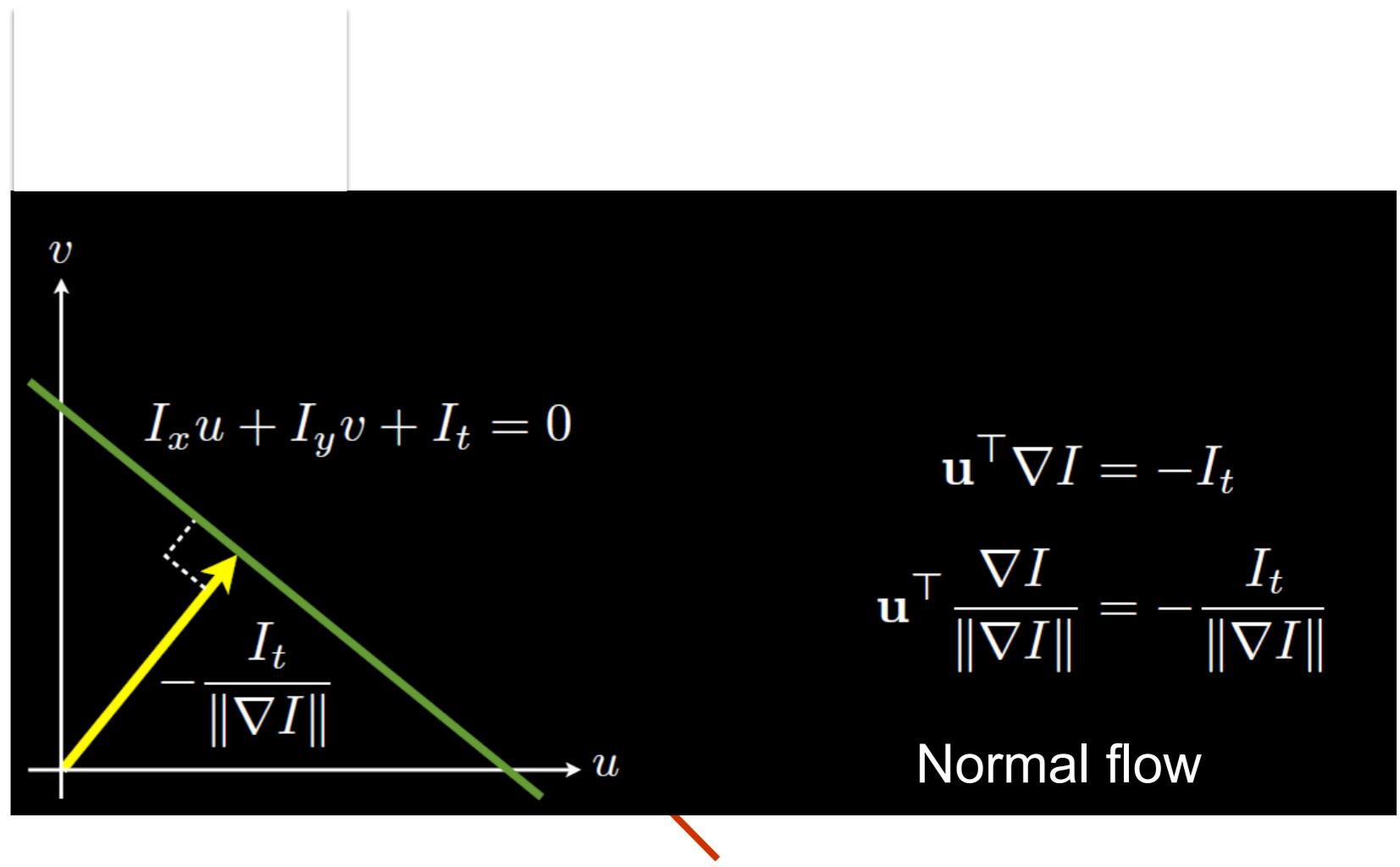
Frame differencing

Brightness constancy equation

Solution lies on a straight line

$$I_x u + I_y v + I_t = 0$$

many combinations of u and v will satisfy the equality

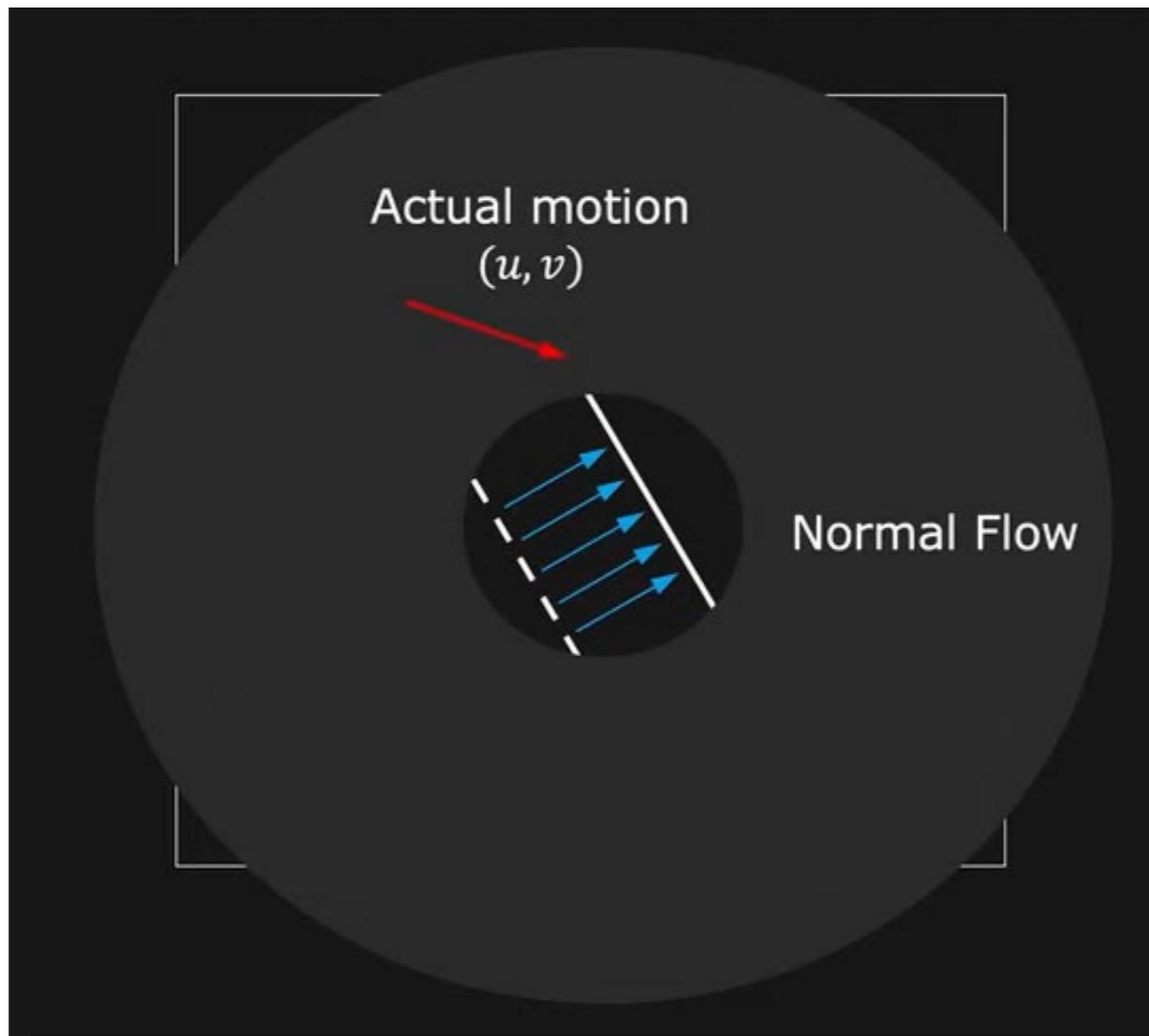


Optical flow can be split in two components: normal flow and parallel flow



The solution cannot be determined uniquely with a single constraint (a single pixel). The solution is underconstrained
Where do we get more equations (constraints)?

Aperture problem



Locally we can only determine normal flow

Two main methods

- **Lucas-Kanade Optical Flow (1981)**
method of differences
'constant' flow (flow is constant for all pixels)
local method (sparse)

- **Horn-Schunck Optical Flow (1981)**
brightness constancy, small motion
'smooth' flow (flow can vary from pixel to pixel)
global method (dense)

Lucas-Kanade Optical Flow

Assumptions

Assume that the surrounding patch has '**constant flow**'

Neighboring pixels have same displacement: i.e for each pixel assume motion field, and hence optical flow (u, v), is constant within a small neighborhood.

Using a 5×5 image patch, gives us 25 equations

$$I_x(\mathbf{p}_1)u + I_y(\mathbf{p}_1)v = -I_t(\mathbf{p}_1)$$

$$I_x(\mathbf{p}_2)u + I_y(\mathbf{p}_2)v = -I_t(\mathbf{p}_2)$$



:

$$I_x(\mathbf{p}_{25})u + I_y(\mathbf{p}_{25})v = -I_t(\mathbf{p}_{25})$$

n^2 (25) equations,
2 unknown: find least
square solution

Least square solution

Solve linear system $Ax=b$

- that it is equivalent to write (least squares using pseudo-inverse)

'Lucas-Kanade Optical Flow'

$$\begin{matrix} A^\top A & x & A^\top b \\ \left[\begin{array}{cc} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{array} \right] & \left[\begin{array}{c} u \\ v \end{array} \right] = - \left[\begin{array}{c} \sum_{p \in P} I_x I_t \\ \sum_{p \in P} I_y I_t \end{array} \right] \\ 2 \times 2 & 2 \times 1 & 2 \times 1 \end{matrix}$$

where the summation is over each pixel p in patch P

$$x = (A^\top A)^{-1} A^\top b$$

When does optical flow estimation work?

$$x = (A^T A)^{-1} A^T b$$

$A^T A$ should be invertible ($\det \neq 0$)

$A^T A$ should be well conditioned

λ_1 and λ_2 should not be too small (both are significant enough)

λ_1 should not be too large respect to λ_2 (λ_1 =larger eigenvalue)

When does optical flow estimation work?

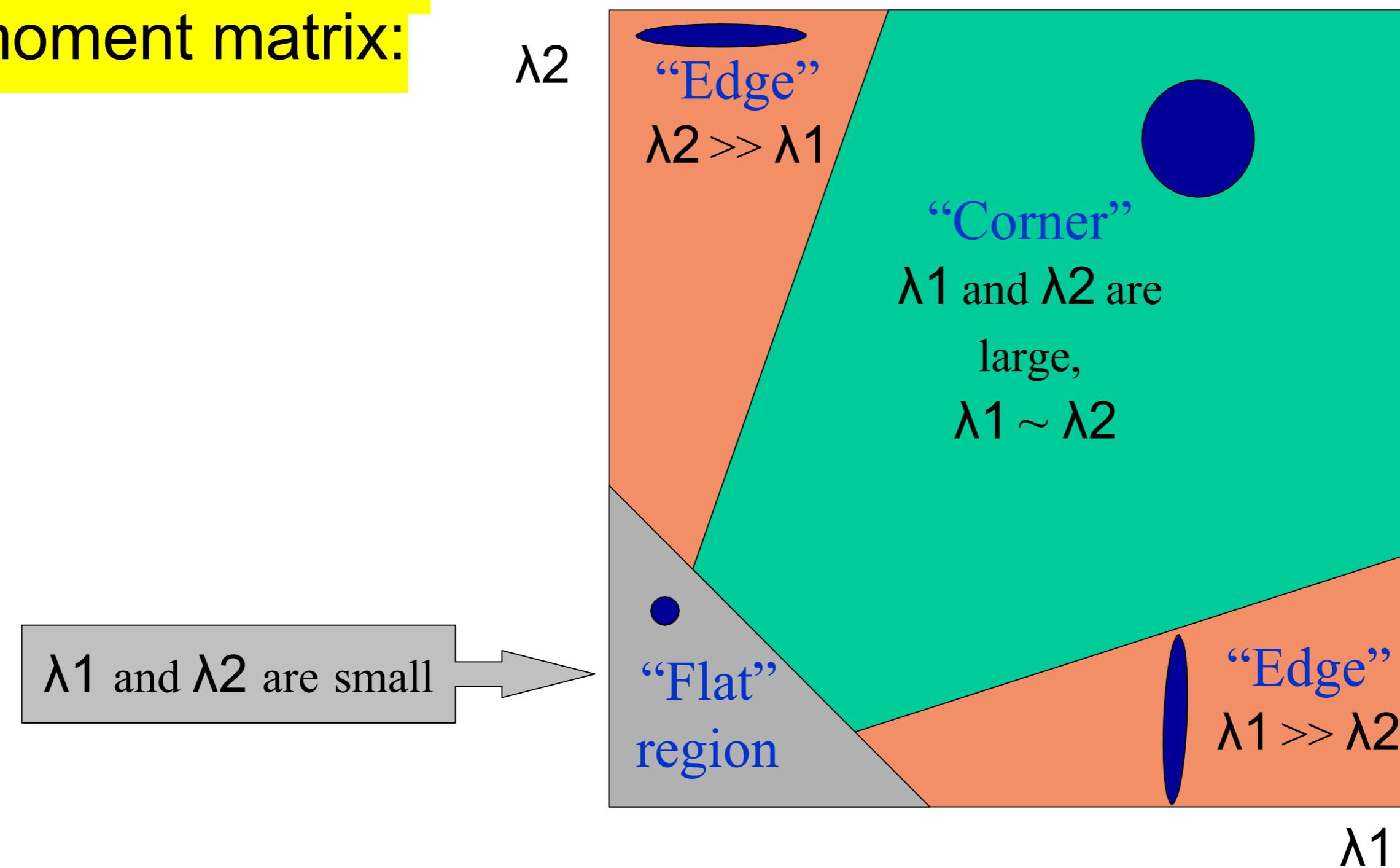
$A^T A$ is the *second moment matrix* (Harris corner detector)

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

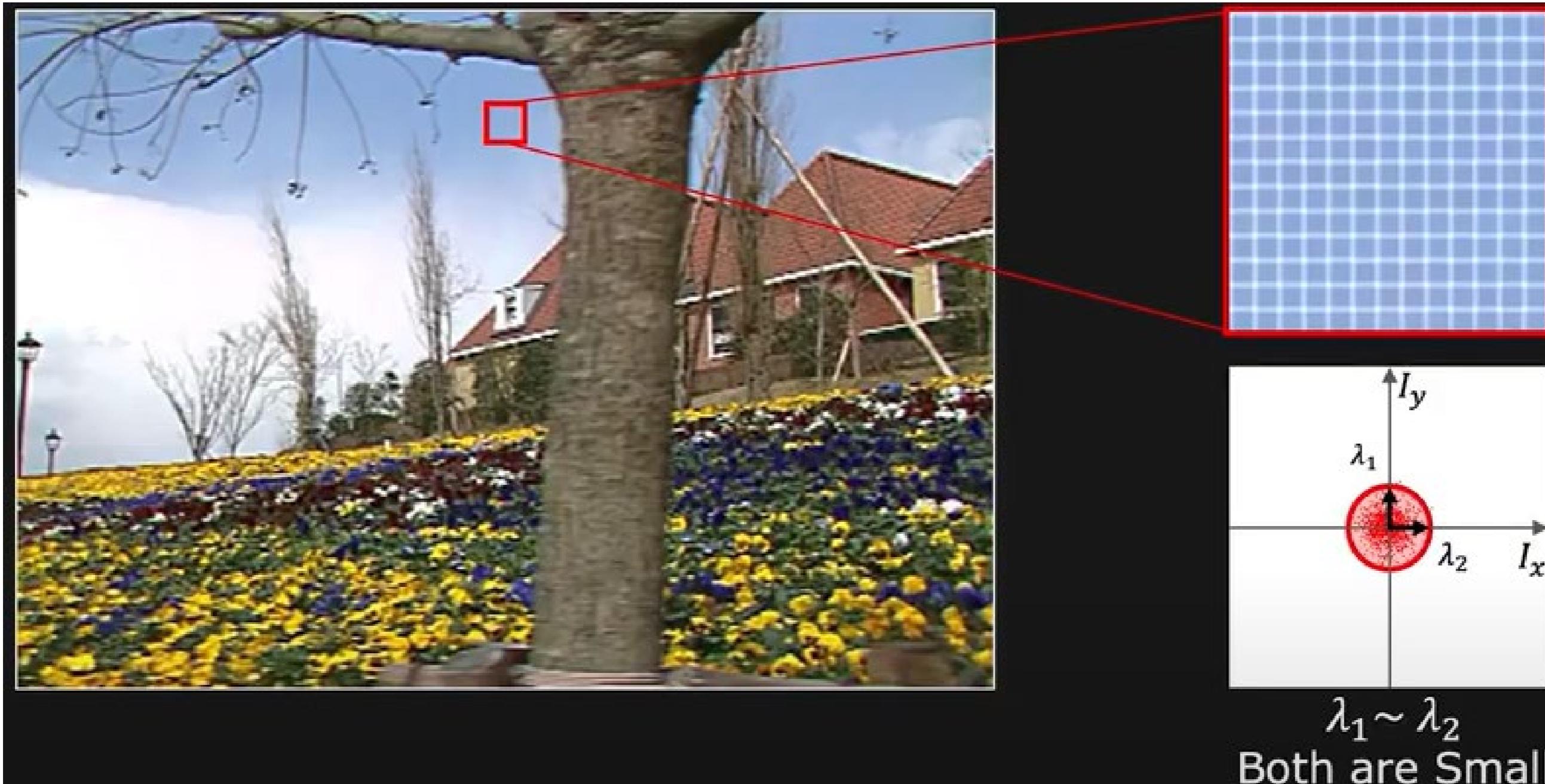
- Eigenvectors and eigenvalues of $A^T A$ relate to edge direction and magnitude
 - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change
 - The other eigenvector is orthogonal to it

Interpreting the eigenvalues

Classification of image points
using eigenvalues of the
second moment matrix:

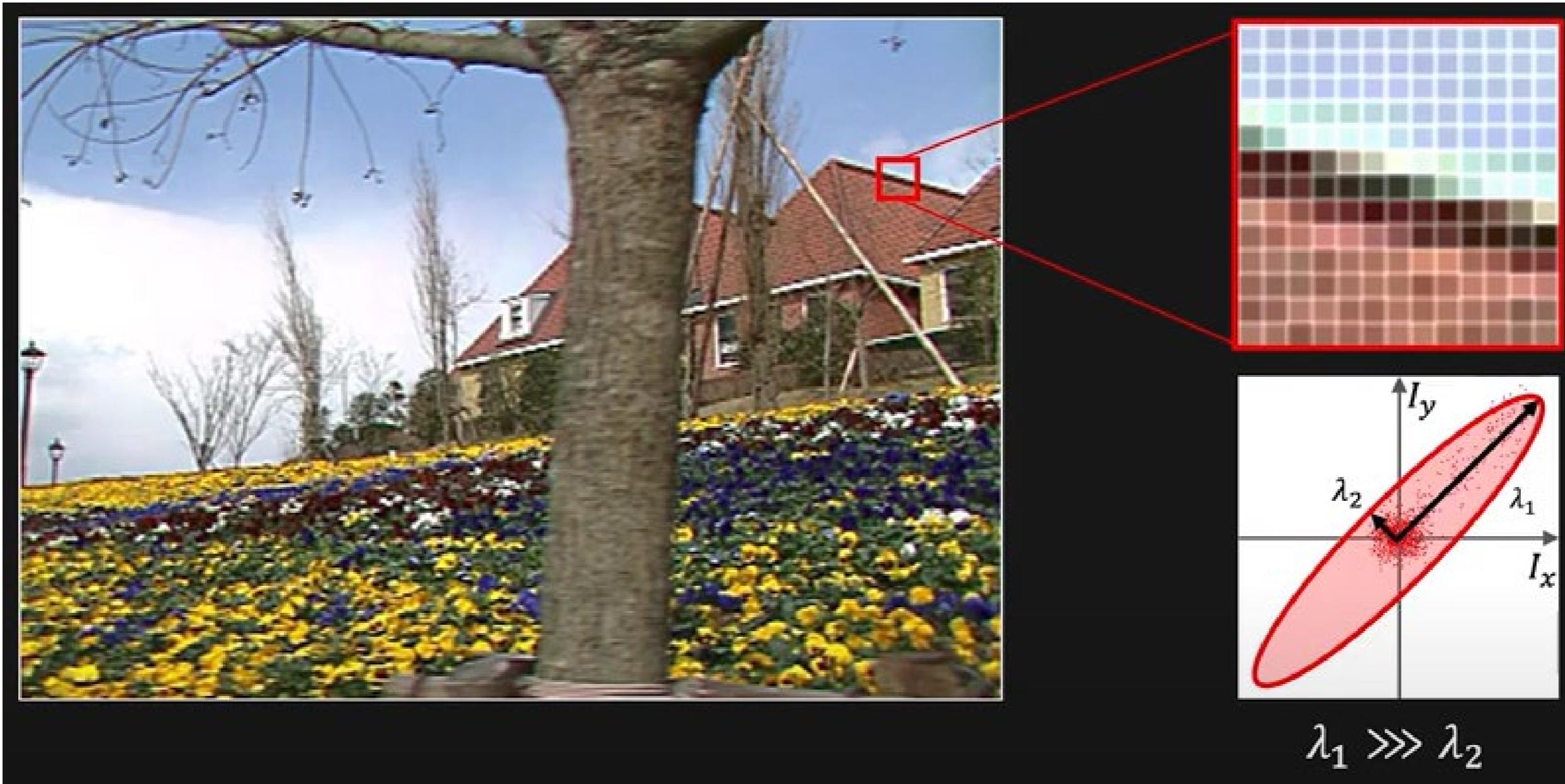


Low-texture region



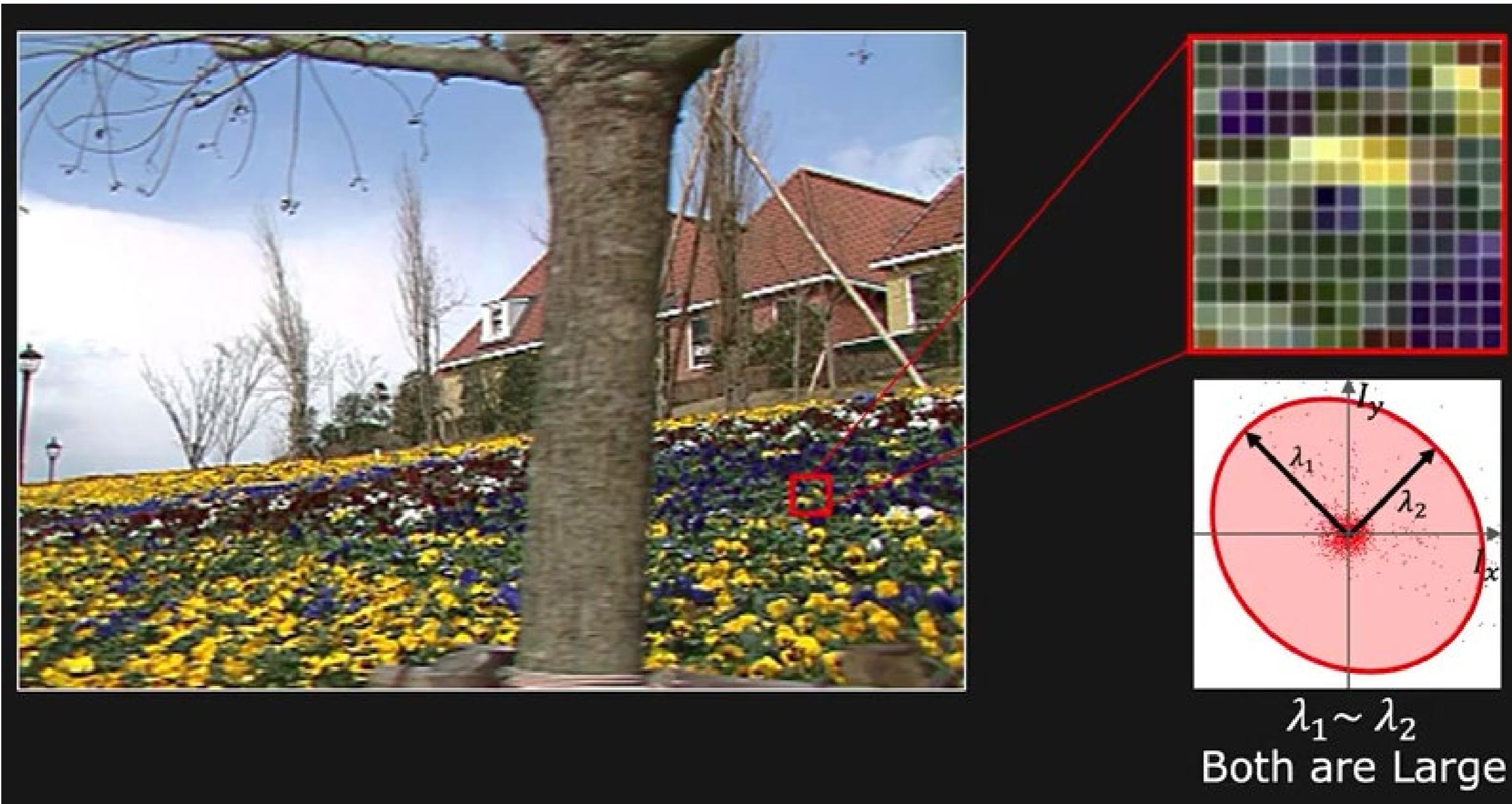
- Equations for all pixels in window are more or less the same
- Gradients have small magnitude
- Can not reliably compute the flow

Edges



- gradients very large or very small
- prominent gradient in one direction \rightarrow badly conditioned
- can not reliably compute the flow

High-texture region



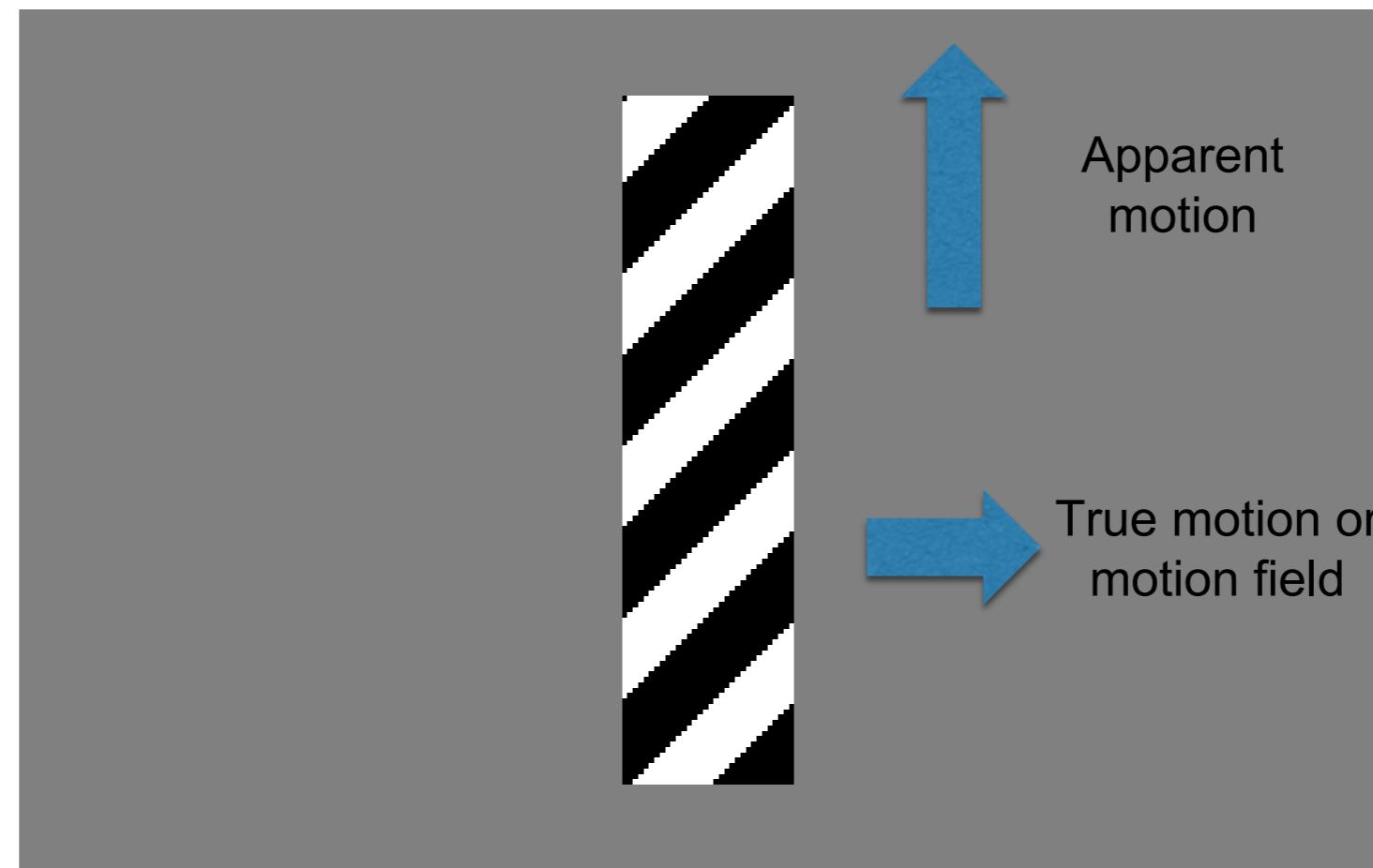
- gradients are different, large magnitudes
- well conditioned
- Can reliably compute optical flow

Implications

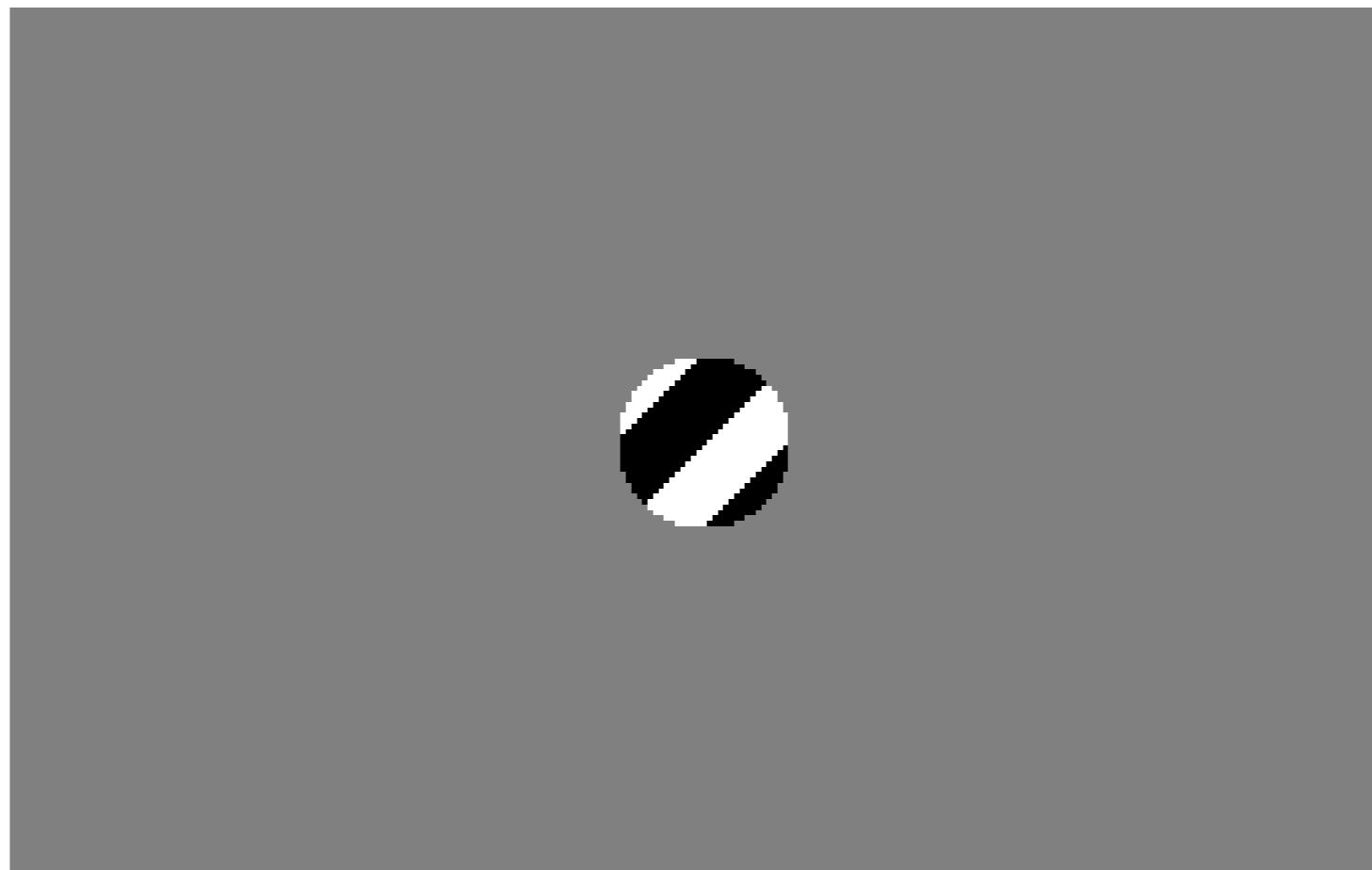
- Corners and high texture regions are when λ_1, λ_2 are big; this is also when Lucas-Kanade optical flow works best
- Corners and high texture regions are regions with two different directions of gradient (at least)
- Corners and high texture regions are good places to compute flow



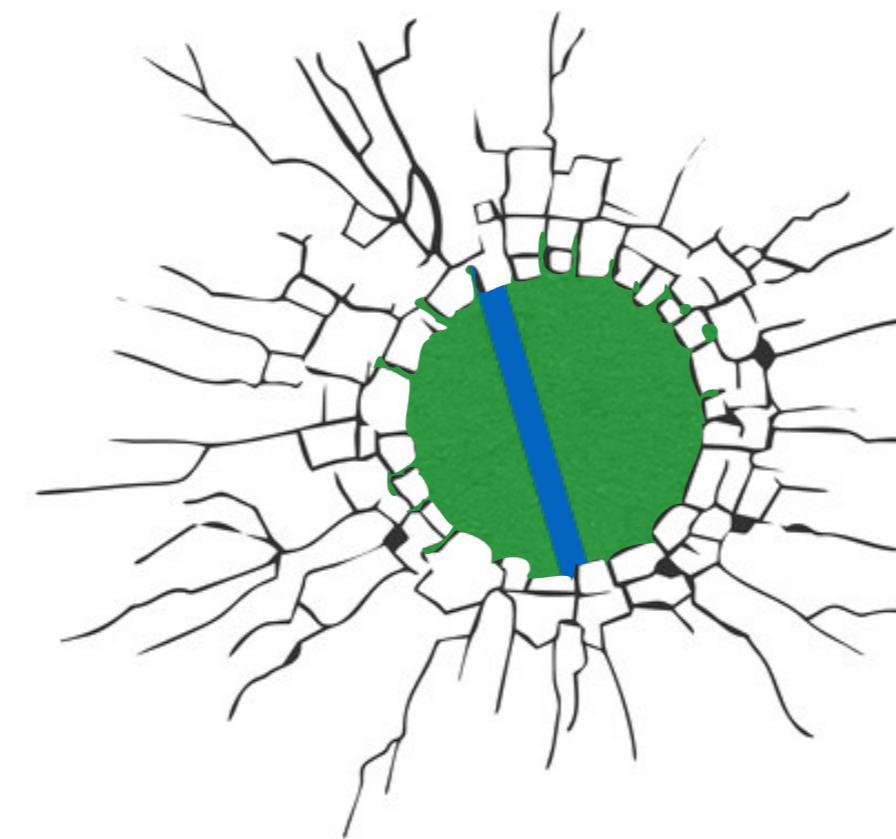
Barber's pole illusion



Barber's pole illusion



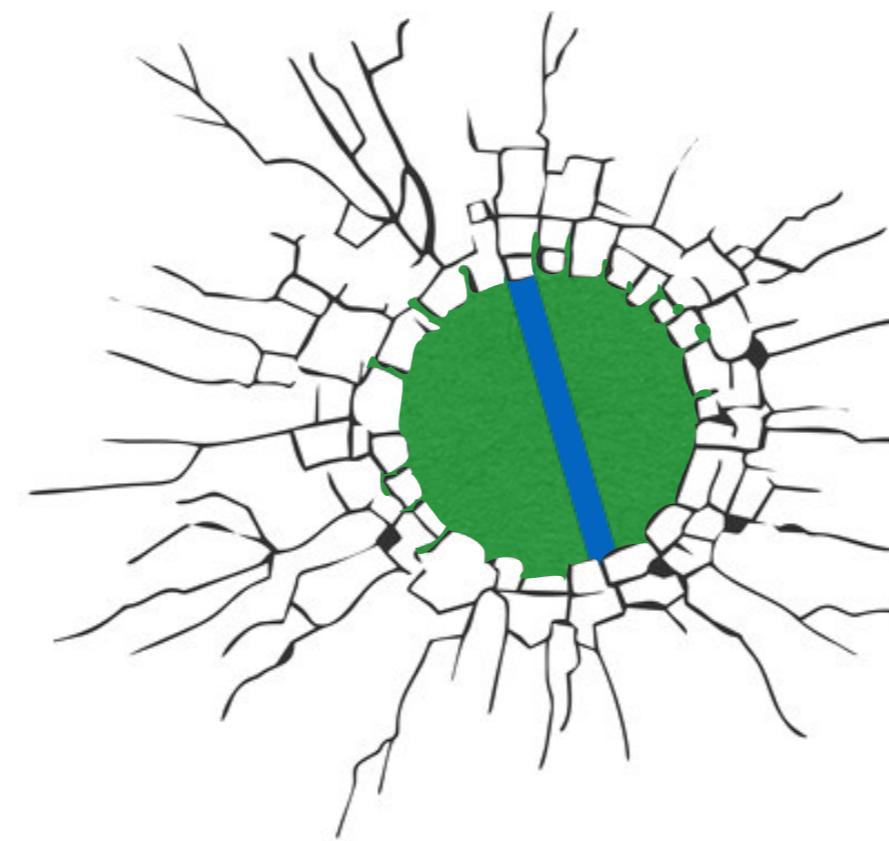
Aperture Problem



small visible image patch

In which direction is the line moving?

Aperture Problem

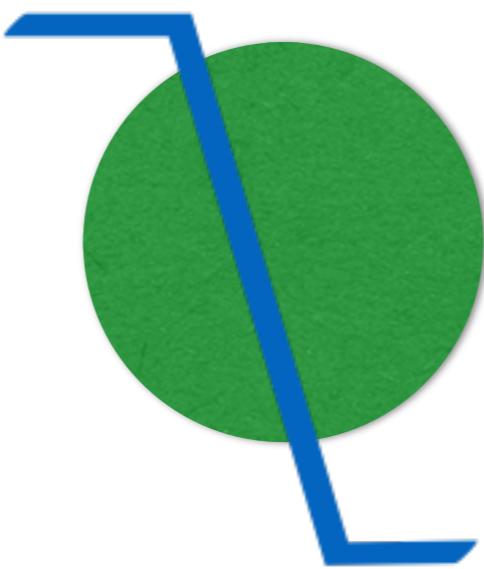


small visible image patch

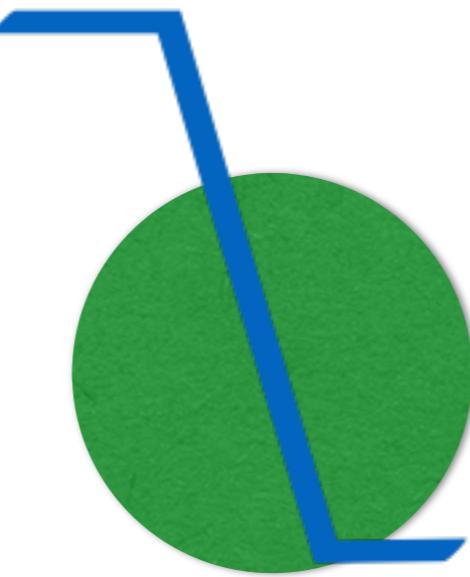


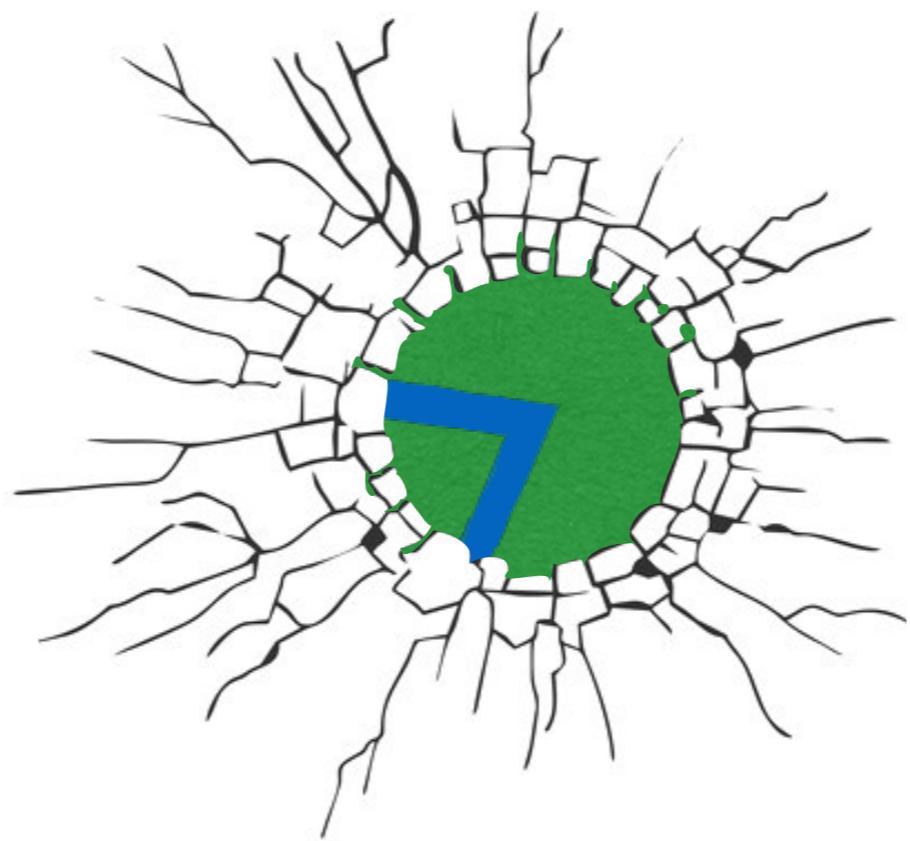
In which direction is the line moving?

Aperture Problem

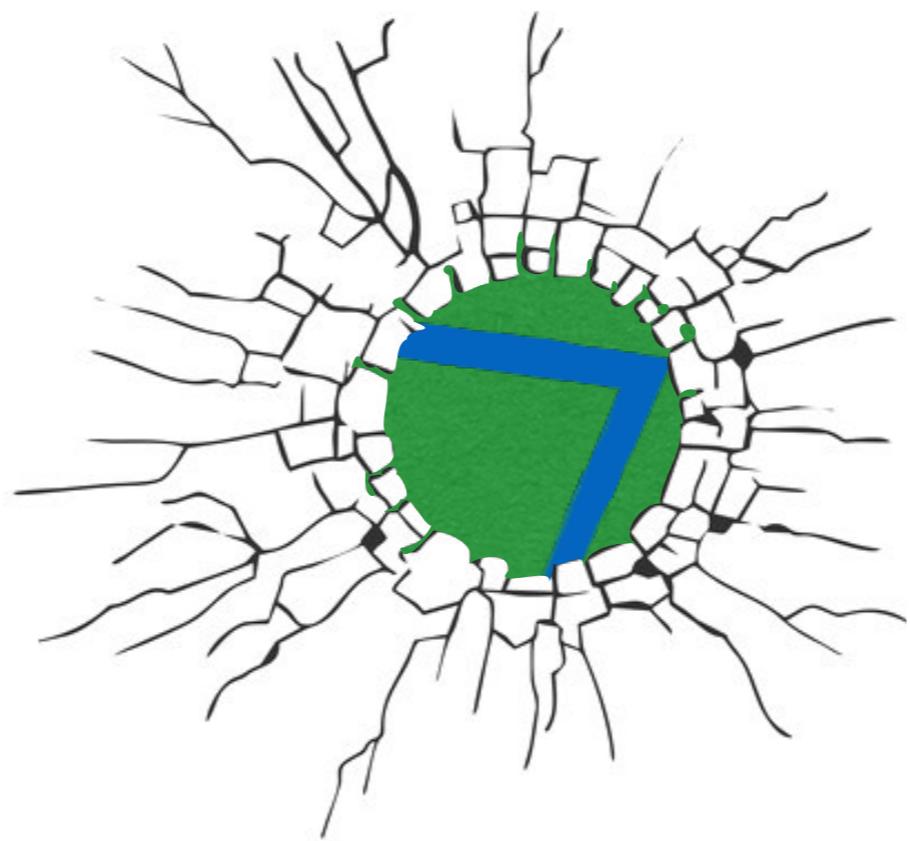


Aperture Problem



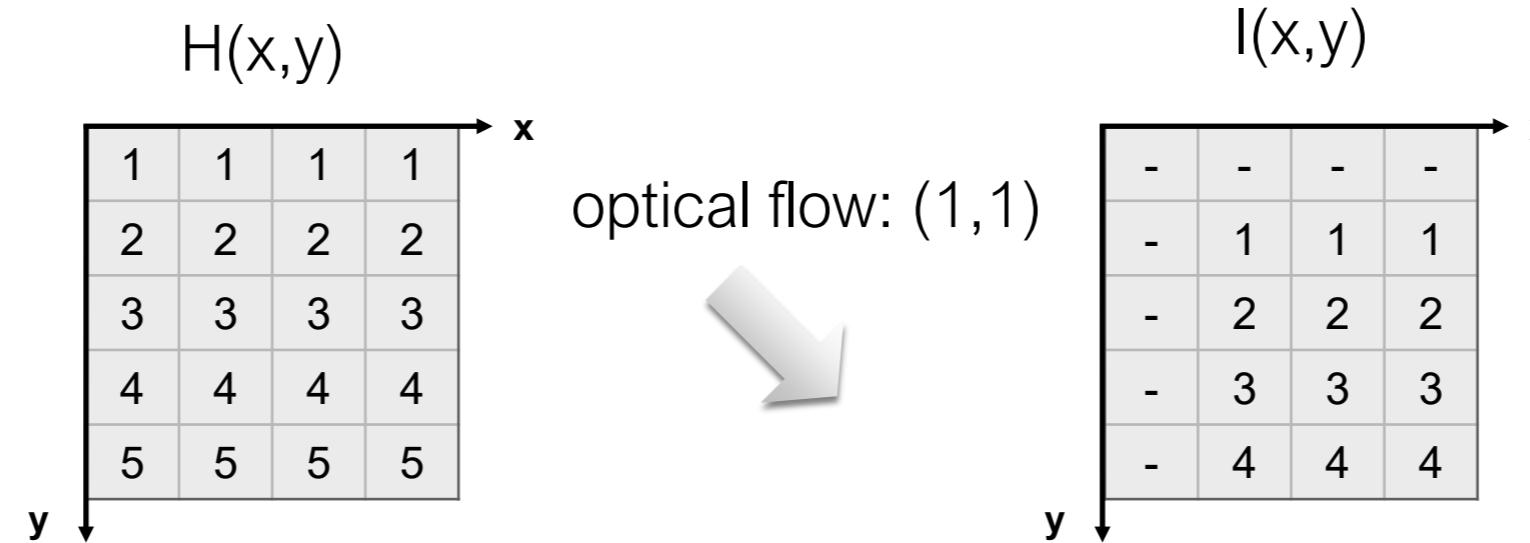


Want patches with different gradients to avoid the aperture problem



Want patches with different gradients to avoid the aperture problem

Aperture Problem: example



$$\cancel{I_x u + I_y v + I_t = 0}$$

Compute gradients

$$I_x(3,3) = 0$$

$$I_y(3,3) = 1$$

$$I_t(3,3) = I(3,3) - H(3,3) = -1$$

Solution:

$$\rightarrow v = 1$$

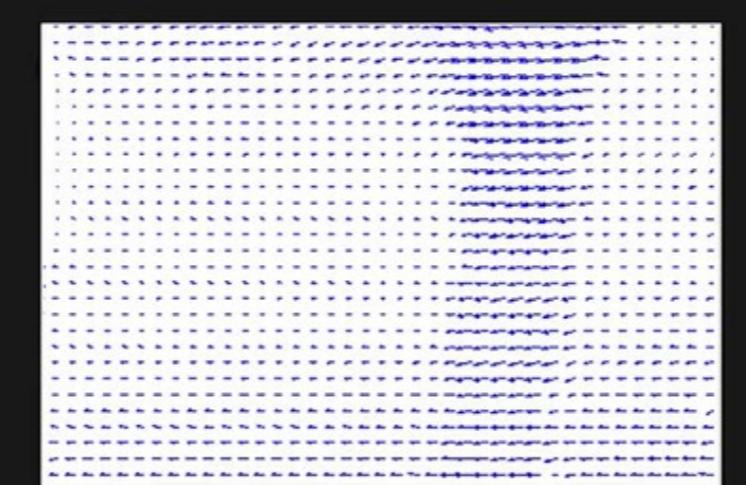
We recover the v of the optical flow but not the u .

Lukas-Kanade method

- Decide for local neighborhood of $W \times W$ pixels and apply uniformly in frames (smoothing frame first with Gaussian filter with a small standard deviation sigma=1.5)
- At frame t , $t+1$ calculate the derivatives I_x , I_y , I_t
- For each couple of frames obtain equational system and solve in the least square sense calculating the eigenvalues
- Plot the optical flow vectors (directions and magnitude)



Image Sequence
(2 frames)



Optical Flow

Two main methods

- **Lucas-Kanade Optical Flow (1981)**
method of differences
'constant' flow (flow is constant for all pixels)
local method (sparse)

- **Horn-Schunck Optical Flow (1981)**
brightness constancy, small motion
'smooth' flow (flow can vary from pixel to pixel)
global method (dense)

Key idea

(of Horn-Schunck optical flow)

In order to compute optical flow:

Enforce
brightness constancy

Enforce
smooth flow field

Enforce brightness constancy

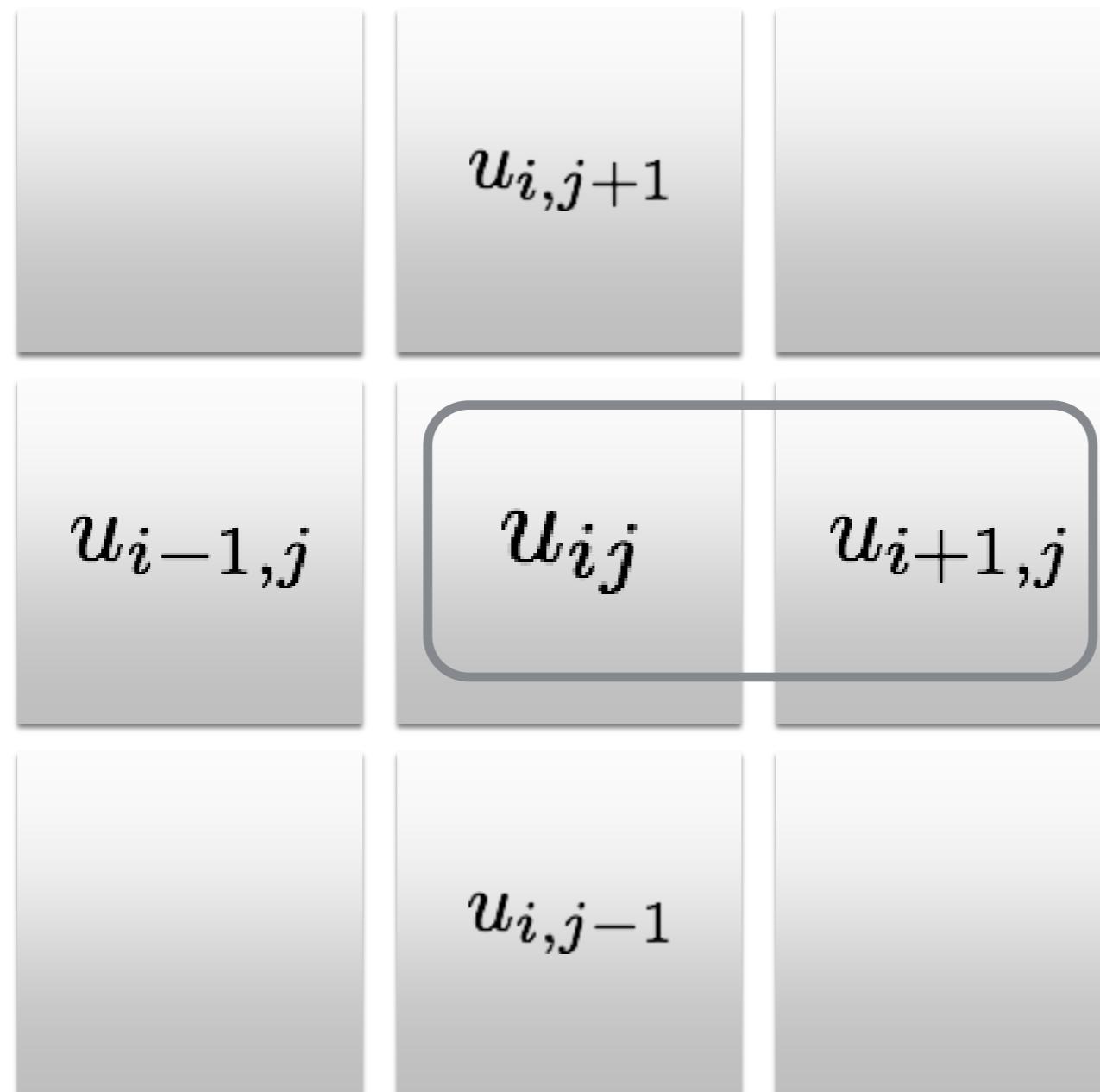
$$I_x u + I_y v + I_t = 0$$

For every pixel, we need to solve the second moment matrix that is approximated by a quadratic form as you remember from HCD

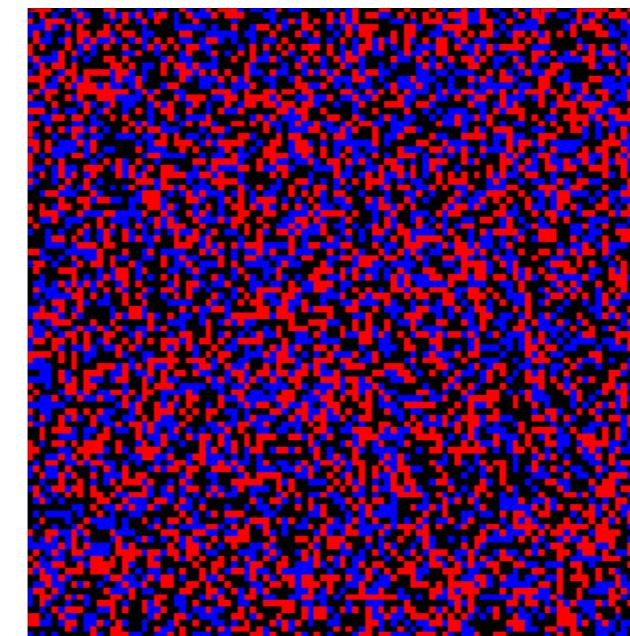
$$\min_{u,v} \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$



Enforce smooth flow field



$$\min_{\mathbf{u}} (u_{i,j} - u_{i+1,j})^2$$



- we expect optical flow fields to be smooth
- **most objects in the world are rigid or deform elastically moving together coherently**

Horn-Schunck (HS) optical flow



$$\min_{\mathbf{u}, \mathbf{v}} \sum_{i,j} \left\{ E_s(i, j) + \lambda E_d(i, j) \right\}$$

↑
brightness constancy
smoothness
weight

Larger values lead to a smoother flow

Optimization problem

- HS algorithm assume smoothness in the flow over the whole image
- The flow is formulated as a global energy function which is then sought to be minimized
- Compute partial derivative, derive update equations (gradient descent!)

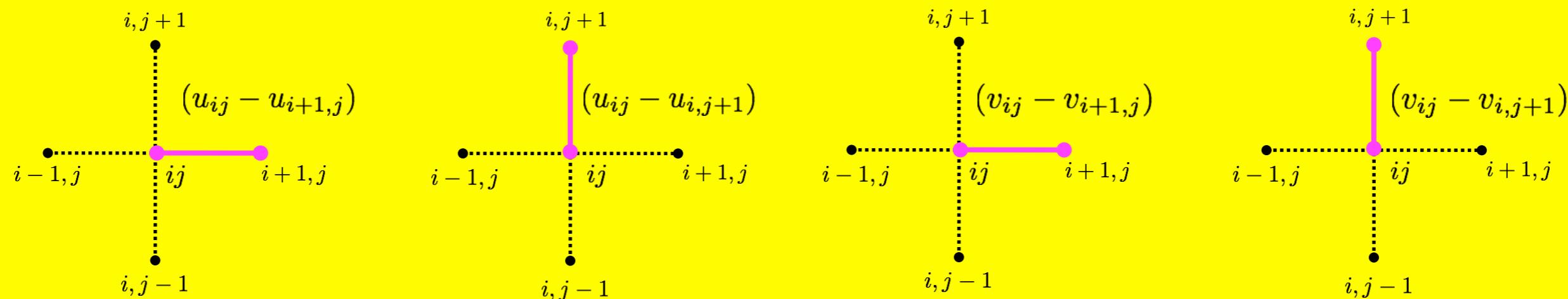
HS optical flow objective function

Brightness constancy

$$E_d(i, j) = \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

Smoothness

$$E_s(i, j) = \frac{1}{4} \left[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right]$$



Horn-Schunck Optical Flow Algorithm

1. Precompute image gradients

$$I_y \quad I_x$$

2. Precompute temporal gradients

$$I_t$$

3. Initialize flow field

$$\mathbf{u} = \mathbf{0}$$

$$\mathbf{v} = \mathbf{0}$$

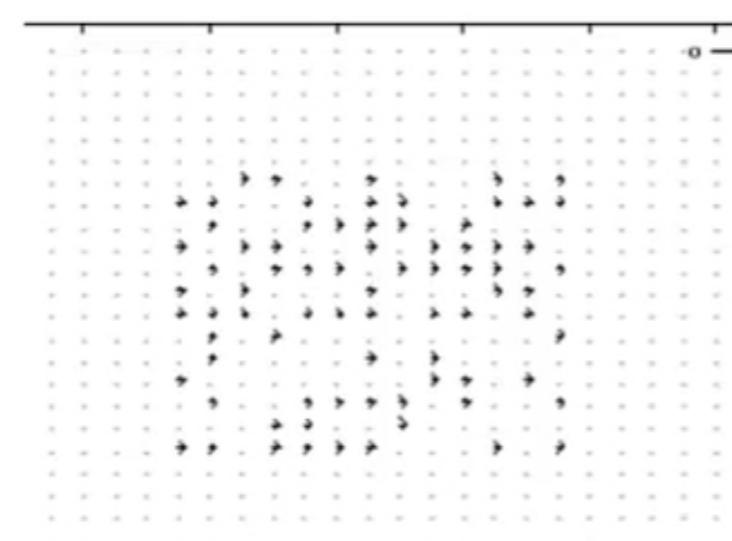
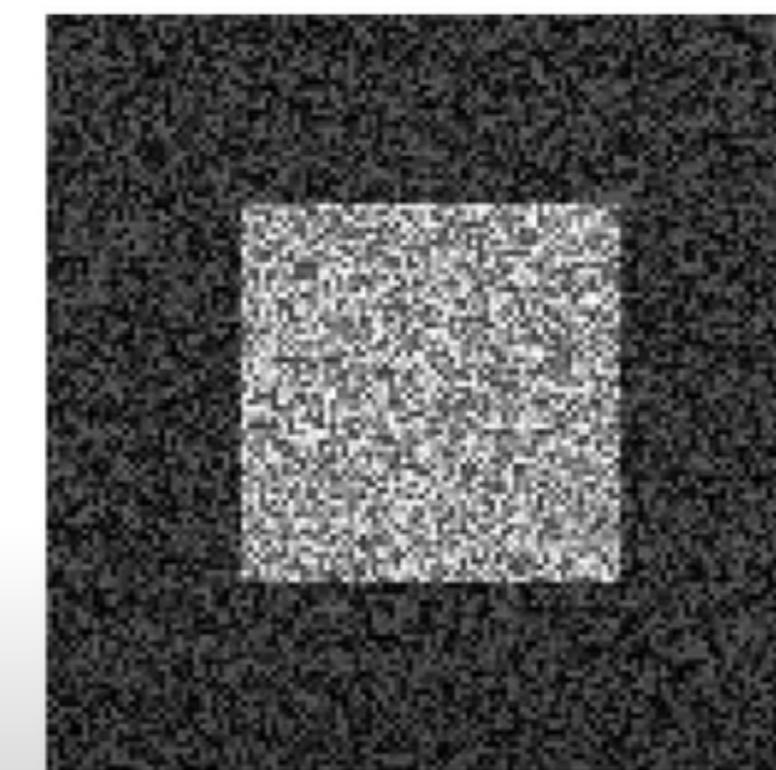
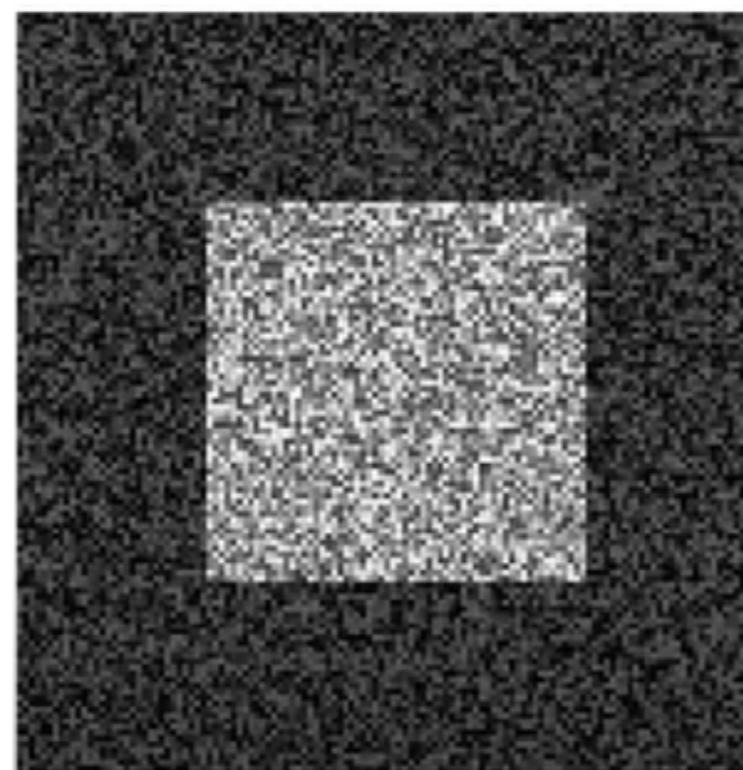
4. While not converged

Compute flow field updates for each pixel:

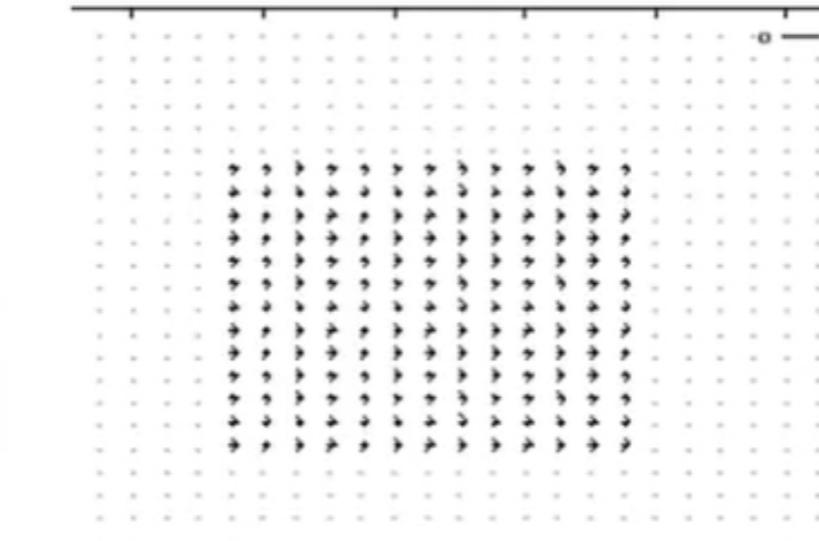
$$\hat{u}_{kl} = \bar{u}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_x$$

$$\hat{v}_{kl} = \bar{v}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_y$$

Horn – Shunck example



One iteration



10 iterations

Recap

Key assumptions

- **Small motion:** points do not move very far
- **Brightness constancy:** projection of the same point looks the same in every frame
- **Spatial coherence:** points move like their neighbors

Revisiting the small motion assumption



Taylor Series approximation of
 $I(x + \delta x, y + \delta y, t + \delta t)$ is not valid

Our simple linear
constraint equation not valid

$$I_x u + I_y v + I_t \neq 0$$

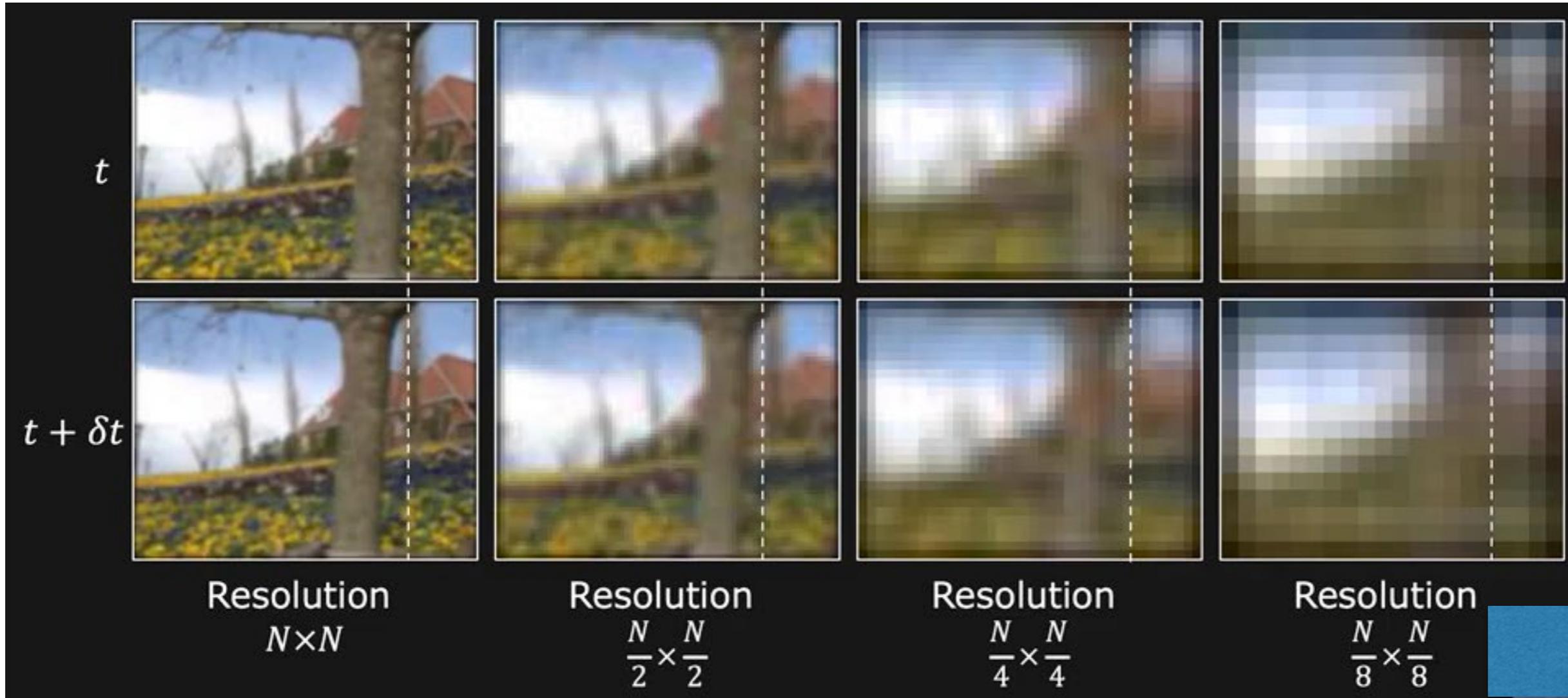
Case of large motion



Is this motion small enough?

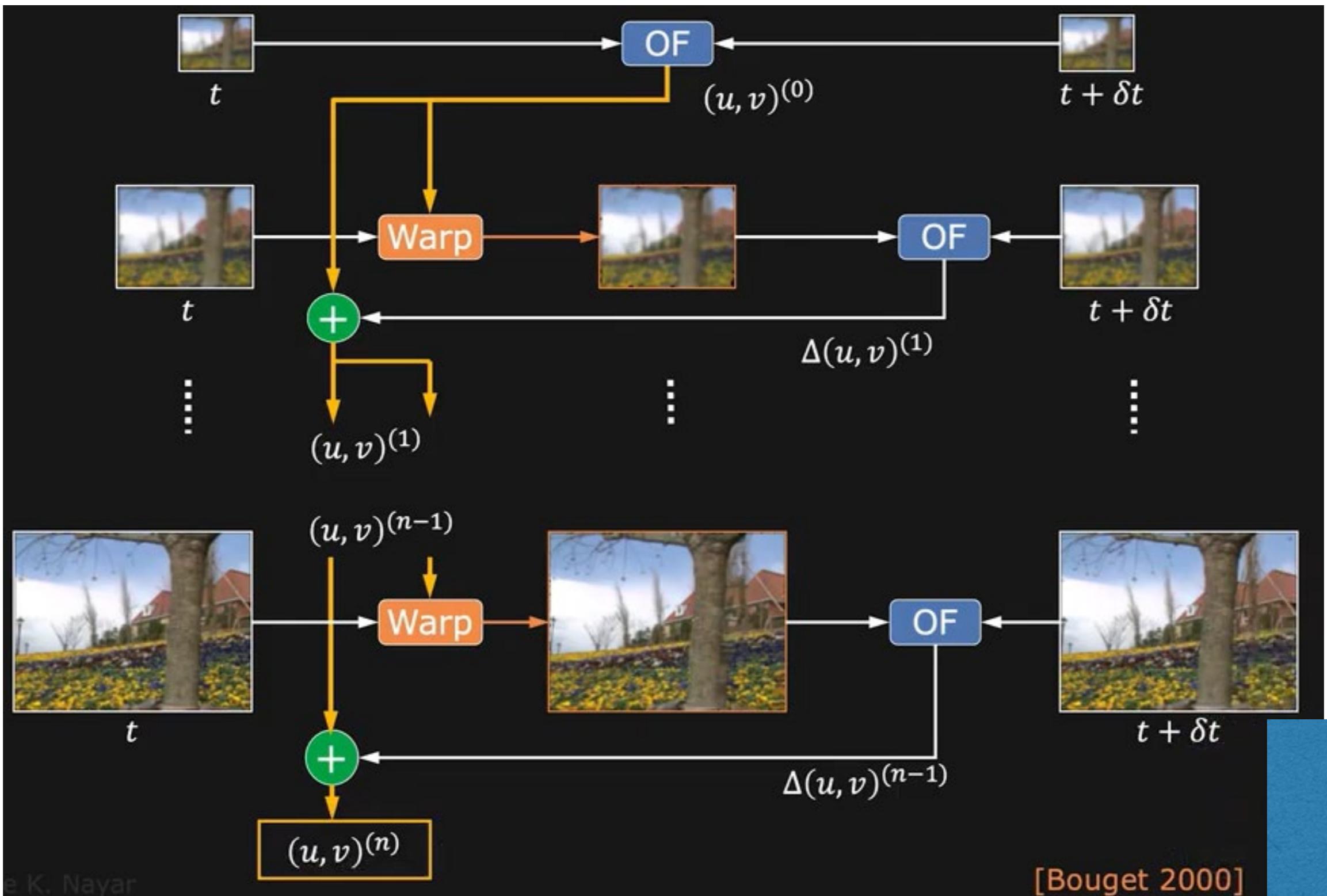
- Probably not: it's much larger than one pixel (2nd order terms dominate)
- How might we solve this problem?

Reduce the resolution

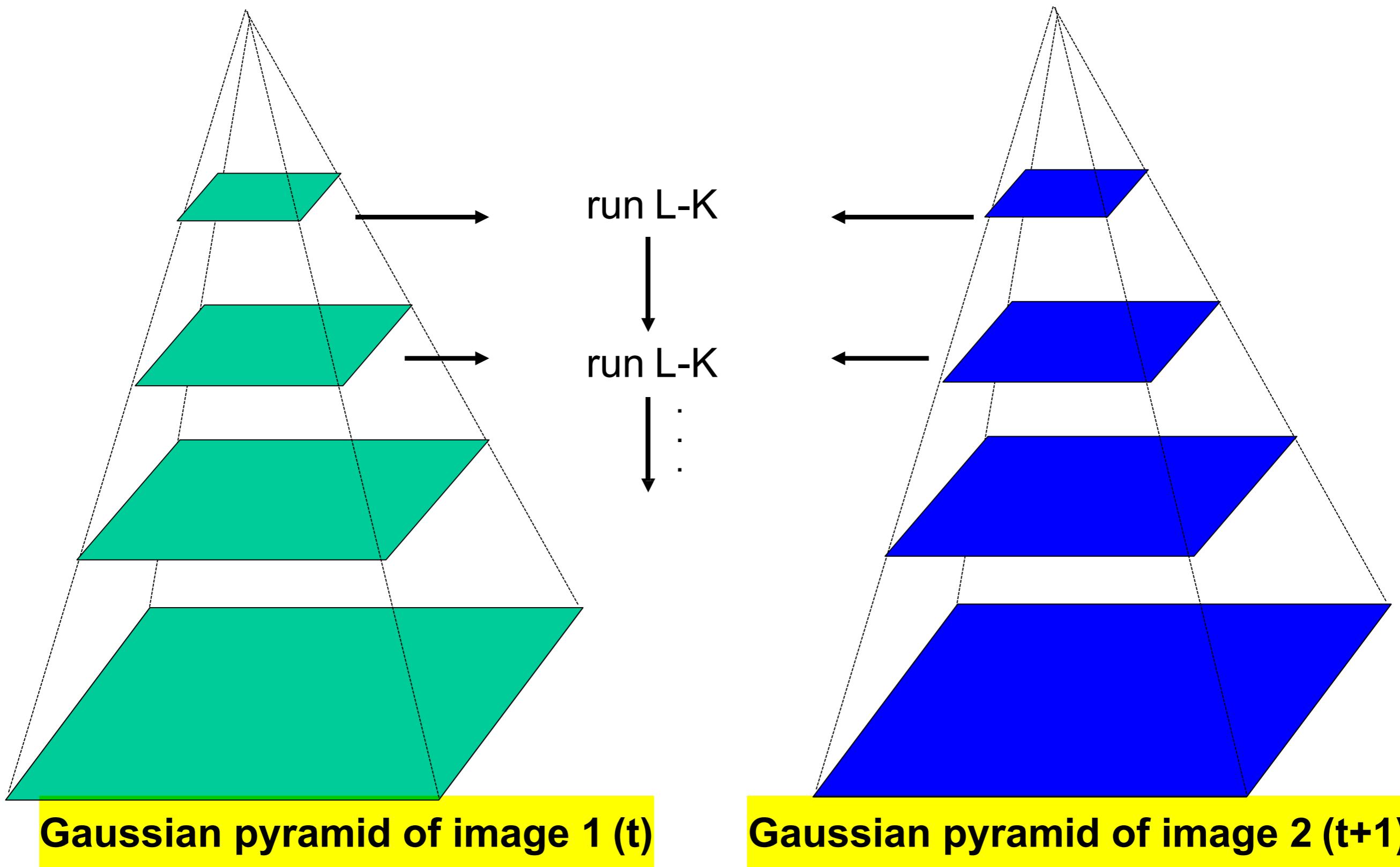


At lowest resolution, motion ≤ 1 pixel

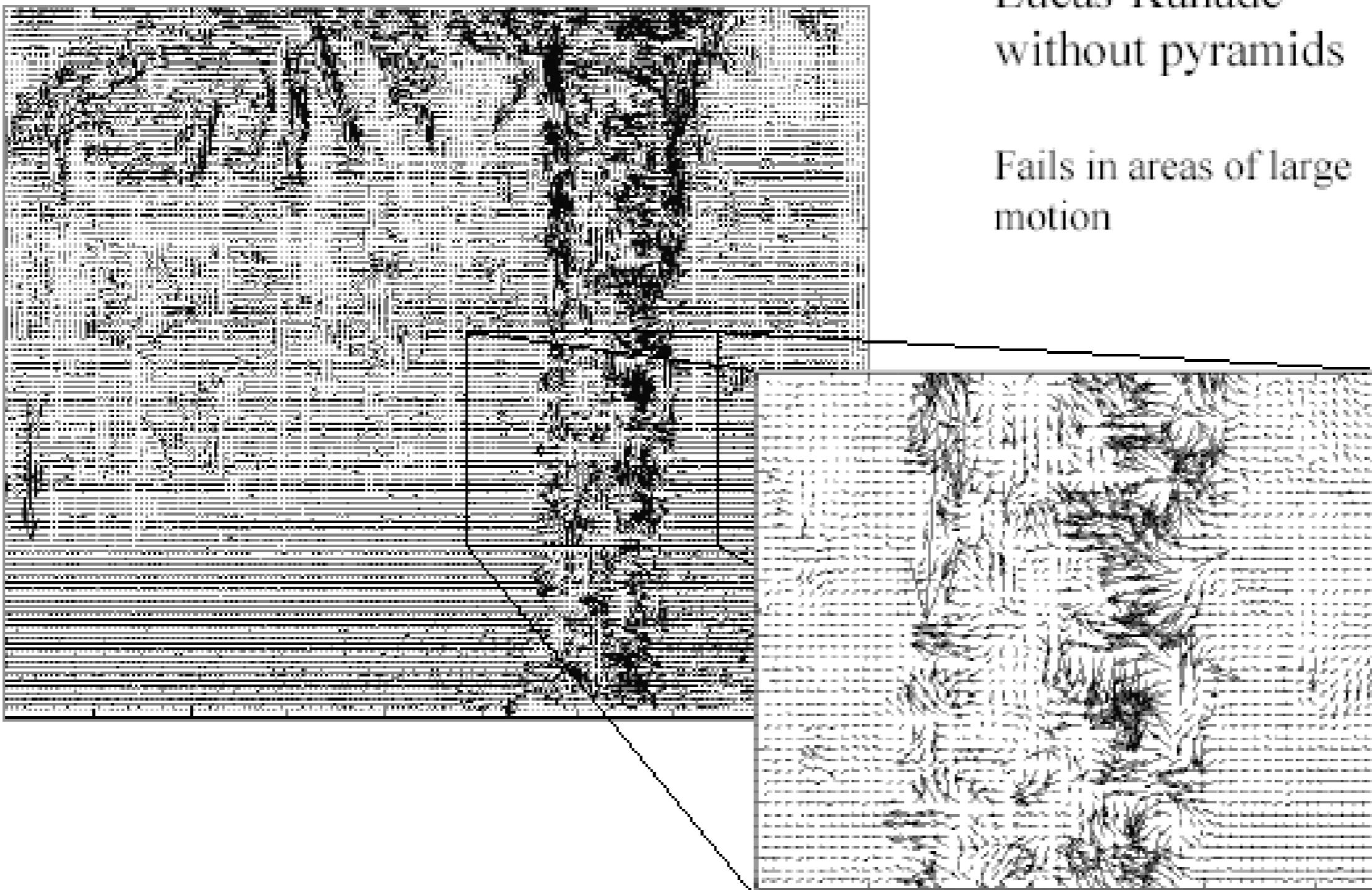
Coarse-to-fine Flow Estimation



Coarse-to-fine optical flow estimation



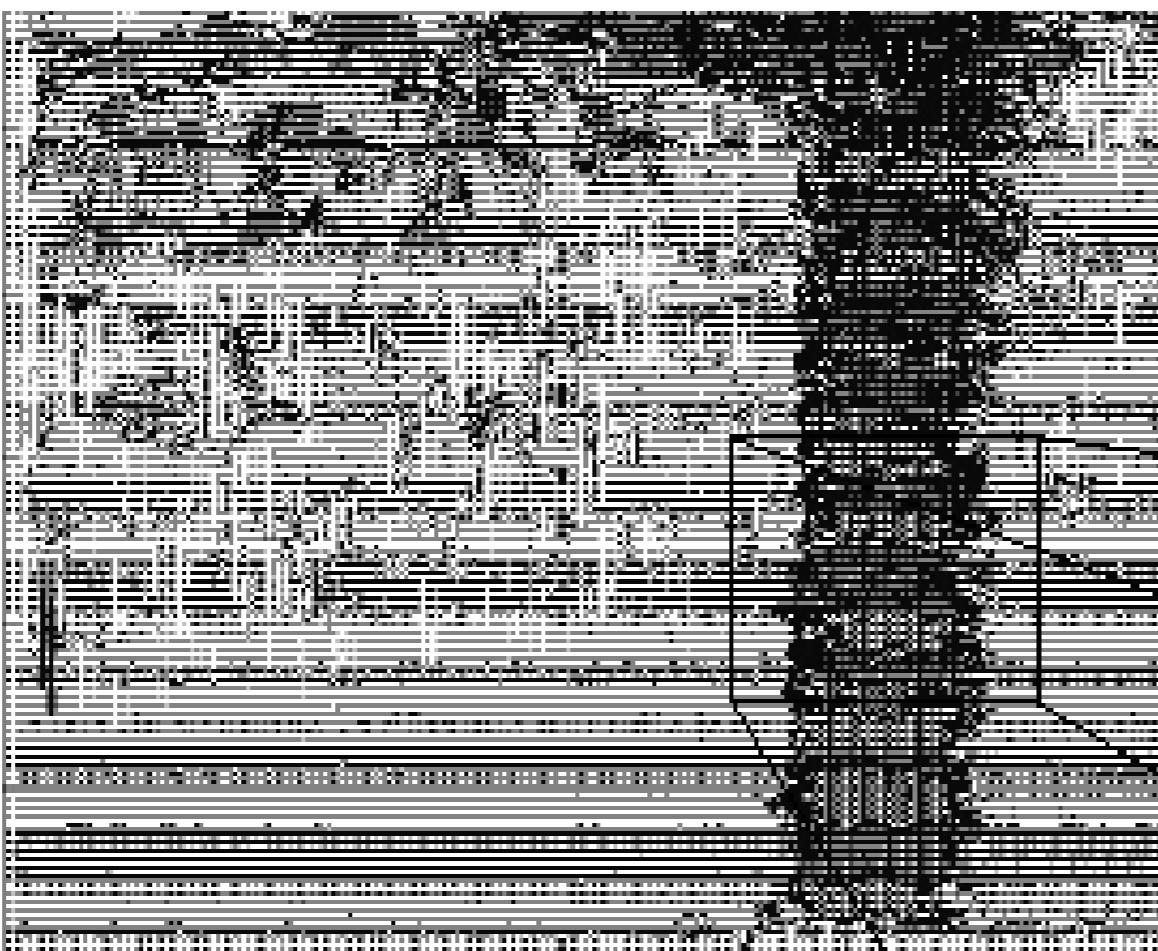
Optical Flow Results



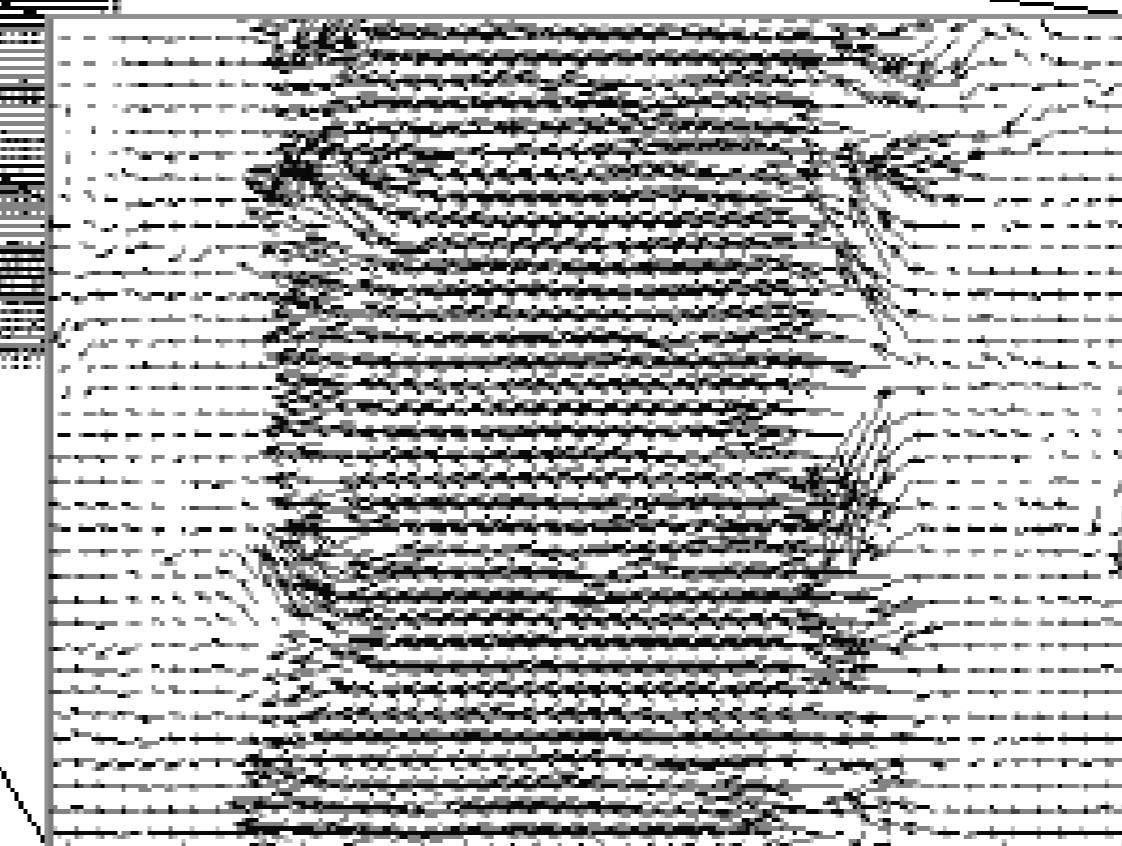
Lucas-Kanade
without pyramids

Fails in areas of large
motion

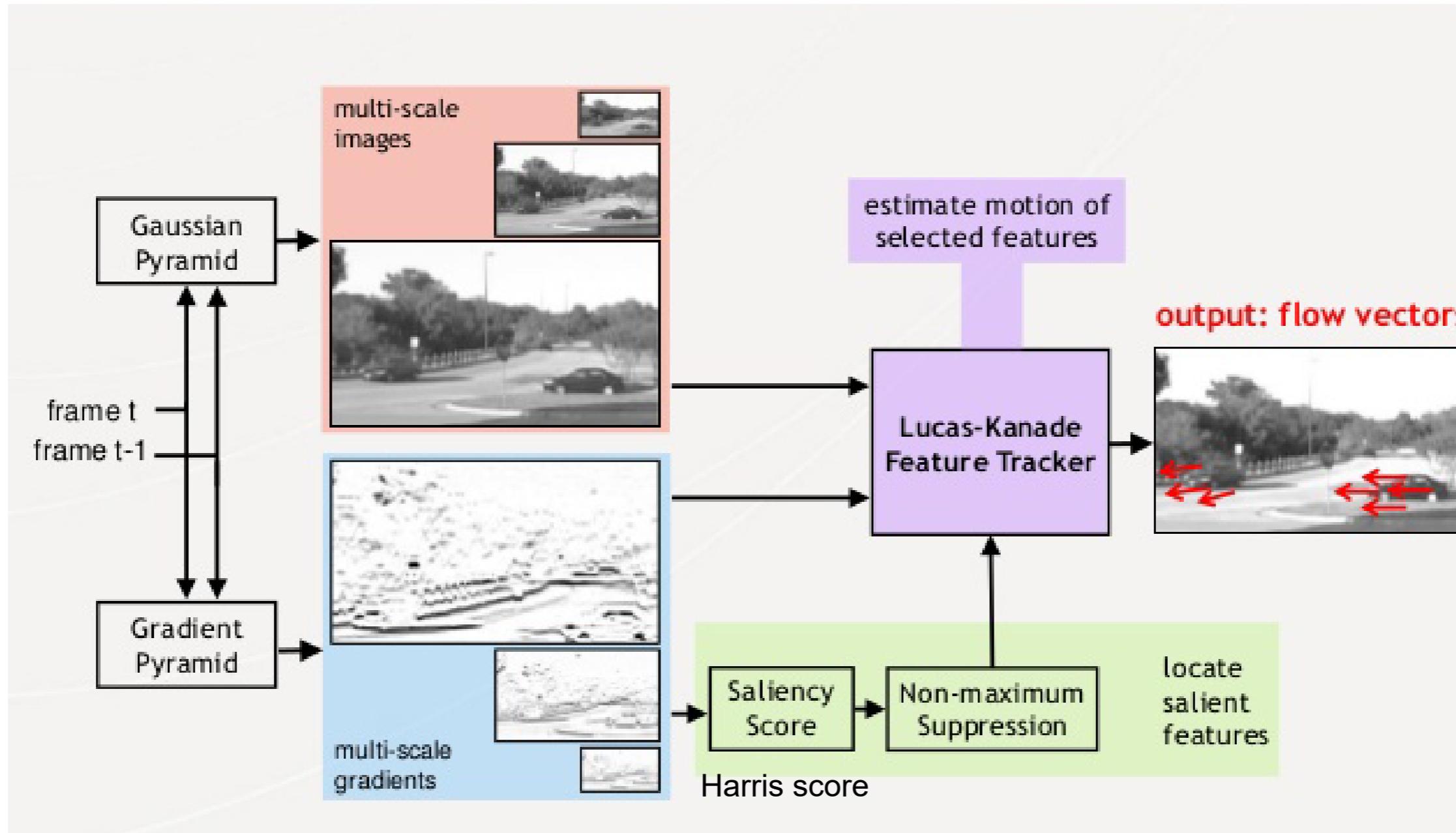
Optical Flow Results



Lucas-Kanade with Pyramids



Lucas- Kanade feature tracker



Recap

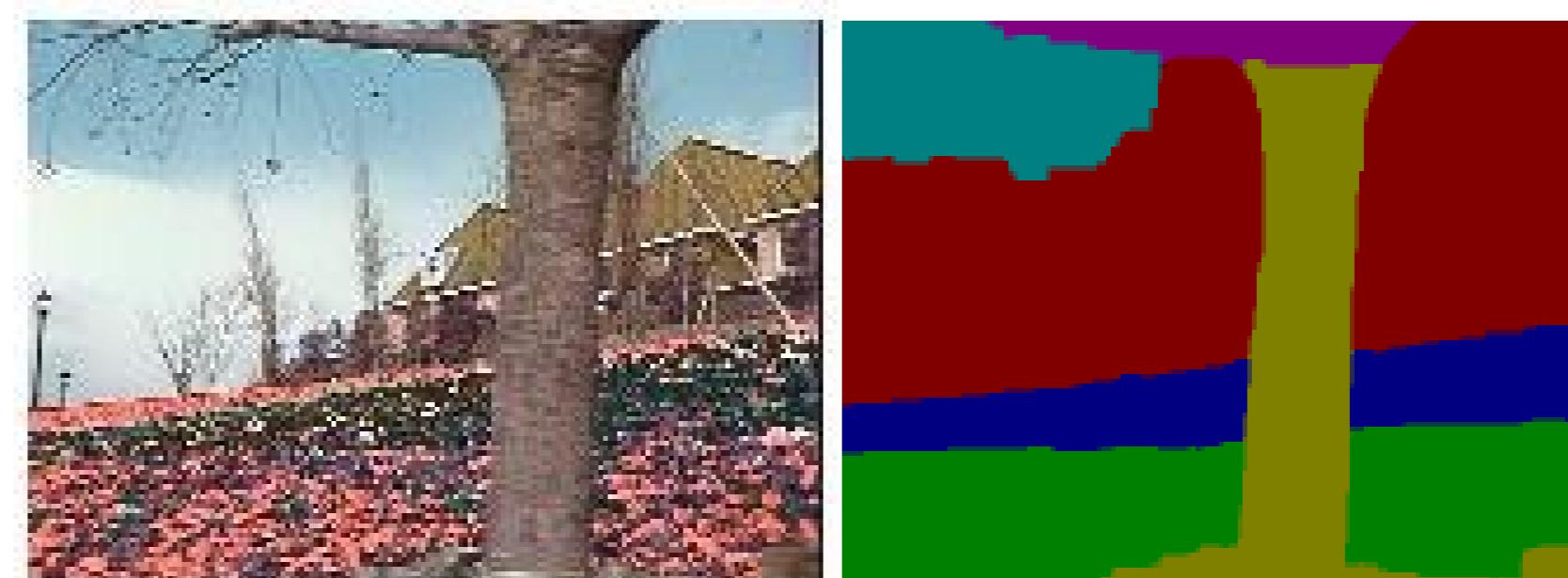
Key assumptions

- **Small motion:** points do not move very far
- **Brightness constancy:** projection of the same point looks the same in every frame
- **Spatial coherence:** points move like their neighbors

Motion segmentation

How do we represent the motion in this scene?

- Break image sequence into “layers” each of which has a coherent (affine) motion



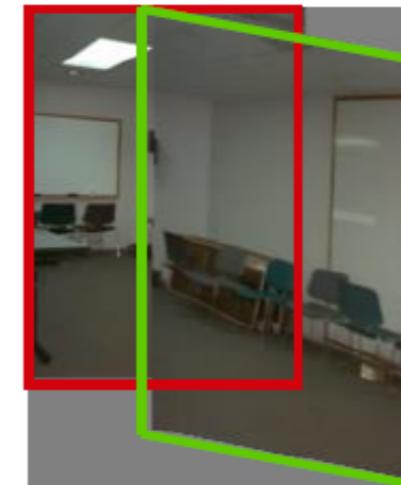
Affine motion

$$u(x, y) = a_1 + a_2x + a_3y$$

$$v(x, y) = a_4 + a_5x + a_6y$$

Substituting into the brightness constancy equation:

$$I_x \cdot u + I_y \cdot v + I_t \approx 0$$



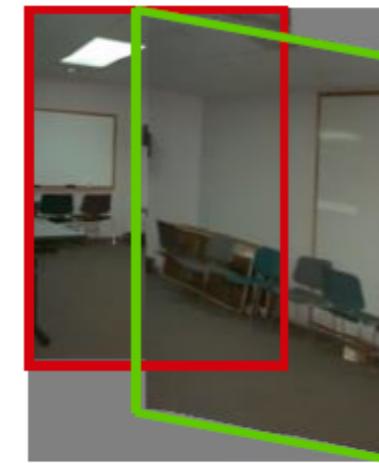
- An affine model is used to approximate the flow patterns consistent with all types of camera motion
- Affine parameters a_1, \dots, a_6 are calculated by minimizing the least squares error of the motion vectors

Affine motion

$$u(x, y) = a_1 + a_2x + a_3y$$

$$v(x, y) = a_4 + a_5x + a_6y$$

Substituting into the brightness constancy equation:



$$I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t \approx 0$$

- Each pixel provides 1 linear constraint in 6 unknowns
- If we have at least 6 pixels in a neighborhood, $a_1 \dots a_6$ can be found by least squares minimization:

$$Err(\vec{a}) = \sum [I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t]^2$$

How do we estimate the layers?

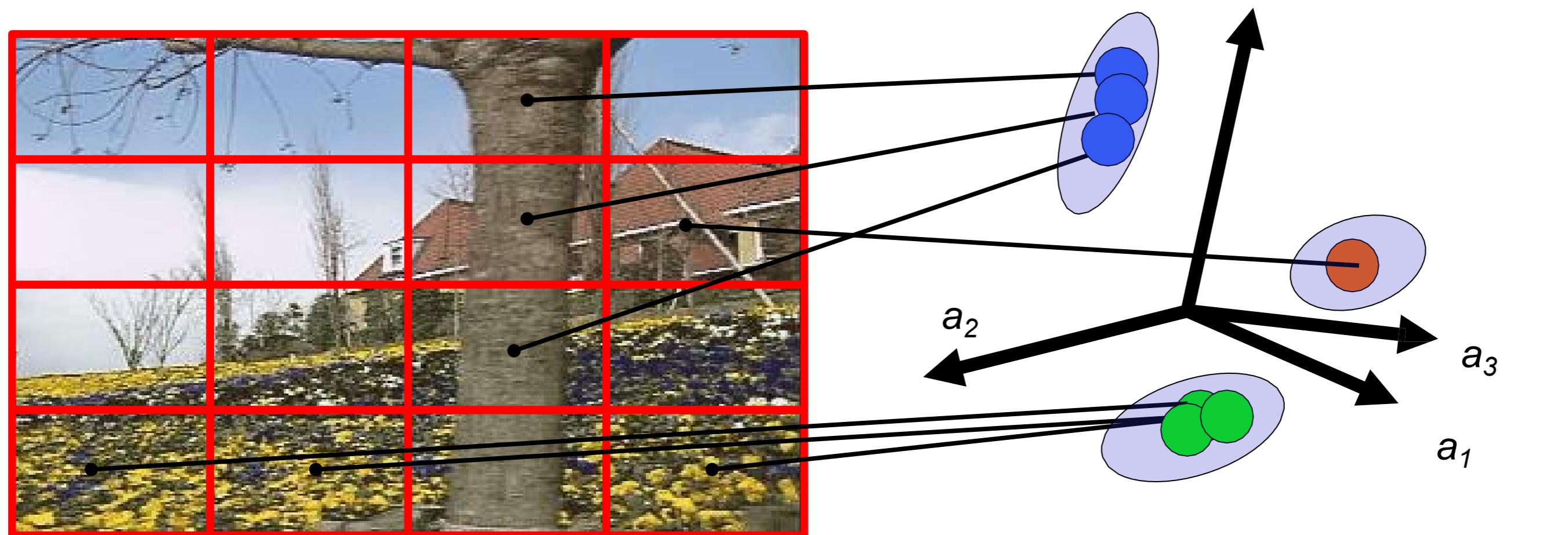
1. Obtain a set of initial affine motion hypotheses

- Divide the image into blocks and estimate affine motion parameters in each block by least squares
 - Eliminate hypotheses with high residual error

2. Map into motion parameter space

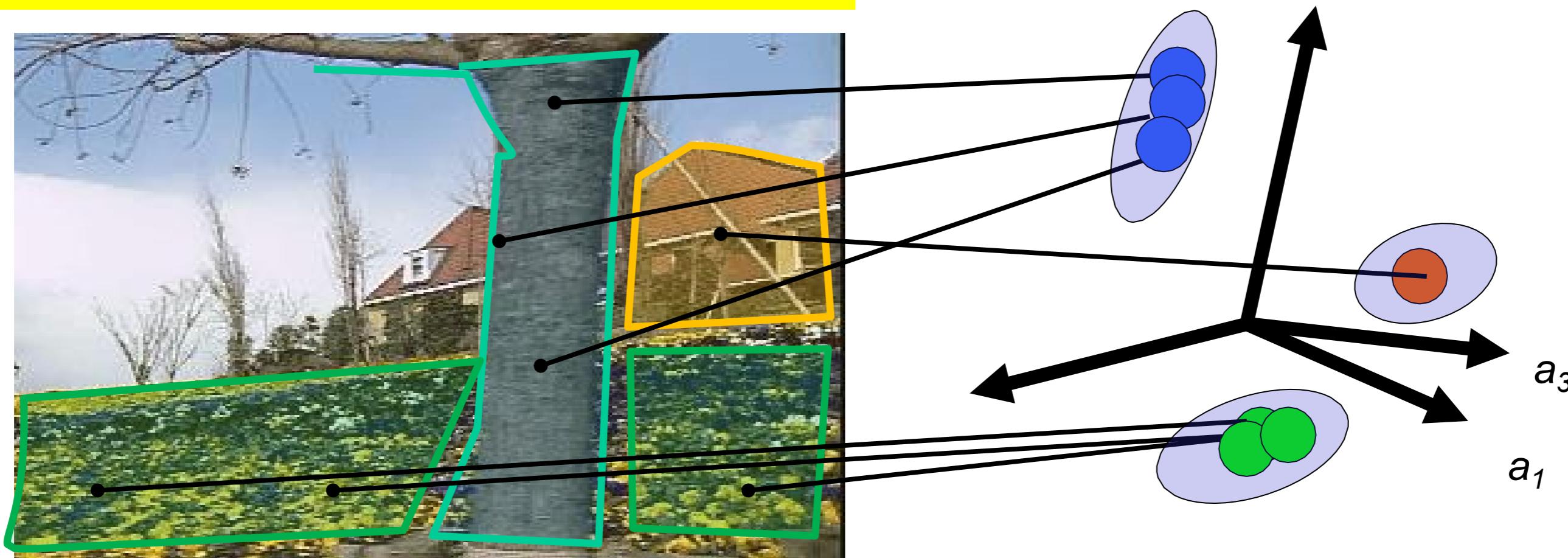
3. Perform k-means clustering on affine motion parameters

- Merge clusters that are close and retain the largest clusters to obtain a smaller set of hypotheses to describe all the motions in the scene

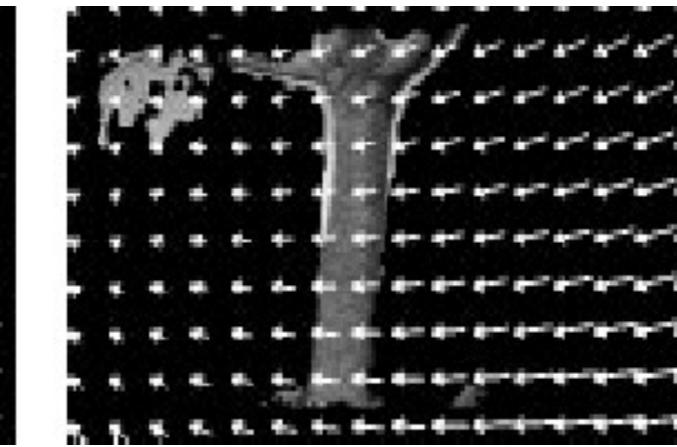
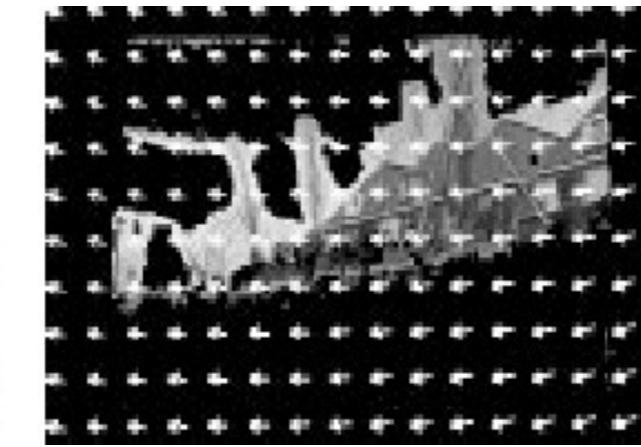
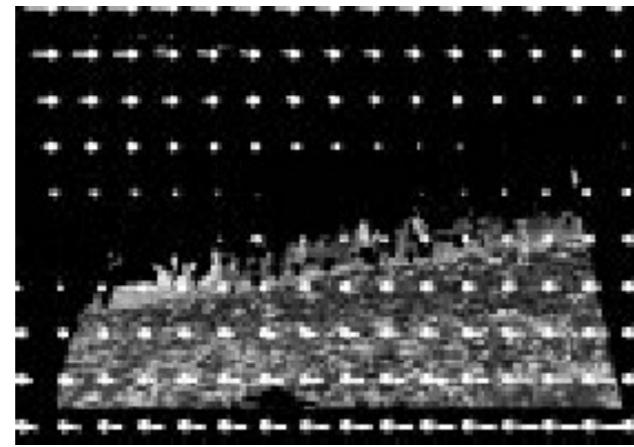
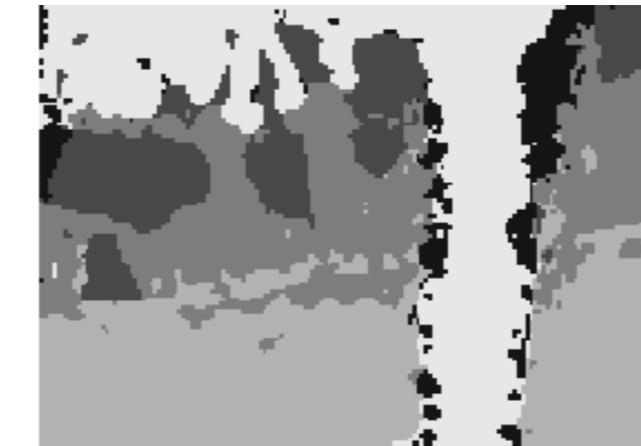
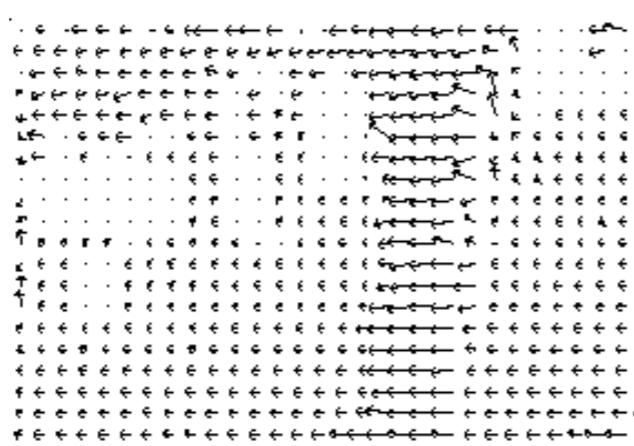


How do we estimate the layers?

1. Obtain a set of initial affine motion hypotheses
 - Divide the image into blocks and estimate affine motion parameters in each block by least squares
 - Eliminate hypotheses with high residual error
2. Map into motion parameter space
3. Perform k-means clustering on affine motion parameters
 - Merge clusters that are close and retain the largest clusters to obtain a smaller set of hypotheses to describe all the motions in the scene
4. Assign each pixel to best hypothesis --- iterate



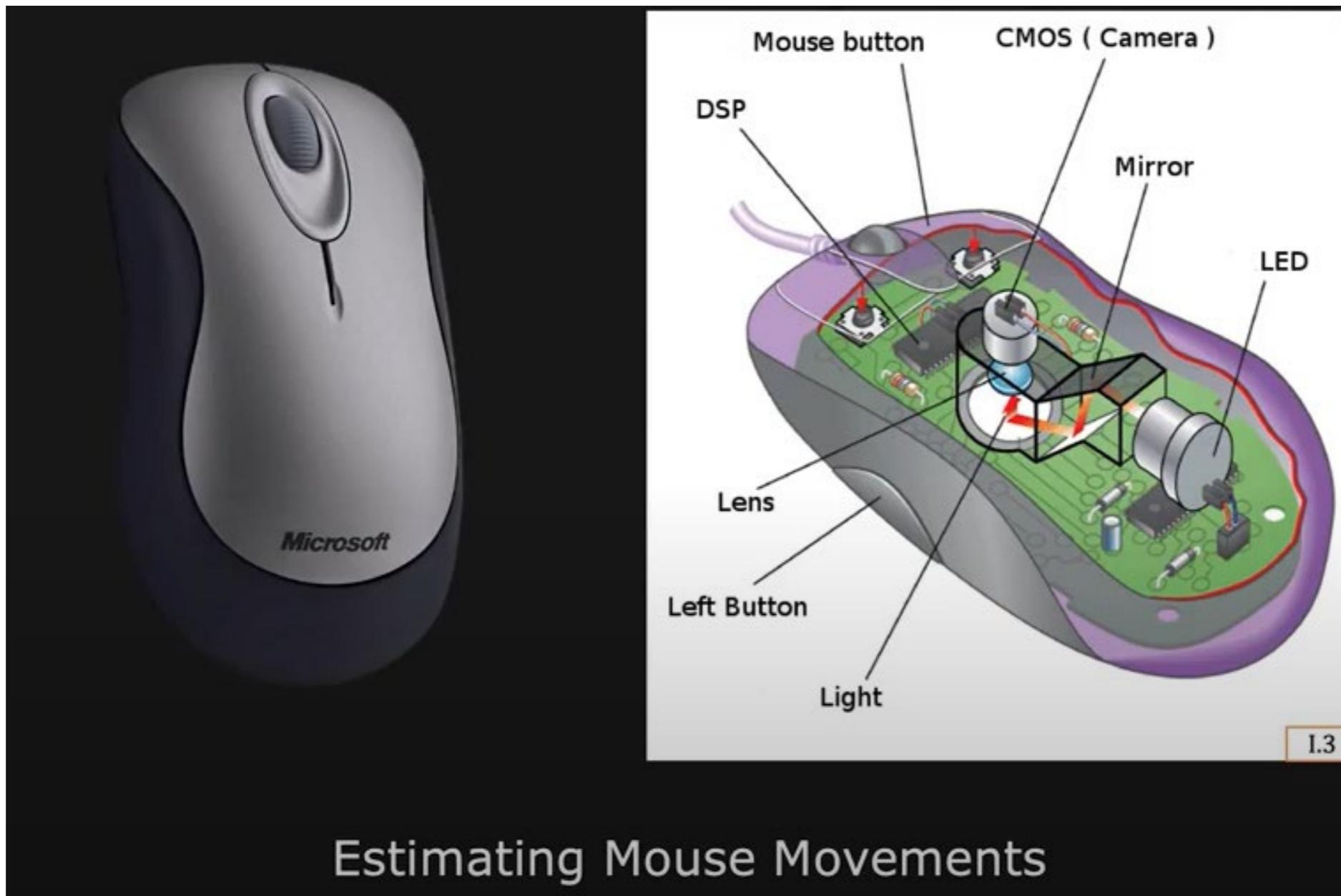
Example result



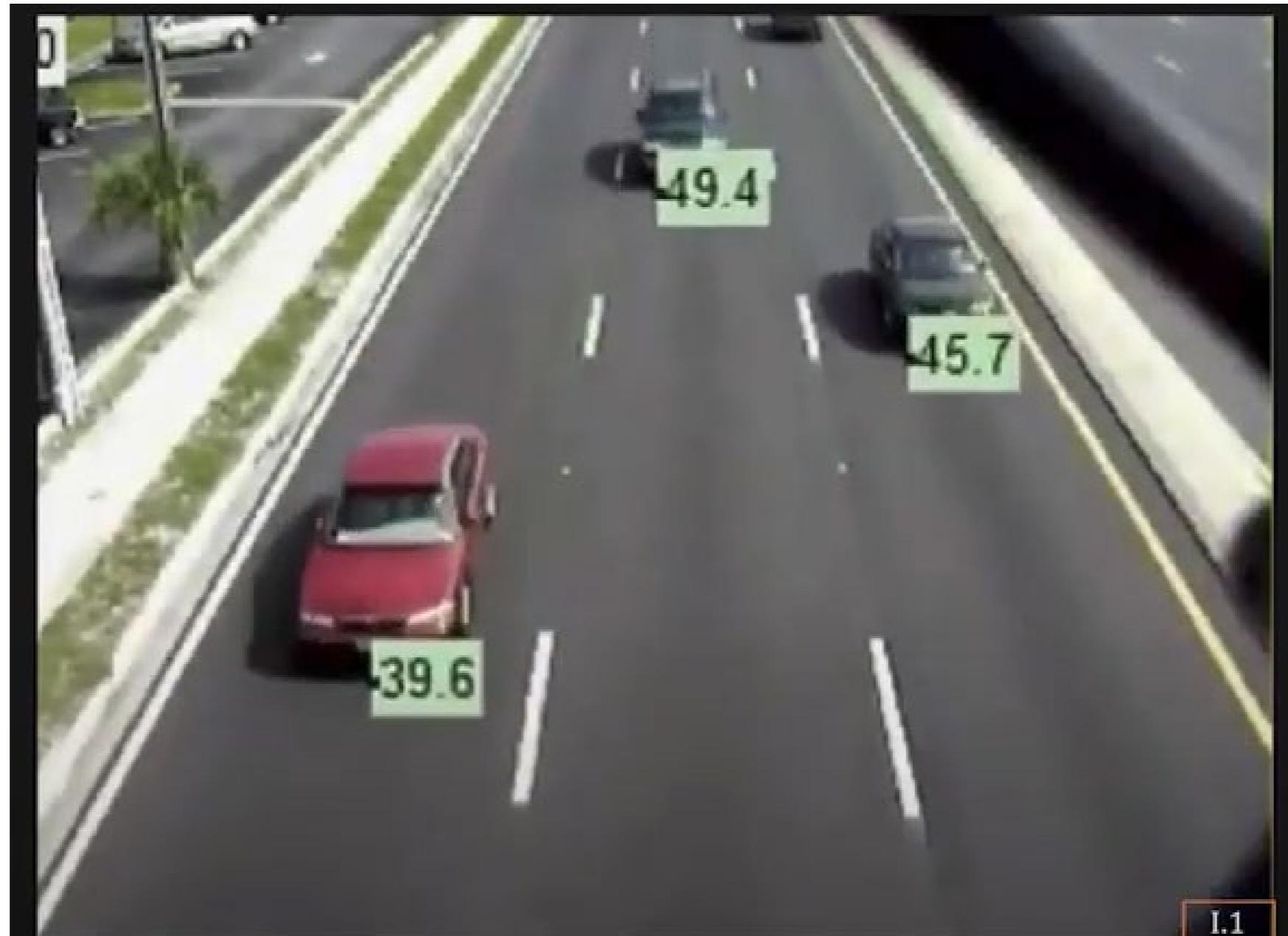
Applications of optical flow

- Video retiming (determine intermediate frames to produce slow motion effects)
- Image stabilization (removing camera shake)
- Face tracking (i.e. eye blinking)
- Games (flow based player interaction)

Optical Mouse



Traffic monitoring



Finding Velocities of Vehicles

Acknowledgement: some slides and material from Bernt Schiele, Mario Fritz, Michael Black, Bill Freeman, Fei-Fei, Justin Johnson, Serena Yeung, R. Szeliski, Fabio Galasso, Ioannis Gkioulekas, Noah Snavely, Abe Davis, Kris Kitani, Xavier Giró-i-Nieto, Shree Nayar