"you want zero-mean unit-variance activations? just make them so."

consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

Input: $x: N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \text{Per-channel mean,} \\ \text{shape is D}$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \text{Per-channel var,} \\ \text{shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \qquad \text{Normalized x,} \\ \text{Shape is N x D}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$
 Output, Shape is N x D

Batch Normalization: Test-Time

Estimates depend on minibatch; can't do this at test-time!

Input: $x: N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \text{Per-channel mean,} \\ \text{shape is D}$$

$$\sigma_j^2 = rac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$
 Per-channel var, shape is D

$$\hat{x}_{i,j} = rac{x_{i,j} - \mu_j}{\sqrt{\sigma_i^2 + \varepsilon}}$$
 Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$
 Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x: N \times D$

$$\mu_j=rac{ ext{(Running) average of}}{ ext{values seen during training}}$$

Per-channel mean, shape is D

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

During testing batchnorm becomes a linear operator!
Can be fused with the previous fully-connected or conv layer

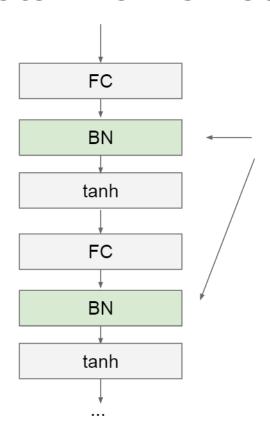
$$\sigma_j^2 = {}^{ ext{(Running)}} \, {}^{ ext{average of values seen during training}}$$

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \qquad \text{N}$$

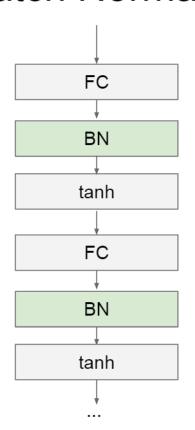
$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D



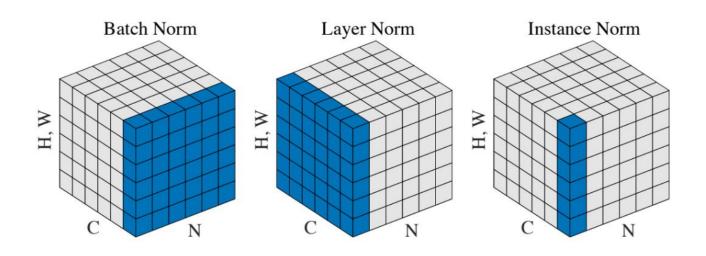
Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$



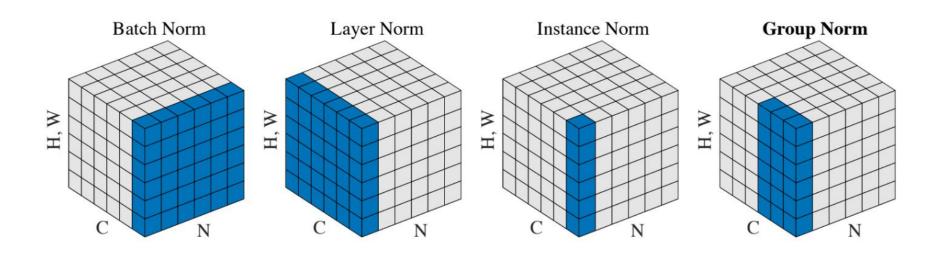
- Makes deep networks much easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Behaves differently during training and testing: this
 is a very common source of bugs!

Comparison of Normalization Layers



Wu and He, "Group Normalization", ECCV 2018

Group Normalization



Wu and He, "Group Normalization", ECCV 2018