point.

$$\mathbb{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ in } HC \Rightarrow \begin{pmatrix} x_1/x_3 \\ x_2/x_3 \end{pmatrix} \text{ in } \mathbb{R}^2.$$

line

$$ax+by+c=0 \text{ in } \mathbb{R}^2 \rightarrow l = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ in } HC$$

$\mathbb{X} l^T = 0$ a point $\mathbb{X}$ on l.

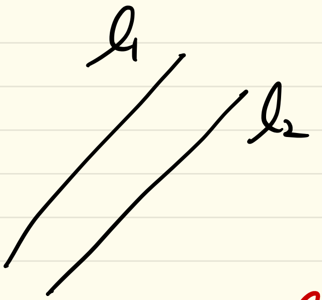
points and lines

$\mathbb{X} = l_1 \times l_2$ cross product.

$l = X_1 \times X_2$

Ideal points

$$l_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad l_2 = \begin{pmatrix} a \\ b \\ c' \end{pmatrix}$$



Ideal.

$$x = l_1 \times l_2 = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}$$

line at infinity

Connect two ideal points

$$l_\infty = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} c \\ d \\ 0 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Conic equation

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Circle, ellipse, parabola, hyperbola, point, line

$$\underbrace{[x_1\ x_2\ x_3]\begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{C} = 0$$

C : Conic C

$$X^T C X = 0 \qquad X^T l = 0$$

$$CX = l \leftarrow \text{tangential line on } C \text{ at } X.$$

$$X^T C X = 0 \qquad l^T C^* l = 0$$

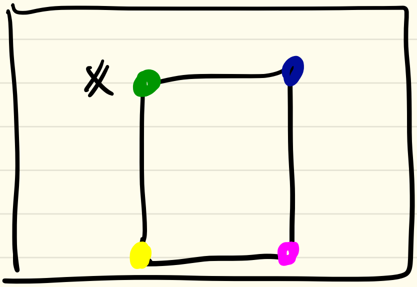
Degenerate conic

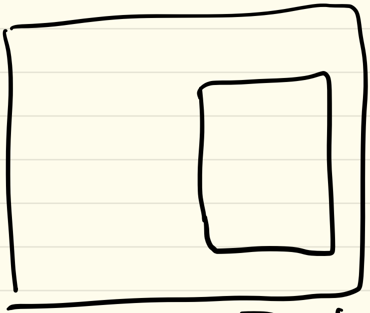
C is not full rank

$$x^T C x = x^T (l m^T + m l^T) x = 0.$$

$$l^T C^* l = l^T (x y^T + y x^T) l = 0$$

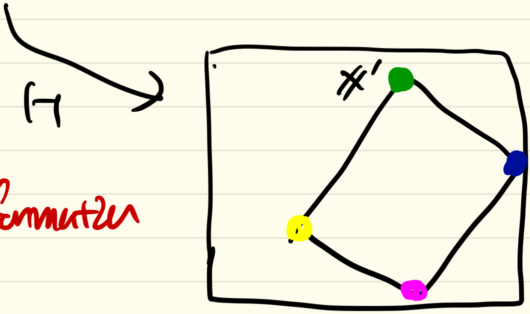
original





Translation

$$\underline{\underline{x' = Hx}}$$



Translation + Rotation

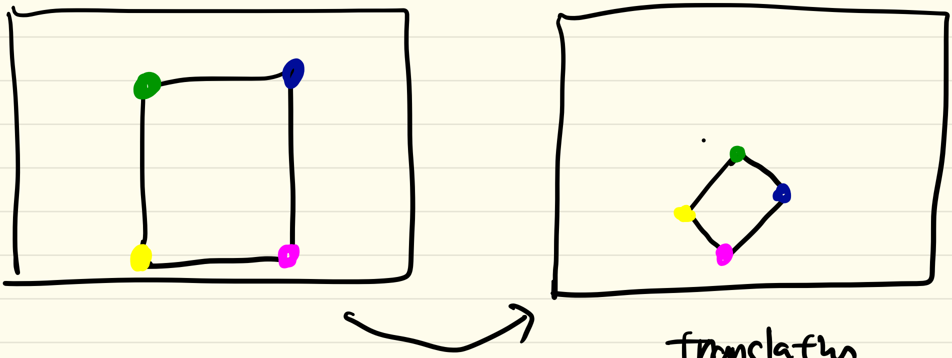
Euclidean Transformation

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\theta, t_x, t_y}{3}$$

Rigid Body Motion

original



H

translation
rotation
scaling

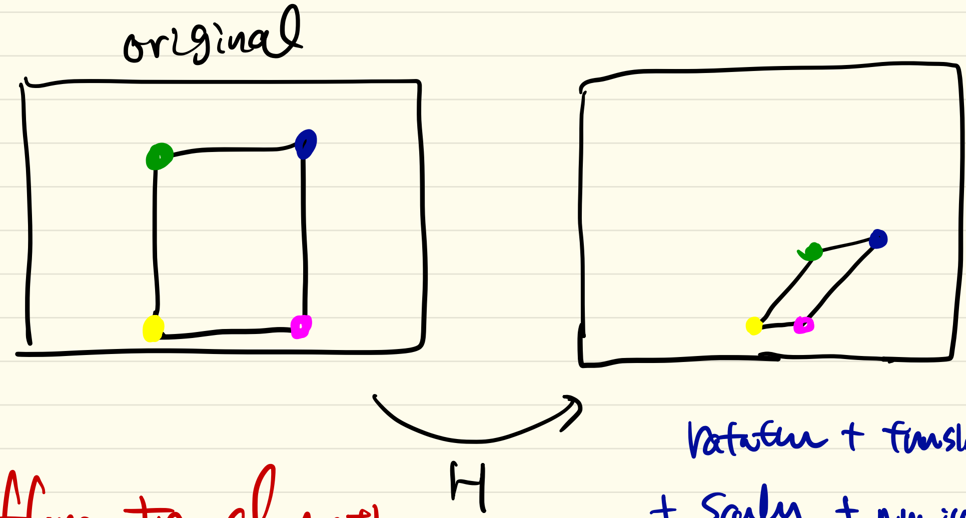
Similarity Transformation

$$H = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

s, θ, t_x, t_y

$= 4$

Preserving Shape



rotation + translation + scaling + non-isotropic scaling.

Affine transformation

$$H = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

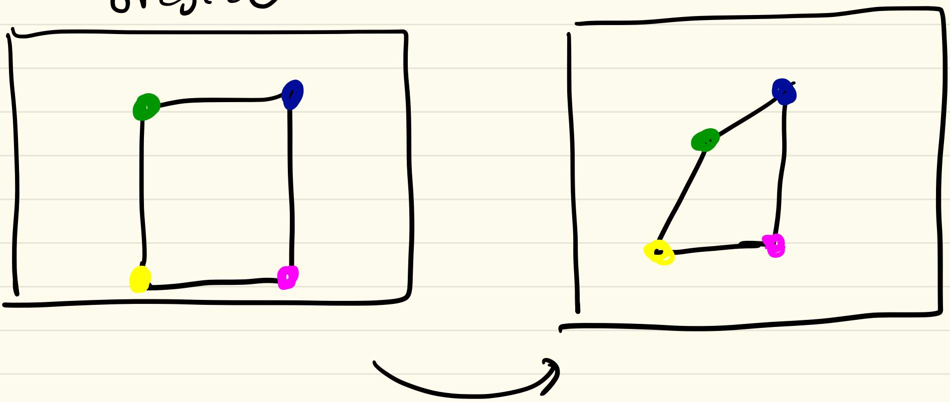
a_{11} a_{12} a_{21} a_{22}

t_x t_y

6.

Keep parallel lines are parallel.

original



projective transformation

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

$$\underline{\begin{matrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & \end{matrix}}$$

8

point go through the same optical center.

$$H = \underbrace{\begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix}}_{H_1} \underbrace{\begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix}}_{H_2} \underbrace{\begin{bmatrix} I & 0 \\ V^T & v \end{bmatrix}}_{H_3}$$

$$= \begin{bmatrix} sRK & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ V^T & v \end{bmatrix}$$

$$= \begin{bmatrix} sRK & \begin{bmatrix} t_1 v_1 \\ t_2 v_2 \, . \end{bmatrix} \\ V^T & v \end{bmatrix}$$

Given H, $\left\{ \text{H can be decomposed to above three matrices } H_1, H_2, H_3 \right.$

$$H^{-1} = \begin{bmatrix} 1 & 0 \\ v^T & v \end{bmatrix}^{-1} \begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix}^{-1} \begin{bmatrix} sR & t \\ \vec{0} & 1 \end{bmatrix}^{-1}$$

Inveritable.

$$H' = H^{-1} = \begin{bmatrix} 1 & 0 \\ v'^T & v' \end{bmatrix} \begin{bmatrix} K' & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} s'R' & t' \\ 0 & 1 \end{bmatrix}$$

$$\qquad\qquad H_1 \qquad\qquad H_2 \qquad\qquad H_3$$

$$B = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} B^{-1} & \cancel{t}A^{-1} \\ 0 & 1 \end{bmatrix}$$

H' can be represented by H_1, H_2, H_3

H_1 has projection transformation term

$H_2 \to$ affine

$H_3 \to$ similarity.

Affine Rectification

$$l = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} \qquad H$$

$$X' = HX \qquad l' = H^{-T} l.$$

Sms l_1 l_2 l_3

$$HH = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ l_1 & l_2 & l_3 \end{bmatrix} \qquad HH^{-T} l = \ ??$$

Circular point