Class 15.

$$ax + by + c = 0$$

$$ax_1 + by_1 + c = 0$$

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$\hookrightarrow = A$

$\text{null}(A)$

$$xh_{11} + yh_{12} + h_{13} - xx'h_{31} - x'y\,h_{32} - x'h_{33} = 0$$

$$xh_{11}/h_{33} + y\,h_{12}/h_{33} + h_{13}/h_{33} - xx'\,h_{31}/h_{33} - x'y\,h_{32}/h_{33} - x' = 0$$

$$x h_{11}/h_{33} + y h_{12}/h_{33} + h_{13}/h_{33} - x x' h_{31}/h_{33} - x' y h_{32}/h_{33} - x' = 0$$

$$\underbrace{x h_{11}/h_{33}}_{h_{11}'} + \underbrace{y h_{12}/h_{33}}_{h_{12}'} + \underbrace{h_{13}/h_{33}}_{h_{13}'} - x x' \underbrace{h_{31}/h_{33}}_{h_{31}'} - \underbrace{x' y h_{32}/h_{33}}_{h_{32}'} = x'$$

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -xx' & -x'y \\ & & & & & & & \end{bmatrix} \begin{bmatrix} h_{11}' \\ h_{12}' \\ h_{13}' \\ h_{21}' \\ h_{22}' \\ h_{23}' \\ h_{31}' \\ h_{32}' \end{bmatrix} = \begin{bmatrix} x' \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

$$A x = b.$$

Least Square

$$y_1 - mx_1 - b = 0$$

$$y_2 - mx_2 - b = 0$$

$$\vdots$$

$$y_1 = mx_1 + b$$

$$y_2 = mx_2 + b$$

$$\vdots$$

known

known.

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix}$$

$$A \quad x = b$$

How to solve.

$$E = \|Y - XB\|^2$$

$$= (Y - XB)^T (Y - XB)$$

$$= Y^T Y - 2(XB)^T Y + (XB)^T (XB).$$

$$\boxed{\frac{\partial E}{\partial B} = 0}$$

$$\frac{\partial E}{\partial B} = 2X^T XB - 2X^T Y$$

$$X^T X B = X^T Y.$$

pseudo inverse.

$$B \approx X^* Y$$

$$\boxed{X^* = (X^T X)^{-1} X^T}$$

If X has full column rank, then its columns are linearly independent. The product Xv with a vector v is a linear combination of the columns of X. Therefore, if $v \neq 0$ (v is not the zero vector) then $y := Xv \neq 0$ (Xv is a non-zero vector, since it is a non-trivial linear combination). Therefore, given any $v \neq 0$ we get

$$v^t X^t X v = (Xv)^t Xv = y^t y = \sum_i y_i^2 > 0$$

which is the implication of positive definite.

$X^T X$ is positive defite, invertible!!

You have more than 4 partent corresponding having Gaussian noise (typical measurement error), how can you estimate homography??