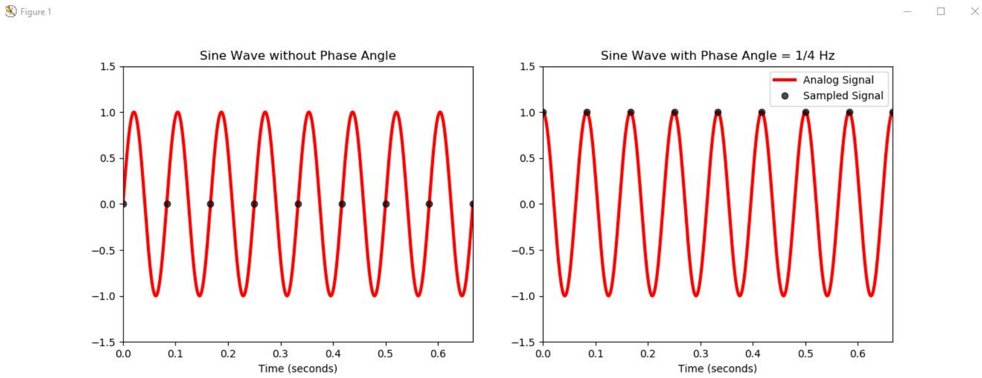
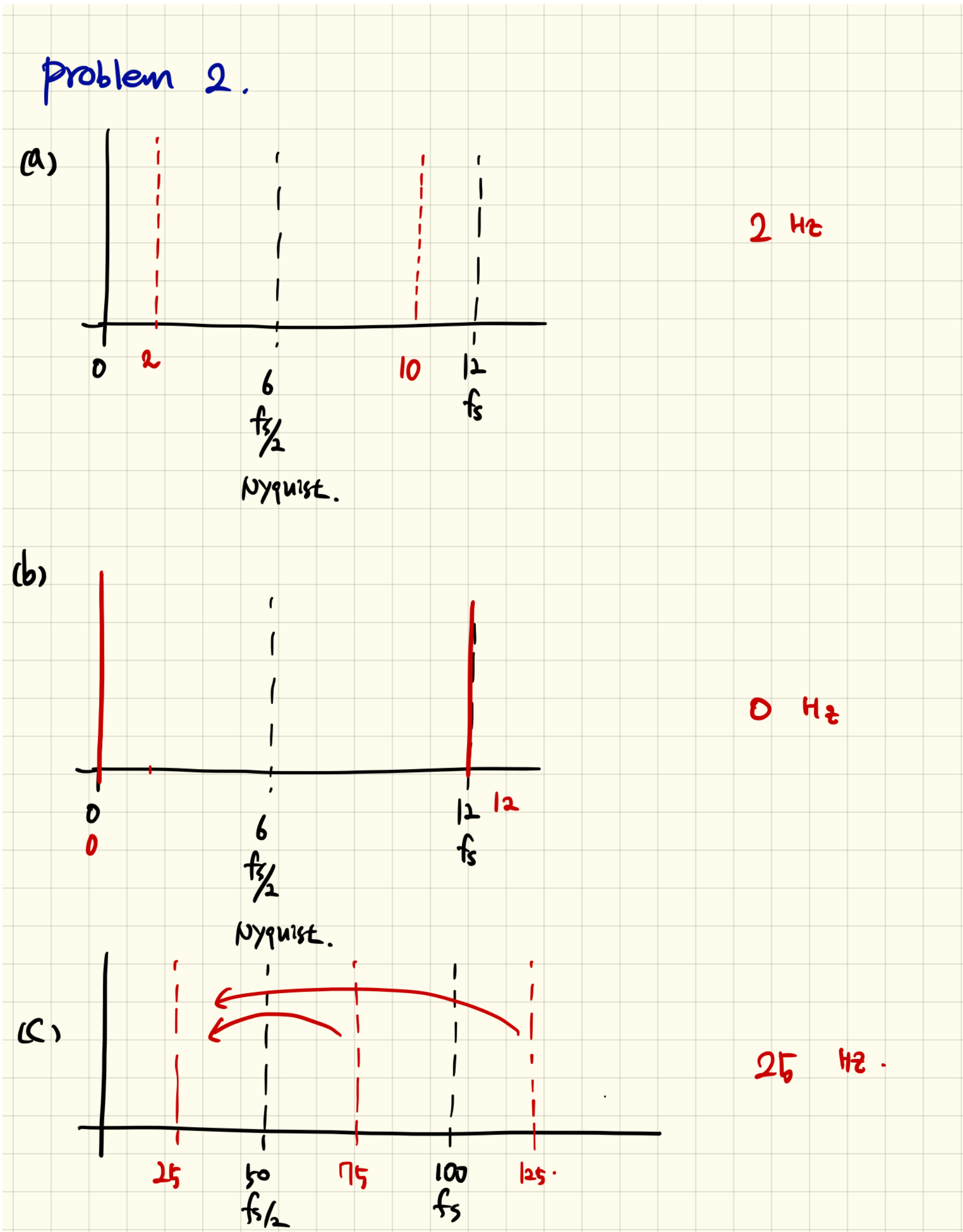
Problem 1

(d) Plotting 6 Hz analog signal with 12Hz sampling frequency **(incorrect)**

(d) Plotting 12 Hz analog signal with 12Hz sampling frequency



Problem 2



Problem 3 (Neil, Juan, Laurent)

(a) Please explain a quantization error. When do they occur? How to avoid? (Laurent)

Quantization error is an error stemming from the conversion of a continuous signal to a discrete signal. There is only a finite range of values which can be represented in a discrete signal, and so information is lost about what happens "between" the discrete points taken from the analog signal. This occurs in all digital sampling cases - one would need infinite memory to fully suppress this error. In practice, the error can be effectively minimized by choosing a high enough resolution for the discrete signal to ensure that any data of interest is not missed between captured points.

No □ □ **Sampling rate**, **high resolution sensor**, **Oversampling**

(b) Please explain a clipping error. When do they occur? How to avoid? (Laurent)
Clipping occurs when the amplitude of a signal is higher than what the sensor can handle. For instance, in a photo, clipping occurs when too much light hits the sensor (overexposure) and everything simply appears white, with loss of information. This can be avoided by choosing a sensor appropriate for the data being captured, with a working range large enough for the analog signal.

No □ □ **Sensor**

(c) Please explain an oversampling issue. When do they occur? What are the consequence of the oversampling? (Neil)

- Oversampling means to sample with a sampling frequency that is (significantly) higher than the required nyquist_frequency+eps.
- The amount of data is increased --> Vectorlength increased.
- **The SNR is increased (see https://en.wikipedia.org/wiki/Oversampling)**

Problem 4 (Laurent)

Problem 4

$$\boxed{\begin{aligned} T_p &= 1/f_0 \\ y(t) &= y(t + nT_p) \\ &= y(t + n/f_0) \\ n&: \text{integer.} \end{aligned}}$$

(b)

$$y(t) = \sin^2(2\pi f_0 t) = \frac{1}{2}(1 - \cos 4\pi f_0 t) = \frac{1}{2}\left(1 - \cos\frac{2\pi \cdot 2t}{T_p}\right)$$

orthogonality!!

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T_p}\right) + b_n \sin\left(\frac{2\pi n t}{T_p}\right)$$

$a_0 = 1$, $b_n = 0$ for all n, $a_n = \begin{cases} -\frac{1}{2} & n = 2 \\ 0 & \text{otherwise.} \end{cases}$

(C)

orthogonality trigonometric function.

$$C_n = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} \frac{1}{2}(1 - \cos 4\pi f_0 t)\, e^{-i 2\pi n f_0 t}$$

$$n = 0 \Rightarrow C_0 = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} \frac{1}{2} dt - \frac{1}{2T_p}\int_{-T_p/2}^{T_p/2} \underbrace{\cancel{\cos 4\pi f_0 t}}_{\to 0}\, dt = \frac{1}{2}$$

periodic

$n \neq 0 \Rightarrow \quad C_n = -\frac{1}{2T_p}\int_{-T_p/2}^{T_p/2} \cos 4\pi f_0 t \, e^{-i 2\pi f_0 t n} \, dt$

$$C_n = \underbrace{-\frac{1}{2T_p}\int_{-T_p/2}^{T_p/2} \cos 4\pi f_0 t \cos 2\pi f_0 n t}_{\text{all } 0 \text{ expet } n=2,-2} - \cancel{i \cos 4\pi f_0 t \sin 2\pi f_0 n t} \, dt$$

$\rightarrow 0$ orthogonality

$$C_{2,-2} = -\frac{1}{2T_p}\int_{-T_p/2}^{T_p/2} \frac{1}{2}(1 + \cos 8\pi f_0 t) \, dt$$

$$= -\frac{1}{4T_0}\int_{-T_p/2}^{T_p/2} \cancel{\cos 8\pi f_0 t \, dt} = -\frac{1}{4}$$

$\rightarrow 0$

$$C_0 = \frac{1}{2}, \quad C_n = \begin{cases} -\frac{1}{4} & n = 2, -2 \\ 0 & \text{otherwise.} \end{cases}$$

Another aproch.

$$C_n = \frac{a_n + b_n i}{2}. \qquad C_0 = \frac{a_0}{2} = \frac{1}{2}$$

$$C_2 = \frac{a_2}{2} = -\frac{1}{4} \qquad C_{-2} = -\frac{1}{4}$$

(d) $y(t) = \sin(2\pi f_0 t)\sin(2\pi f_0 t) + 5$

Non-perioetic term

$b_n = 0$ for all n, $a_n = \begin{cases} -\frac{1}{2} & n=2 \\ 0 & \text{otherwise} \end{cases}$

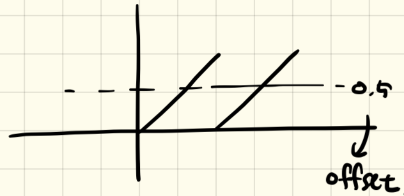
non-periodic term $\Rightarrow$ $\frac{1}{2} + 5 = \frac{a_0}{2}$ $\therefore a_0 = 11$

offset, DC term.

Problem 5 (Juan, Laurent)

problem 5 (Juan)

$T_p = 1 \qquad x(t) = t.$



offset.

$0.5 = \frac{a_0}{2}$

(b)

$a_0 = 1$

$$a_n = 2\int_0^1 t\cos 2\pi n t\,dt = 2\left[\underbrace{\cancel{t\frac{\sin 2\pi n t}{2\pi n}\Big|_0^1}}_{0} - \frac{1}{2\pi n}\underbrace{\cancel{\int_0^1 \sin 2\pi n t\,dt}}_{0} \right]$$

$= 0$

$\sin 2\pi = \sin 0 = 0$

Integration one period.

$$b_n = \frac{2}{T_p}\int_0^{T_p} t\sin 2\pi n t\,dt = 2\left[-\frac{t\cos 2\pi n t}{2\pi n}\Big|_0^1 - \frac{1}{2\pi n}\underbrace{\cancel{\int_0^1 \cos 2\pi n t\,dt}}_{0} \right]$$

$= -\frac{1}{\pi n}$

$a_0 = 1$, $a_n = 0$ for all n, $b_n = -\frac{1}{n\pi}$.

(c) $C_n = \int_0^1 t\, e^{-i2\pi n t}\,dt$

$$C_n = \frac{t\, e^{-i2\pi n t}}{-i\,2\pi n}\Bigg|_0^1 - \int_0^1 \frac{1}{i\,2\pi n} \underbrace{\cancel{e^{-i2\pi n t}\, dt}}_{0}$$

integration one period.

$$= \left(\frac{1}{-i\,2\pi n}\right) e^{-i2\pi n}$$

$$= \left(\frac{1}{-i2\pi n}\right)\Big(\underbrace{\cos 2\pi n}_{1} - i\, \underbrace{\cancel{\sin 2\pi n}}_{0}\Big)$$

for $n \neq 0$

$C_0 = \frac{1}{2}$

$$= \frac{i}{2\pi n}$$

$$\therefore \quad C_n = \begin{cases} \frac{1}{2} & n = 0 \\ \frac{i}{2\pi n} & n \neq 0 \end{cases}$$

Problem 6 and 7 (Steven)

Problem 6 (Steven)

(a) cosine wave (Steven)

$$x(t) = \cos(2\pi p_0 t)$$

$$X(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft} \cos(2\pi p_0 t)\mathrm{d}t = \int_{-\infty}^{\infty} e^{-i2\pi ft} \frac{e^{i2\pi p_0 t} + e^{-i2\pi p_0 t}}{2} \mathrm{d}t = \frac{1}{2} \int_{-\infty}^{\infty} [e^{-i2\pi(f-p_0)t} + e^{-i2\pi(f+p_0 t)}]\mathrm{d}t$$

$$\because X(f) = \frac{1}{2}[\delta(f-p_0) + \delta(f+p_0)]$$

(b) cosine wave + dc (direct current) wave

$$x(t) = \cos(2\pi p_0 t) + d$$

$$X(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft}[\cos(2\pi p_0 t) + d]\mathrm{d}t = \int_{-\infty}^{\infty} e^{-i2\pi ft}[\frac{e^{i2\pi p_0 t} + e^{-i2\pi p_0 t}}{2} + d]\mathrm{d}t$$

$$\because X(f) = \frac{1}{2} \int_{-\infty}^{\infty} [e^{-i2\pi(f-p_0)t} + e^{-i2\pi(f+p_0 t)}]\mathrm{d}t + \int_{-\infty}^{\infty} de^{-i2\pi ft}\mathrm{d}t = \frac{1}{2}[\delta(f-p_0) + \delta(f+p_0)] + d\delta(f)$$

(c) Gaussian function (Steven)

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x)^2/2\sigma^2}$$

$$X(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft}[\frac{1}{\sigma\sqrt{2\pi}} e^{-(x)^2/2\sigma^2}]\mathrm{d}t = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i2\pi ft}[e^{-(x)^2/2\sigma^2}]\mathrm{d}t$$

$$\because X(f) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[x^2 - i4\pi\sigma^2 fx]}\mathrm{d}t = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[x^2 - 2(i2\pi\sigma^2 f)t + (i2\pi\sigma^2 f)^2 - (i2\pi\sigma^2 f)^2]}\mathrm{d}t$$

$$\because X(f) = \frac{1}{\sigma\sqrt{2\pi}}[\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(t - i2\pi\sigma^2 f)^2}\mathrm{d}t] \cdot e^{\frac{1}{2\sigma^2}(i2\pi\sigma^2 f)^2} = \frac{1}{\sigma\sqrt{2\pi}}[\sigma\sqrt{2\pi}] \cdot e^{-(\sqrt{2}\pi\sigma f)^2} = e^{-2\pi^2\sigma^2 f^2}$$

Problem 7: Fourier Transformation 2

(a) Compute the Fourier transformation (integral) of the following function (Steven)

$$y = e^{-a|t|}(b \cdot \cos(2\pi f_1 t) + c \cdot \cos(2\pi f_2 t)$$

$$T(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft}[e^{-a|t|}(b \cdot \cos(2\pi f_1 t) + c \cdot \cos(2\pi f_2 t)]\mathrm{d}t$$

$$\because T(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft}[e^{-a|t|}(b \cdot \cos(2\pi f_1 t)]\mathrm{d}t + \int_{-\infty}^{\infty} e^{-i2\pi ft}[e^{-a|t|}c \cdot \cos(2\pi f_2 t)]\mathrm{d}t$$

$$\because T(f) = b\int_{-\infty}^{\infty} e^{-a|t|}\frac{1}{2}(e^{i2\pi f_1 t} + e^{-i2\pi f_1 t})e^{-i2\pi ft}\mathrm{d}t + c\int_{-\infty}^{\infty} e^{-a|t|}\frac{1}{2}(e^{i2\pi f_2 t} + e^{-i2\pi f_2 t})e^{-i2\pi ft}\mathrm{d}t$$

$$\because T(f) = \frac{b}{2}\int_{-\infty}^{\infty} e^{-a|t|}(e^{-i2\pi(f-f_1)t} + e^{-i2\pi(f-f_1)t})\mathrm{d}t + \frac{c}{2}\int_{-\infty}^{\infty} e^{-a|t|}(e^{-i2\pi(f-f_2)t} + e^{-i2\pi(f-f_2)t})\mathrm{d}t$$

$$\because T(f) = \frac{b}{2}\int_{-\infty}^{\infty} e^{-a|t|}(e^{-i2\pi(f-f_1)t} + e^{-i2\pi(f-f_1)t})\mathrm{d}t + \frac{c}{2}\int_{-\infty}^{\infty} e^{-a|t|}(e^{-i2\pi(f-f_2)t} + e^{-i2\pi(f-f_2)t})\mathrm{d}t$$

$$\because T(f) = \frac{ab}{a^2 + [2\pi(f-f_1)]^2} + \frac{ab}{a^2 + [2\pi(f+f_1)]^2} + \frac{ac}{a^2 + [2\pi(f-f_2)]^2} + \frac{ac}{a^2 + [2\pi(f+f_2)]^2}$$