Structure From Motion

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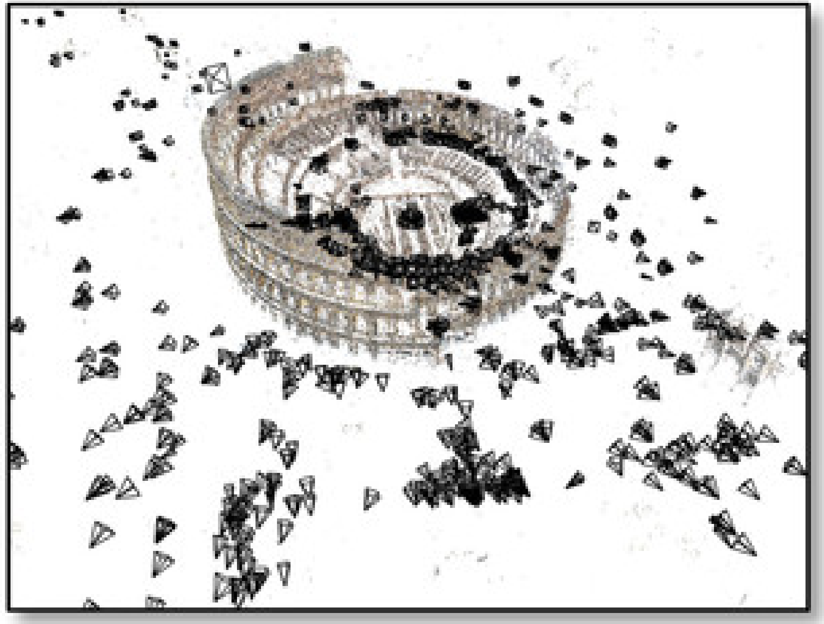
What is Structure from Motion (SfM) ?

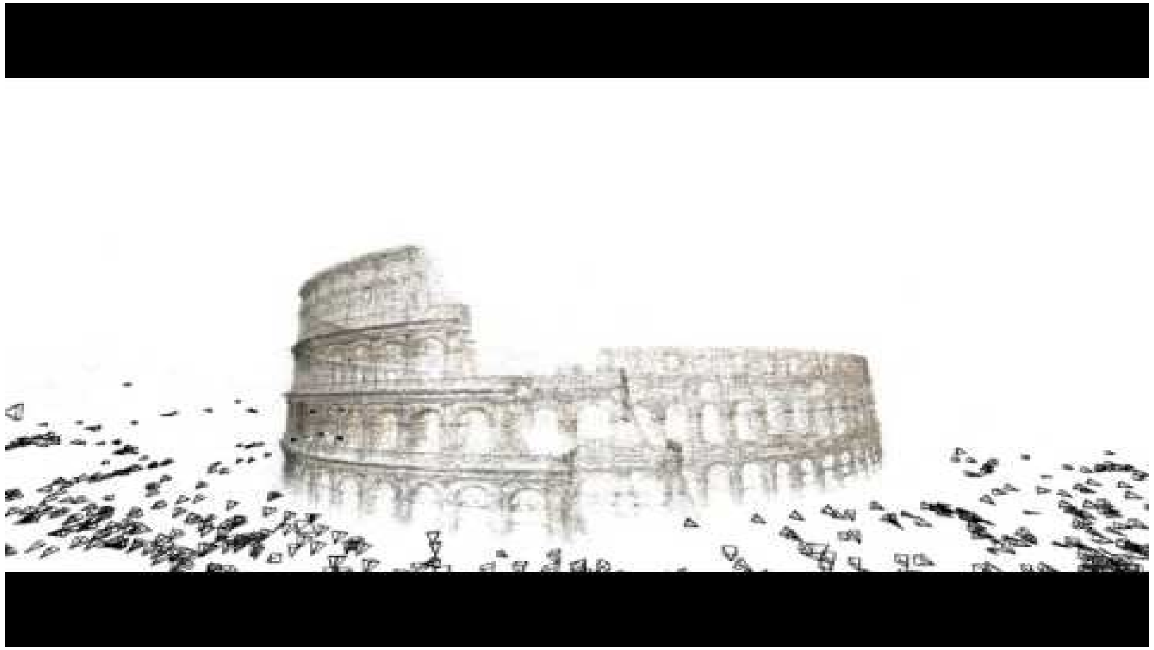
Pictures



Scene structure & camera locations and parameters

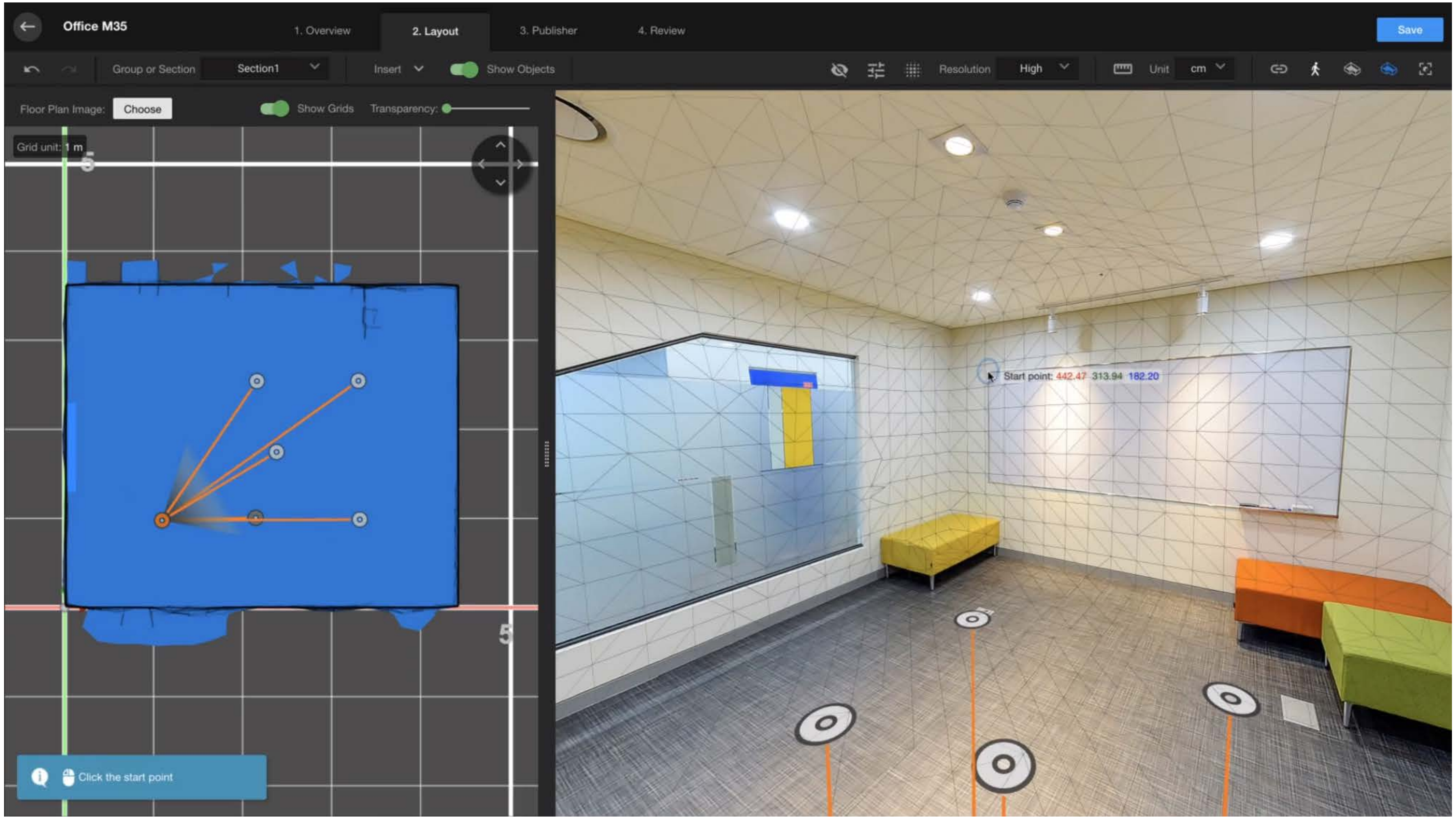
http://www.cs.cornell.edu/projects/bigsfm/

Example: BigSfM - Reconstructing the World from Internet Photos



http://www.cs.cornell.edu/projects/bigsfm/

Example: Cupix



https://www.cupix.com/

Example: Automated Progress Monitoring Using Images and BIM

http://raamac.cee.illinois.edu/progressmonitoring

Example: RESCUAV in Globalmedic

https://globalmedic.ca/rescuav/

Projection Matrix

i : Image number

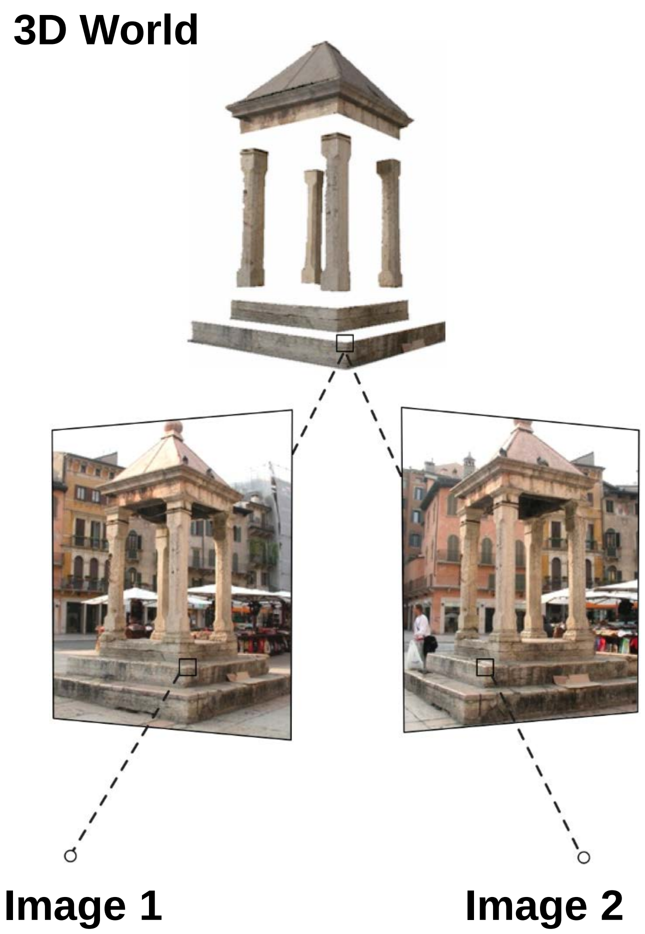
$$\mathbf{x} = \mathbf{P_i}\mathbf{X}$$

2D point **3D point**

- **If we knew a projective matrix in each image, we can compute the image point corresponding to the world point**

- **If we knew more two image points indicating same world point, we can compute the location of the world point. (triangulation)**

FAQ in Camera (Multi-view) Geometry



Q1. Can we compute a 3D location of a single image point?

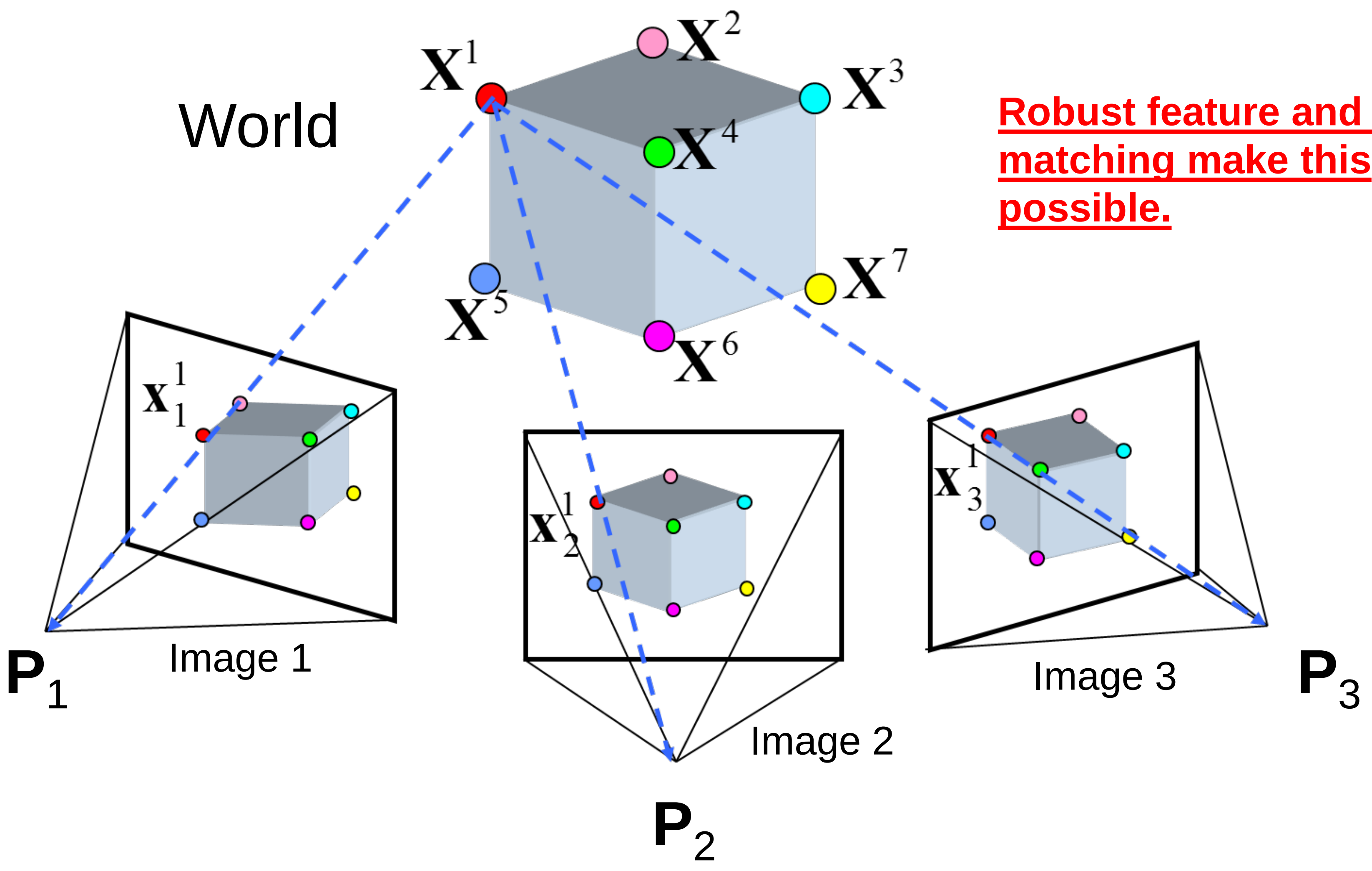
Q2. Why do we need to know a camera model?

Q3. Can we compute a 2D point on a image if we know 3D points?

Q4. Can we measure a real-distance from images?

Q5. What is the role of GPS data?

Multiview Geometry (More than Two Images)



Robust feature and feature matching make this possible.

Revisit the SfM Problem

	$\mathbf{X}^1$	$\mathbf{X}^2$	....	$\mathbf{X}^M$
Image 1	$\mathbf{x}_1^{\,1} = \mathbf{P}_1\mathbf{X}^1$		....	$\mathbf{x}_1^{\,M} = \mathbf{P}_1\mathbf{X}^M$
Image 2		$\mathbf{x}_2^{\,2} = \mathbf{P}_2\mathbf{X}^2$	....	$\mathbf{x}_2^{\,M} = \mathbf{P}_2\mathbf{X}^M$
⋮	⋮	⋮	⋱	⋮
Image N	$\mathbf{x}_N^{\,1} = \mathbf{P}_N\mathbf{X}^1$	$\mathbf{x}_N^{\,2} = \mathbf{P}_N\mathbf{X}^2$	....	$\mathbf{x}_N^{\,M} = \mathbf{P}_N\mathbf{X}^M$

Input
Observed 2D image position

Output
Unknown Camera Parameters (with some guess)
Unknown Point 3D coordinate (with some guess)

Robust feature and feature matching make this possible.

Bundle Adjustment

Observation

$$\begin{array}{ccc} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{array} \quad = $$

Features matching

Re-projection

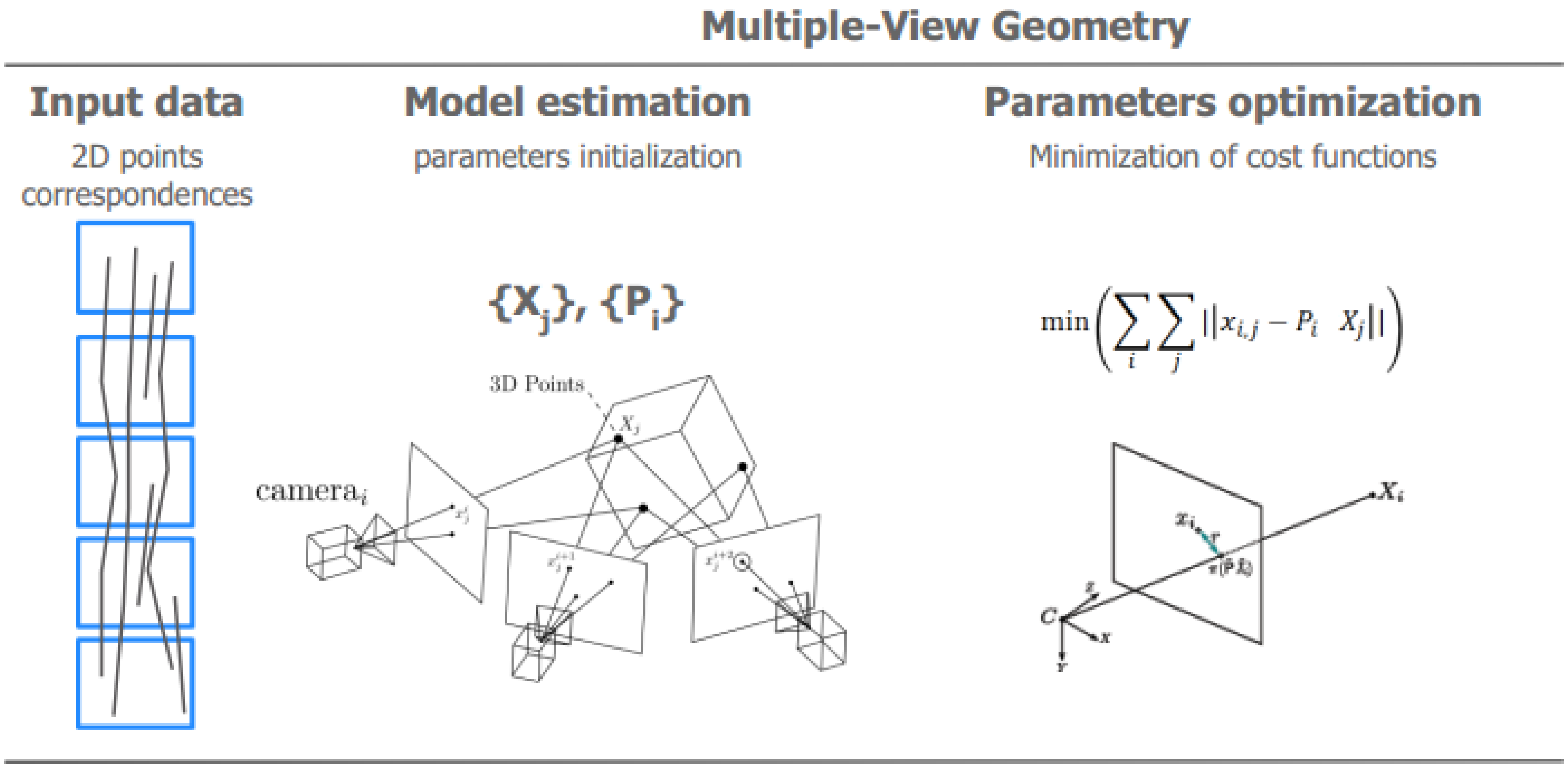
$\mathbf{x}_1^{\ 1} = \mathbf{P}_1\mathbf{X}^1$		....	$\mathbf{x}_1^{\ M} = \mathbf{P}_1\mathbf{X}^M$
	$\mathbf{x}_2^{\ 2} = \mathbf{P}_2\mathbf{X}^2$	....	$\mathbf{x}_2^{\ M} = \mathbf{P}_2\mathbf{X}^M$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathbf{x}_N^{\ 1} = \mathbf{P}_N\mathbf{X}^1$	$\mathbf{x}_N^{\ 2} = \mathbf{P}_N\mathbf{X}^2$	....	$\mathbf{x}_N^{\ M} = \mathbf{P}_N\mathbf{X}^M$

$$\boxed{\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^{\ j} - \mathbf{P}_i \mathbf{X}^j \right)^2}$$

Optimization problem

A valid solution must let the re-projection close to the observation.

SfM is an Optimization Problem



Known parameters	**Unknown** parameters	**The** “math” **problem**
Camera intrinsics: {K_i} **2D correspondences**: {{x_{i,j}}	**Poses**: {R_i|C_i} **3D Structure**: {Xi;{{x_{i,j}}	Minimization of a residual error: sum_(i,j)(ϵ)

What Makes This Problem Challenging?

- **Not enough overlaps across the images**
- **Not enough features on the scene in the world**
- **$O(N^2)$ complexity (matching)**
- **Wrong matching**

Problem Statement: Image-based Measurement

i : Image number

$$\mathbf{x} = \mathbf{P_i X}$$

Input Input Output

Obtain the projective matrices
from SfM software

Triangulation Methods (3D Position from 2D Points on Images)

$$\mathbf{x}_1 = \mathbf{P}_1\mathbf{X} \qquad \mathbf{x}_2 = \mathbf{P}_2\mathbf{X}$$

- Write as linear equations in $\mathbf{X}$

$$\begin{bmatrix} x\mathbf{p}^{3\top} - \mathbf{p}^{1\top} \\ y\mathbf{p}^{3\top} - \mathbf{p}^{2\top} \\ x'\mathbf{p}'^{3\top} - \mathbf{p}'^{1\top} \\ y\mathbf{p}'^{3\top} - \mathbf{p}'^{2\top} \end{bmatrix} \mathbf{X} = \mathbf{0}$$

- Solve for $\mathbf{X}$.
- Generalizes to point match in several images.
- Minimizes no meaningful quantity – not optimal.

Find a null vector of A when Ax=0

Triangulation Methods (Derivation)

The linear triangulation method is the most common one, described, for instance, in [8]. Suppose that $\mathbf{u} = P\mathbf{x}$. We write in homogeneous coordinates $\mathbf{u} = w(u, v, 1)^\top$, where (u, v) are the observed point coordinates and w is an unknown scale factor. Now, denoting by $\mathbf{p}_i^\top$ the ith row of the matrix P, the equation $\mathbf{u} = P\mathbf{x}$ may be written as

$$wu = \mathbf{p}_1^\top\mathbf{x}, \quad wv = \mathbf{p}_2^\top\mathbf{x}, \quad w = \mathbf{p}_3^\top\mathbf{x}.$$

Eliminating w using the third equation, we arrive at

$$\begin{aligned} u\mathbf{p}_3^\top\mathbf{x} &= \mathbf{p}_1^\top\mathbf{x} \\ v\mathbf{p}_3^\top\mathbf{x} &= \mathbf{p}_2^\top\mathbf{x}. \end{aligned} \tag{8}$$

https://perception.inrialpes.fr/Publications/1997/HS97/HartleySturm-cviu97.pdf

Slide Credits and References

- Lecture notes: JianXiong Xiao. “Multi-view 3D Reconstruction for Dummies”. Princeton Vision Group
- CVPR 2015 Tutorial: SfM Pipelines
- http://vision.princeton.edu/courses/SFMedu/
- http://cs.brown.edu/courses/cs143/
- http://people.csail.mit.edu/torralba/courses/6.869/6.869.computervision.htm
- http://www.cs.utexas.edu/~grauman/courses/fall2009/schedule.htm
- http://graphics.cs.cmu.edu/courses/15-463/2010_spring/463.html
- https://courses.engr.illinois.edu/cee598vsc/sp2015/lecturenotes/
- VisualSfM: http://ccwu.me/vsfm/doc.html
- Pix4D: https://support.pix4d.com/hc/en-us/sections/200591059-Manual#gsc.tab=0