1. SIFT on web site

2. Collinearity (H Computation). in Task 5

3. Evaluation Date (March 26).
   Please bring your laptop.
   (Finish the class earlier.)

4. Prob 3(a): No rotation
   $ax^2 + by^2 + cx + dy + e = 0$
   Distance Computation.

5. back-projecting $F = [e_2]_\times P_2 P_1^{-1}$ (previous lecture)

$$X \xrightarrow{H_\pi} X'$$

Given X, H_π, is X' unique??

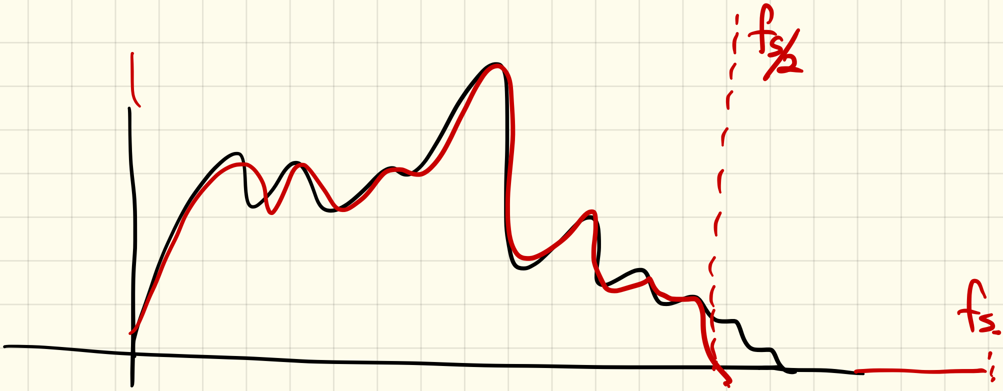
Given x, X', is H_π unique??

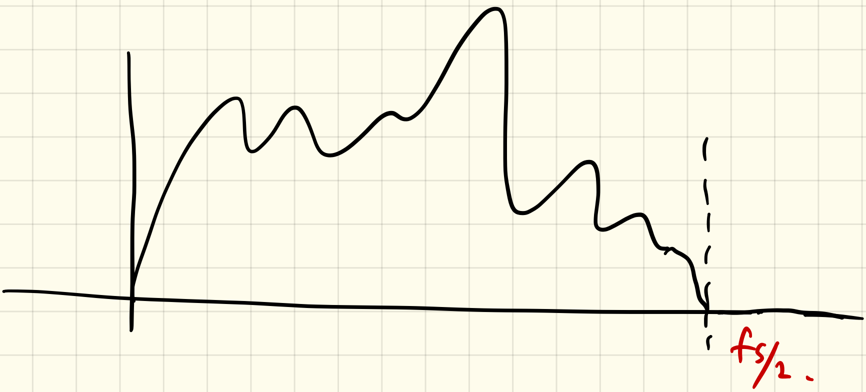
8. Task 6.

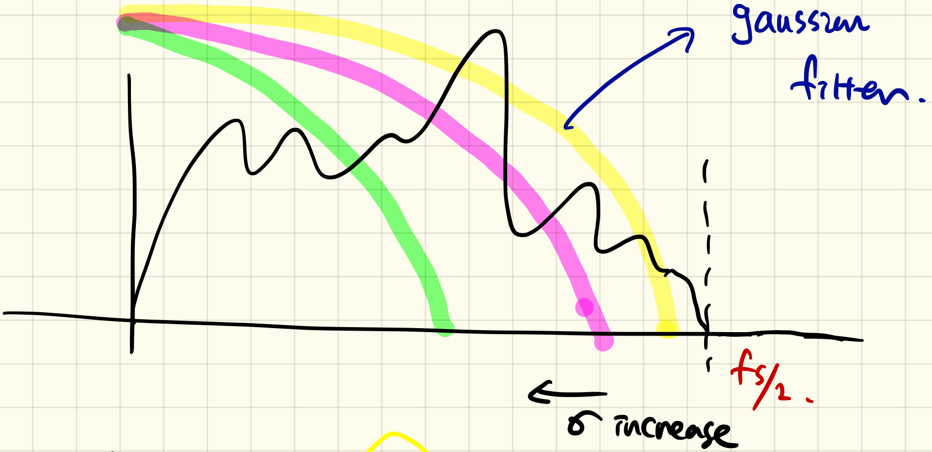
6. SfM Lecture

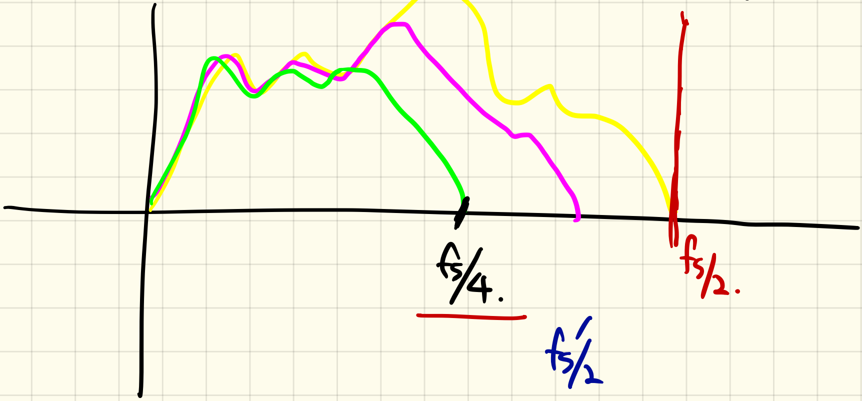
7. Final presentation

Capture it using your image sensor.

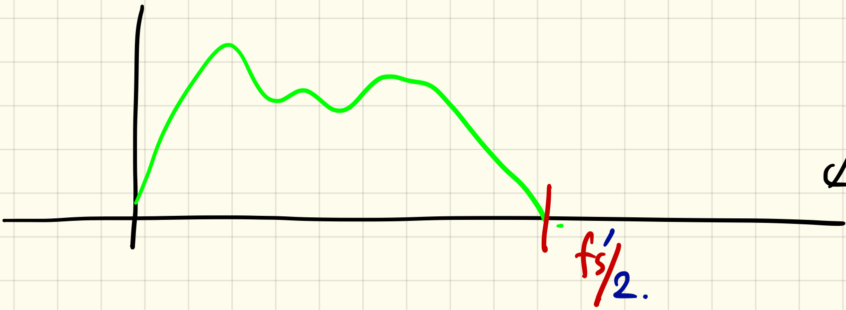




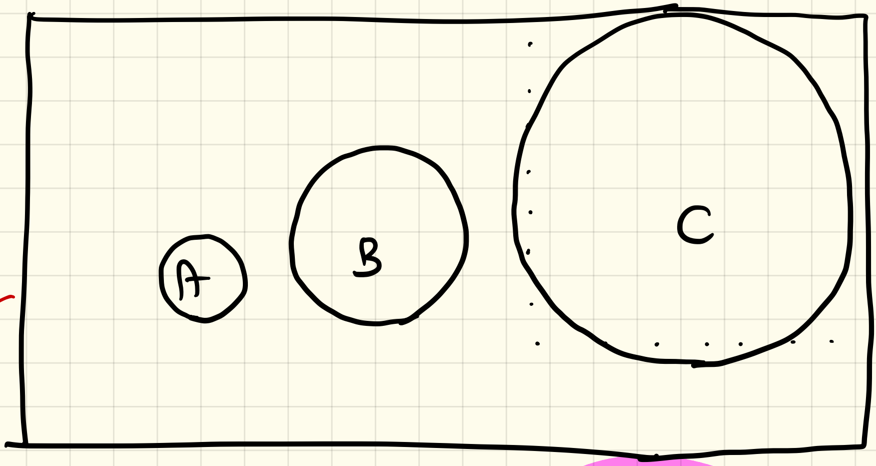




Resampling (increase octave).

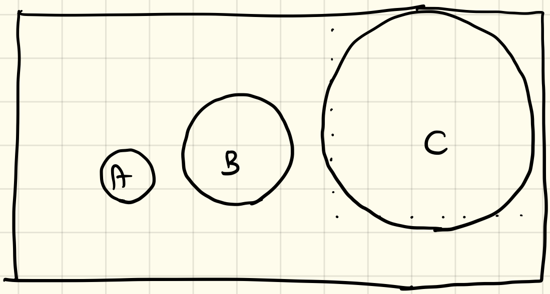


Use same size operators



A: 2×2 B = 4×4 C = 8×8.

resize (0.5)



8 8

A': 1×1 B' = 2×2 C': 4×4.

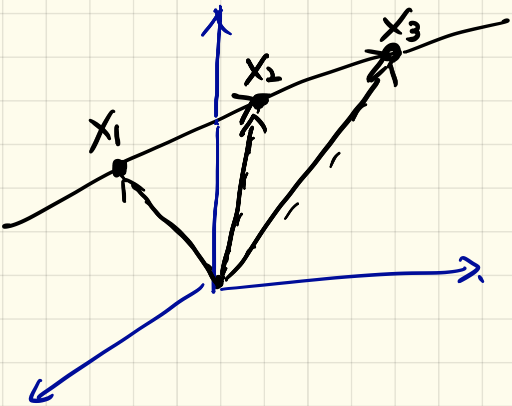
Applying C' on the resized image is more efficient than computing with a large C.

Collinearity : Collinearity of a set of points is the property of them lying on a single line

$$ax + by + cz + d = 0$$ ← plane equation

(not in HC)

$$\underbrace{\begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$



$$X_1 - X_2 = \lambda(X_3 - X_2)$$

$$\boxed{\begin{aligned} \mathrm{Rank}(A) &= \dim(\mathrm{Col}(A)) \\ &= \dim(\mathrm{row}(A)) \\ &= 2 \end{aligned}}$$

$$A = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 & 0 \\ x_2 & y_2 & z_2 & 0 \\ x_3 - x_2 & y_3 - y_2 & z_3 - z_2 & 0 \end{bmatrix}$$

nullity(A) $= \underline{2}$.

not 1

Triangulation

row vector (1×4) 　　 $4\times1.$

$$PX = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}}_{X} = \begin{bmatrix} p_1X \\ p_2X \\ p_3X \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

scalar.

$$P'X = \begin{bmatrix} p_1'X \\ p_2'X \\ p_3'X \end{bmatrix} = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

from PX

$$\begin{cases} x_3\, p_1 X = x_1 x_3 \\ x_1\, p_3 X = x_1 x_3 \end{cases}$$

$$x_3 p_1 X - x_1 p_3 X = 0$$

$$x_3 p_2 X - x_2 p_3 X = 0$$

$$\begin{cases} x_3' p_1' X - x_1' p_3 X = 0 \\ x_3' p_2' X - x_2' p_3 X = 0 \end{cases}$$

from $p'X$

1×4 　　 4×1

$$\therefore \quad \underbrace{\begin{bmatrix} x_3 p_1 - x_1 p_3 \\ x_3 p_2 - x_2 p_3 \\ x_3' p_1 - x_1' p_3 \\ x_3' p_2 - x_2' p_3 \end{bmatrix}}_{4\times4} X = 0$$

Singular value decomposition of an $m \times n$ matrix is a factorization $U \Sigma V^T$.

V^T → orthonormal matrix

$$v_i^T v_j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$1\times 4,\ 4\times 1$

$$A = U \Sigma V^T \qquad \underbrace{A}_{3\times 4.} \in \mathbb{R}^{3\times 4}$$

$U : 3\times 3 \qquad \Sigma : 3\times 4 \qquad V : 4\times 4$

$\sigma_1 > \sigma_2 > \sigma_3$

$$A = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & 0 \\ & & \sigma_3 & 0 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \\ v_4^T \end{bmatrix}$$

1×4

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T$$

scalar $\quad 3\times 1 \quad 1\times 4 \Rightarrow 3\times 4.$

$$A v_4 = \sigma_1 u_1 v_1^T v_4 + \sigma_2 u_2 v_2^T v_4 + \sigma_3 u_3 v_3^T v_4$$

$3\times 4 \quad 4\times 1 \qquad 3\times 1 \quad 1\times 4 \quad 4\times 1 \qquad 3\times 1.$

$$= 0 + 0 + 0$$

$$= 0$$

$A v_4 = 0$ → $\boxed{\text{null}(A) = v_4}$

$$A = U\Sigma V^T \qquad A \in \mathbb{R}^{3\times 3}$$

$$U : 3\times 3 \qquad \Sigma : 3\times 3 \qquad V : 3\times 3$$

$$A = [\underset{3\times 1}{u_1}\ u_2\ u_3] \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_1^T \\ v_3' \end{bmatrix} \quad \xrightarrow{} 1\times 3$$

$$A = \underset{\text{scalar}}{\sigma_1}\, \underset{3\times 1}{u_1}\, \underset{1\times 3}{v_1^T} + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T$$

$$= \sigma_1 \cdot \begin{bmatrix} 3\times 3 \end{bmatrix} + \sigma_2 \cdot \begin{bmatrix} 3\times 3 \end{bmatrix} + \sigma_3 \cdot \begin{bmatrix} 3\times 3 \end{bmatrix}$$

$$Av_3 = 0 + 0 + \sigma_3 u_3$$

if $\sigma_3 \approx 0$, $\boxed{Av_3 \approx 0.}$ → approximate null space.

A is almost singular matrix