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State-Space Model Conversion of a System with State Delay

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ABSTRACT

A new approach to model conversion of a state-delay system is introduced in this paper. The discrete-time model is especially derived from the continuous-time system, that it has a wider range for the sampling period in real implementation. A simple method is also proposed to determine the unknown state-delay time from the available discrete-time model, and the continuous-time model is established through some important results demonstrated in this paper.

Key Word: state delay, state-space model, model conversion

I. Introduction

The time delay phenomenon in control paths or state variables is unavoidable in many physical systems. Time delays are common in real systems, for example, chemical processes and biological systems, and the design of controllers for such systems depends critically on knowledge of the delays. It is well known that control system behavior is more sensitive to time delay than to other linear system parameters. In fact, a closed loop control system may be unstable or may exhibit unacceptable transient response characteristics if the time delay used in the system model for controller design does not coincide with the actual process time delay. Ignorance of the computation delay during analysis and design of digital control systems may lead to unpredictable and unsatisfactory system performance.

Generally speaking, there are two types of time delay: input delay and state delay. Input delay is caused by the transmission of a control signal over a long distance, or the delay is sometimes built into a system deliberately for control purposes (Jury, 1964). State delay is due to transmission or transport delay among interacting elements in a dynamic system (Tsien, 1954; Amstrong and Tripp, 1981). The input delay system has been developed completely and has been reported by Amstrong and Tripp (1981), Astrom and Wittenmark (1990), and Chen and Chang (1998). As for the state delay system, Manitius and Olbort (1979) proposed a finite spectrum assignment method to solve the design problem. Alekal (1969), Chyung and Lee (1966), Eller *et al*. (1969), Muller (1971), and Ross (1971) studied the optimal control theory for the state delay system. However, most methods were developed for continuous-time systems rather than discrete-time systems and seem to be impractical, not only the implementation difficulty, but also because developed control laws or design approaches which involve too much analytical mathematics.

Based on the developed discrete-time model for a type of state delay, Amstrong and Tripp (1981) used the discrete-time control theory to design a digital controller that can be easily realized by means of microprocessors. However, the developed method is restricted in that the sampling period must be shorter than the state delay time. This is not convenient for real implementation. Recently, Chen and Chang (1998) proposed an improved step to avoid this limitation, but the general formation can not be obtained via such a method, also it can not simplify the digital implementation. In this paper, a general counterpart of the discrete-time model is derived, where the sampling period can be longer than the state delay time, and where the general form of the microprocessor application is more effective and convenient in real implementation. Applying these discrete-time techniques to state delay systems is one of the main goals of this paper. Another main purpose of this paper is to determine the unknown state delay from the derived discrete-time model. Based on the geometric-series method (Shieh *et al.*, 1980) and the principal nth-root of a matrix (Shieh *et al.*, 1986), the state-delay model conversion can be obtained in state-space form, which will be useful in future applications. Since the proposed method is derived based on the state-space model, it can be easily extended for the MIMO (multi-input and multi-output) system.

II. Continuous-to-Discrete Model Conversion

A linear time-invariant system with state delay is described by

$$\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{A}_1x(t-\tau)+\boldsymbol{B}u(t), \tag{1}$$

where $\boldsymbol{A}_1\in R^{n\times n}$ is the system matrix for the state delay. $\boldsymbol{A}\in R^{n\times n}$ and $\boldsymbol{B}\in R^{n\times p}$ are the system matrix and input matrix, respectively. The discrete-time description of Eq. (1) with Z.O.H. (zero-order hold) can be obtained as

$$x(kT+T)=\boldsymbol{G}x(kT)+\boldsymbol{F}x(t-\tau)+\boldsymbol{H}u(kT), \tag{2a}$$

where T is the sampling period, and

$$\boldsymbol{G}=e^{AT}, \tag{2b}$$

$$\boldsymbol{F}=\int_{kT}^{kT+T}e^{A(kT+T-\lambda)}\boldsymbol{A}_1d\lambda=[\boldsymbol{G}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{A}_1, \tag{2c}$$

$$\boldsymbol{H}=\int_{kT}^{kT+T}e^{A(kT+T-\lambda)}\boldsymbol{B}d\lambda=[\boldsymbol{G}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{B}. \tag{2d}$$

It is seen that the main problem of Eq. (2a) depends on whether $x(t-\tau)$ in $kT\sim kT+T$ can be represented as piecewise-constant data. Utilizing the mean-value method, $x(t-\tau)$ in $kT\sim kT+T$ becomes

$$x(t-\tau)\approx\frac{1}{2}[x(kT+T-\tau)+x(kT-\tau)], \tag{3}$$

and by Eq. (3), we can obtain Eq. (2a) as

$$x(kT+T)=\boldsymbol{G}x(xT)+\frac{1}{2}\boldsymbol{F}[x(kT+T-\tau)+x(kT-\tau)]$$
$$+\boldsymbol{H}u(kT). \tag{4}$$

1. Case 1: $\tau<T$

It seems to be too difficult to calculate $x(kT+T-\tau)$ and $x(kT-\tau)$. We can evaluate $x(kT)$ using the mean value of $x(kT+\tau)$ and $x(kT-\tau)$:

$$x(kT)=\frac{1}{2}[x(kT+\tau)+x(kT-\tau)]; \tag{5a}$$

thus,

$$x(kT-\tau)=2x(kT)-x(kT+\tau). \tag{5b}$$

From Eq. (5b), the main problem appears to be $x(kT+\tau)$, so the solution $x(kT-\tau)$ can be evaluated as sampled data. The simple derivation of $x(kT+\tau)$ is given in the following.

Let us consider the state transition equation in Eq. (2a) by using a time substitution to obtain

$$x(kT+T+\tau)$$
$$=e^{\boldsymbol{A}(T+\tau)}x(kT)+\int_{kT}^{kT+T+\tau}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{A}_1x(\lambda-\tau)d\lambda$$
$$+\int_{kT}^{kT+T+\tau}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{B}u(kT)d\lambda. \tag{6}$$

Multiplying Eq. (2a) by $e^{\boldsymbol{A}\tau}$ to yields

$$e^{\boldsymbol{A}t}x(kT+T)$$
$$=e^{\boldsymbol{A}(T+\tau)}x(kT)+\int_{kT}^{kT+T}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{A}_1x(\lambda-\tau)d\lambda$$
$$+\int_{kT}^{kT+T}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{B}u(kT)d\lambda. \tag{7}$$

Subtracting Eq. (7) from Eq. (6) yields

$$x(kT+T+\tau)-e^{A\tau}x(kT+T)$$
$$=\int_{kT+T}^{kT+T+\tau}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{A}_1x(\lambda-\tau)d\lambda$$
$$+\int_{kT+T}^{kT+T+\tau}e^{\boldsymbol{A}(kT+T+\tau-\lambda)}\boldsymbol{B}u(kT)d\lambda. \tag{8}$$

Simplifying the integral in Eq. (8), we can obtain

$$x(kT+T+\tau)-\boldsymbol{G}_1x(kT+T)$$
$$=\frac{1}{2}\boldsymbol{P}_1[x(kT+T+\tau)+x(kT+T)]+\Gamma_1u(kT), \tag{9a}$$

where

$$\boldsymbol{G}_1=e^{At}, \tag{9b}$$

$$\boldsymbol{P}_1=\int_{kT+T}^{kT+T+\tau}e^{A(kT+T+\tau-\lambda)}\boldsymbol{A}_1d\lambda=[e^{A\tau}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{A}_1, \tag{9c}$$

$$\Gamma_1=\int_{kT+T}^{kT+T+\tau}e^{A(kT+T+\tau-\lambda)}\boldsymbol{B}d\lambda=[e^{A\tau}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{B}, \tag{9d}$$

and $\boldsymbol{I}_n$ denotes an $n\times n$ identity matrix. By rearranging Eq. (9), we can obtain

$$x(kT+T+\tau)=(\boldsymbol{G}_1+\frac{1}{2}\boldsymbol{P}_1)x(kT+T)+\frac{1}{2}\boldsymbol{P}_1x(kT+T-\tau)+\Gamma_1u(kT). \quad \textbf{(10)}$$

If we move backwards one sampling period T, Eq. (10) becomes

$$x(kT+\tau)=(\boldsymbol{G}_1+\frac{1}{2}\boldsymbol{P}_1)x(kT)+\frac{1}{2}\boldsymbol{P}_1x(kT-\tau)+\Gamma_1u(kT-T). \quad \textbf{(11)}$$

Using Eq. (11), the above Eq. (5b) can be obtained as

$$x(kT-\tau)=2x(kT)=(\boldsymbol{G}_1+\frac{1}{2}\boldsymbol{P}_1)x(kT)-\frac{1}{2}\boldsymbol{P}_1x(kT-\tau)-\Gamma_1u(kT-T), \quad \textbf{(12a)}$$

and $x(kT-\tau)$ is

$$x(kT-\tau)=(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}(2\boldsymbol{I}_n-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1)x(kT)-(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}\Gamma_1u(kT-T). \quad \textbf{(12b)}$$

If we move forward one sampling period T, Eq. (12b) becomes

$$x(kT+T-\tau)=(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}(2\boldsymbol{I}_n-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1)x(kT+T)-(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}\Gamma_1u(kT). \quad \textbf{(13)}$$

Using Eq. (12b) and Eq. (13), the discrete-time model of Eq. (4) becomes

$$\begin{aligned}
&x(kT+T)\\
&=\boldsymbol{G}x(kT)+\frac{1}{2}\boldsymbol{F}[x(kT+T-\tau)+x(kT-\tau)]+\boldsymbol{H}u(kT)\\
&=e^{AT}x(kT)+\frac{1}{2}\boldsymbol{F}[(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}(2\boldsymbol{I}_n-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1)x(kT+T)\\
&\quad-(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}\Gamma_1u(kT)+(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}(2\boldsymbol{I}_n-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1)\\
&\quad\bullet x(kT)-(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}\Gamma_1u(kT-T)]+\boldsymbol{H}u(KT),
\end{aligned}$$

and the final result becomes

$$\begin{aligned}
&x(kT+T)\\
&=\boldsymbol{Q}(\boldsymbol{G}+\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\boldsymbol{S}_2)x(k\mathrm{T})+\boldsymbol{Q}(\boldsymbol{H}-\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\Gamma_1)u(kT)\\
&\quad-\frac{1}{2}\boldsymbol{Q}\boldsymbol{F}\boldsymbol{S}_1\Gamma_1u(kT-T), \quad \textbf{(14a)}
\end{aligned}$$

where

$$\boldsymbol{S}_1=(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{P}_1)^{-1}, \quad \textbf{(14b)}$$

$$\boldsymbol{S}_2=(2\boldsymbol{I}_n-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1), \quad \textbf{(14c)}$$

$$\boldsymbol{Q}=(\boldsymbol{I}_n+\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\boldsymbol{S}_2)^{-1}. \quad \textbf{(14d)}$$

2. Case 2: $\tau>T$

May other researchers have proposed versions of the state-delay model that are suitable for $\tau>T$. Amstrong and Tripp (1981) used the trapezoidal integration to derived a general discrete-time model as

$$x(kT+T)=\boldsymbol{G}x(kT)+\frac{T}{2}e^{AT}\boldsymbol{A}_1x(kT-mT)+\frac{T}{2}\boldsymbol{A}_1x(kT+T-mT)+\boldsymbol{H}u(kT). \quad \textbf{(15)}$$

Chen and Chang (1998) used the block pulse function to derive a general discrete-time model as

$$x(kT+T)=\boldsymbol{G}x(kT)+\frac{1}{2}\boldsymbol{F}[x(kT+T-mT)+x(kT-mT)]+\boldsymbol{H}u(kT). \quad \textbf{(16)}$$

Comparing Eqs. (4), (15), and (16), we find that they all assume that $\tau\approx mT$, and that their sampled-data models are all the same. It is hard to determine which model is more accurate (Chen and Chang, 1998).

III. Discrete-to-Continuous Conversion

Assume that the state-delay $\tau=mT+\tau'$ (m is an integer, $m\geq 1$, $\tau'<T$). To evaluate the integral term in Eq. (2c), we can split $kT\sim kT+T$ into $kT\sim kT+\tau'$ and $kT+\tau'\sim kT+T$, respectively. Then we have

$$x(kT+T)=\boldsymbol{G}x(kT)+\boldsymbol{F}_1x(kT-T-mT)+\boldsymbol{F}_0x(kT-mT)+\boldsymbol{H}u(kT), \quad \textbf{(17a)}$$

where

$$\boldsymbol{F}_1=\int_{kT}^{kT+\tau'}e^{A(kT+T-\lambda)}\boldsymbol{A}_1d\lambda=[\boldsymbol{G}-\boldsymbol{G}^{(1-\gamma)}]\boldsymbol{A}^{-1}\boldsymbol{A}_1, \quad \textbf{(17b)}$$

$$\boldsymbol{F}_0=\int_{kT+\tau'}^{kT+T}e^{A(kT+T-\lambda)}\boldsymbol{A}_1d\lambda=[\boldsymbol{G}^{(1-\gamma)}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{A}_1, \quad \textbf{(17c)}$$

$$\gamma=\frac{\tau'}{T}. \quad \textbf{(17d)}$$

Let $(1-\gamma)\equiv q/l$ be a fractional number, where $\boldsymbol{G}^{(1-\gamma)}=$

$(\sqrt[l]{G})^q$ represents the q's power of the principal lth-root (Shieh *et al.*, 1986) of the matrix $\boldsymbol{G}$. If $\boldsymbol{A}$ in Eq. (17) is singular, the computations of $\boldsymbol{F}_1$ and $\boldsymbol{F}_0$ can be carried out using the geometric-series method (Shieh *et al.*, 1980). It is noted that $\boldsymbol{F}$ can be rewritten as

$$\boldsymbol{F}=\boldsymbol{F}_1+\boldsymbol{F}_0. \tag{18}$$

Let us consider the discrete-time system ($\boldsymbol{G}$, $\boldsymbol{F}_1$, $\boldsymbol{F}_0$) as shown in Eq. (17a). We wish to find the associated $\boldsymbol{A}$, $\boldsymbol{A}_1$, $\boldsymbol{B}$ and τ in the continuous-time system as shown in Eq. (1). The relationship between the matrices $\boldsymbol{A}$ and $\boldsymbol{G}$ is

$$\boldsymbol{A}=\frac{1}{T}\ln \boldsymbol{G}. \tag{19}$$

From Eq. (2c) and Eq. (18), we can obtain

$$\boldsymbol{A}_1=\boldsymbol{A}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}\boldsymbol{F}=\boldsymbol{A}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}(\boldsymbol{F}_1+\boldsymbol{F}_0). \tag{20}$$

From Eq. (2d), we can obtain

$$\boldsymbol{B}=\boldsymbol{A}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}\boldsymbol{H}. \tag{21}$$

The determination of the state-delay time τ can be described as follows. Let us consider the following infinite series:

$$\begin{aligned}
&\boldsymbol{G}^{1-\gamma}\\
&=e^{(1-\gamma)AT}\\
&=\boldsymbol{I}_n+(1-\gamma)T\boldsymbol{A}+\frac{1}{2!}[(1-\gamma)T\boldsymbol{A}]^2+\frac{1}{3!}[(1-\gamma)T\boldsymbol{A}]^3+\ldots\\
&=\boldsymbol{I}_n+(1-\gamma)T\boldsymbol{A}+\frac{1}{2!}[(1-\gamma)T\boldsymbol{A}]^2+\sum_{j=3}^{\infty}\frac{1}{j!}[(1-\gamma)T\boldsymbol{A}]^j\\
&\approx\boldsymbol{I}_n+(1-\gamma)T\boldsymbol{A}+\frac{1}{2!}[(1-\gamma)T\boldsymbol{A}]^2+\sum_{j=3}^{\infty}\frac{1}{2^{j-1}}[(1-\gamma)T\boldsymbol{A}]^j\\
&=\boldsymbol{I}_n+\{\boldsymbol{I}_n+\frac{1}{2}[(1-\gamma)T\boldsymbol{A}]\\
&\quad+\sum_{j=3}^{\infty}\frac{1}{2^{j-1}}[(1-\gamma)T\boldsymbol{A}]^{j-1}\}(1-\gamma)T\boldsymbol{A}\\
&=\boldsymbol{I}_n+\{\boldsymbol{I}_n-\frac{1}{2}(1-\gamma)T\boldsymbol{A}\}^{-1}(1-\gamma)T\boldsymbol{A}\\
&\qquad\text{for } \frac{1}{2}(1-\gamma)T<\frac{1}{\|\boldsymbol{A}\|}
\end{aligned} \tag{22}$$

Substituting Eq. (22) into Eq. (17c) yields

$$\begin{aligned}
\boldsymbol{F}_0&=[\boldsymbol{G}^{(1-\gamma)}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{A}_1\\
&=[\boldsymbol{I}_n+\{\boldsymbol{I}_n-\frac{1}{2}(1-\gamma)T\boldsymbol{A}\}^{-1}(1-\gamma)T\boldsymbol{A}-\boldsymbol{I}_n]\boldsymbol{A}^{-1}\boldsymbol{A}_1\\
&=[\boldsymbol{I}_n-\frac{1}{2}(1-\gamma)T\boldsymbol{A}]^{-1}(1-\gamma)T\boldsymbol{A}\boldsymbol{A}^{-1}\boldsymbol{A}_1\\
&=[\boldsymbol{I}_n-\frac{1}{2}(1-\gamma)T\boldsymbol{A}]^{-1}(1-\gamma)T\boldsymbol{A}_1.
\end{aligned} \tag{23a}$$

By means of some mathematical operations, Eq. (23a) becomes

$$\boldsymbol{F}_0=(1-\gamma)T[\boldsymbol{A}_1+\frac{1}{2}\boldsymbol{A}\boldsymbol{F}_0]=(T-\tau')[\boldsymbol{A}_1+\frac{1}{2}\boldsymbol{A}\boldsymbol{F}_0]\equiv(T-\tau')\boldsymbol{J}, \tag{23b}$$

where $\boldsymbol{J}=\boldsymbol{A}_1\frac{1}{2}\boldsymbol{A}\boldsymbol{F}_0$. By comparing the associated elements of $\boldsymbol{F}_0$ and $\boldsymbol{J}$ in Eq. (23b), the state-delay time τ' can be evaluated as

$$\tau' = T-\frac{1}{n\times p}\sum_{i=1}^{n}\sum_{j=1}^{p}\frac{f_{ij}}{J_{ij}}, \tag{24}$$

where f_{ij} and J_{ij} represent the elements of the ith-row and jth-column of $\boldsymbol{F}_0$ and $\boldsymbol{J}$, respectively. From the viewpoint of estimation structure, $\boldsymbol{F}_1$ and $\boldsymbol{F}_0$ are related to $x(kT-T-mT)$ and $x(kT-mT)$, respectively. Once m is identified, the delay time is found to be $\tau=mT+\tau'$. Using Eqs. (19)-(21) and Eq. (24), we can obtain the continuous-time model in Eq. (1) from the discrete-time model in Eq. (17a).

IV. Examples

1. Example 1

Let us consider a wind tunnel system (Amstrong and Tripp, 1981), where the control input is a unit step function. The state-delay system is described by

$$\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{A}_1x(t-\tau)+\boldsymbol{B}u(t)$$

with

$$x(0)=\begin{bmatrix}-0.1\\8.55\\0\end{bmatrix},$$

where

$$\boldsymbol{A}=\begin{bmatrix}-0.509 & 0 & 0\\ 0 & 0 & 1\\ 0 & -36 & -9.6\end{bmatrix},$$

$$\boldsymbol{A}_1=\begin{bmatrix}0 & -0.006 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix}0\\0\\36\end{bmatrix},$$

and the time delay τ=0.06 [sec]<T=0.2 sec. Thus, the corresponding discrete-time model of Eq. (14) is given by

$$x(kT+T)$$
$$=\boldsymbol{Q}(\boldsymbol{G}+\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\boldsymbol{S}_2)x(kT)+\boldsymbol{Q}(\boldsymbol{H}-\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\Gamma_1)u(kT)$$
$$-\frac{1}{2}\boldsymbol{Q}\boldsymbol{F}\boldsymbol{S}_1\Gamma_1u(kT-T),$$

where

$$\boldsymbol{G}=e^{AT}=\begin{bmatrix}0.9032&0&0\\0&0.6245&0.0701\\0&-2.5247&-0.0488\end{bmatrix},$$

$$\boldsymbol{F}=[\boldsymbol{G}-\boldsymbol{I}_3]\boldsymbol{A}^{-1}\boldsymbol{A}_1=\begin{bmatrix}0&-0.0011&0\\0&0&0\\0&0&0\end{bmatrix},$$

$$\boldsymbol{S}_1=(\boldsymbol{I}_3+\frac{1}{2}\boldsymbol{P}_1)^{-1}=\begin{bmatrix}1.0000&0.0002&0\\0&1.0000&0\\0&0&1.0000\end{bmatrix},$$

$$\boldsymbol{S}_2=[2\boldsymbol{I}_3-\boldsymbol{G}_1-\frac{1}{2}\boldsymbol{P}_1]$$

$$=\begin{bmatrix}1.0301&0.0002&0\\0&1.0534&-0.0446\\0&1.6069&1.4819\end{bmatrix},$$

$$\boldsymbol{Q}=[\boldsymbol{I}_3-\frac{1}{2}\boldsymbol{F}\boldsymbol{S}_1\boldsymbol{S}_2]^{-1}=\begin{bmatrix}1.0000&-0.0006&0.0000\\0&1.0000&0\\0&0&1.0000\end{bmatrix},$$

$$\boldsymbol{H}=[\boldsymbol{G}-\boldsymbol{I}_3]\boldsymbol{A}^{-1}\boldsymbol{B}=\begin{bmatrix}0\\0.3755\\2.5247\end{bmatrix},$$

$$\Gamma_1=[\boldsymbol{G}_1-\boldsymbol{I}_3]\boldsymbol{A}^{-1}\boldsymbol{B}=\begin{bmatrix}0\\0.0534\\1.6069\end{bmatrix};$$

thus,

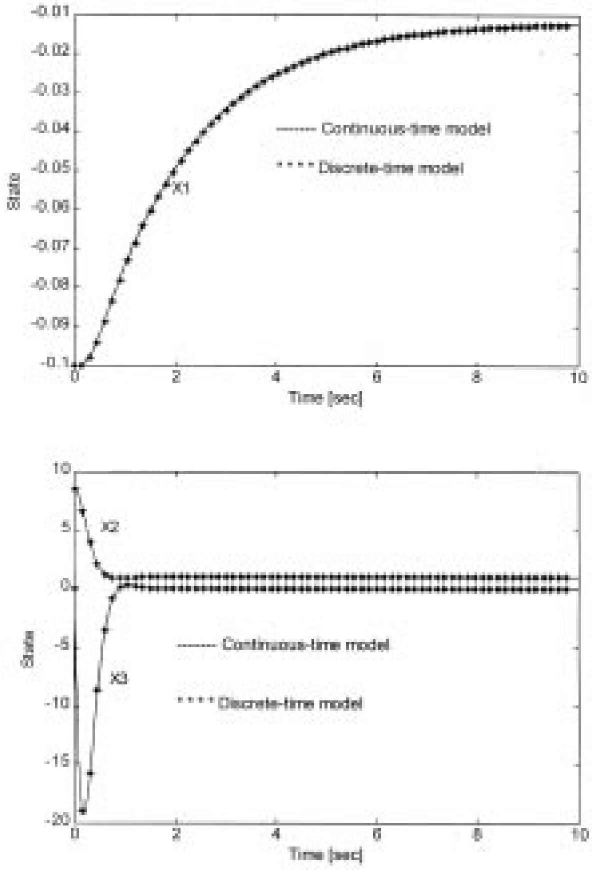

Fig. 1. State trajectories of both the continuous-time model and discrete-time model for τ=0.06 and T=0.2.

$$x(kT+T)=\begin{bmatrix}0.9032&-0.0010&0.0000\\0&0.6245&0.0701\\0&-2.5247&-0.0488\end{bmatrix}x(kT)$$

$$+\begin{bmatrix}-0.0001\\0.3755\\2.5247\end{bmatrix}u(kT)$$

$$+\begin{bmatrix}0.3047\times10^{-4}\\0\\0\end{bmatrix}u(kT-T).$$

The state responses of both the continuous-time (using the Runge-Kutta fourth-order algorithm) and discrete-time models are shown in Fig. 1.

2. Example 2

Consider a wind tunnel continuous-time state-delay model given as

$$\dot{x}(t)=\begin{bmatrix}-0.509 & 0 & 0\\ 0 & 0 & 1\\ 0 & -36 & -9.6\end{bmatrix}x(t)$$

$$+\begin{bmatrix}0 & -0.006 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}x(t-0.06)+\begin{bmatrix}0\\ 0\\ 36\end{bmatrix}u(t).$$

By applying continuous-to-discrete model conversion to Eq. (17a), we can obtain the desired equivalent discrete-time model with a sampling period of T=0.2 sec as

$$x(kT+T)=\boldsymbol{G}x(kT)+\boldsymbol{F}_1x(kT-T-mT)$$

$$+\boldsymbol{F}_0x(kT-mT)+\boldsymbol{H}u(kT),$$

where

$$\boldsymbol{G}=\begin{bmatrix}0.9032 & 0 & 0\\ 0 & 0.6245 & 0.0701\\ 0 & -2.5247 & -0.0488\end{bmatrix},$$

$$\boldsymbol{F}=\begin{bmatrix}0 & -0.0011 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{F}_1=\begin{bmatrix}0 & -0.3302\times 10^{-3} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{F}_0=\begin{bmatrix}0 & -0.8108\times 10^{-3} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{H}=\begin{bmatrix}0\\ 0.3755\\ 2.5247\end{bmatrix}.$$

In the meantime, according to $\boldsymbol{G}$, $\boldsymbol{F}$, $\boldsymbol{F}_1$, $\boldsymbol{F}_0$, the evaluate continuous-time system matrices $\hat{\boldsymbol{A}}$, $\hat{\boldsymbol{A}}_1$ and $\hat{\boldsymbol{B}}$ can be obtained by Eqs. (19)-(21) as

$$\hat{\boldsymbol{A}}=\frac{1}{T}\ln\boldsymbol{G}=\begin{bmatrix}-0.5090 & 0 & 0\\ 0 & 0.0000 & 1.0000\\ 0 & -36.0000 & -9.6000\end{bmatrix},$$

$$\hat{\boldsymbol{A}}_1=\hat{\boldsymbol{A}}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}\boldsymbol{F}=\hat{\boldsymbol{A}}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}(\boldsymbol{F}_1+\boldsymbol{F}_0)$$

$$=\begin{bmatrix}0 & -0.0060 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\hat{\boldsymbol{B}}=\hat{\boldsymbol{A}}(\boldsymbol{G}-\boldsymbol{I}_n)^{-1}\boldsymbol{H}=\begin{bmatrix}0\\ 0.0000\\ 36.0000\end{bmatrix}.$$

Also, by Eqs. (23b) and (24), we can obtain

$$\boldsymbol{J}=\hat{\boldsymbol{A}}_1+\frac{1}{2}\hat{\boldsymbol{A}}\boldsymbol{F}_0=\begin{bmatrix}0 & -0.0058 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

and the state-delay time τ=0.0600. the obtained results are quite satisfactory.

V. Conclusions

The general model of a state-delay system for the case where the delay time is shorter than sampling time has been developed in this paper. In practical applications, a very short sampling period interval will lead to an increase in the computation time and will create a serious dimensionality problem. Based on the model derived in this paper, we can utilize a longer sampling time for sampled-data models. Furthermore, it is more convenient to apply the discrete-time control theory in order to design a practical delay system. On the other hand, discrete-to-continuous conversion can be applied to system identification, and the delay time can be effectively evaluated to determine its corresponding state-space model in the continuous-time domain. By employing the application methods presented in this paper, a more useful and practical control design can be obtained.

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狀態空間延遲系統之模式轉換

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摘　要

本文對狀態延遲系統提出一個新的模式轉換方法。對於有狀態延遲時間之離散時間模式可以用新的方法從連續時間系統中得到，因此對於實際的應用上對取樣時間可以有較大的應用範圍。另外本文也提出一個簡單的方法來從離散時間模式存在的一些關係中計算得知未知的狀態延遲時間，進而回復其連續時間的延遲系統模式。