Про силу притяжения

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У меня получилась следующая система

$$\begin{cases} \ddot{\boldsymbol{r}}_1 = \dfrac{Gm_2(\boldsymbol{r}_2 - \boldsymbol{r}_1)}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|^3} \\ \ddot{\boldsymbol{r}}_2 = \dfrac{Gm_1(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3} \end{cases}$$

Преобразовал в такую систему

$$\begin{cases} \dot{\boldsymbol{r}}_1 = \boldsymbol{a} \\ \dot{\boldsymbol{r}}_2 = \boldsymbol{b} \\ \dot{\boldsymbol{a}} = \dfrac{Gm_2(\boldsymbol{r}_2 - \boldsymbol{r}_1)}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|^3} \\ \dot{\boldsymbol{b}} = \dfrac{Gm_1(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3} \end{cases}$$

Ввёл такие обозначения:

$$\boldsymbol{c} \approx \boldsymbol{r}_1$$

$$\boldsymbol{d} \approx \boldsymbol{r}_2$$

$$\boldsymbol{e} \approx \boldsymbol{a}$$

$$\boldsymbol{f} \approx \boldsymbol{b}$$

По методу Рунге-Кутта:

$$\boldsymbol{c}_{n+1} = \boldsymbol{c}_n + \frac{1}{6}h(\boldsymbol{g}_1 + 2\boldsymbol{g}_2 + 2\boldsymbol{g}_3 + \boldsymbol{g}_4)$$

$$\boldsymbol{d}_{n+1} = \boldsymbol{d}_n + \frac{1}{6}h(\boldsymbol{h}_1 + 2\boldsymbol{h}_2 + 2\boldsymbol{h}_3 + \boldsymbol{h}_4)$$

$$\boldsymbol{e}_{n+1} = \boldsymbol{e}_n + \frac{1}{6}h(\boldsymbol{e}_1 + 2\boldsymbol{e}_2 + 2\boldsymbol{e}_3 + \boldsymbol{e}_4)$$

$$\boldsymbol{f}_{n+1} = \boldsymbol{f}_n + \frac{1}{6}h(\boldsymbol{f}_1 + 2\boldsymbol{f}_2 + 2\boldsymbol{f}_3 + \boldsymbol{f}_4)$$

Для коэффициентов $\boldsymbol{g}_1, .., \boldsymbol{g}_4$:

$$\boldsymbol{g}_1 = \boldsymbol{a}_n$$
$$\boldsymbol{g}_2 = \boldsymbol{a}_n + \frac{1}{2}h\boldsymbol{g}_1$$
$$\boldsymbol{g}_3 = \boldsymbol{a}_n + \frac{1}{2}h\boldsymbol{g}_2$$
$$\boldsymbol{g}_4 = \boldsymbol{a}_n + h\boldsymbol{g}_3$$

Для коэффициентов $\boldsymbol{h}_1, .., \boldsymbol{h}_4$:

$$\boldsymbol{h}_1 = \boldsymbol{b}_n$$
$$\boldsymbol{h}_2 = \boldsymbol{b}_n + \frac{1}{2}h\boldsymbol{h}_1$$
$$\boldsymbol{h}_3 = \boldsymbol{b}_n + \frac{1}{2}h\boldsymbol{h}_2$$
$$\boldsymbol{h}_4 = \boldsymbol{b}_n + h\boldsymbol{h}_3$$

Для коэффициентов $\boldsymbol{k}_1, .., \boldsymbol{k}_4$:

$$\boldsymbol{k}_1 = \frac{Gm_2(\boldsymbol{d}_n - \boldsymbol{c}_n)}{|\boldsymbol{d}_n - \boldsymbol{c}_n|^3}$$
$$\boldsymbol{k}_2 = \frac{Gm_2\left(\left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_1\right) - \left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_1\right)\right)}{\left|\left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_1\right) - \left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_1\right)\right|^3}$$
$$\boldsymbol{k}_3 = \frac{Gm_2\left(\left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_2\right) - \left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_2\right)\right)}{\left|\left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_2\right) - \left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_2\right)\right|^3}$$
$$\boldsymbol{k}_4 = \frac{Gm_2\left((\boldsymbol{d}_n + h\boldsymbol{h}_3) - (\boldsymbol{c}_n + h\boldsymbol{g}_3)\right)}{\left|(\boldsymbol{d}_n + h\boldsymbol{h}_3) - (\boldsymbol{c}_n + h\boldsymbol{g}_3)\right|^3}$$

Для коэффициентов $\boldsymbol{l}_1, ..., \boldsymbol{l}_4$:

$$\boldsymbol{l}_1 = \frac{Gm_1(\boldsymbol{c}_n - \boldsymbol{d}_n)}{|\boldsymbol{c}_n - \boldsymbol{d}_n|^3}$$

$$\boldsymbol{l}_2 = \frac{Gm_1\left(\left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_1\right) - \left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_1\right)\right)}{\left|\left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_1\right) - \left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_1\right)\right|^3}$$

$$\boldsymbol{l}_3 = \frac{Gm_1\left(\left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_2\right) - \left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_2\right)\right)}{\left|\left(\boldsymbol{c}_n + \frac{1}{2}h\boldsymbol{g}_2\right) - \left(\boldsymbol{d}_n + \frac{1}{2}h\boldsymbol{h}_2\right)\right|^3}$$

$$\boldsymbol{l}_4 = \frac{Gm_1\left(\left(\boldsymbol{c}_n + h\boldsymbol{g}_3\right) - \left(\boldsymbol{d}_n + h\boldsymbol{h}_3\right)\right)}{\left|\left(\boldsymbol{c}_n + h\boldsymbol{g}_3\right) - \left(\boldsymbol{d}_n + h\boldsymbol{h}_3\right)\right|^3}$$