

Solving University Course Timetabling Problem Using Ant Colony Optimization: An Example of Mersin University Engineering Faculty

S. ASLAN¹ and C. ACI¹

¹ Mersin University, Department of Computer Engineering, 333434, Mersin, Turkey, aslansemir26@gmail.com
¹ Mersin University, Department of Computer Engineering, 333434, Mersin, Turkey, caci@mersin.edu.tr

Abstract - Building effective schedules in academical institutions considering the wishes and needs of administrative staff, professors and students at the same time is a rather difficult and time-consuming activity for staff involved in this work. Despite improvements in software and hardware technology in recent years, charts are still manually created in many educational institutions and the desired efficiency has not achieved. In this study, the course chart of Mersin University Engineering Faculty was built using Ant Colony Optimization (ACO) technique. While the course schedule was being formed, 9 departments, 24 common classrooms, 105 faculty members, 239 courses, 14,374 students who have attendance obligations and 8286 students who have not attendance obligations were taken into consideration. In the placement of the courses, adaptation to ACO algorithm has been achieved by targeting the maximum lecture minimum classroom usage. The appropriate hours of the lecturers were accepted as strict constraints and other cases were added to soft constraints. All courses of Mersin University Engineering Faculty have placed the course schedule to appropriate classrooms at the rate of 99% using ACO technique, and 17 classrooms of common 24 classrooms were determined to be sufficient for educational activities.

Keywords - Course scheduling; ant colony optimization; timetabling.

I. INTRODUCTION

The problem of course schedule varies from country to country due to the diversity in academic systems, even from universities within the academic system of the same country. Timetabling problems in the class of Np-Hard problems such as other scheduling problems have caught the intense attention of researchers since about 1950's. Many algorithms have been developed for solving these problems [1]. Due to these reasons, the time scheduling problem has been worked intensively for many years in the operational research literature and still maintains its popularity [2]. Generally, the important parameters of schedules in universities are lecturers of the university, number of students who are required to attend classes and classroom capacity. The course schedule is prepared by the lecturer or secretary of the department. Consequently, this implementation causes to interruption on lecturer's work time and reduce their productivity.

In addition to the demands of the lecturers on the problem of scheduling the course, the number of students who have attendance obligations is added to the number of students who have not attendance obligations. The necessity of this process ensures the adequacy of the classroom capacity in case of students who have not attendance obligations willingness to

join the class. Furthermore, it was also aimed to eliminate disruptions that physical conditions could cause. In this study, Ant Colony Optimization (ACO) algorithm has been modeled with MATLAB programming language on Mersin University Engineering Faculty lessons and the course schedule was established.

II. ANT COLONY

Real ants are capable of finding the shortest path from a food source to the nest without using visual cues. Also, they are capable of adapting to changes in the environment, e.g. finding a new shortest path once the old one is no longer feasible due to a new obstacle. Consider Fig. 1A: ants are moving in a straight line that connects a food source to their nest. It is well known that the primary means for ants to form and maintain the line is a pheromone trail. Ants deposit a certain amount of pheromone while walking, and each ant probabilistically prefers to follow a direction rich in pheromone. This elementary behavior of real ants can be used to explain how they can find the shortest path that reconnects a broken line after the sudden appearance of an unexpected obstacle has interrupted the initial path (Fig. 1B). In fact, once the obstacle has appeared, those ants which are just in front of the obstacle cannot continue to follow the pheromone trail and therefore they have to choose between turning right or left. In this situation, we can expect half the ants to choose to turn right and the other half to turn left. A very similar situation can be found on the other side of the obstacle (Fig. 1C). It is interesting to note that those ants which choose, by chance, the shorter path around the obstacle will more rapidly reconstitute the interrupted pheromone trail compared to those who choose the longer path. Thus, the shorter path will receive a greater amount of pheromone per time unit and in turn, a larger number of ants will choose the shorter path. Due to this positive feedback (autocatalytic) process, all the ants will rapidly choose the shorter path (Fig. 1D). The most interesting aspect of this autocatalytic process is that finding the shortest path around the obstacle seems to be an emergent property of the interaction between the obstacle shape and ants distributed behavior: although all ants move at approximately the same speed and deposit a pheromone trail at approximately the same rate, it is a fact that it takes longer to contour obstacles on their longer side than on their shorter side which makes the pheromone trail accumulate quicker on the shorter side. It is the ant's preference for higher pheromone trail

levels which makes this accumulation still quicker on the shorter path[3].

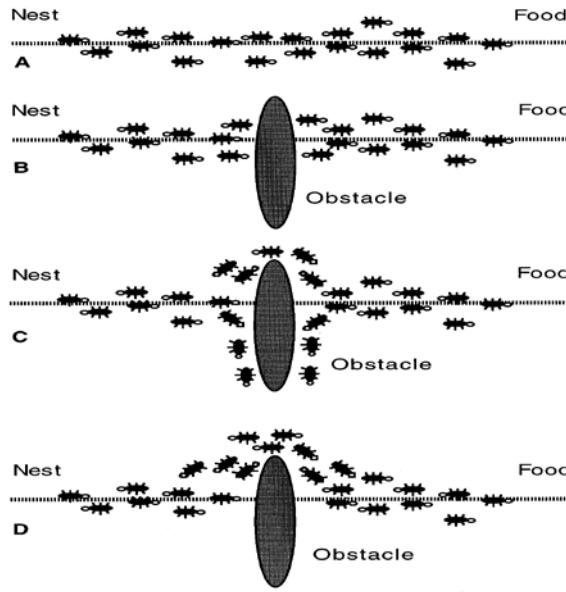


Figure 1: Ant colony food search behavior

III. ANT COLONY OPTIMIZATION ALGORITHM

ACO takes inspiration from the foraging behavior of some ant species. These ants deposit pheromone on the ground in order to mark some favorable path that should be followed by other members of the colony. ACO exploits a similar mechanism for solving optimization problems [4].

Ant System (AS): It is the first ACO algorithm proposed in the literature. Its main characteristic is that, at each iteration, the pheromone values are updated by *all* the m ants that have built a solution in the iteration itself. The pheromone, associated with the edge joining cities i and j , is updated as follows:

$$k\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} + \sum_{c=1}^m \Delta\tau_{ij}^k \quad (1)$$

Where ρ is the evaporation rate, m is the number of ants, and $\Delta\tau_{ij}^k$ is the quantity of pheromone laid on edge (i, j) by ant k .

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{L_k} & \text{if used edge } (i,j) \text{ in this tour,} \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

Where Q is a constant, and L_k is the length of the tour constructed by ant .

In the construction of a solution, ants select the following city to be visited through a stochastic mechanism. When ant k is in city i and has so far constructed the partial solution s^{ρ} , the probability of going to city j is given by:

$$P_{ij}^k = \begin{cases} \frac{\tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}}{\sum_{C_{il} \in N(s^{\rho})} \tau_{il}^{\alpha} \times \eta_{il}^{\beta}} & \text{if } c_{il} \in N(s^{\rho}) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Where $N(s^{\rho})$ is the set of feasible components; that is, edges (i, l) where l is a city not yet visited by the ant k . The parameters α and β control the relative importance of the pheromone versus the heuristic information, which is given by:

$$\eta_{ij} = \frac{1}{d_{i,j}}, \quad (4)$$

Where $d_{i,j}$ is the distance between cities i and j [4].

IV. ADAPTATION OF ACO SYSTEM TO SCHEDULING PROBLEM

When ACO algorithm is applied, appropriate hours of the lecturers, course-hour eligibility and lecturer-course hour parameters are added to the equation. The roulette wheel method has been applied for selection and it is added in ACO algorithm. The resulting equation and the meanings of the parameters according to this equation are shown below.

1. Choosing Classroom

When modeling is done, the course-classroomdistance matrix is normalized to 0-1 (Figure 2). A random course is selected from unselected courses. According to the selected course, (3) is applied for classroom selection. In Equation 3, t is the classroom which is not yet visited by the ant k . Appropriate classrooms are selected.

	Classroom																							
	239x24 double																							
1	0.0395	0.0353	0.0318	0.0185	0.0367	0.0339	0.0339	0.0216	0.0185	0.0185	0.0339	0.0367	0.0255	0.0339	0.0185	0.0185	0.0216	0.0353	0.0367	0.0216	0.0185	0.0185	0.0311	0.0367
2	-0.00...	0.0382	0.0347	0.0214	0.0396	0.0368	0.0368	0.0246	0.0214	0.0214	0.0368	0.0396	0.0284	0.0368	0.0214	0.0214	0.0246	0.0382	0.0396	0.0246	0.0214	0.0214	0.0340	0.0396
3	0.0374	0.0332	0.0297	0.0164	0.0346	0.0318	0.0318	0.0195	0.0164	0.0164	0.0318	0.0346	0.0234	0.0318	0.0164	0.0164	0.0195	0.0332	0.0346	0.0195	0.0164	0.0164	0.0290	0.0346
4	0.0374	0.0332	0.0297	0.0164	0.0346	0.0318	0.0318	0.0195	0.0164	0.0164	0.0318	0.0346	0.0234	0.0318	0.0164	0.0164	0.0195	0.0332	0.0346	0.0195	0.0164	0.0164	0.0290	0.0346
5	-0.00...	1	0.0365	0.0232	-0.0014	0.0386	0.0386	0.0263	0.0232	0.0232	0.0386	-0.0014	0.0302	0.0386	0.0232	0.0232	0.0263	1	-0.00...	0.0263	0.0232	0.0232	0.0358	-0.00...
6	0.0379	0.0337	0.0302	0.0169	0.0351	0.0323	0.0323	0.0200	0.0169	0.0169	0.0323	0.0351	0.0239	0.0323	0.0169	0.0169	0.0200	0.0337	0.0351	0.0200	0.0169	0.0169	0.0295	0.0351
7	-0.00...	-0.00...	0.0388	0.0255	-0.0037	-8.75...	-8.75...	0.0286	0.0255	0.0255	-8.75...	-0.0037	0.0325	-8.75...	0.0255	0.0255	0.0286	-0.0023	-0.00...	0.0286	0.0255	0.0255	0.0381	-0.00...
8	-0.00...	0.0398	0.0363	0.0230	-0.0012	0.0384	0.0384	0.0262	0.0230	0.0230	0.0384	-0.0012	0.0300	0.0384	0.0230	0.0230	0.0262	0.0398	-0.00...	0.0262	0.0230	0.0230	0.0356	-0.00...
9	0.0367	0.0325	0.0290	0.0157	0.0339	0.0311	0.0311	0.0188	0.0157	0.0157	0.0311	0.0339	0.0227	0.0311	0.0157	0.0157	0.0188	0.0325	0.0339	0.0188	0.0157	0.0157	0.0283	0.0339
10	0.0396	0.0354	0.0319	0.0186	0.0368	0.0340	0.0340	0.0218	0.0186	0.0186	0.0340	0.0368	0.0256	0.0340	0.0186	0.0186	0.0218	0.0354	0.0368	0.0218	0.0186	0.0186	0.0312	0.0368
11	-0.02...	-0.01...	-0.0133	1	-0.0182	-0.01...	-0.0154	-0.0032	1	1	-0.01...	-0.0182	-0.0070	-0.0154	1	1	-0.00...	-0.0168	-0.01...	-0.0032	1	1	-0.00...	-0.01...
12	-0.01...	-0.01...	-0.0072	0.0339	-0.0121	-0.00...	-0.0093	0.0370	0.0339	0.0339	-0.00...	-0.0121	-8.751...	-0.0093	0.0339	0.0339	0.0370	-0.0107	-0.01...	0.0370	0.0339	0.0339	-0.00...	-0.01...
13	-0.01...	-0.01...	-0.0089	0.0356	-0.0138	-0.01...	-0.0110	0.0388	0.0356	0.0356	-0.01...	-0.0138	-0.0026	-0.0110	0.0356	0.0356	0.0388	-0.0124	-0.01...	0.0388	0.0356	0.0356	-0.00...	-0.01...
14	-0.02...	-0.01...	-0.0133	1	-0.0182	-0.01...	-0.0154	-0.0032	1	1	-0.01...	-0.0182	-0.0070	-0.0154	1	1	-0.00...	-0.0168	-0.01...	-0.0032	1	1	-0.00...	-0.01...
15	-0.01...	-0.01...	-0.0119	0.0386	-0.0168	-0.01...	-0.0140	-0.0018	0.0386	0.0386	-0.01...	-0.0168	-0.0056	-0.0140	0.0386	0.0386	0.0386	-0.0124	-0.01...	-0.0018	0.0386	0.0386	-0.00...	-0.01...

Figure 2: Course-Classroom Matrix

2. Choosing Course

Equation 5 is applied to assign the remaining courses in the selected classroom. Where l is a classroom not yet visited by the ant k . Parameters are added to the equation and calculated. Selected courses are assigned to the classroom.

This is the symbolic representation and the added parameters.

$$P_{ij}^k = \begin{cases} \frac{\tau_{ij}^\alpha \times \eta_{ij}^\beta \times \mu^\theta \times \lambda^\delta \times v_{:h}^\gamma}{\sum_{C_{il} \in N(s^P)} \tau_{ij}^\alpha \times \eta_{ij}^\beta \times \mu^\theta \times \lambda^\delta \times v_{:h}^\gamma} & \text{if } c_{il} \in N(s^P) \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

τ_{ij} : The pheromone matrix between classroom i and lecture j ,

η_{ij} : The visibility matrix between classroom i and course j ,

μ : Matrix of theoretical lecture hours,

λ : Weekly total free time of lecturers,

$v_{:h}$: The matrix of daily leisure hours of the faculty members,

α : Relative importance of τ_{ij}

β : Relative importance of η_{ij}

θ : Relative importance of the μ ,

δ : Relative importance of the λ ,

γ : Relative importance of the $v_{:h}$

V. DATA USED IN IMPLEMENTING THE PROGRAM

The details of data used are shown in Figure 3. Data were taken from Information Technologies Center of Mersin University. The numbers belong to the spring semester of Engineering Faculty in 2015-2016 Academic year.

University	Mersin University
Department	Engineering Faculty
Department Number	9
Common Classrooms	24
Faculty Members	105
Courses	239
Compulsory Course	159
Elective Course	50
Students Who Have Attendance Obligations	14.374
Students Who Have Not Attendance Obligations	8286

Figure 3: Data properties

VI. RUNNING THE PROGRAM

Data that were taken from Information Technologies Center of Mersin University are imported to the program by being set it to be convenient to MATLAB. MATLAB is an interactive computer program that serves as a convenient "laboratory" for

computations involving matrices [5]. The first course selection has been done randomly and a random classroom has been chosen from the course-classroom matrix. The reason for this choice is to make sure that the algorithm can select different classrooms without getting stuck on local optima. After the classroom is selected, course hours are started according to (5) of suitable courses and it is sent to the function of applied roulette wheel selection. A course is chosen in order to place it in the classroom. After the classroom is chosen, the same process is carried out for another course selection and until all of the courses are placed or all the classrooms are visited, the process continues. All the ants do this process. After all processes have finished, the best course schedule is recorded and the global pheromone is updated. The next iteration is passed. After all of the iterations are completed, the course schedule is saved and the program ends.

The pseudo-code of the algorithm that is given information above is presented in Figure 4.

Figure 4: Pseudo code for course schedule ACO algorithm.

```

-----
for maxIt
    for maxAnt
        rand(course)
        for maxClassroomNumber
            apply equation (3)
            roulette wheel selection for Classroom
                for weeklyPeriod
                    apply equation (5)
                    roulette wheel selection for the
                    course
                end
            end
        end
    end
global pheromone update
print course schedule
end
-----
```

- One week is divided into 50 time slot.
- It is accepted as there will be 5 courses before noon and 5 courses afternoon every day.
- One lesson period is accepted as 40 minutes.
- 1-10 defines Monday, 11-20 defines Tuesday, 21-30 defines Wednesday, 31-40 defines Thursday and 41-50 defines Friday.
- Courses that have been placed in a classroom can be seen in Figure 5.

Course Information																		
	Fields	DersId	SubId	DersAdi	DersKredi	DersKodu	DersSaat	DersMe	HocaAdi	HocaSoyadi	Hocalid	SubeAdi	Bolumu	Bolum	DerslikKap	DerslikAdi		
Time slot	1	69498	368817	'YÖNETİM VE ORGAN...	3	'ŞL206'	3	71' 'BERİL'	'DÖNMEZ'	2652 'A Şubesı'	'ELEKTRIK - ELEKTR...	320	88 'MF301'					
	2	69498	368817	'YÖNETİM VE ORGAN...	3	'ŞL206'	3	71' 'BERİL'	'DÖNMEZ'	2652 'A Şubesı'	'ELEKTRIK - ELEKTR...	320	88 'MF301'					
	3	69498	368817	'YÖNETİM VE ORGAN...	3	'ŞL206'	3	71' 'BERİL'	'DÖNMEZ'	2652 'A Şubesı'	'ELEKTRIK - ELEKTR...	320	88 'MF301'					
	4	1838	352339	'TARIHSEL JELOJİ'	2	'JM228'	2	73' 'NURDAN'	'İNAN'	371 'A Şubesı'	'JELOJİ MÜHENİDIS...	620	88 'MF301'					
	5	1838	352339	'TARIHSEL JELOJİ'	2	'JM228'	2	73' 'NURDAN'	'İNAN'	371 'A Şubesı'	'JELOJİ MÜHENİDIS...	620	88 'MF301'					
	6	101359	366791	'BİLGİSAYAR DESTEKL...	2	'MM102'	1	68' 'KADER'	'METİN'	797 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
	7	83224	357195	'YAPI STATİĞİ I'	4	'İNŞ 322'	2	77' 'SUAT'	'KAVAS'	731 'A Şubesı'	'İNSAAT MÜHENİDIS...	530	88 'MF301'					
	8	83224	357195	'YAPI STATİĞİ II'	4	'İNŞ 322'	2	77' 'SUAT'	'KAVAS'	731 'A Şubesı'	'İNSAAT MÜHENİDIS...	530	88 'MF301'					
	9	102685	366818	'MATEMATİK 2'	4	'MAT106'	2	87' 'ÖZGÜR'	'MIZRAK'	1182 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
	10	102685	366818	'MATEMATİK 2'	4	'MAT106'	2	87' 'ÖZGÜR'	'MIZRAK'	1182 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
Classroom	11	101362	366793	'FİZİK II'	4	'FZK 122'	3	71.5000' 'SERDAR'	'YILMAZ'	1057 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
	12	101362	366793	'FİZİK II'	4	'FZK 122'	3	71.5000' 'SERDAR'	'YILMAZ'	1057 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
	13	101362	366793	'FİZİK II'	4	'FZK 122'	3	71.5000' 'SERDAR'	'YILMAZ'	1057 'A Şubesı'	'METALURİ VE MAL...	910	88 'MF301'					
	14	5776	352375	'JEOFİZİK'	2	'JM246'	2	60.5000' 'MUSTAFA'	'ERYILMAZ'	382 'A Şubesı'	'JELOJİ MÜHENİDIS...	620	88 'MF301'					
	15	5776	352375	'JEOFİZİK'	2	'JM246'	2	60.5000' 'MUSTAFA'	'ERYILMAZ'	382 'A Şubesı'	'JELOJİ MÜHENİDIS...	620	88 'MF301'					
	16	70648	352471	'FİZİK II'	4	'FZK122'	3	32.5000' 'ŞEREF'	'KAPLAN'	246 'A Şubesı'	'JELOJİ MÜHENİDIS...	610	88 'MF301'					
	17	70648	352471	'FİZİK II'	4	'FZK122'	3	32.5000' 'ŞEREF'	'KAPLAN'	246 'A Şubesı'	'JELOJİ MÜHENİDIS...	610	88 'MF301'					
	18	70648	352471	'FİZİK II'	4	'FZK122'	3	32.5000' 'ŞEREF'	'KAPLAN'	246 'A Şubesı'	'JELOJİ MÜHENİDIS...	610	88 'MF301'					
	19	133229	368835	'SAĞLIK İKİLYASAM'	2	'FFM384'	2	76' 'EVREN'	'DEĞIRMENCI'	1554 'A Şubesı'	'FI FKTRIK - FI FKTR'	230	88 'MF301'					
	20																	

Figure 5: Classroom course schedule

A sample of Engineering Faculty's course scheduling is seen in Figure 6. Course Identity is written on the figure.

	Time Slot																																										
Classroom	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
	2	1329...	132...	5783	5783	5783	1038...	103...	1038...	0	103566	103566	1035...	0	0	103893	1038...	70634	70634	0	10485	10485	10485	1599	0	5782	5782	5782	1005...	1005...	103828	103828	70640										
	3	1005...	100...	70625	70625	0	1863	1863	100...	1005...	100526	10494	10494	10494	1617	1617	19549	19549	0	0	1332...	1332...	133226	103568	103568	19518	19518	0	0	0	0	0	10474										
	4	98276	98276	98276	103...	103...	0	0	0	83205	0	71952	71952	71952	0	0	83637	0	98273	98273	98273	70636	0	71946	71946	71946	0	71934	71934	71934	1005...	1005...	1894	0	0								
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
	6	16040	16040	16040	0	0	71909	71909	71909	0	69245	69245	69245	0	0	71926	71926	1038...	0	71935	71935	71935	1890	1890	103...	103843	1709	1709	69399	103855	103855	103855	103855	103855									
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
	8	98275	98275	98275	1609	1609	98271	98271	98271	0	0	83225	83225	83225	18276	18276	18276	0	103574	69233	69233	69233	102685	102685	6668	6668	1709	1709	71948	1863	1863	0	0	19762									
	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
Classroom	11	1013...	101...	1038...	103...	1598	103...	103...	1038...	69414	103843	103843	1888	1888	0	71936	71936	71936	71927	71927	1850	1850	7132	7132	0	15901	15901	15901	0	0	100593	100593	100593										
	12	0	0	100...	100...	0	69441	69441	69441	69251	0	69459	69459	69459	0	0	0	85355	85355	85355	85355	1608	1608	1872	1872	1872	17532	17532	1034...	1034...	102597	83642	83642										
	13	69498	69498	69498	1838	1838	1013...	132...	132...	83224	83224	1026...	102685	101362	101362	1013...	5776	5776	70648	70648	1332...	69396	69396	103894	103894	83225	83637	83637	101361	101361	100535	100535	100535										
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
	15	83219	83219	2062	2062	0	11427	11427	11427	83239	83239	0	71945	71945	71945	694...	694...	69250	69250	10534	10534	83220	83220	83220	83219	83219	1706	1706	69493	69493	0	0	71938										
	16	1652	1652	1652	0	0	1005...	100...	100...	69401	69401	83212	83212	83212	0	0	10495	10495	10495	69400	0	100533	100533	83214	83214	132...	16716	16716	0	0	1587	1587	1587										
	17	0	103...	0	0	0	0	0	0	70636	70636	70636	0	0	0	0	0	19782	19782	19782	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					
	18	1036...	100...	100...	100...	100...	100...	100...	100...	132...	132...	132...	100588	100588	100588	100588	100588	100588	100588	100588	100588	100588	0	70630	70630	70630	70630	70630	1910	1910	0	102598	70635	70635	0	19503	19503	19503	19503	19503			
	19	7953	7953	100...	100...	100...	1035...	103...	103...	1601	1601	0	19777	19777	19777	719...	719...	71957	71957	71957	71943	71943	1035...	103575	103575	0	0	10534	10534	1611	0	69398	69398	100583	100583	100583	100583	100583					
	20																																										

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