ASSIGNMENT

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1. Solve the following recurrence relations.

a) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

$x(1) = x(1-0)+5$

$x(1) = 0+5$

$x(1) = 0$

$x(2) = x(2-1)+5$

$= x+5 = 0+5 = 5$

$x(3) = x(2)+5$

$= 5+5 = 10$

$x(4) = x(3)+5$

$= 10+5 = 15$

Hence $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

$x(1) = 4$

$x(2) = 3x(2-1)$

$= 3x(1)$

$= 3\times4 = 12$

$x(3) = 3x(3-1)$

$= 3\times12$

$= 36$

$x(4) = 3x(3)$

$= 3(36)$

$= 108$

Hence

$x(n) = x(1) \cdot r^{n-1}$

Here $r = 3$

$x(n) = 4 \cdot 3^{(n-1)}$

c) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$ (solve for $n = 2^k$)

$n = 2^k$

then,

$x(2^k) = x(2^{k-1}) + 2^k$

$x(1) = 1$

$x(2) = x(1) + 2$

$= 1 + 2 = 3$

$x(4) = x(2) + 4$

$= 3 + 4 = 7$

$x(8) = x(4) + 8$

$= 7 + 8 = 15$

Hence $x(2^k) = 2^{k+1} - 1$

b) setup a recurrence relation for the algorithm basic operation count and solve it.

The comparison temp <= A[n-1]

The recursive call min1 (A [0 ... n-2])

For n = 1

$T(1) = C$, where C is constant

For n > 1

$T(n) = T(n-1) + D$ where D is constant

$T(n) - T(n-1) = D$

$T(2) = T(1) + D$

hence

$T(n) = O(n)$

② Evaluate the following recurrences completely.

i) $T(n) = T(n/2) + 1$ where $n = 2^k$ for all $k \geq 0$

$T(2^k) = T(2^{k-1}) + 1$

$T(1) = 1 + c$

$T(2) = T(1) + 1$
$= 2 + c$

$T(4) = T(2) + 1$
$= 2 + 1 = 3 + c$

$T(8) = T(4) + 1$
$= 3 + 1 = 4 + c$

Hence
$T(2^k) = c + k$

$T(n) = T(2^k) = c + k = c + \log_2(n)$

$\therefore T(n) = c + \log_2^n$.

ii) $T(n) = T(n/3) + T(2n/3) + cn$, where c is a constant and n is the input size.

Master's theorm

$a = 2$
$b = 3$

$\log_3^2 = 0.67$

therefore $\log_a^b < k$

$T(n) = \Theta(f(n)) = \Theta(cn) = \Theta(n)$

Thus,

$T(n) = \Theta(n)$

③ Consider the following recursion algorithm

Min 1 (A[0 ..... n-1])

if n=1 return A[0]

Else temp = min 1 (A[0 ..... n-2])

if temp <= A[n-1] return temp

Else

Return A[n-1]

a) what does this algorithm compute?

This algorithm computes the minimum value in the array A[0 ..... n-1]. it does so by recursively finding the minimum value in the subarray A[0 ..... n-2] and then comparing this minimum value to A[n-1]. it returns the smaller of two values.

$x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$, for $n = 3^k$

$n = 3^k$

$x(3^k) = x(3^{k-1}) + 1$

$x(1) = 1$

$x(3) = x(1) + 1$

$= 2$

$x(9) = x(3) + 1 = 2 + 1$

$= 3$

$x(27) = x(9) + 1 = 3 + 1 = 4$

Hence =

$x(3^k) = k + 1$

Analyze the order of growth

i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$ use $\Omega(g(n))$ notation.

$2n^2$

To prove :- $2n^2 + 5 > 7n(c)$

$2n^2 + 5 > 7n \cdot c$

$2n > 7c$

$\frac{2n}{7} > c$

$c = \frac{2n_0}{7}$

$c = \frac{2 \cdot 1}{7} = \frac{2}{7}$

for $n \geq 1$:

$2n^2 + 5 \geq \frac{2}{7} \cdot 7n$

$2n^2 + 5 \geq 2n$

For $n \geq 1$

$2n^2 + 5 > \frac{2}{7} \cdot 7n$

$2n^2 + 5 > 2n$

Thus,
$F(n) = 2n^2 + 5$ is indeed $7(n)$.
$F(n) = \Omega(g(n))$ where, $g(n) = 7n$, $c = \frac{2}{7}$ & $n_0 = 1$