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ANALYTICAL ASSIGNMENT-2

① If $t(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. Prove the assertions.

$t_1(n) < c_1 g_1(n)$ for $n \geq n_1$

$t_2(n) < c_2 g_2(n)$ for $n \geq n_2$

Sum of Inequities:-

$$t_1(n) + t_2(n) \leq c_1 g_1(n) + c_2 g_2(n)$$

hence,

$$g_1(n) \leq \max\{g_1(n), g_2(n)\}$$
$$g_2(n) \leq \max\{g_1(n), g_2(n)\}$$

therefore,

$$c_1 g_1(n) \leq c_1 \max\{g_1(n), g_2(n)\} \quad \text{—— ①}$$
$$c_2 g_2(n) \leq c_2 \max\{g_1(n), g_2(n)\} \quad \text{—— ②}$$

from ① & ②

$$c_1 g_1(n) + c_2 g_2(n) \leq (c_1 + c_2) \max\{g_1(n), g_2(n)\}$$

Final Inequity

$$t_1(n) + t_2(n) \leq c_1 g_1(n) + c_2 g_2(n) + c_2 g_2(n) \leq$$
$$(c_1 + c_2) \max\{g_1(n), g_2(n)\}$$

$$\boxed{t_1(n) + t_2(n) \in \Theta(\max\{g_1(n), g_2(n)\})}$$ Hence proved.

2) Find the time complexity of below recurrence equation.

i) $T(n) = \{_1\ 2T(n/2)+1$ if $n>1$

$a=2 \quad b=2$

$\log_a^b = \log_2^2 = 1$

$f(n)=1 \Rightarrow n^k \log_a^b = 1$

$= \theta(n^0)$ and $0<1$

$f(n) = \theta(n^0)$ and $0 < \log_2^2$

$T(n) = \theta(n^k \log_a^b) = \theta(n^1) = \theta(n)$

$\therefore T(n) = \theta(n)$

ii) $T(n) = \{2T(n-1)$ if $n>0$

$T(n) = 2T(n-1)$

$T(n) = 2T(n-1)$

$T(n-1) = 2T(n-2)$

$T(n-2) = 2T(n-3)$

Hence,

$T(n) = 2T(n-1) = 2\cdot 2T(n-2) = 2^2 \cdot 2T(n-3)$

$= 2^3(T(n-3))$.

hence -

$T(n) = 2^k T(n-k)$

Base case, when $k=n$

$T(n) = 2^n T(0)$

Gn, $T(0)=1 \Rightarrow T(n) = 2^n(1) = 2^n$

$\therefore T(n) = O(2^n)$.

5) Big O notation :- show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$

Big O $\Rightarrow f(n) = c \cdot g(n)$

$f(x) = n^2 + 3n + 5$

Let us assume $g(x) = g(x^2)$

$= n^2 + 3n + 5 < g(n^2$

$\Rightarrow 1 + \frac{3}{n} + \frac{5}{n^2} < c$

when $n = 1 \Rightarrow 1^2 + 3 + 5 = g(1)^2$

$\Rightarrow 9$

$\frac{3}{n} \leq 3$ for all $n \geq 1$

$5/n^2 \leq 5$ for all $n \geq 1$

Have to $n \geq 1$

$1 + 3/n + 5/n^2 \leq 1 + 3 + 5 = 9$ $\therefore c = 9$

when $n = 2$

$= 2^2 + 3(2) + 5 = 9(4) = 36$

$15 < 36$

when $n = 3$

$3^2 + 3(3) + 5 = 9(9) = 81$

$23 < 81$

$\therefore f(x) \leq c \cdot g(n)$

$\therefore$ Big (O) is satisfied.

6) Big omega notation : prove that $g(n) = n^3 + 2n^2 + 4$ in $\Omega(n^3)$.

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

$\frac{2}{n}$ & $\frac{4}{n^3}$ becomes smaller.

For all $n \geq 1$: $2/n > 0$

$4/n^2 > 0$

Hence for $n \geq 1$:

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1$$

$\therefore$ $c = 1$ for $n \geq 1$

$\therefore$ $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$ with $c = 1$ and $n_0 = 1$

Hence proved.

7) Big theta notation Determine whether $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not.

$$4n^2 + 3n \leq c_1 \cdot n^2$$

Divide both side by n^2.

$$4 + 3/n \leq c_1$$

$$4 + 3/n \leq 4 + 3 = 7$$

$\therefore$

$$c_1 = \cancel{7} = 7 \; \& \; n_1 = 1$$

$$4n^2 + 3n \geq 7n^2$$

$$\therefore h(n) \geq c_2 \cdot n^2$$

$$4n^2 + 3n \geq c_2 \cdot n^2$$

$$4 + 3/n \geq c_2.$$

$$4 + 3/n \geq 4$$

$$\therefore c_2 = 4 \; \& \; n_2 = 1$$

$$4n^2 + 3n \geq 4n^2.$$

$\therefore$ $h(n) = 4n^2 + 3n$ is both $\theta(n^2)$ & $\Omega(n^2)$.

$$h(n) = 4n^2 + 3n \simeq \theta(n^2).$$

8) Let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n))$

$f(n) \geq c \cdot g(n)$

$n^3 - 2n^2 + n \geq c \cdot (-n^2) \Rightarrow n^3 - 2n^2 + n \geq -cn^2$

$n^3 + (c-2)n^2 + n \geq 0$

Consider $c = 3$

$n^3 + (3-2)n + n = n^3 + n^2 + n$

For all $n \geq 1$

$n^3 + n^2 + n \geq 0$

Thus $c = 3$ & $n_0 = 1$

$f(n) = n^3 - 2n^2 + n \geq 3(-n^2) = -3n^2$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(-n^2)$

$\therefore$ the statement $f(n) = \Omega(g(n))$ is true.

9. Determine whether $h(n) = n\log n + n$ is in $\Theta(n\log n)$

Prove a righieous proof.

$c \cdot g(n) < h(n) < c_2 \cdot g(n)$

where $g(n) = n\log n$.

$c_1 \cdot n\log n \leq n\log n + n \leq c_2 \cdot n\log n$.

where $g(n) = n\log n$.

$c_1 \cdot n\log n \leq n\log n + n \leq c_2 \cdot n\log n$

$n\log n + n \leq c \cdot n\log n$.

$n \log n + n \leq c_2 \cdot n \log n$

$\log n + 1 \leq c_2 \log n$

$\div$ both side by $\log n$

$1 + \frac{1}{\log n} \leq c_2$

as $c_2 = 2$.

$\log n + 1 \leq 2 \log n$

$c_1 \cdot n \log n \leq n \log n + n$

$c_1 \cdot n \log n \leq n \log n + n$

$c_1 \log n \leq \log n + 1$

$(c_1 - 1) \log n \leq 1$

$c_1 - 1 \leq \frac{1}{\log n}$

therefore, $h(n) = n \log n + n$ is $\theta (n \log n)$.

10. Solve the following recurrence relations & tied the order of growth for solutions $T(n) = 4T(n/2) + n^2, T(1) = 1$

$a = 4, b = 2, f(n) = n^2$

$f(n) = n^k \log^p n$

$= n^2 \log^p n$

$\log_b^a = 2$

$f(n) = n^2$

$f(n) = \theta(n^2)$

If $f(n) = \theta(n^k \log^b_a)$

$T(n) = \theta(n^k \log \log n)$

Thus $- T(n) = \theta(n^2 \log n)$.

array [4, −2, 5, 3, 10, −5, 2, 8, −3, 6, 7, −4, 1, 9, −1, 0, −6, −8, 11, −9]. find max & min product that can be obtained by multiplying two integer

Sorted Array:-

[−9, −8, −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

max = 10 × 11
$\boxed{= 110}$

min = −9 × 11
$\boxed{= -99}$

12. Demonstrate Binary search method to search Key = 23 from arr[] = {2, 5, 8, 12, 16, 23, 38, 56, 76, 91}.

1. Iteration 1

left = 0, right = 9, mid = 4

$\frac{0+9}{2} \approx 4$

arr [4] = 16, update left = 5

2. Iteration 2

left = 5, right = 9

$\frac{5+9}{2} \sim 7$ arr[7] = 56

update right = 6

3. Iteration 3:

left = 5 , right = 6

$\frac{5+6}{2} = \approx 5$

$\boxed{arr[5] = 23}$

hence key found.

13. Apply merge sort and order the list of 8 elements. d= (45, 67, -12, 5, 22, 30, 50, 20). Set up a recurrence relation for the no. of key comparisions made.

45 67 -12 5 22 30 50 20

$\frac{0+7}{2} = 3.5$

$\boxed{45 \quad 67 \quad -12 \quad 5}$ $\boxed{22 \quad 30 \quad 50 \quad 20}$

$\boxed{45 \quad 67}$ $\boxed{-12 \quad 5}$ $\boxed{22 \quad 30}$ $\boxed{50 \quad 20}$

45 67 -12 5 22 30 50 20

$\boxed{45 \quad 67}$ $\boxed{-12 \quad 5}$ $\boxed{22 \quad 30}$ $\boxed{20 \quad 50}$

$\boxed{-12 \quad 5 \quad 45 \quad 67}$ $\boxed{20 \quad 22 \quad 30 \quad 50}$

Final :- -12 5 20 22 30 45 50 67

Recurrence Relation:-

$$T(n) = 2T(n/2) + (n-1)$$

. Find the no. of times to perform swapping for selection sort. Also estimate the time complexity for the order of notation Set S (12, 7, 5, -2, 18, 6, 13, 4)

$\Rightarrow$ 12 7 5 -2 18 6 13 4

$\Rightarrow$ -2 7 5 12 18 6 13 4

$\Rightarrow$ -2 4 5 12 18 6 13 7

$\Rightarrow$ -2 4 5 6 18 12 13 7

$\Rightarrow$ -2 4 5 6 7 12 13 18

sorted.

Total no. of swaps :- 4

hence -

$$(n-1)+(n-2)+ \cdots +1 = \frac{(n-1)n}{2}$$

time complexity = $O(n^2)$

15. Find the index for the target value 10 using binary search from following list of elements [2, 4, 6, 8, 10, 12, 14, 16, 18, 20].

left = 0 right = 9

$$\frac{0+9}{2} = 4.5 \sim 4$$

1st iteration -

arr[4] = 10

Hence found.

16. sort the following elem... divide & conquer stategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5] analyse complexity of algorithm.

38 27 43 3 9 82 10 15 88 52 60 5

| 38 27 43 3 9 82 | | 10 15 88 52 60 5 |

| 38 27 43 | | 3 9 82 | | 10 15 88 | | 52 60 5 |

| 38 | | 27 43 | | 3 | | 9 82 | | 10 | | 15 88 | | 52 | | 60 5 |

| 27 38 43 | | 3 9 82 | | 10 15 88 | | 5 52 60 |

3 5 8 9 10 15 27 38 43 52 60 82 88

⇒ 3 5 8 9 10 15 27 38 43 52 60 82 88

Analysising time complexity:-

$$T(n) = 2T(n/2) + O(n)$$

$a = 2 \quad b = 2$

$\log_2^2 = 1$

hence

$\log_b^a > k$

hence $T(n) = \Theta(n \log n)$

Sort the array 64, 34, 25, 12, 22, 11, 90 using Bubble sort. what is time complexity of selection sort in the best, worst & average cases?

Phase 1 -

64	34	25	12	22	11	90
34	64	25	12	22	11	90
34	25	64	12	22	11	90
34	25	12	64	22	11	90
34	25	12	22	64	11	90
34	25	12	22	11	64	90

Phase 2 -

25	34	12	22	11	64	90
25	12	34	22	11	64	90
25	12	22	34	11	64	90
25	12	22	11	34	64	90

Phase 3 -

12	25	22	11	34	64	90
12	25	11	22	34	64	90

Phase 4 -

12	11	25	22	34	64	90
11	12	25	22	34	64	90

Phase 5 - (final)

11	12	22	25	34	64	90

Time complexity of selection sort :-

Best case - $O(n^2)$

worst case = $O(n^2)$

Average case = $O(n^2)$.

(18) Sort the array 64, 25, 12, 22, 11 using selection sort. What is the time complexity of selection sort.

64	25	12	22	11
11	25	12	22	64
11	12	25	22	64
11	12	22	25	64

Time complexity = $O(n^2)$.

19. Sort the following using insertion sor using Brute force Approach stategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5] and analyze complexity of the algorithm.

38	27	43	3	9	82	10	15	88	52	60	5
27	38	43	3	9	82	10	15	88	52	60	5
27	38	3	43	9	82	10	15	88	52	60	5
27	3	38	43	9	82	10	15	88	52	60	5
3	27	38	43	9	82	10	15	88	52	60	5

38 9 43 82 10 15 88 52 60 5

27 9 38 43 82 10 15 88 52 60 5

3 9 27 38 43 82 10 15 88 52 60 5

3 9 27 38 43 10 82 15 88 52 60 5

3 9 27 38 10 43 82 15 88 52 60 5

3 9 27 10 38 43 82 15 88 52 60 5

3 9 10 27 38 43 15 82 88 52 60 5

Final :- 3 5 9 10 15 27 38 43 52 60 82 88

Analysing complexity of insertion sort

Best case $= O(n)$

worst case $= O(n^2)$

Average case $= O(n^2)$

20. Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers, sort the following elements using insertion sort using Brute force Appro.

4 -2 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-2 4 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-2 3 4 5 16 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-2 3 4 5 1,6 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-5 -2 2 3 4 5 10 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

Final :- [-9 ; -8 , -6 , -5 , -4 , -3 , -2 , -1 , 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10 , 11] .