

# Cellular Fundamentals 2: Frequency Planning

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- Frequency Reuse
- Co-channel Interference
- Adjacent Channel Interference

# Review of Previous Lectures

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- **Cellular Concept Architecture**
  - **Frequency Reuse**
- **Functionality of architectural components**
  - **MSC, HLR, VLR, BS and MS, Channels**
- **AMPS spectrum allocation – 30KHz**

# Frequency Planning

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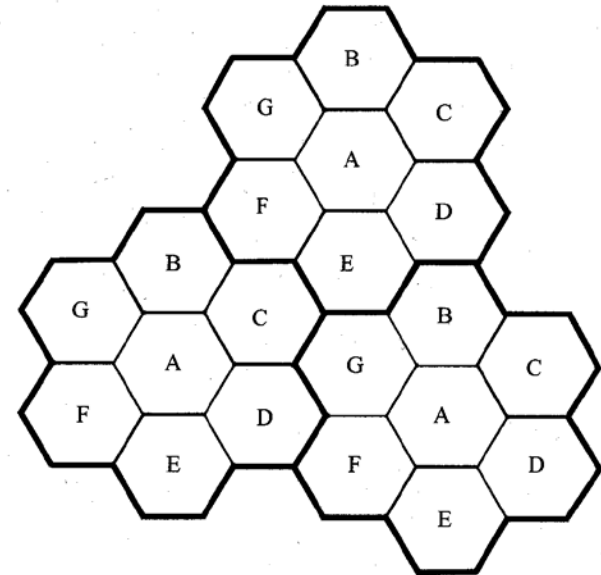
- The allocation of frequency channels depends on several factors:
  - Cellular geometry
  - Signal propagation characteristics
  - Interference
- In the Cellular Concept (AMPS)
  - The channel allocation was fixed
    - ♦ A set of frequency channels was statically allocated to a cell

# Frequency Reuse

- Each cellular base station is allocated a group of radio channels within a small geographic area called a *cell*.
- Neighboring cells are assigned different channel groups.
- By limiting the coverage area to within the boundary of the cell, the channel groups may be reused to cover different cells.
- Keep interference levels within tolerable limits.
- Frequency reuse or frequency planning
  - seven groups of channel from A to G
  - footprint of a cell - actual radio coverage
  - omni-directional antenna v.s. directional antenna

Why hexagonal cells?

It is proved that hexagons are the most efficient cells fill up an area without gaps.



# Frequency Reuse Factor

- Consider a cellular system which has a total of  $S$  duplex channels.
- Each cell is allocated a group of  $k$  channels,  $k < S$
- The  $S$  channels are divided among  $N$  cells.
- The total number of available radio channels

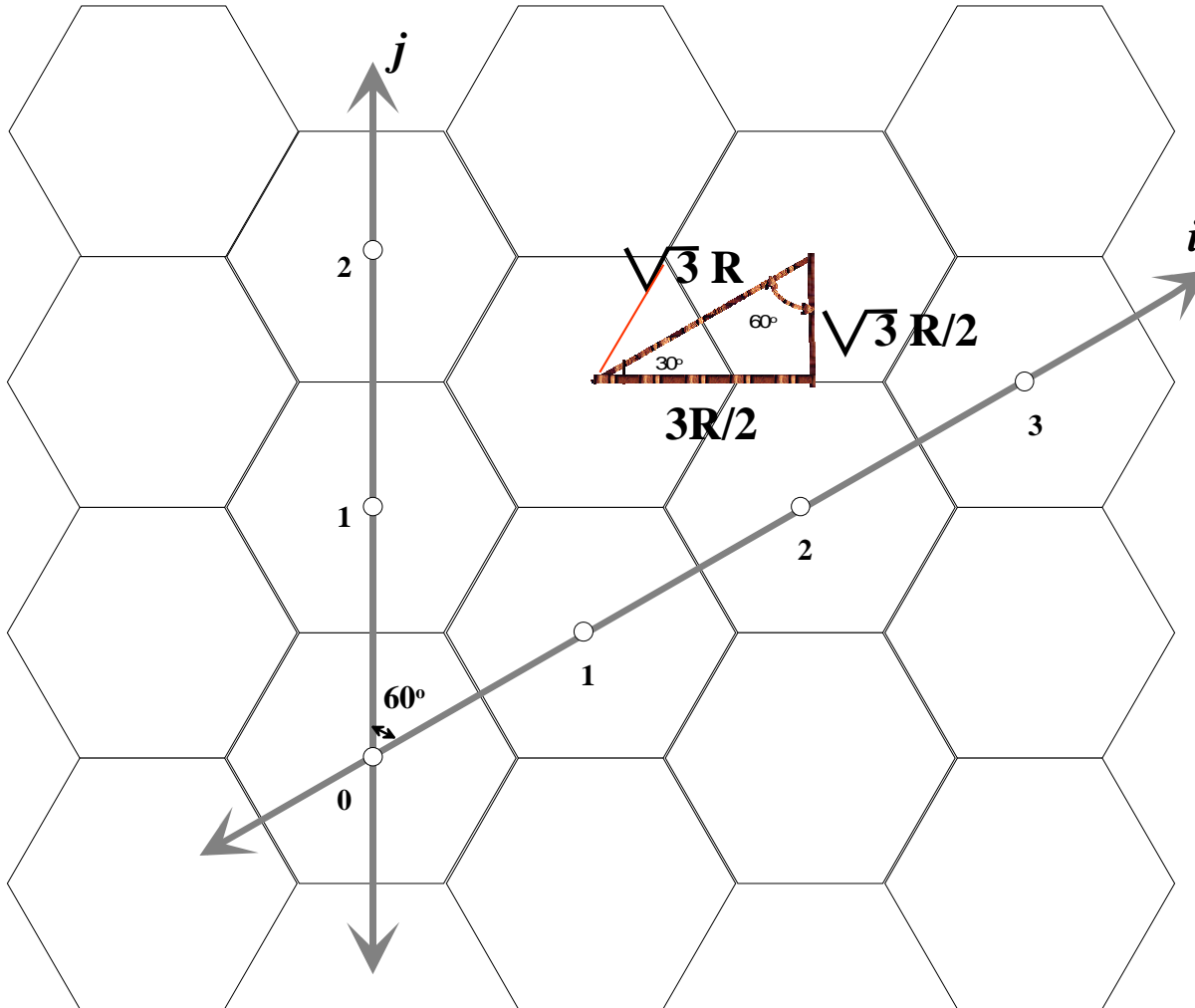
$$S = kN$$

- The  $N$  cells which use the complete set of channels is called *cluster*.
- The cluster can be repeated  $M$  times within the system. The total number of channels,  $C$ , is used as a measure of capacity

$$C = MkN = MS$$

- The capacity is directly proportional to the number of replication  $M$ .
- The cluster size,  $N$ , is typically equal to 3, 4, 7, 9 or 12.
- Small  $N$  is desirable to maximize capacity as  $k = S/N$ .
- The frequency reuse factor is given:  $1/N$

# Hexagonal Geometry



Assuming cell radius:  $R$

The distance between two adjacent cell centres  $d$  is:

$$d = 2 \times R \times \cos 30^\circ$$

$$= 2 \times R \times \frac{\sqrt{3}}{2} = \sqrt{3}R(1.1)$$

Cell area:

$$area = 6 \times \left( \frac{1}{2} R \times \frac{\sqrt{3}}{2} R \right)$$

$$= \frac{3R^2 \sqrt{3}}{2} (1.2)$$

# Hexagonal Geometry

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- The positive halves of the coordinates  $i$  and  $j$  intersect at a  $60^\circ$  angle
- The unit distance along either axis is  $\sqrt{3}$  times the cell radius ( $R$ )
- The cell radius is defined as the distance from the centre of the hexagon to any of its vertices
- The vectors from the centre of any arbitrary cell and the six adjacent cells are separated from each other by  $60^\circ$  angle, the same observation is valid for the vectors to the co-channel cells

# Hexagonal Coordinates

- Distance between two points in the Orthogonal Coordinates

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

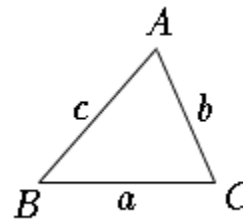
- Distance between two points in the Hexagonal Coordinates

$$d = \sqrt{(i_2 - i_1)^2 + (i_2 - i_1)(j_2 - j_1) + (j_2 - j_1)^2}$$

- Why?

- Trigonometry

- ♦ *The Law of cosines*



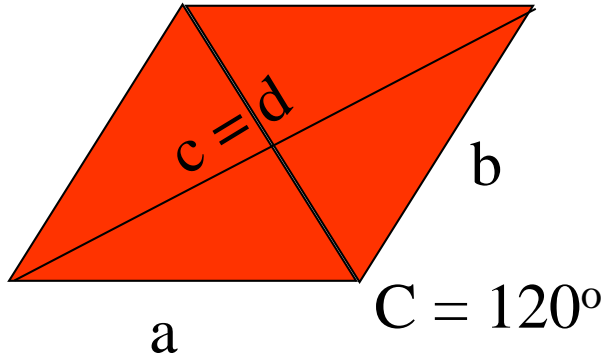
$$a^2 = b^2 + c^2 - 2bccosA$$

$$b^2 = c^2 + a^2 - 2cacosB$$

$$c^2 = a^2 + b^2 - 2abcosC$$



# Hexagonal Coordinates



The distance between the origin to any cell centre, **in hexagonal coordinate system**, is given by:

$$d = (i^2 + ij + j^2)^{1/2} \quad (1.3)$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = a^2 + b^2 - 2ab \cos 120^\circ$$

$$c^2 = a^2 + b^2 - 2ab \left( -\frac{1}{2} \right)$$

$$c^2 = a^2 + b^2 + ab$$

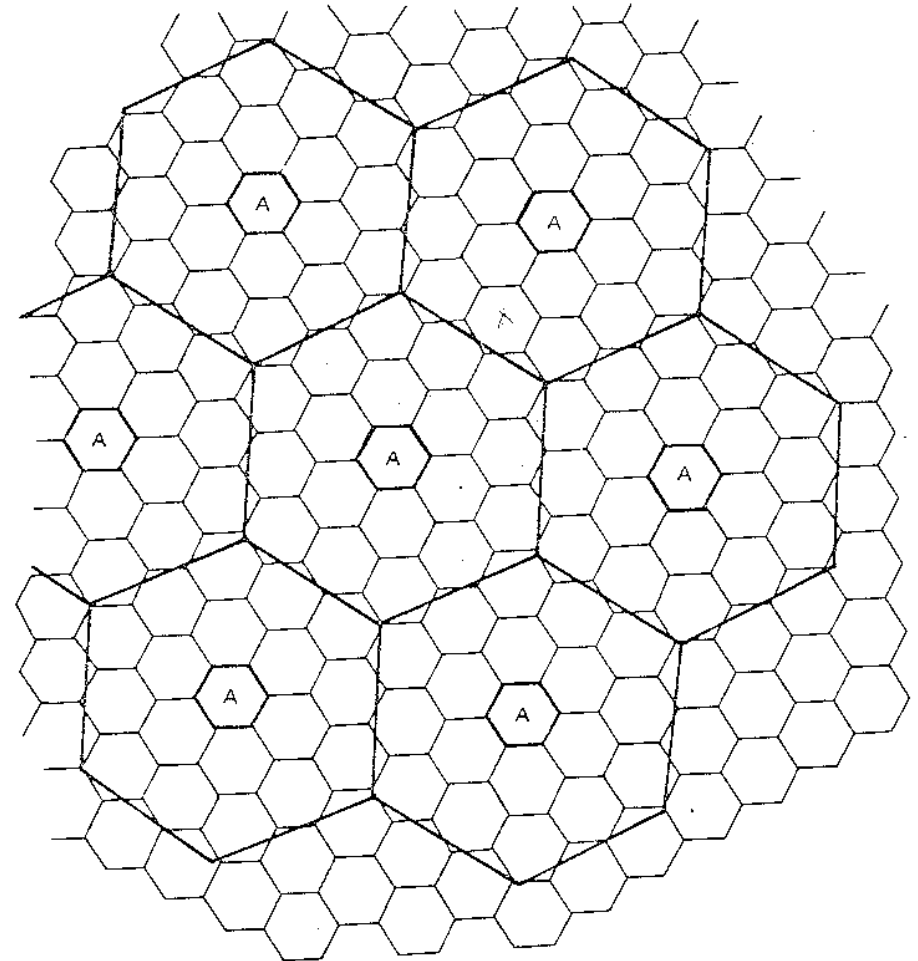
$$c = \sqrt{a^2 + ab + b^2}$$

# Construction of the large hexagons [Mac 79]

- A cluster of contiguous cells can be visualized as a large hexagon (actually a large hexagon can have the same area of a cell cluster) and the distance between centres of large hexagons is

$$(\bar{i}^2 + ij + \bar{j}^2)^{1/2}$$

- D is the frequency reuse distance – between the centre of two co-channel cells



# Construction of the large hexagons

- The pattern of the large hexagons can be seen as an enlargement of the original cell size using an enlargement scale of  $(i^2 + ij + j^2)^{1/2}$
- The number of cell areas contained inside the large hexagon is:

$$N = i^2 + ij + j^2 \quad (1.4)$$

- Therefore, the area of the larger hexagon is N times greater than the smaller hexagon, only if N complies with the values given by 1.7
- Valid number of cells per cluster are: 3, 4, 7, 9, 12, 13, 19, etc.

Proof:

$$small\_ha = \frac{3R_{sh}^2\sqrt{3}}{2}$$

$$R_{bh} = R_{sh}\sqrt{i^2 + ij + j^2}$$

$$\begin{aligned} big\_ha &= \frac{3\left(R_{sh}\sqrt{i^2 + ij + j^2}\right)^2\sqrt{3}}{2} \\ &= small\_ha(i^2 + ij + j^2) \\ &= small\_ha \times N \end{aligned}$$

$$N = i^2 + ij + j^2$$

# Finding the number of cells per cluster

- The relation between the **co-channel reuse ratio:**  $D/R$  and the **number of cells per cluster** can be found by combining the equations 1.1 and 1.3 :

$$\frac{D}{R} = \sqrt{3N} \quad (1.5)$$

Proof: owing to (1.1)

$$\begin{aligned} D &= \sqrt{3}R \times \sqrt{i^2 + ij + j^2} \\ &= R\sqrt{3} \times \sqrt{N} \end{aligned}$$

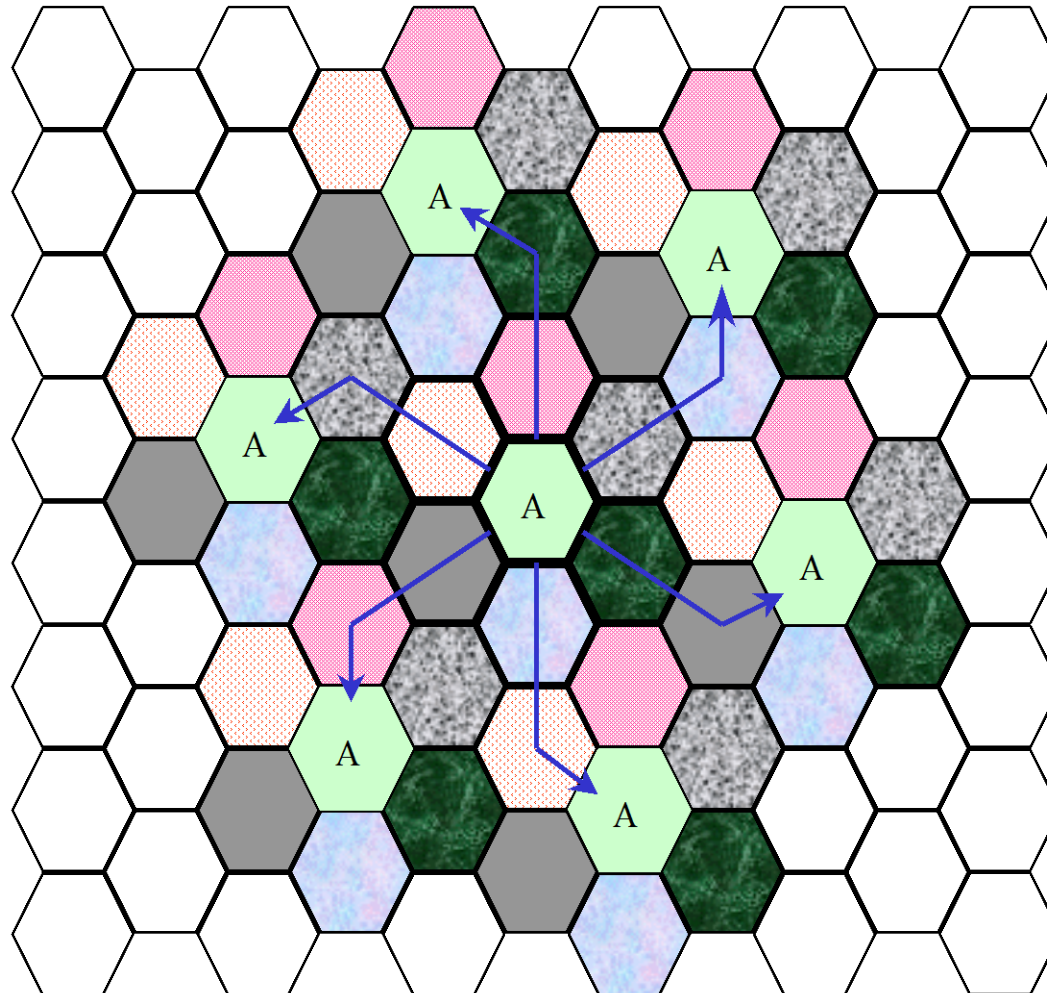
$$\frac{D}{R} = \sqrt{3N}$$

# Finding co-channel cells

- The frequency reuse layout of the cellular system is easily assembled following a scheme that finds the nearest co-channel cells of any cell of the network
- In the scheme,  $i$  and  $j$  are called *shift parameters*, depending on their values different patterns are formed
- The chosen cell and its N-1 surrounding cells form a pattern known as a *compact pattern*
- The steps of the scheme are as follow:
  - Choose any cell as reference;
  - For each side of the hexagon: move  $i$  cells along that side, turn anti-clockwise 60 degrees and then move  $j$  cells on this new direction
  - Repeat the scheme to the surrounding cells of the initial reference cell which are not found co-channel cells
- Next slide shows seven compact patterns in a 7-cell cluster cellular network (where  $i=2$  and  $j=1$ )

# Finding co-channel cells

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# Frequency Planning - Summary

- This same set of channels is used in other distant cells (*co-channel cells*)
  - The distance between co-channel cells, called ***co-channel reuse distance  $D$*** , is such that users can utilise the same channels and have a good quality connection with acceptable levels of interference
  - A group of cells using different sets of frequency channels is called a *cell cluster* (there is no frequency reuse inside a cluster)
  - *The cluster* is repeated inside the cellular network
- \* More details about signal propagation, cellular geometry and channel interference are needed in order to understand how the channels are allocated to cells

# Propagation Path Loss

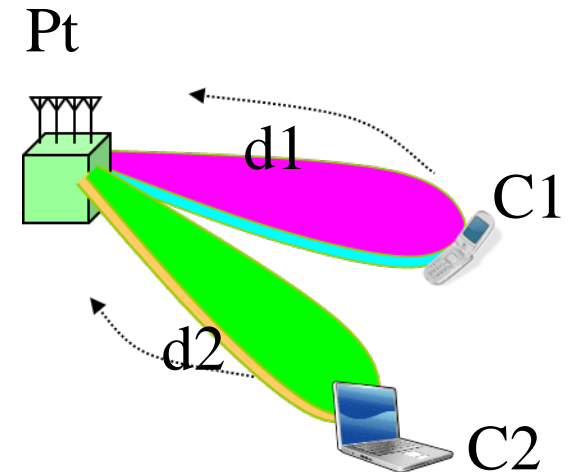
- The propagation path loss is a function of several factors: environment, antenna type, antenna height, antenna position
- Given a transmitted power ( $P_t$ ), the received power ( $P_r$ ) can be roughly calculated as [Lee95]:

$$P_r = P_t d^{-\gamma} \quad (1.6)$$

- The ratio between the received power in two different locations  $d_1$  and  $d_2$  can be roughly estimated as:

$$C_2/C_1 = d_2^{-\gamma}/d_1^{-\gamma} \quad (1.7)$$

- $C_1$  is the signal level in receiver 1
- $C_2$  is the signal level in receiver 2
- $d_1$  is the distance between the transmitter and receiver 1
- $d_2$  is the distance between the transmitter and receiver 2





# Co-channel Interference

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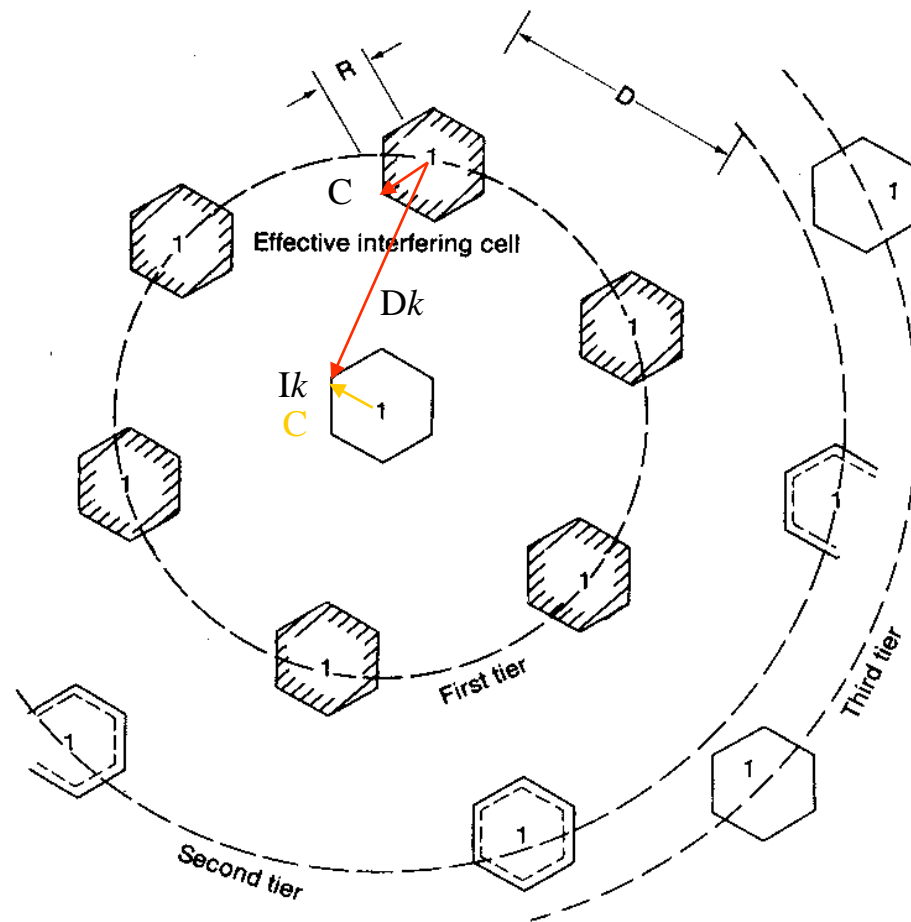
- What is the minimum reuse distance ( $D$ ) a cellular system needs to leave between co-channel cells in order to have acceptable levels of co-channel interference?
  - The co-channel interference occurs as a result of the multiple use of the same frequency carrier
  - The *carrier-to-interference ratio* ( $C/I$ ) is used to measure the amount of interference over a specific carrier
  - In an ideal situation, where all cells have the same radius and coverage area (assuming all BSs transmit with power  $P$ ), the co-channel interference is independent of the transmitted power [Lee 95] therefore:

# Six Effective Interfering cells of cell 1 [Lee 95]

- $K_I$  is the number of co-channel interfering cells in the first tier (the interference caused by the second tier of interfering cells can be neglected [Lee95])

- Total co-channel interfering signal at the cell edge using Eq.(1.7), assuming all co-channel transmitting at the level C.

$$I = \sum_{k=1}^{K_I} I_k \quad I_k = C \frac{D_k^{-\gamma}}{R^{-\gamma}}$$



# Co-channel Interference

- Assuming the local noise can also be neglected in relation to the signal strength, the **C/I** ratio can be expressed by equation 1.8
  - $\gamma$  is the propagation path loss factor
  - $D$  is the distance from BS<sub>*i*</sub>
  - $R$  is the cell radius, defined as the distance from the centre of the cell to its edge
  - Assuming, omni directional antennas and  $D_k = D$  for all  $K_I$ , we get expression 1.9 (in decibel) [Far 96]
  - $D$  is the reuse distance
- $D/R$  is the co-channel reuse ratio

$$\frac{C}{I} = \frac{C}{\sum_{k=1}^{K_I} I_k}$$

$$\frac{C}{I} = \frac{1}{\sum_{k=1}^{K_I} \left( \frac{D_k}{R} \right)^{-\gamma}} \quad (1.8)$$

$$\frac{C}{I} = 10 \log \left[ \frac{1}{K_I} * \left( \frac{D}{R} \right)^{\gamma} \right] \quad (1.9)$$

# Co-channel Interference

- After several field tests and consideration of other propagation loss dependent factors, the designers of AMPS conclude that a minimum C/I and *signal-to-noise ratio* (SNR) of 18 dB would be acceptable as good quality communication
- Solving 1.9 for C/I equals to 18 dB, 6 interfering cells and propagation path loss factor of 4, we have:
  - $D/R = 4.41$
- After practical simulations and taking into consideration the specified 75% of the mobile users saying that the voice quality is good or excellent in 90% of the coverage area, led a value of  $D/R$  of 4.6 as described in the Bell labs publication [Mac79]
- Using 1.5, we can calculate the number of cells needed to obtain a 4.6 D/R using omni-directional antennas:

$$N=7$$

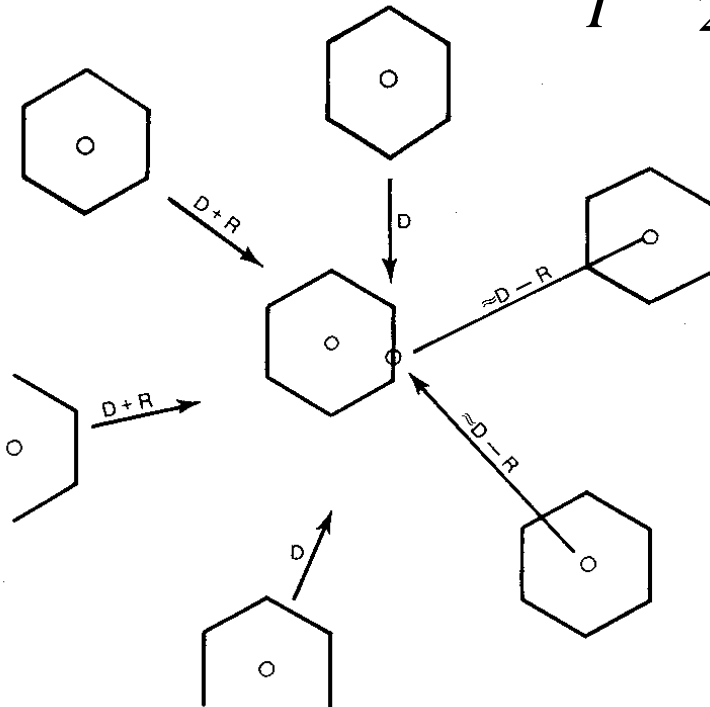
# Mobile Radio Propagation Effects

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- However, in practical systems when omni directional antennas are used a cell cluster of 9 or 12 cells is implemented (Why???)
- Actually, a 7-cell cluster does not provide a sufficient frequency reuse distance separation even when an ideal condition of flat terrain is assumed
- To understand why, we need to study the worst case scenario for the mobile station
- the mobile station is at the boundary  $R$ , where it would receive the weakest signal from its own base station
- The distances from all six co-channel cells are: two distances of  $D - R$ , two distances of  $D$ , and two distances of  $D + R$

# Mobile Radio Propagation Effects

$$\frac{C}{I} = \frac{R^{-4}}{2(D-R)^{-4} + 2D^{-4} + 2(D+R)^{-4}} \quad (1.10)$$



$$\frac{C}{I} = \frac{1}{\frac{2(D-R)^{-4}}{R^{-4}} + \frac{2D^{-4}}{R^{-4}} + \frac{2(D+R)^{-4}}{R^{-4}}}$$

$$\frac{C}{I} = \frac{1}{2\left(\frac{D}{R} - 1\right)^{-4} + 2\left(\frac{D}{R}\right)^{-4} + 2\left(\frac{D}{R} + 1\right)^{-4}}$$

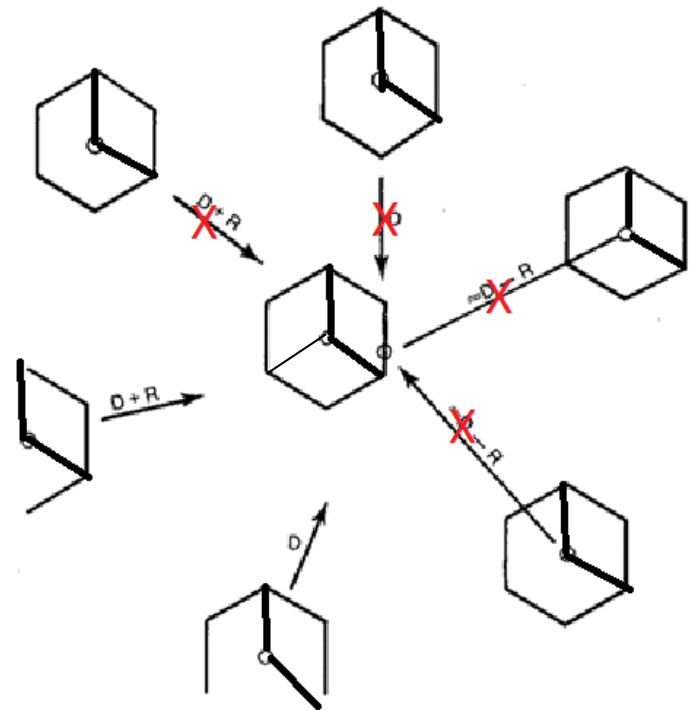
# Mobile Radio Propagation Effects [Lee 95]

- For a D/R of 4.6, the value of C/I is 54 or 17 dB, which is lower than 18 dB
- In real systems as the site locations are imperfect and the terrain is not flat, the C/I received is always worse than 17 dB and could be lower than 14 dB
- Therefore, in an omni directional cell system, a cell cluster of 9 or 12 would be a correct choice, because, even considering the shortest distance of D-R for all six interferers as worst case, the values of C/I would be greater than 18 dB

N	D/R	C/I Eq(1.10)	C/I (dB)
7	4.6	54	17
9	5.2	84.5	19.25
12	6	179.33	22.54

# Cell Sectoring (Capacity Expansion Technique)

- The use of directional antennas can improve the C/I without the need to increase the number of cells in the cluster
- The left diagram illustrates the situation for a cell site with three 120-degree directional antennas
- The front-to-back ratio of a sectored antenna is at least 10 dB
  - therefore the interference can be considered in only one direction
- For the 3-sector cell, the number of interfering cells is reduced to two

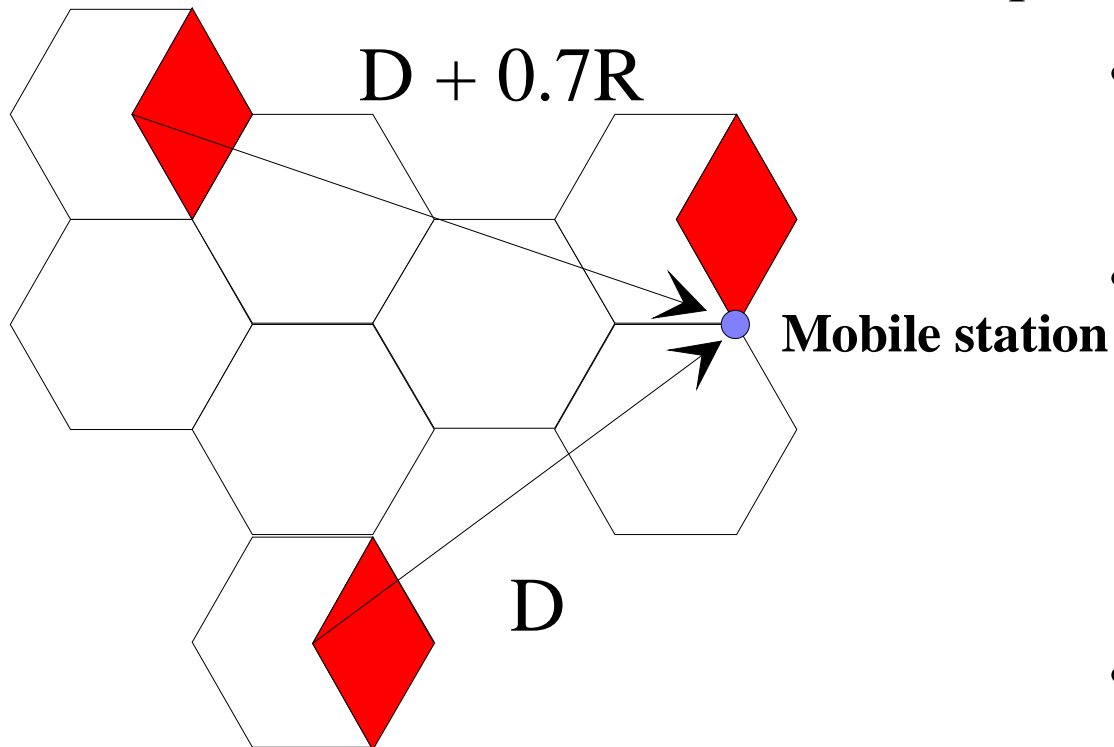




# Cell Sectoring (Capacity Expansion Technique)

Assuming the values of the distance of the interfering cells to the mobile station are  $D+0.7R$  and  $D$ , then:

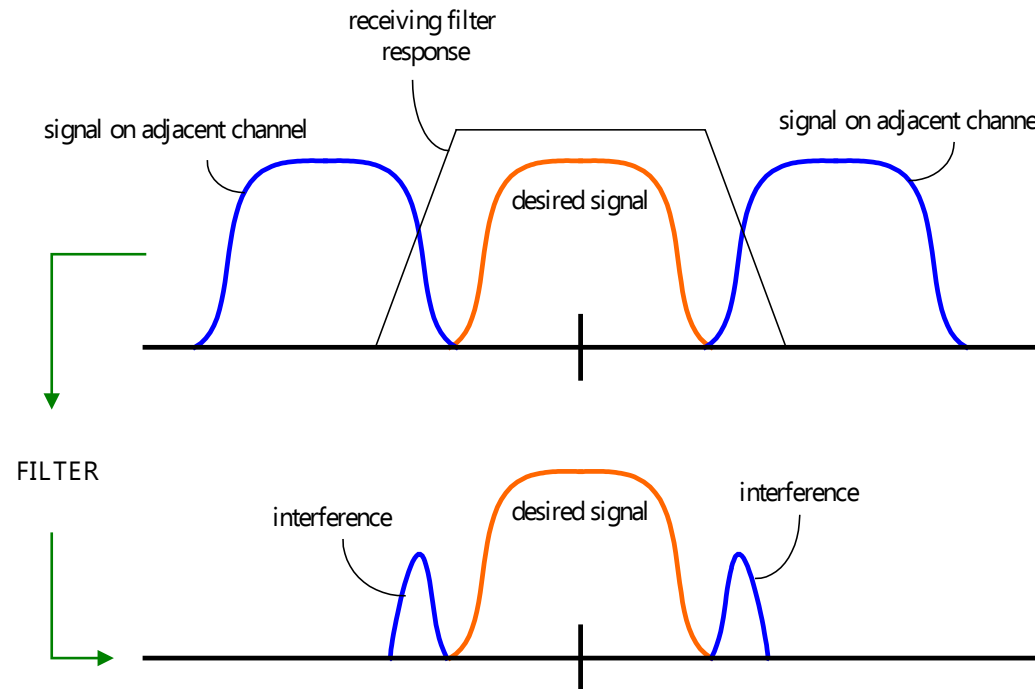
$$\frac{C}{I} = \frac{R^{-4}}{(D+0.7R)^{-4} + D^{-4}} \quad (1.11)$$



- Applying the reuse distance of 4.6 to the equation, results in a C/I of 24.5 dB > 18 dB
- In real systems, the C/I could be 6 dB weaker in a heavy traffic area as a result of irregular terrain contour and imperfect site locations [Lee95]
- but in this case the C/I still would be adequate

# Adjacent Channel Interference

- Adjacent channel interference: interference from adjacent in frequency to the desired signal.
  - Imperfect receiver filters allow nearby frequencies to leak into the passband
  - Performance degrade seriously due to *near-far* effect.



# Adjacent Channel Interference

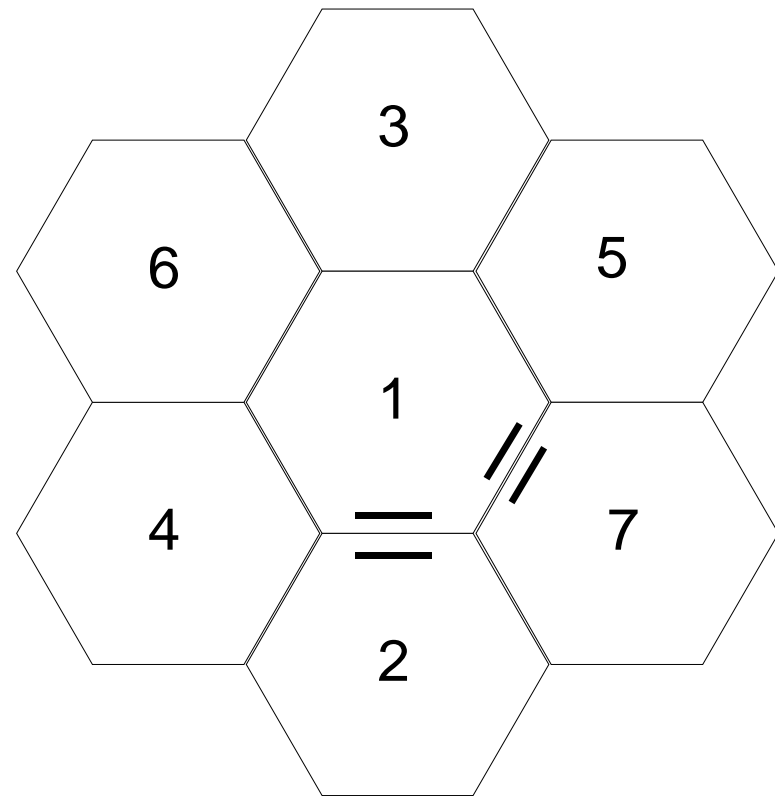
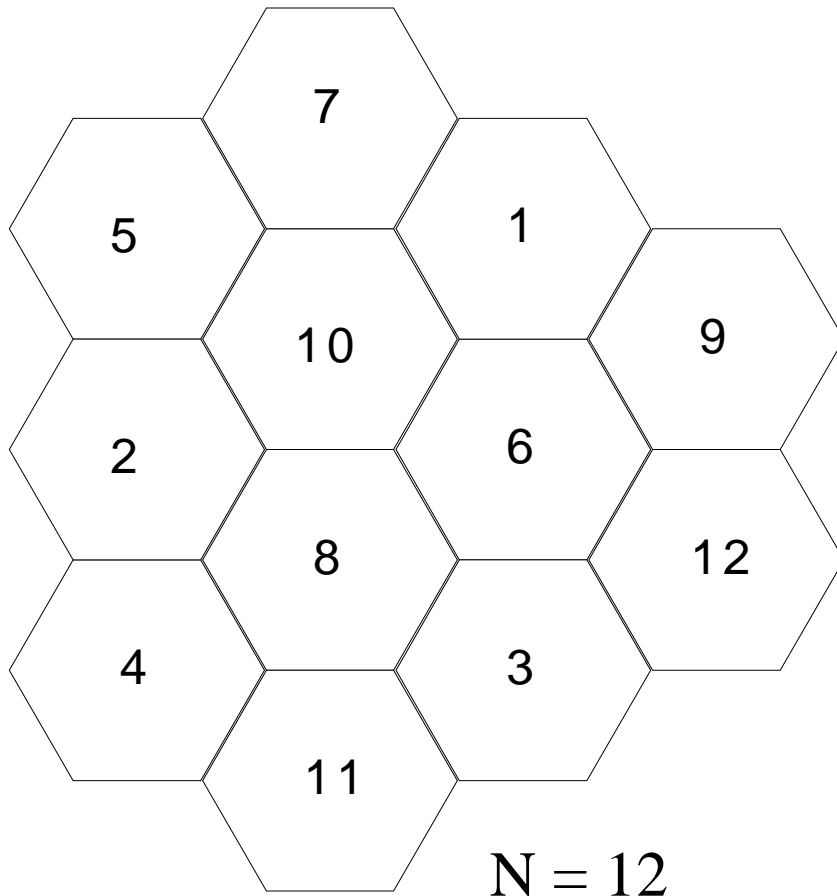
- frequency filters
  - require a substantial spectral guard band to reject adjacent frequencies adequately
  - For example, in AMPS, the spacing of 30 kHz , and peak deviation of 12 kHz in AMPS are not enough for a correct rejection
- The assignment of channels in a cell is kept as far apart as possible. Method:
  - If the number of cells per cluster is  $N$ , then  $N$  disjoint channel sets are deployed, and the  $n$ th set would contain channels  $n, n + N, n + 2N$ , etc.
  - For example, if  $N = 7$ , the first set would contain channels 1, 8, 15, etc.

Give one example of the separation of channels table:

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21 ...

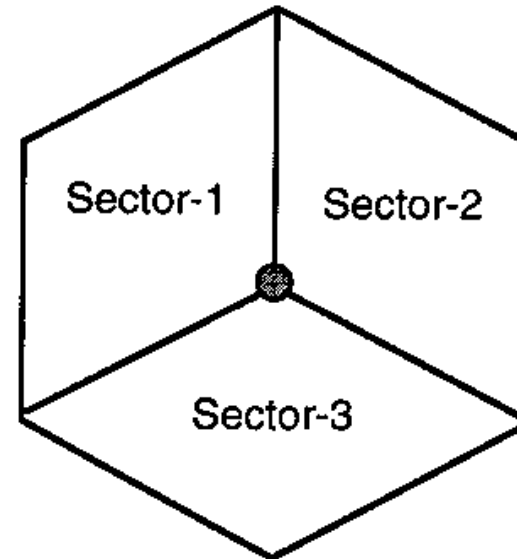
# Adjacent Channel Interference

- Second source of adjacent channel interference  
geographically adjacent cells

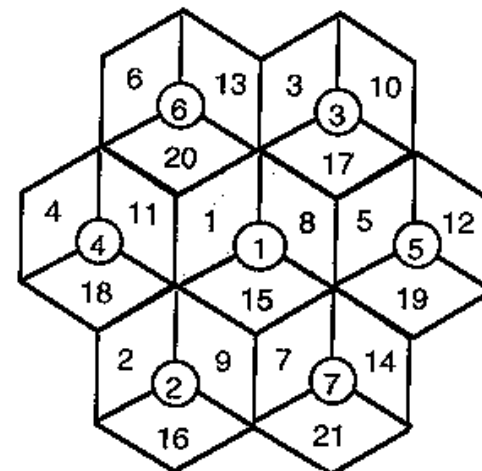


# Adjacent Channel Interference

- It is impossible to avoid adjacent interference in a 7-cell cluster
- The use of directional antennas allows to overcome this problem



(a)



(b)

# Class Quiz

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- What is the co-channel reuse ratio? How is the co-channel reuse ratio related to the number of cells in a cluster?
- What is the co-channel interference? How do we determine how many cells in a cluster? Why can cell sectoring reduce the co-channel interference?
- What is the adjacent channel interference? How can the adjacent channel interference be mitigated?

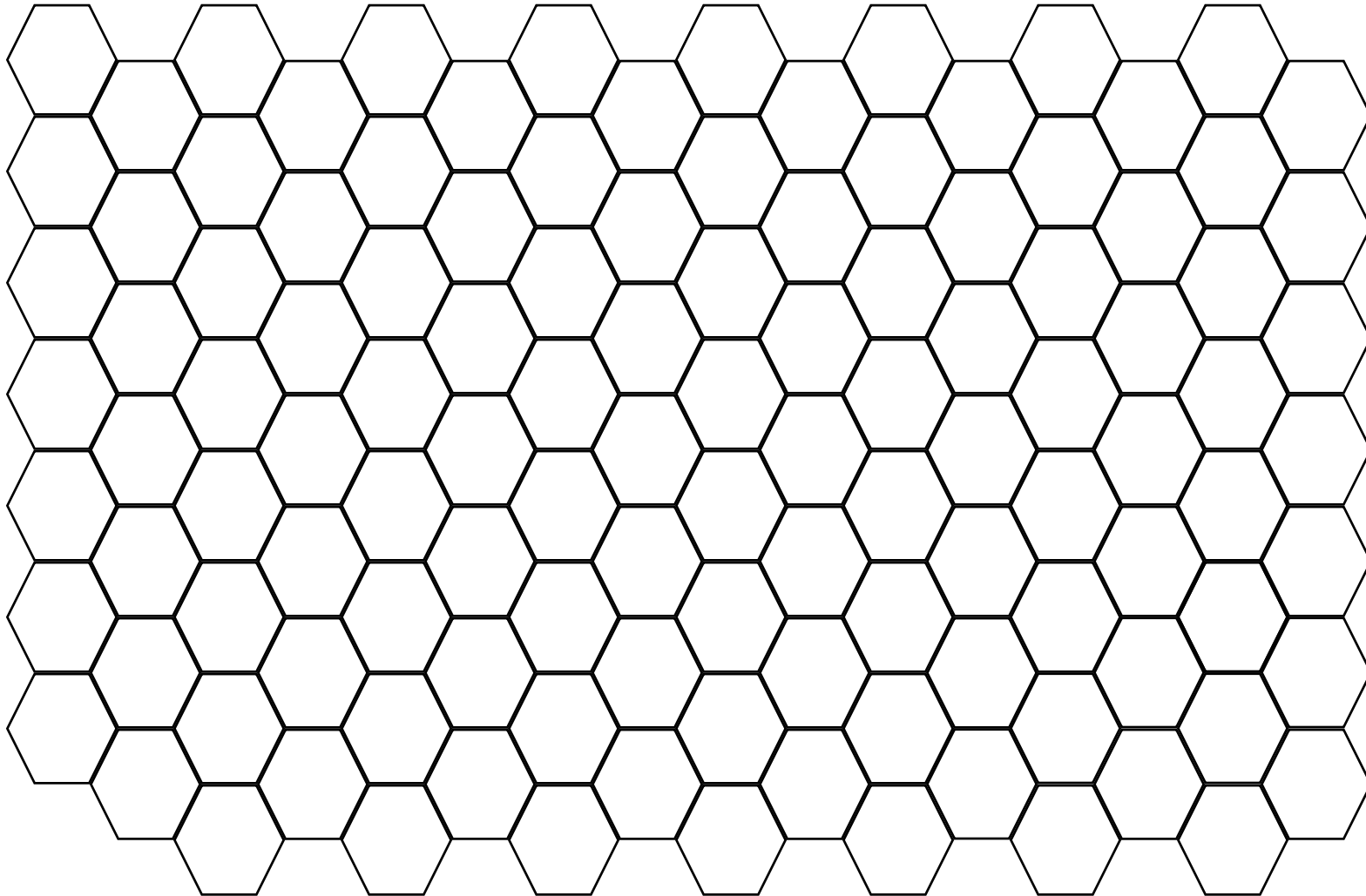
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# Network Grid

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# Network Grid

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