CS390

511883

lag compensator

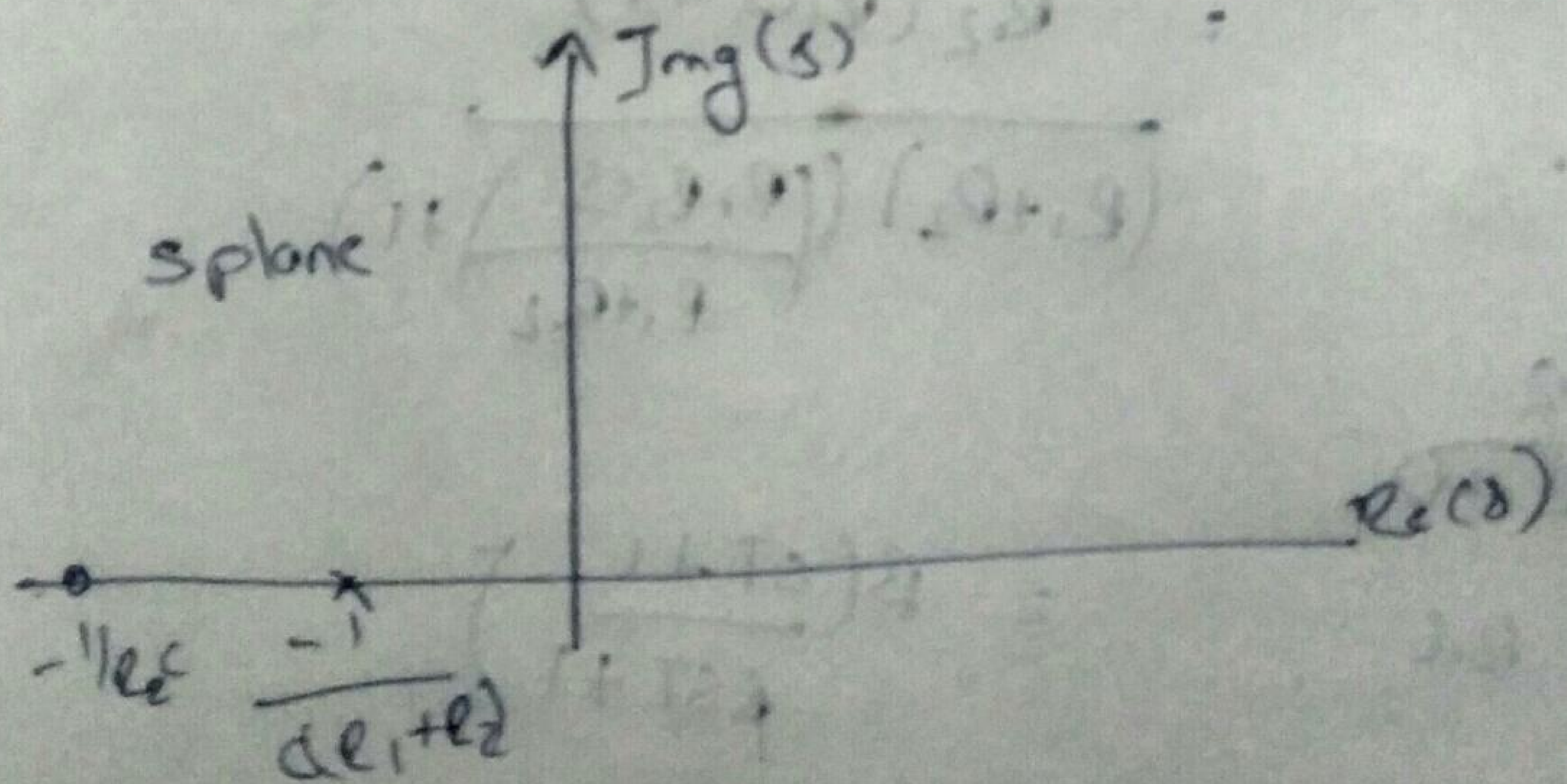
$$V_o(s) = \frac{V_1(s)\left[R_2 + 1/sC\right]}{R_1 + R_2 + 1/sC}$$

$$T(s) = \frac{V_o(s)}{V_1(s)}$$

$$T(s) = \frac{R_2 + 1/sC}{R_1 + R_2 + 1/sC} = \frac{1 + R_2 sC}{C(R_1 + R_2)s + 1}$$

Pole $s \Rightarrow -\frac{1}{C(R_1+R_2)}$

zero $s \Rightarrow \frac{-1}{R_2 C}$



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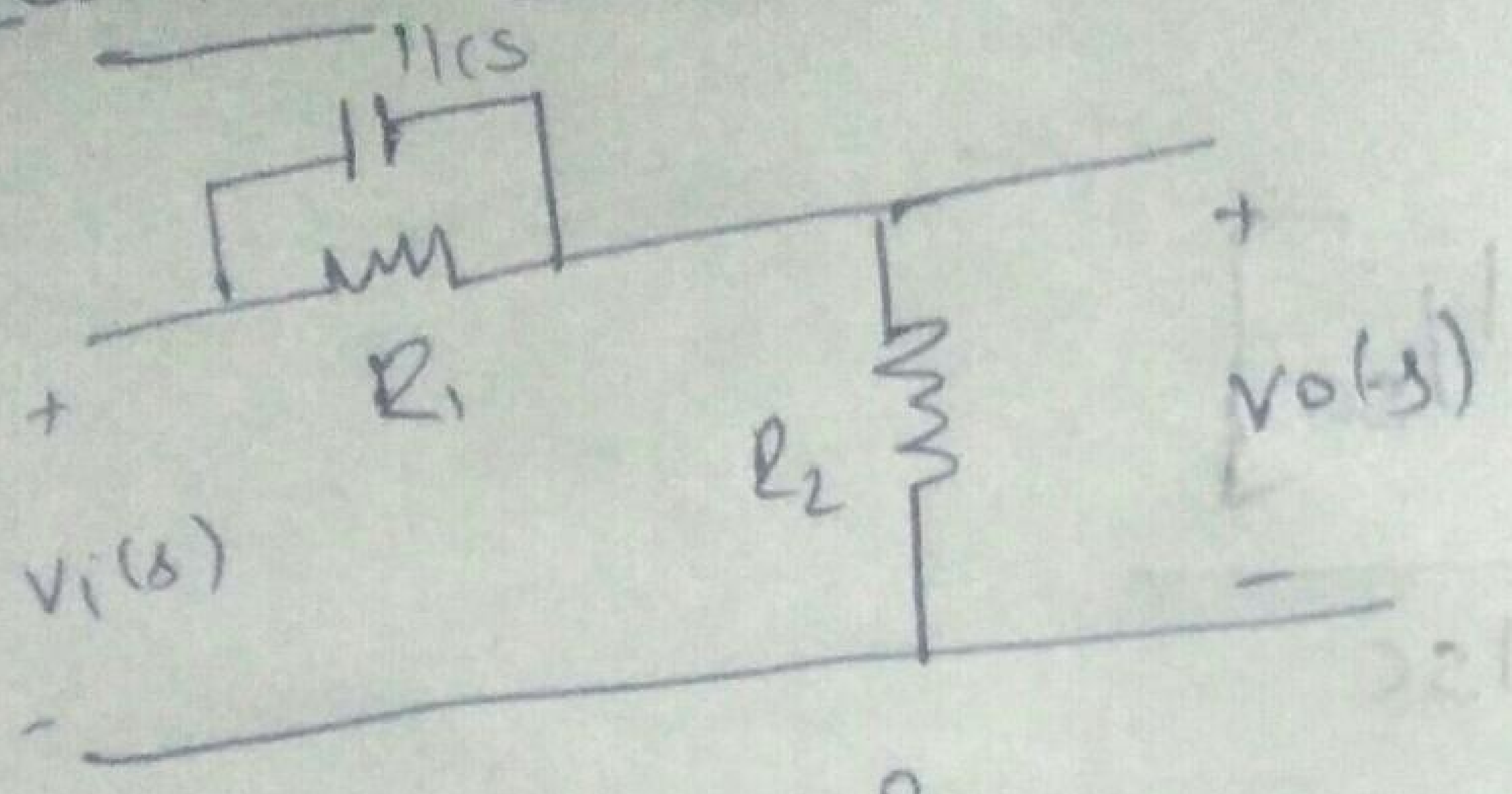
Lead compensator

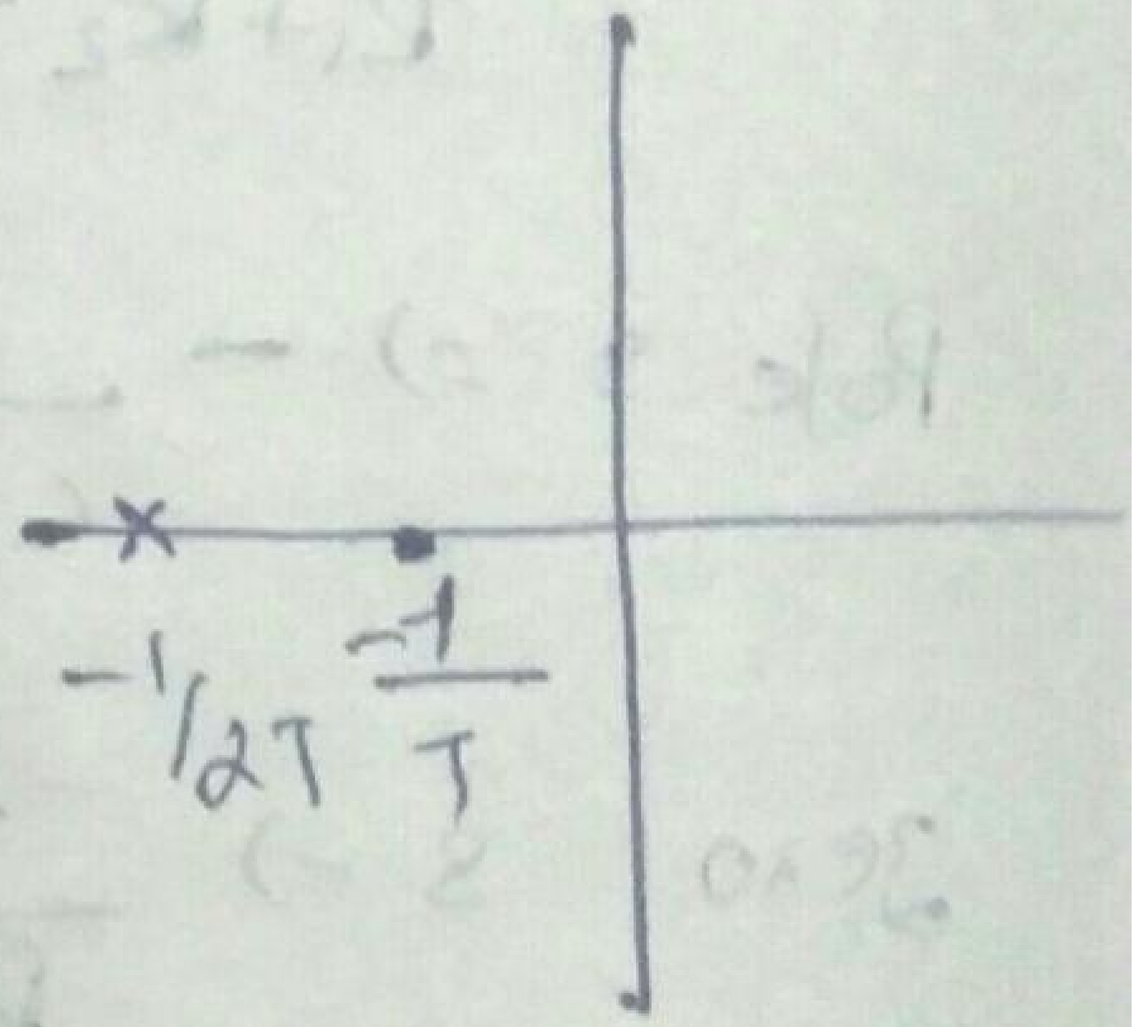


$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \left(\frac{1}{cs} \| R_1\right)}$$

$$= \frac{R_2}{R_2 + \frac{R_1/cs}{R_1 + 1/cs}}$$

$$\alpha = \frac{R_1+R_2}{R_2}$$

$$= \frac{R_2(R_1cs+1)}{R_2(R_1cs+1)+R_1}$$

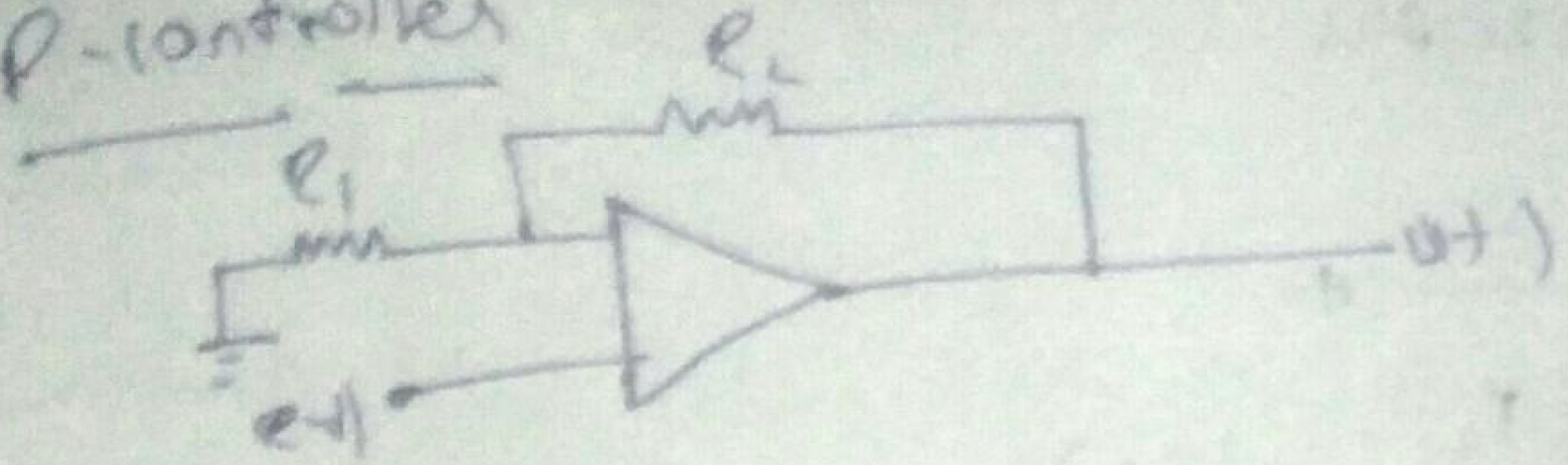


$$= \frac{R_2(R_1cs+1)}{(R_1+R_2)\left(\frac{R_1R_2cs}{R_1+R_2}+1\right)}$$

$$\beta = \frac{R_2}{R_1+R_2}$$

$$T = R_1C \qquad = \beta\left(\frac{sT+1}{\beta sT+1}\right)$$

P-controller



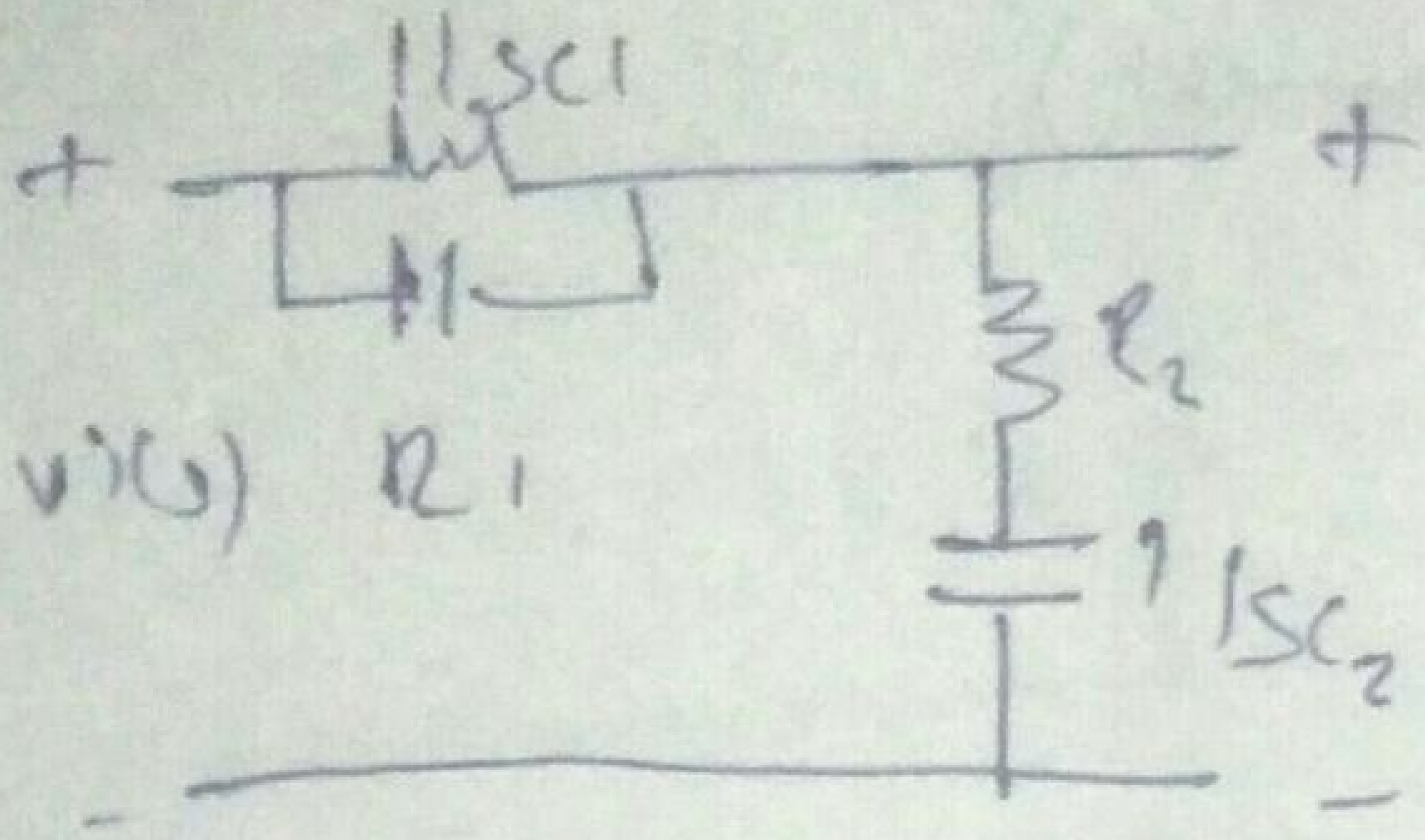
By voltage division rule

$$e(t) = \frac{R_1}{R_1+R_2} \qquad \therefore v(t) = \frac{R_1+R_2}{R_1} e(t)$$

$$V(s) = \frac{R_1+R_2}{R_1} E(s)$$

$$\boxed{\frac{V(s)}{E(s)} = \frac{R_1+R_2}{R_1}}$$

lag lead compensator



$$V_i(s) = \left(\frac{R_1}{R_1 s C_1 + 1} + \frac{R_2 s C_2 + 1}{s C_2} \right) I(s)$$

$$V_o(s) = \frac{(R_2 s C_2 + 1)\, I(s)}{s C_2}$$

$$TF = \frac{V_o(s)}{V_i(s)} = \frac{R_2 s C_2 + 1 / s C_2}{\dfrac{R_2 s C_2 + (R_1 s C_1 + 1)(R_2 s C_2 + 1)}{s C_2 (R_1 s C_1 + 1)}}$$

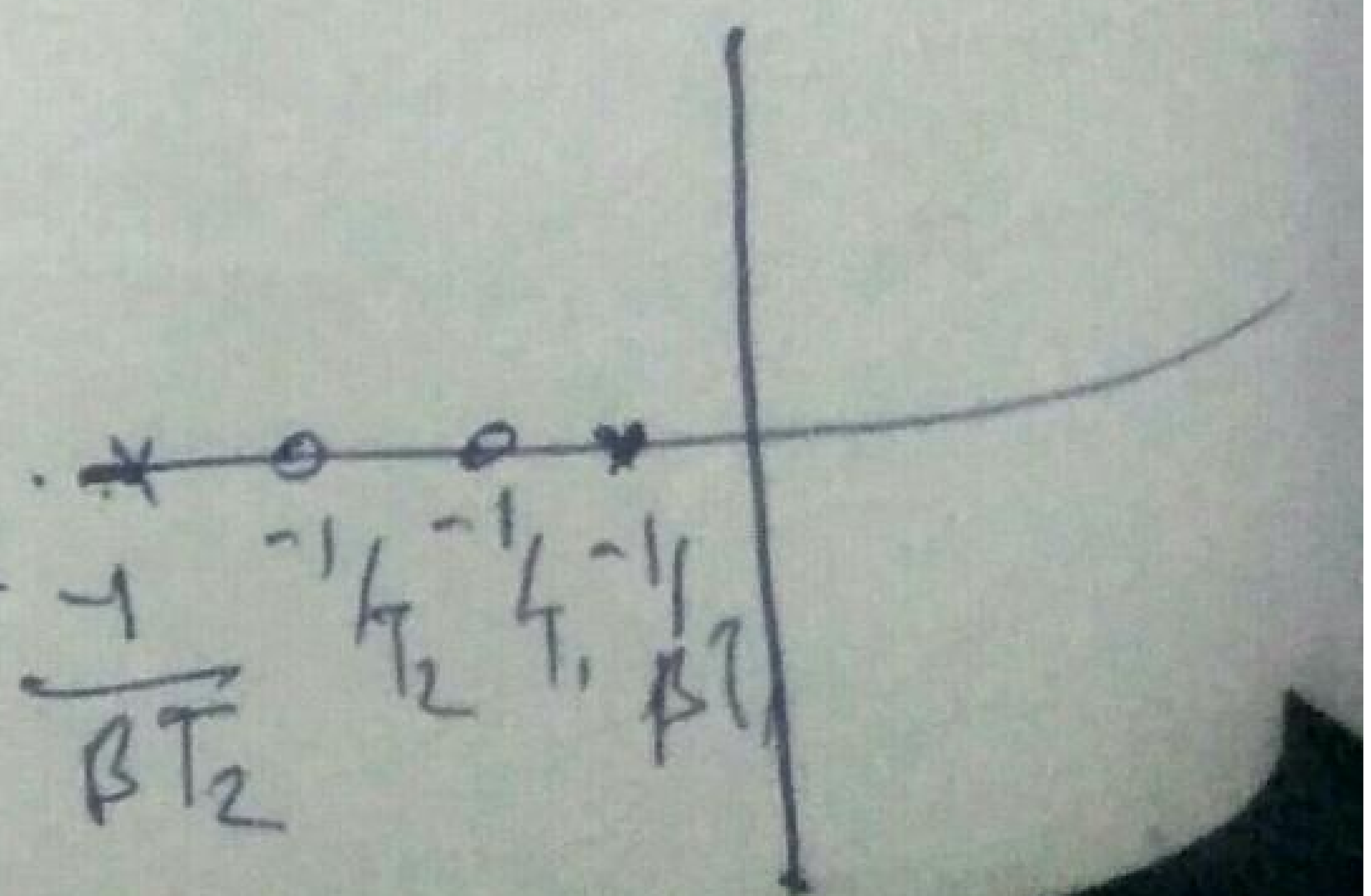
$$TF\ \text{lag-lead} = \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

$\alpha T_1 = R_1 C_1$

$R_2 C_2 = \beta T_2$; $R_1 R_2 C_1 C_2 = \alpha \beta T_1 T_2$

$T_1 T_2 = R_1 R_2 C_1 C_2$

$$TF = \frac{(1 + \alpha T_1 s)(1 + \beta T_2 s)}{(1 + T_1 s)(1 + T_2 s)}$$



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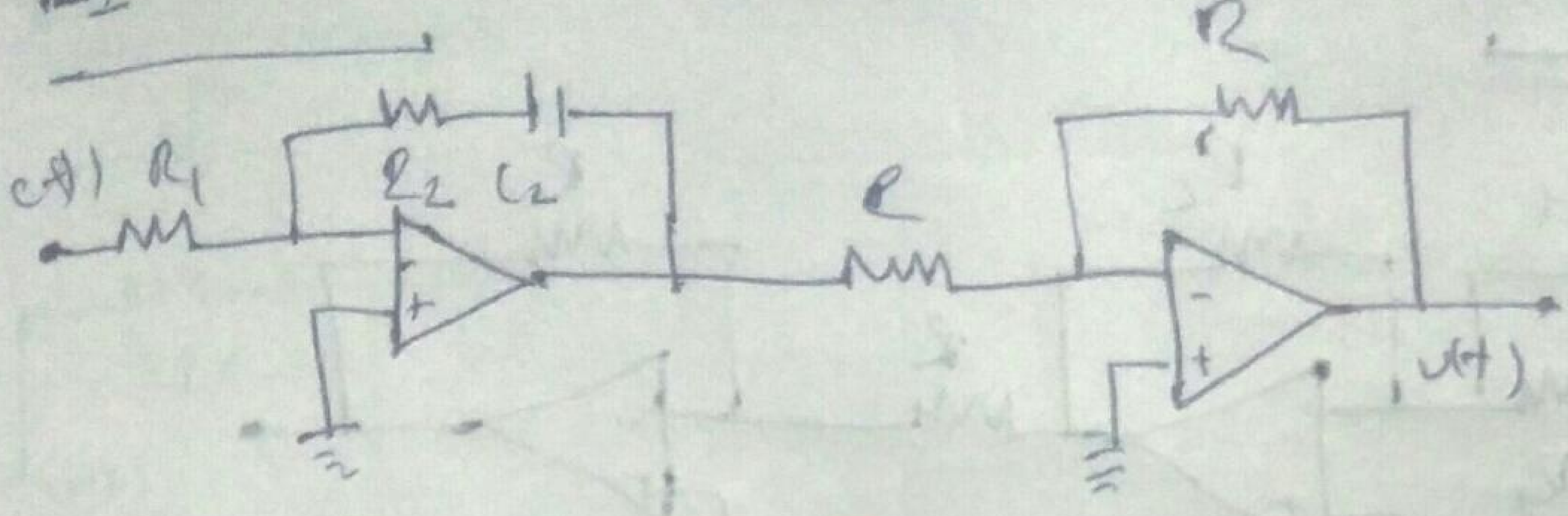
PI controller



$$u(t) \propto \left[e(t) + \int e(t)\,dt \right] \quad \therefore u(t) = k_p e(t) + \frac{k_p}{T_1} \int e(t)\,dt$$

where k_p = Proportional gain

T_1 = Integral time

$$\frac{U(s)}{E(s)} = k_p \left[1 + \frac{1}{T_1 s} \right].$$

$$U(s) = k_p E(s) + \frac{k_p}{T_1} \frac{E(s)}{s}.$$

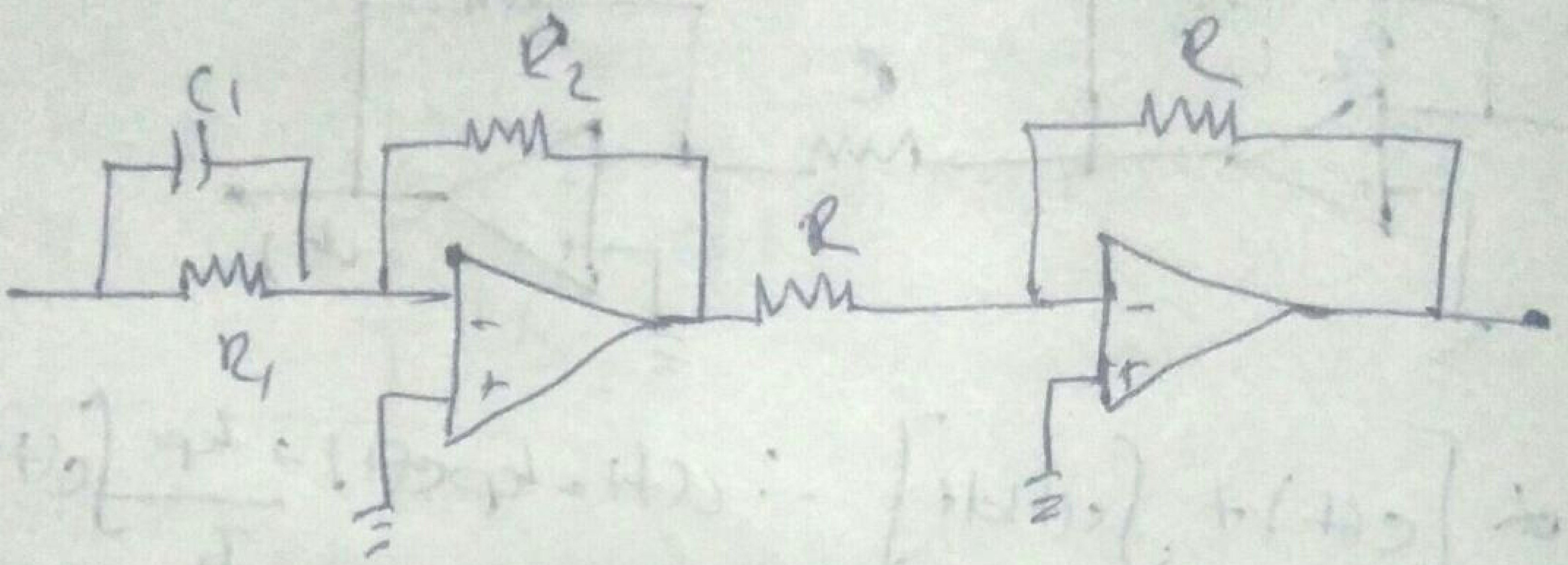
In above example

$$-u(t) = \frac{-e(t)}{R_1} R_2 - \frac{1}{C_2} \int \frac{e(t)}{R_1}\,dt$$

$$\frac{U(s)}{E(s)} = \frac{R_2}{R_1} \left[1 + \frac{1}{R_2 C_2 s} \right].$$

$$\therefore k_p = \frac{R_2}{R_1} \qquad T_1 = R_2 C_2$$

PD controller



$u(t) \propto e(t) + \frac{d}{dt} e(t)$

$u(t) = k_p e(t) + k_p T_d \frac{d}{dt} e(t)$

$U(s) = k_p E(s) + k_p T_d \, s E(s)$

$$\boxed{TF = k_p(1 + T_d s)}$$

Above example

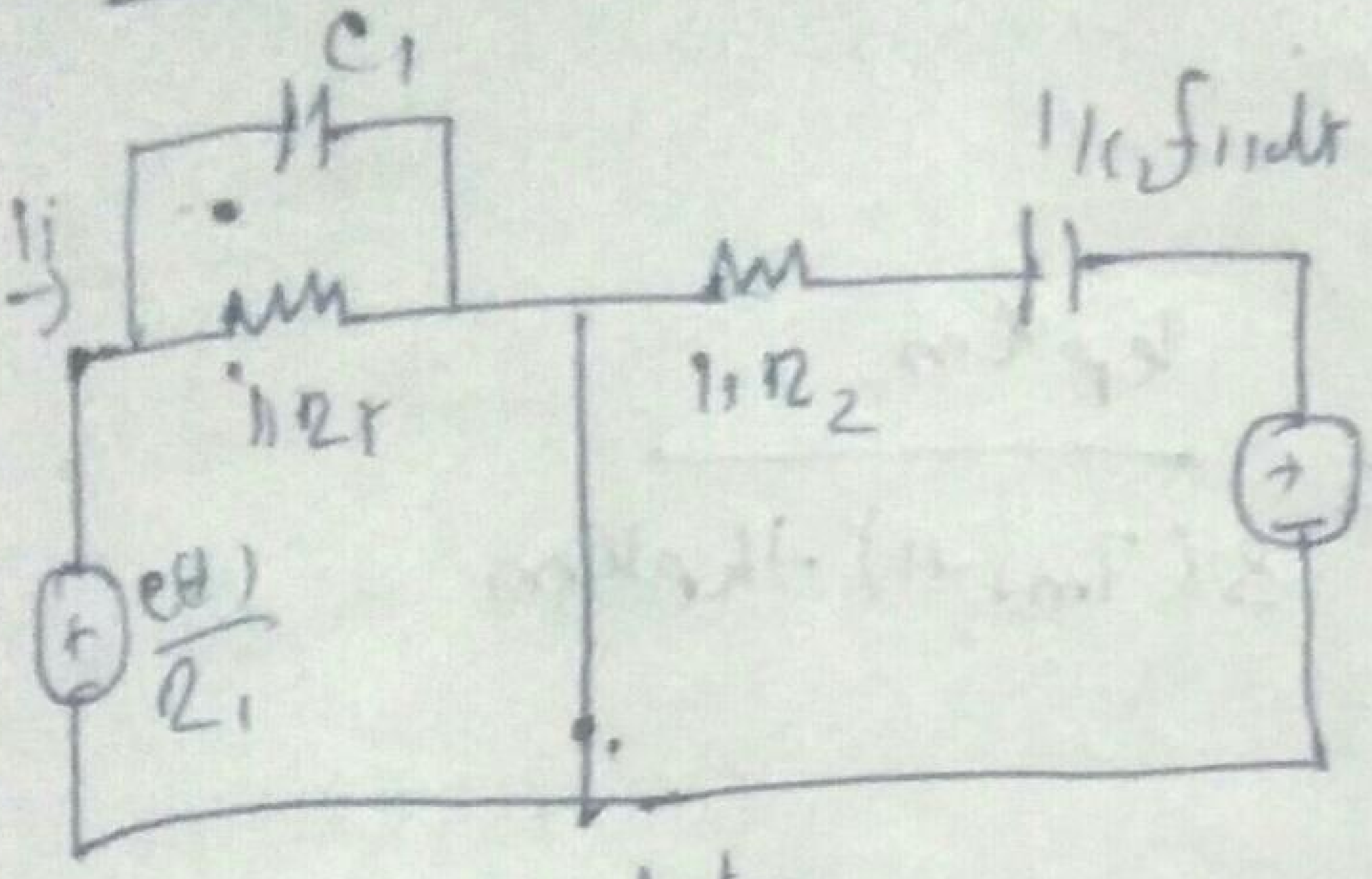
$-u(t) = \frac{R_2}{R_1} e(t) + R_2 C_1 \frac{d}{dt} e(t)$

$u(t) = \frac{R_2}{R_1} e(t) + R_2 C_1 \frac{d}{dt} e(t)$

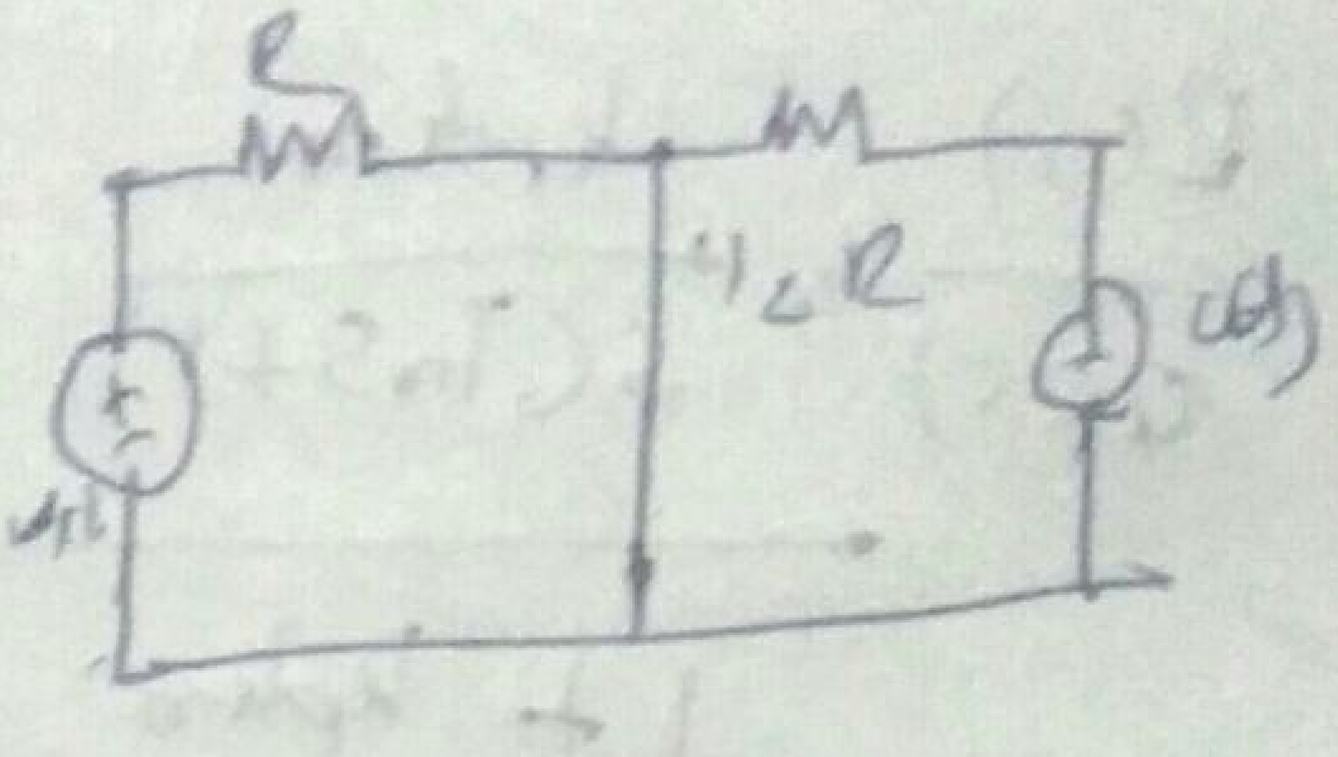
$\frac{U(s)}{E(s)} = \frac{R_2}{R_1}(1 + R_1 C_1 s)$

$k_p = \frac{R_2}{R_1} \quad T_d = R_1 C_1$

PID controller



amplifier



sign changer

$$I_1 = \frac{e(t)}{R_1} + C_1 \frac{de(t)}{dt}$$

$$I_1(s) = \frac{1}{R_1} E(s) + C_1 s E(s)$$

$$I_1(s) = \left(\frac{1}{R_1} + C_1 s\right) E(s)$$

$$I_1(s)\left(R_2 + \frac{1}{C_2 s}\right) = -U_1(s)$$

$$-\left[\frac{R_2}{R_1} + \frac{C_1}{C_2} + \frac{1}{R_1 C_2 s} + R_2 C_1 s\right] E(s) = U(s)$$

$$\frac{C(s)}{E(s)} = \frac{R_2}{R_1}\left[\frac{R_2C_2 + R_1C_1}{R_2C_2} + \frac{1}{R_2C_2 s} + R_1C_1 s\right]$$

$T_1 = R_2 C_2$

$K_p = \frac{R_2}{R_1}$ $\quad T_d = R_1 C_1$

$$\frac{R_1C_1 + R_2C_2}{R_2C_2} = 1$$

Servo Motor

$$\frac{R(s)}{Q(s)} = \frac{\frac{k_p k_m}{s(T_m s+1)}}{1+\frac{k_p k_m}{s(T_m s+1)}} = \frac{k_p k_m}{s(T_m s+1)+k_p k_m}$$

$$1+G(s)H(s)=0$$

$$T_m s^2 + s + k_p k_m = 0$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4T_m k_p k_m}}{2T_m}$$

poles of the Transfer function depends on the values of k_p & k_m //

h) Time domain Specifications

$$\text{c/s eq}^n = s^2 + \frac{s}{T_m} + \frac{k_p k_m}{T_m}$$

$$\omega_n = \sqrt{\frac{k_p k_m}{T_m}}$$

$$2\xi\omega_n = \frac{1}{T_m}$$

$$\xi = \frac{1}{2T_m}\sqrt{\frac{T_m}{k_p k_m}}$$

$$\xi = \frac{1}{2\sqrt{T_m k_p k_m}}$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= \frac{\sqrt{4T_m k_p k_m - 1}}{2T_m}$$

$$t_r = \frac{2T_m\left[\pi - \frac{\pi}{180}\,105^\circ 8\right]}{\sqrt{4T_m k_p k_m - 1}}$$

$$t_p = \frac{2\pi T_m}{\sqrt{4T_m k_p k_m - 1}}$$

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$$M_o = e^{\frac{-\pi \xi_1}{\sqrt{1-\xi_1^2}}}$$

$$= e^{\frac{-\pi \, 1/2 \sqrt{T_m k_p k_m}}{\sqrt{1 - \frac{1}{4 T_m k_m k_p}}}}$$

$$M_o = e^{-\pi / \sqrt{4 k_p k_m T_m - 1}}$$

<u>Settling Time</u> :-

$$2\% \text{ EB } t_{set} = \frac{4}{\xi_1 \omega_n} = 4(1/2 T_m) = 8 T_m$$

$$5\% \text{ EB } t_{set} = \frac{3}{\xi_1 \omega_n} = \frac{3}{1/2 T_m} = 6 T_m$$

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Difference between first order and second order systems

1st order

-) In the system in which as input changes output also changes but not immediately is called first order system.

-) The system takes some delay but without oscillations. [does not exhibit ripples]

2nd order

-) 2 independent energy storage element.

-) may or ~~not~~ may not exhibit oscillatory behaviour

-) Natural frequency and damping ratio play a important role.