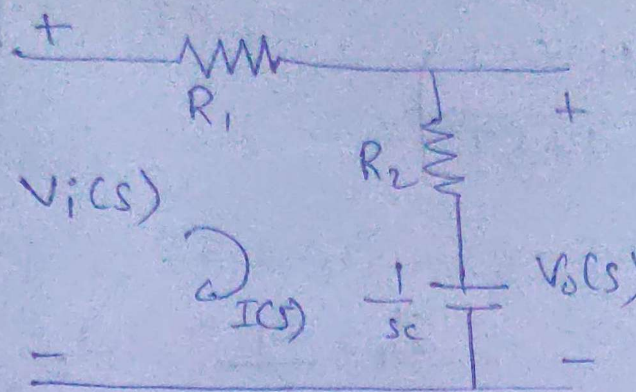
1. Design and Study of lag, lead and lag-lead Compensator networks?

(a) lag Compensator



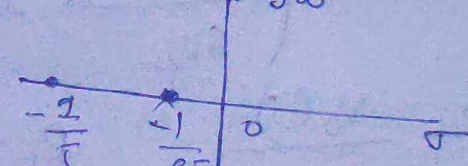
$$V_i(s) = \left(R_1 + R_2 + \frac{1}{sc}\right) I(s),$$

$$V_o(s) = \left(R_2 + \frac{1}{sc}\right) I(s)$$

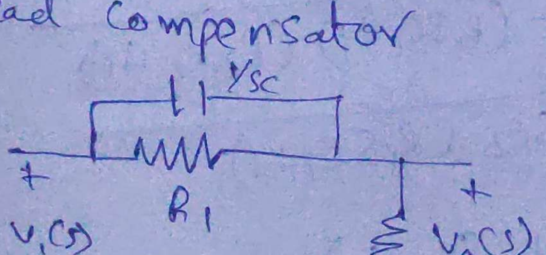
$$TF = \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sc}}{R_1 + R_2 + \frac{1}{sc}} = \frac{R_2 sc + 1}{R_1 sc + R_2 sc + 1}$$

$T = R_2 C$, $\beta = \frac{R_2 + R_1}{R_1}$

$$TF = \frac{1 + Ts}{1 + \beta T s}$$



(b) lead compensator



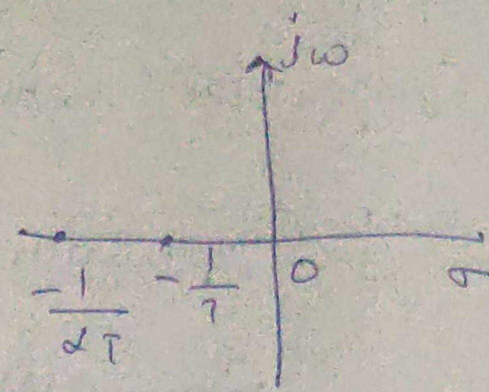
$$V_i(s) = (Z_1 + Z_2) I(s) = \left(\frac{R_1}{R_1 sc + 1} + R_2\right) I(s)$$

$$V_o(s) = R_2 I(s)$$

$$TF = \frac{V_o(s)}{V_i(s)} = \frac{R_2 (R_1 sc + 1)}{R_1 + R_2 + R_1 R_2 sc} = \frac{1 + R_1 R_2 sc}{R_1 + R_2 + R_1 R_2 sc}$$

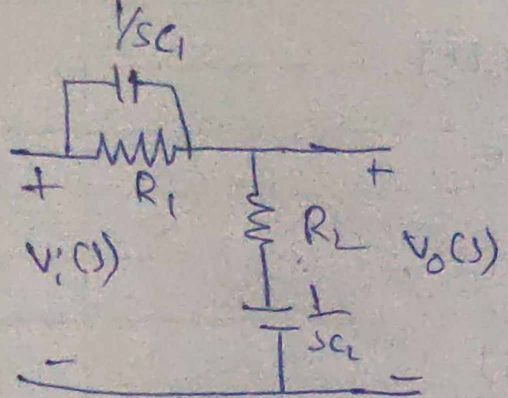
$\alpha = \dfrac{R_1 + R_2}{R_2}$, $T = \dfrac{R_1 R_2}{R_1 + R_2}$

$$TF = \frac{R_2}{R_1+R_2}\left(\frac{1+sCR_1}{1+\dfrac{sCR_1R_2}{R_1+R_2}}\right)$$



$$= \frac{1}{\alpha}\left(\frac{1+\alpha sT}{1+sT}\right)$$

c)



$$Z_1 = \frac{\dfrac{R_1}{sC_1}}{R_1 + \dfrac{1}{sC_1}} = \frac{R_1}{R_1sC_1 + 1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2sC_2 + 1}{sC_2}$$

$$V_i(s) = \left(\frac{R_1}{R_1sC_1+1} + \frac{R_2sC_2+1}{sC_2}\right) I(s)$$

$$V_o(s) = \left(\frac{R_2sC_1+1}{sC_2} I(s)\right)$$

$$TF = \frac{V_o(s)}{V_i(s)} = \frac{R_2sC_2+1 \,/ sC_2}{\dfrac{R_2sC_2 + (R_1sC_1+1)(R_2sC_2+1)}{sC_2(R_1sC_1+1)}}$$

$$TF = \frac{(R_2sC_2+1)(R_1sC_1+1)}{R_2sC_2 + (R_1sC_1+1)(R_2sC_2+1)}$$

$$= \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}}$$

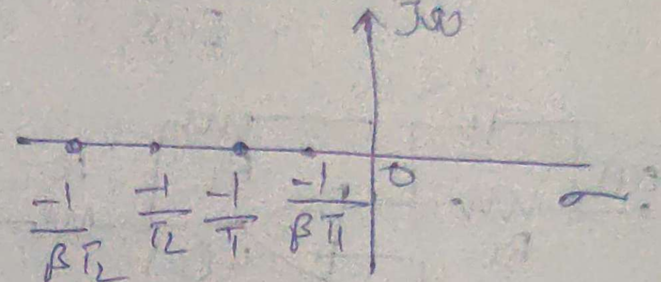
$$TF(\text{lag-lead}) = \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$$

$\alpha T_1 = R_1C_1$

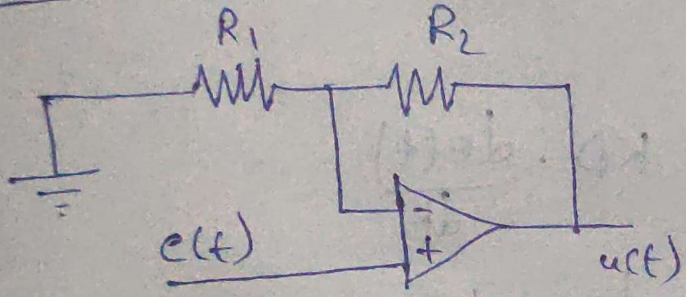
$R_2C_2 = \beta T_2$; $R_1R_2C_1C_2 = \alpha\beta T_1T_2$

$T_1T_2 = R_1R_2C_1C_2$

$$TF = \frac{(1+\alpha T_1 s)(1+\beta T_2 s)}{(1+T_1 s)(1+T_2 s)}$$



d) P- Controller:



$e(t) \rightarrow$ Error signal

$u(t) \propto e(t)$

$u(t) = K_p \cdot e(t)$

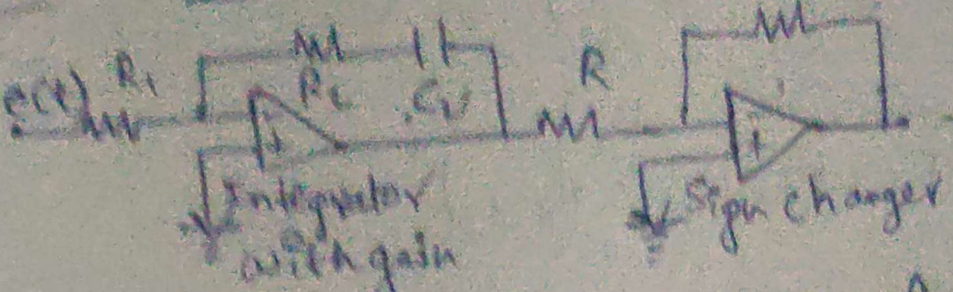
Applying laplace transform

$U(s) = K_p E(s)$

$$\frac{U(s)}{E(s)} = K_p$$

Transfer function - K_p

e) PI Collector:-



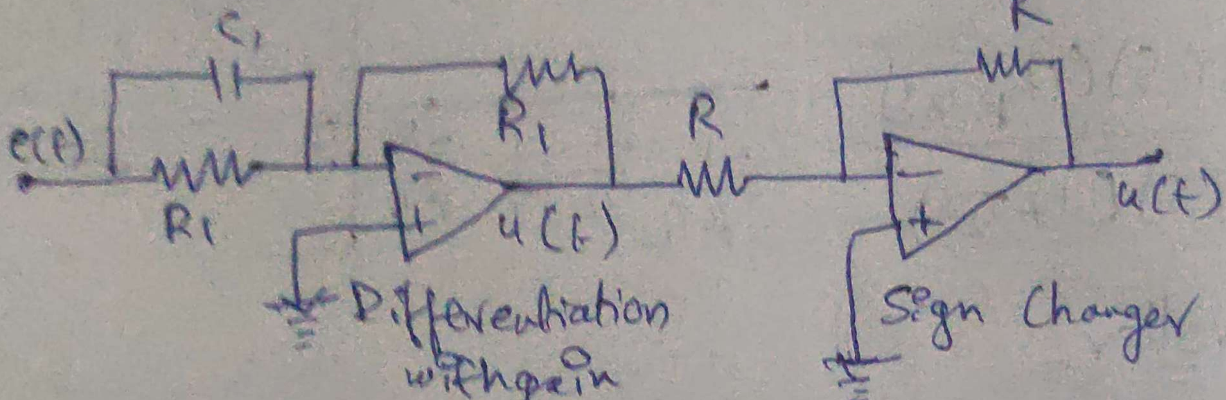
$$u(t) = e(t) \cdot K_p + K_I \int e(t)\,dt$$

Applying laplace transform

$$u(s) = \left(K_p + \frac{K_I}{s}\right) E(s)$$

$$T.F = \frac{U(s)}{E(s)} = K_p + \frac{K_I}{S}$$

f)



PD Controller

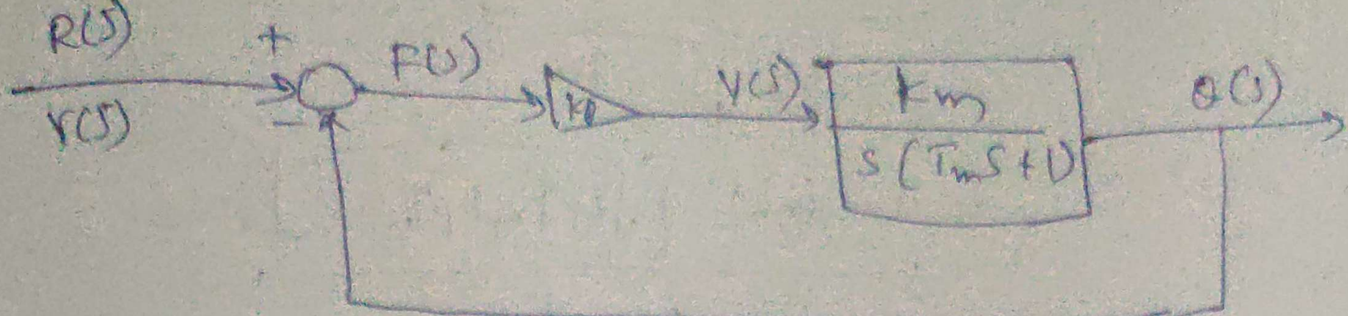
$$u(t) = e(t) K_p + K_D \cdot \frac{de(t)}{dt}$$

Applying laplase transform

$$U(s) = [K_p + K_D \cdot S] \cdot E(s)$$

$$T.F = \frac{U(s)}{E(s)} = K_p + K_D \cdot S$$

g) Servomotor:



$$T.F = \frac{\theta(s)}{R(s)} = \frac{\frac{Kp.\,km}{s(Tms+1)}}{1+\frac{kp.\,km}{s(Tm.s+1)}}$$

h) Time domain Specification of above model :-

$$T.F = \frac{kp.\,km}{s^2 Tm + s + kpkm}$$

$$= \frac{\frac{kp\,km}{Tm}}{s^2 + \frac{1}{Tm}.s + \frac{kpkm}{Tm}}$$

Comparing to $\frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$

$$\omega_n^2 = \frac{kpkm}{Tm}$$

$$\boxed{\omega_n = \sqrt{\frac{kp\,km}{Tm}}}$$

$$2\xi\omega_m = \frac{1}{Tm}$$

$$\xi = \frac{1}{2Tm}\sqrt{\frac{Tm}{kpkm}}$$

$$\boxed{\xi = \frac{1}{2\sqrt{Tm\,kp\,km}}}$$

Rise time $t_r = \dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n\sqrt{1-\xi^2}}$

$$= \frac{\pi - \tan^{-1}\left[\sqrt{4K_m K_p T_m - 1}\right]}{2K_p \cdot K_m}$$

delay Time $t_d = \dfrac{1+0.7\xi}{\omega_n}$

$$= \frac{1 + \dfrac{0.35}{\sqrt{T_m K_p K_m}}}{\sqrt{\dfrac{K_p K_m}{T_m}}}$$

$$= \frac{\sqrt{T_m K_p K_m} + 0.35}{K_p K_m}$$

Peak Time $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$$= \frac{\pi}{2K_p K_m}$$

Peak Over shoot $M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$

$$= e^{\frac{-\pi}{\sqrt{4T_m K_p K_m - 1}}}$$

Settling time

$$ks = \frac{4}{\xi \omega_n} \quad (\text{for } 2\% \text{ error})$$

$$= \frac{4}{\left(\frac{1}{2} T_m\right)} = 8 T_m$$

$$k_s = \frac{3}{\xi \omega_n} \quad (\text{for } 5\% \text{ Error})$$

$$k_s = 6 T_m$$

(2) Write down basic difference between 1st order and 2nd order system

Ans:- The system order in a ~~transform~~ function is the degree of the polynomial.

First order System:-

When input changes, output also changes but not immediately. The system takes some delay but without oscillation.

Second order System:-

When input changes, output changes with some delay and with oscillation