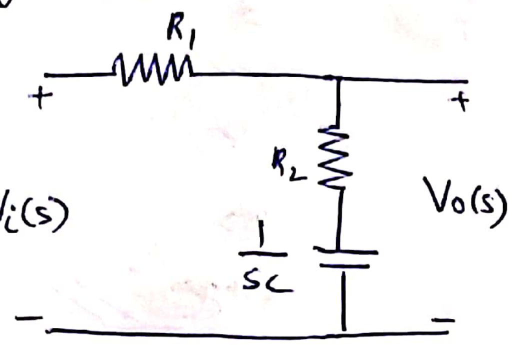
Q1] a) Lag Compensator

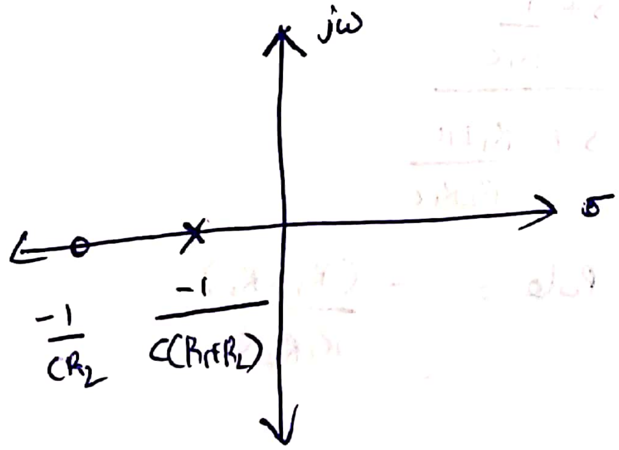


$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{1 + R_2 Cs}{(R_1+R_2)Cs + 1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 \cancel{C}\left(s + \frac{1}{R_2 C}\right)}{\cancel{C}(R_1+R_2)\left[s + \frac{1}{(R_1+R_2)C}\right]} = \frac{s + \frac{1}{R_2 C}}{\alpha\left(s + \frac{1}{(R_1+R_2)C}\right)}$$

$$\alpha = \frac{R_1 + R_2}{R_2} > 1 \text{ for lag compensator}$$

$$\text{zero} = \frac{-1}{R_2 C} \qquad \text{Pole} = \frac{-1}{(R_1+R_2)C}$$



$$T = R_2 C$$

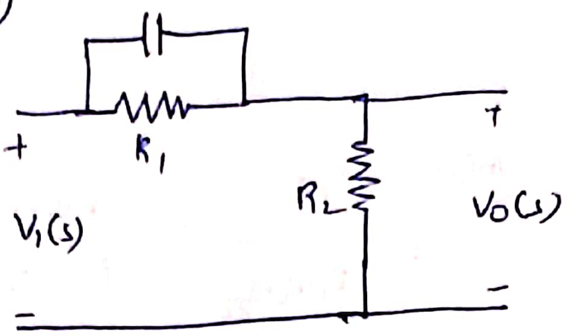
$$\alpha T = (R_1+R_2)C$$

$$\frac{1}{\alpha T} = \frac{1}{(R_1+R_2)C}$$

$$\because \alpha > 1 \Rightarrow \frac{1}{\alpha T} < \frac{1}{T}$$

The pole $\frac{-1}{T\alpha}$ is nearer to origin $\leftarrow$ $\Rightarrow \frac{-1}{\alpha T} > \frac{-1}{T}$

(b) **Lead Compensator**



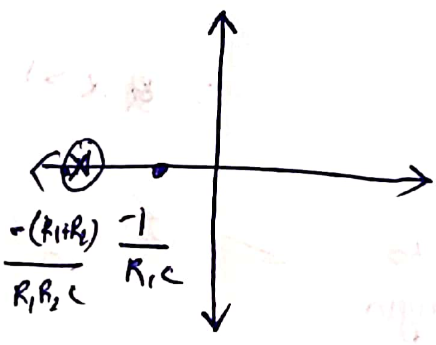
$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \dfrac{R_1}{1+R_1sC}} = \frac{R_2(1+R_1sC)}{R_1+R_2+sR_1R_2C}$$

$$TF = \frac{R_2 + R_2R_1sC}{R_1+R_2+sR_1R_2C}$$

$$= \frac{R_2R_1C\left(s+\dfrac{1}{R_1C}\right)}{R_1R_2C\left(s+\dfrac{R_1+R_2}{R_2R_1C}\right)}$$

$$= \frac{s+\dfrac{1}{R_1C}}{s+\dfrac{R_1+R_2}{R_2R_1C}}$$

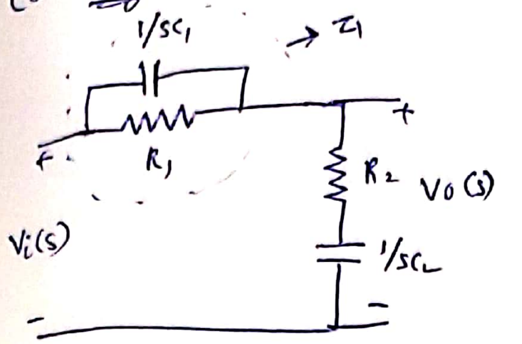
Zero $= \dfrac{-1}{R_1C}$ Pole $= -\dfrac{(R_1+R_2)}{R_1R_2C}$



Here the pole is far away from the origin

$\because \dfrac{+(R_1+R_2)}{R_1R_2C} > \dfrac{1}{R_1C}$

(c) Lag Lead compensator



$$Z_1 = \frac{R_1\left(\frac{1}{sC_1}\right)}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1+sR_1C_1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2} + \frac{R_1}{1+sR_1C_1}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(\frac{1+sR_2C_2}{\cancel{sC_2}}\right)}{\left[\frac{R_1 sC_2 + R_2(sC_2)(1+sR_1C_1) + 1 + sR_1C_1}{\cancel{(sC_2)}(1+sR_1C_1)}\right]}$$

$$TF = \frac{(1+sR_1C_1)(1+sR_2C_2)}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

$$= \frac{1 + s^2R_1R_2C_1C_2 + sR_2C_2 + sR_1C_1}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

<u>Zeroes</u> :-

$$s^2(R_1R_2C_1C_2) + s(R_2C_2 + R_1C_1) + 1 = 0$$

$$s = \frac{-(R_1R_2C_1C_2) \pm \sqrt{(R_1C_1 + R_2C_2)^2 - 4R_1R_2C_1C_2}}{2R_1R_2C_1C_2}$$

<u>Poles</u> :-

$$s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1 = 0$$

$$s = \frac{-(R_1C_1 + R_1C_2 + R_2C_2) \pm \sqrt{(R_1C_1 + R_1C_2 + R_2C_2)^2 - 4R_1R_2C_1C_2}}{2R_1R_2C_1C_2}$$

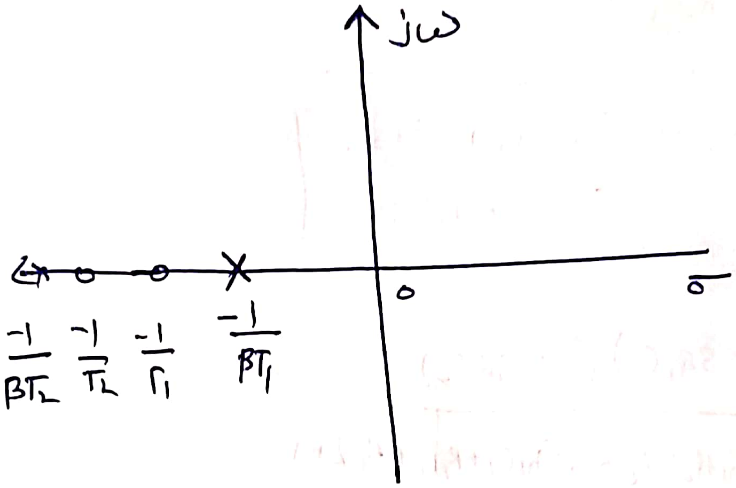
$$TF = \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1} = \frac{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right) + \frac{1}{R_1R_2C_1C_2}}$$

Let $\alpha T_1 = R_1 C_1$

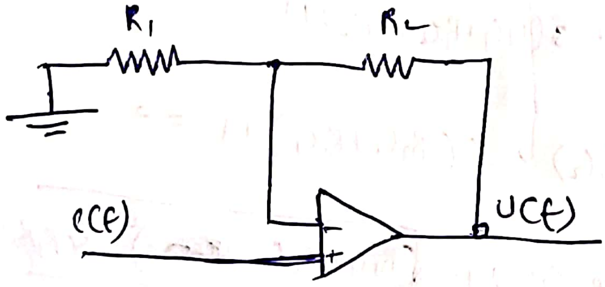
$\beta T_2 = R_2 C_2$

$\Rightarrow \quad R_1 R_2 C_1 C_2 = T_1 T_2 \times \alpha\beta$

$$\Rightarrow TF = \frac{(1+\alpha T_1 s)(1+\beta T_2 s)}{(1+T_1 s)(1+T_2 s)}$$



(d) <u>P- Controller</u>



The voltage in inverting terminal is same as the one in non-inverting due to virtual short.

$$\frac{c(t)-0}{R_1} + \frac{c(t)-u(t)}{R_2} = 0$$

$$c(t)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{u(t)}{R_2}$$

$$c(t) = u(t) \times \frac{R_2}{R_1+R_2}$$

By using Laplace Transform

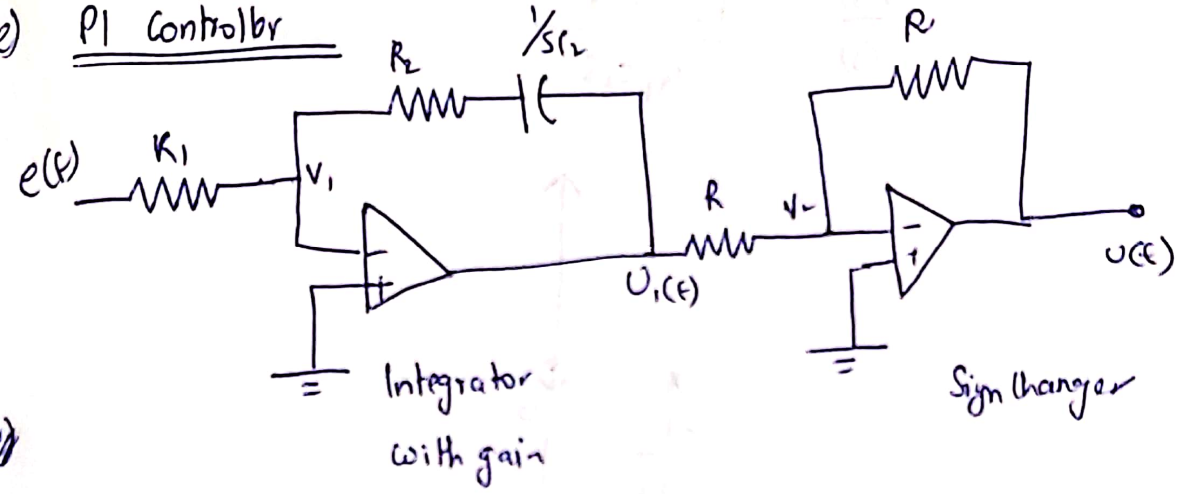
$$C(s) = \frac{(R_1+R_2)}{R_1} U(s)$$

$$\boxed{\frac{U(s)}{C(s)} = \frac{R_1}{R_1+R_2}}$$

The TF is constant in P-controller.

~~P Controller~~

(e) <u>PI Controller</u>



Due virtual short $V_1 = V_2 = 0$

At V_1 using nodal

$$\frac{V_1 - U_1(s)}{R_2 + \frac{1}{sC_2}} + \frac{V_1 - E(s)}{R_1} = 0$$

$$-\frac{E(s)}{R_1} - \frac{U_1(s)}{R_2+\frac{1}{C_2 s}} = 0 \quad [\because V_1 = 0]$$

$$\frac{E(s)}{R_1} = \frac{-U_1(s)}{R_2+\frac{1}{C_2 C}} \Rightarrow \frac{U_1(s)}{E_1(s)} = \frac{-\left(R_2+\frac{1}{sC_2}\right)}{R_1} \quad -(1)$$

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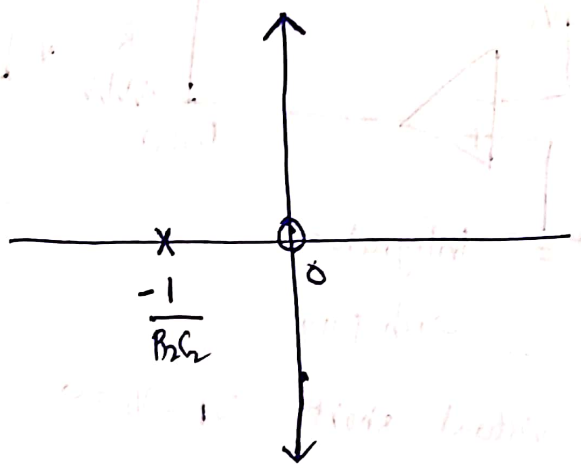
$$\frac{V_2 - U_1(s)}{R} + \frac{V_2 - U(s)}{R} = 0$$

$$U_1(s) = -U(s) \quad \text{— (2)}$$

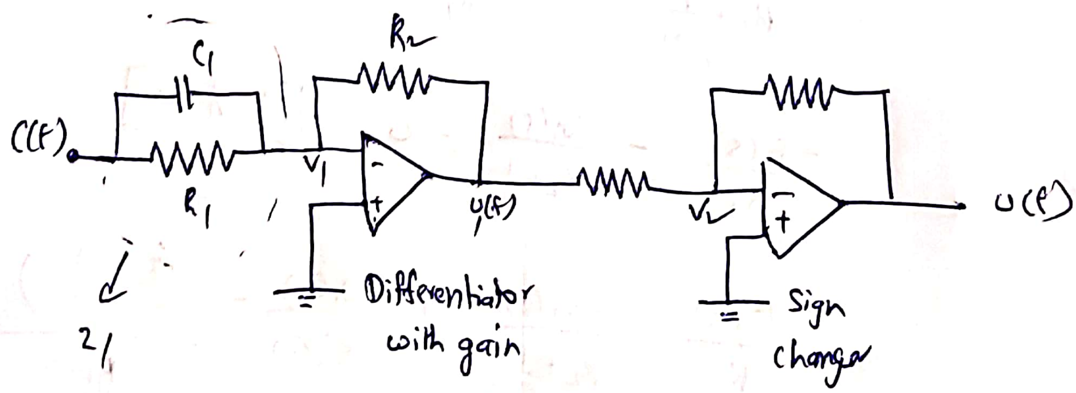
Subbing (1) in (2)

$$\Rightarrow \boxed{\frac{U(s)}{E(s)} = \frac{R_2}{R_1} + \frac{1}{R_1 C_2 s}}$$

$$\frac{U(s)}{E(s)} = \frac{R_2 C_2 s + 1}{R_1 C_2 s}$$



(f) PD Controller



$$Z_1 = \frac{R_1\left(\frac{1}{C_1 s}\right)}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s + 1}$$

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$V_1 = V_2 = 0$ Due to virtual short

$$\frac{0 - C(s)}{\frac{R_1}{R_1C_1s+1}} + \frac{0 - U_1(s)}{K} = 0$$

$$\frac{-K(R_1C_1s+1)C(s)}{R_1} = U_1(s) \quad \text{--- (1)}$$

$$\frac{0 - U_1(s)}{K} + \frac{0 - U(s)}{K} = 0$$

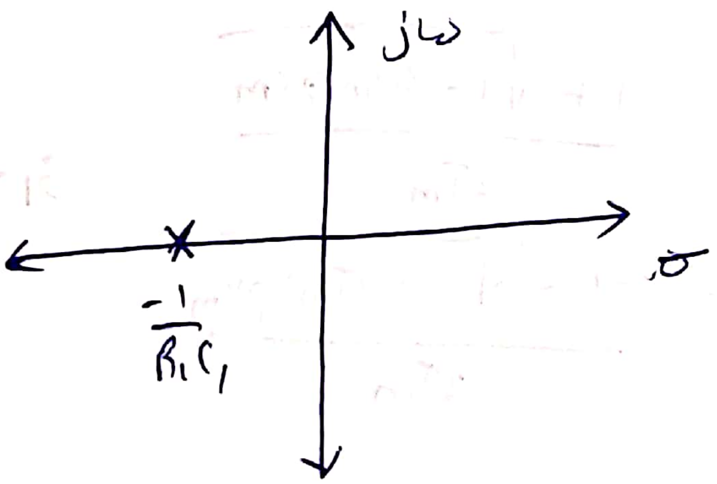
$$\boxed{U_1(s) = U(s)} \quad \text{--- (2)}$$

Subbing (1) in (2)

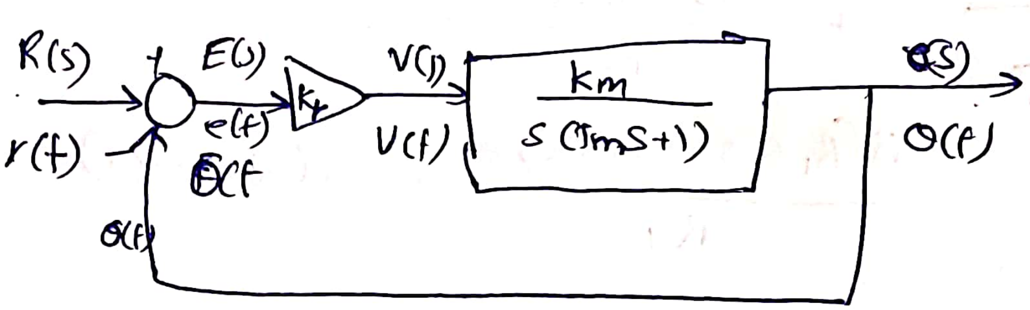
$$+\frac{R_2}{R_1}(R_1C_1s+1)C(s) = U(s)$$

$$\frac{U(s)}{C(s)} = \frac{R_2}{R_1}(R_1C_1s+1)$$

$$\frac{U(s)}{C(s)} = R_2C_1\left(s+\frac{1}{R_1C_1}\right)$$



(g) Servo Motor



$$\frac{R(s)}{\theta(s)} = \frac{\dfrac{K_p K_m}{s(T_m s+1)}}{1 + \dfrac{K_p K_m}{s(T_m s+1)}} = \frac{K_p K_m}{s(T_m s+1) + K_p K_m}$$

Characteristic eqn:

$$1 + G(s)H(s) = 0$$

$$T_m s^2 + s + K_p K_m = 0$$

$$s = \frac{-1 \pm \sqrt{1 - 4T_m K_p K_m}}{2T_m}$$

$$S_1 = \frac{-1 + \sqrt{1 - 4T_m K_p K_m}}{2T_m} \qquad S_1 > S_2$$

$$S_2 = \frac{-1 - \sqrt{1 - 4T_m K_p K_m}}{2T_m}$$

$n j\omega$,

(H) Time domain ~~sp~~ specification

$$TF = \frac{K_p \cdot K_m}{s^2 T_m + s + K_p K_m} = \frac{K_p K_m / T_m}{s^2 + \frac{1}{T_m} s + \frac{K_p K_m}{T_m}} \equiv \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{K_p K_m}{T_m} \qquad 2\xi\omega_n = \frac{1}{T_m}$$

$$\xi = \frac{1}{2\sqrt{T_m K_p K_m}}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n \sqrt{1-\xi^2}} = \frac{\pi - \tan^{-1}\left[\sqrt{4 K_p K_m T_m - 1}\right]}{2 K_p K_m}$$

$$t_d = \frac{1 + 0.7\xi}{\omega_n} = \frac{\sqrt{T_m K_p K_m} + 0.35}{K_p K_m}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} = \frac{\pi}{2 K_p K_m}$$

Peak overshoot $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$$= e^{-\pi/\sqrt{4 K_p K_m T_m - 1}}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{\frac{1}{2T_m}} = 8T_m \quad (2\% \text{ error})$$

for 5% error

$$t_s = \frac{3}{\xi\omega_n} = 6T_m$$

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Q2] → In first order system, there is only one independent energy storage element present, whereas in second order there are two

→ The response of first order doesn't exhibit ripples irrespective of the pole position.

→ Second order system may (or) may not exhibit oscillatory behaviour.