

Frequency response of second order system

Aim :- To obtain frequency response of a second order system calculation of frequency domain specifications.

Apparatus :- PC, with matlab software

Theory :-

Frequency response is defined as the magnitude and phase difference between the input and output sinusoids. It is obtained by applying a sinusoidal input with different frequency and observing magnitude and phase of output. If  $G(j\omega)$  is transfer function the  $G(j\omega)$  vs  $\omega$  given frequency plot.

Frequency response can be obtained by using Bode plot, Polar plot and Nyquist plot.

This gain and phase magnitudes are obtained from the plots helps to decide the

Frequency domain specifications are the measures of performance and characteristics of control system

1. Resonant frequency ( $\omega_r$ ) :-

The frequency at which system has maximum magnitude is called resonant frequency

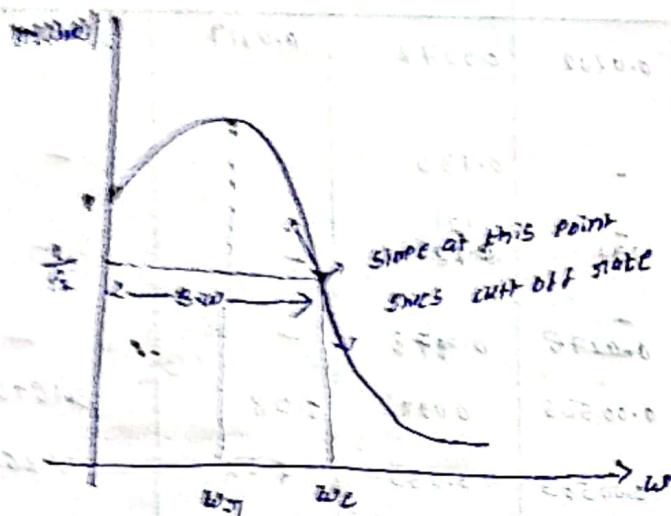
$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

2. Resonant peak magnitude ( $M_r$ ) :-

The resonant peak magnitude is the maximum value of the closed loop frequency response

$$M_r = \frac{1}{2\zeta \sqrt{1 - \zeta^2}}$$

$$|M(\omega)| = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$



3. Bandwidth ( $\omega_b$ ):-

For feedback control system the range of frequencies over which  $|M(j\omega)|$  is equal to (or) greater than  $|M(0)/2|$  is defined as the bandwidth

$$\omega_b = \omega_n \left\{ (1 - 2\xi^2) + \sqrt{(1 - 2\xi^2)^2 + 1} \right\}^{1/2}$$

procedure

1. open matlab command window

2. open new script file

3. Enter the code and obtain its bode plot

4. Run the program and obtain PM and GM from bode plot and frequency domain specifications

5. Tabulate the observations theoretically and practically and observe

6. observe the wave form and exit

$\xi = 0.2$ (2nd) (1st)		$\xi = 0.5$ (2nd) (1st)		$\xi = 0.7$ (2nd) (1st)	
Position	Amplitude	Position	Amplitude	Position	Amplitude
0	0	0	0	0	0
1	1	1	1	1	1
1.5	1.5	1.5	1.5	1.5	1.5
2	2	2	2	2	2
2.5	2.5	2.5	2.5	2.5	2.5
3	3	3	3	3	3
3.5	3.5	3.5	3.5	3.5	3.5
4	4	4	4	4	4
4.5	4.5	4.5	4.5	4.5	4.5
5	5	5	5	5	5



## 6. Theoretical values

For second order system

$$1) T.F = \frac{121}{s^2 + 11s + 121}$$

$$\omega_n^2 = 121 \quad \omega_n = 11$$

$$2\xi\omega_n = 11 \quad \xi = 1/2$$

$$\boxed{\xi < 1}$$

underdamped system

2) Resonant frequency

$$1) \omega_r = \omega_n \sqrt{1 - 2\xi^2}$$

$$= 11 \sqrt{1 - 2(1/2)^2}$$

$$= 11/\sqrt{2} = 7.77$$

$$\boxed{\omega_r = 7.77 \text{ rad/sec}}$$

3) Resonant peak magnitude

$$1) M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$= \frac{1}{2 \times \frac{1}{2} \sqrt{1 - (1/2)^2}}$$

$$\boxed{M_r = 1.1547}$$

$$2) T.F = \frac{15}{(s+3)(s+15)} \Rightarrow \frac{15}{s^2 + 18s + 45}$$

$$\omega_n^2 = 15 \quad \omega_n = \sqrt{15}$$

$$2\xi\omega_n = 18$$

$$\xi = \sqrt{\frac{16}{15}}$$

$$\boxed{\xi > 1}$$

overdamped system

2) For overdamped system

resonant peak is obtained at initial position only

$$\therefore \boxed{\omega_r = 0}$$

$$2) M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\boxed{M_r = 1}$$

3) Band width:

$$\omega_b = \omega_n \left[ (1 - 2\xi^2) + \sqrt{(1 - 2\xi^2)^2 + 1} \right]^{1/2}$$

$$= 11 \sqrt{1 - 2(1/2) + \sqrt{(1 - (1/2)^2)^2 + 1}}$$

$$= 13.92$$

$$\boxed{\omega_b = 13.92}$$

$$\text{at } \omega_c = 11 \text{ rad/sec}$$

$$\angle G(j\omega) + 11(j\omega) = -\tan^{-1} \left( \frac{11\omega}{121 - \omega^2} \right)$$

$$= -90^\circ$$

$$PM = 180 + \angle G(j\omega)H(j\omega)$$

$$PM = 180 + (-90)$$

$$\boxed{PM = 90^\circ}$$

observation table

$$2) \omega_b = \sqrt{15} \sqrt{1 - 2(1/3) + \sqrt{(1 - (1/3)^2)^2 + 1}}$$

$$= \sqrt{15} \sqrt{-1/3 + \sqrt{289/285} + 1}$$

$$= \sqrt{15} \sqrt{-1/3 + 1.1511}$$

$$\boxed{\omega_b = 2.2814}$$

$$\omega_b = 2.2814$$

$$\text{at } \omega_c = 0 \text{ rad/sec}$$

$$\angle G(j\omega) + 11(j\omega)$$

$$= -\tan^{-1}(1/3) - \tan^{-1}(4/5)$$

$$PM = 180 + \angle G(j\omega)H(j\omega)$$

$$PM = 180 + 0$$

$$\boxed{PM = 180^\circ}$$

	$TF = \frac{121}{s^2 + 11s + 121}$		$TF = \frac{15}{(s+3)(s+5)}$	
Parameters	Theoretical	Practical	Theoretical	Practical
$\omega_n$	7.77		0	
$\zeta\omega_n$	1.547		1	
$\omega_b$	13.92		2.38M	
Gain (cross over freq)	11 rad/sec		0 rad/sec	
Phase (cross over frequency)	infinity		infinity	
Gain margin	infinity		infinity	
Phase margin	90°		180°	

Conclusion:-

The degree of stability depends on gain margin and phase margin. The bode plot gives clear distinction of steady state response of the system for different frequencies.

Result:-

The frequency response of system has been studied for 2nd order system.