

Batch 8

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Exp 2:- Frequency response of Second order System

Aims:- To determine frequency response of Second order system and Evaluation of frequency domain specification

Theory:- The frequency response of a system is the steady state response of a system to the sinusoidal input signal. The commonly used methods to sketch frequency response for given system are

→ Bode plot

→ Polar plot

→ Nyquist plot

Frequency domain Specification:-

These are the measures of the performance characteristics of a control system

(i) Resonant peak magnitude ( $M_r$ ):-

It is the maximum value of closed loop frequency response

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

(ii) Resonant frequency ( $\omega_r$ ):-

The frequency at which the system magnitude is maximum

$$\omega_r = \omega_n \sqrt{1-2\xi^2}$$



Cut off frequency ( $\omega_c$ ):

The frequency at which the system magnitude  $|M(j\omega)|$  is  $[\frac{1}{\sqrt{2}}]$  times its maximum value

Band width:-

It is the range of frequencies upto cutoff frequency

$$\omega_b = \omega_n \sqrt{(1-2\xi^2) + \sqrt{4\xi^4 - 4\xi^2 + 2}}$$

Gain margin:-

It is the factor by which gain can be raised before instability occurs

$$GM = \frac{1}{G(j\omega_c)} \text{ at } \omega_c = \omega_{PCO}$$

$\omega_{PCO} \rightarrow$  The phase crossover frequency is the frequency at which

$$|G(j\omega)| \text{ is } -180^\circ$$

Phase margin:-

The phase in degrees which can be raised in a system without making it unstable.

$$PM = 180^\circ + \angle G(j\omega_c) \quad \text{where } \omega_c = \omega_{GC}$$

$\omega_{GC} \rightarrow$  the gain crossover frequency is the frequency at which  $|G(j\omega)| = 1$



## Theoretical Calculations:

$$\textcircled{1} T.f = \frac{100}{s^2 + 10s + 100}$$

Comparing this with  $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\omega_n = 10$$

$$2\xi\omega_n = 10$$

$$\xi = 0.5$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.15$$

$$\omega_r = \omega_n \sqrt{1-2\xi^2} = 7.07 \text{ rad/sec}$$

$$\omega_b = \omega_n \sqrt{(1-2\xi^2) + \sqrt{4\xi^2 - 4\xi^4 + 2}}$$

$$\omega_b = 12.72 \text{ rad/sec}$$

$$\textcircled{2} T.f = \frac{12}{(s+3)(s+4)} = \frac{12}{s^2 + 7s + 12}$$

$$\omega_n = \sqrt{12} = 3.46$$

$$2\xi\omega_n = 7$$

$$\xi = \frac{7}{2 \times \sqrt{12}} \quad \xi = 1.01$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\omega_r = \sqrt{\omega_n^2 (1-2\xi^2)}$$

These Equations are valid only

$$\text{where } 0 \leq \xi \leq 0.707$$