Exponential distribution

In probability theory and statistics, the **exponential distribution** (known as the **negative exponential distribution**) is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution, and it has the key property of being memoryless. In addition to being used for the analysis of Poisson point processes it is found in various other contexts.

Probability density function

The probability density function (pdf) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Cumulative distribution function

The cumulative distribution functionis given by

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Mean, variance, moments and median

The mean or expected value of an exponentially distributed random variable X with rate parameter λ is given by

$$\mathbf{E}[X] = \frac{1}{\lambda} = \beta, \text{ see above.}$$

In light of the examples given above, this makes sense: if you receive phone calls at an average rate of 2 per hour, then you can expect to wait half an hour for every call.

The variance of X is given by

$$\mathrm{Var}[X] = \frac{1}{\lambda^2} = \beta^2,$$

so the standard deviation is equal to the mean.

Example

An average of 4 trucks arrived per hour to be unloaded at a warehouse. Find the probability that the time between the arrival of successive trucks is less than 5 minutes

Solution

$$f(x) = 3e^{-3x}, x > 0$$

5 minutes $=\frac{1}{12}$ hours

$$p(x < \frac{1}{12}) = \int_0^{\frac{1}{12}} 3e^{-3x} dx$$

$$= -e^{-3x} \mid_0^{\frac{1}{12}}$$

$$= 1 - e^{-\frac{1}{4}} = 0.221$$

And the probability that it will exceed 45 minutes is

$$p(x > \frac{3}{4}) = \int_{\frac{3}{4}}^{\infty} 3e^{-3x} dx$$

$$= -e^{-3x} \mid_{\frac{3}{4}}^{\infty}$$

$$= 0 + e^{-\frac{9}{4}} = e^{-\frac{9}{4}} = 0.105$$