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# Bandwidth optimization of information application system under fine integral method of fuzzy fractional order ordinary differential equations



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#### KEYWORDS

Fine integral method; Software defined network; Example simulation analysis; Numerical accuracy; Network channel bandwidth **Abstract** In order to solve the problems that the network bandwidth of previous information application systems can't guarantee the quality of big data transmission, resulting in low transmission efficiency and slow data processing, etc., an improved information application system is proposed by optimizing the fine integration method of fuzzy fractional ordinary differential equations and combining with software-defined networking (SDN). Firstly, the fractional-order fuzzy differential equation fine integration method of Pade approximation is derived. Then, based on the error analysis theory, the optimization formula for the adaptive selection of weighted parameter N and expanded item number q is obtained. Combined with SDN, an improved information application system is designed, and the improved algorithm and system are detected by example simulation and performance test. The results show that the numerical accuracy and the computational efficiency of the improved algorithm are higher than that of the improved algorithm. The improved port data merge rate and task completion efficiency under different bandwidth are also significantly higher than the original system. It shows that the information application system proposed in this research can better solve the problems of insufficient bandwidth and low communication speed of traditional systems, and provide a new perspective for bandwidth optimization of big data.

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#### 1. Introduction

Fractional calculus is a pure theoretical knowledge, which has been studied and applied for many years, mainly including the calculation methods of arbitrary derivatives and fine integrals

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[1]. Fuzzy fractional-order ordinary differential equations are calculus equations that combine fuzzy characteristics with fractional-order calculus, which have the advantages of non-locality and memorization of fractional-order derivatives and are widely used [2]. According to the basic idea of finite difference, many scholars have studied the numerical calculation methods of fuzzy fractional-order ordinary differential equations and put forward a lot of calculation methods, such as new-mark method, Runge-Kutta method and so on. However, many finite-difference methods can't simultaneously take into

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account the three very important performance of the algorithm: computational efficiency, computational stability and computational accuracy [3]. Therefore, in recent years, a numerical method for precise integration of matrix exponentials is proposed, which can effectively avoid the truncation bias caused by fine division and upgrade the numerical solution of matrix exponential to the accuracy of computer algorithm. After being proposed, the fine integral method has been widely used in many fields such as dynamic response of structures, optimal mechanical control, random vibration, thermal conduction, radio wave transmission, elastic–plastic analysis of complex structures, calculation of analytic solution values, rigidity problem, partial differential equation solution, etc. [4].

Since the 20th century, with the rapid development of computer Internet technology and the popularity of highperformance computers, people's network life has become increasingly colorful, and the Internet traffic data model under the background of cloud computing and big data technology has also undergone revolutionary changes [5]. Under the impact of such a huge number of network users and huge and complicated traffic data, the innovation of Internet application technology also faces severe challenges [6]. Network traffic processing efficiency and network bandwidth problems arise with the rise of information system performance and cluster architecture. How to make the system run with the highest efficiency and shortest completion time has become an issue that affects the overall function of the Internet. The lack of network bandwidth in the computer data processing center is the main factor leading to this problem [7,8]. Thereby, in the big data communication environment, how to solve the problem that the massive data in the network communication channel takes up too much bandwidth, which leads to that the traditional communication network channel bandwidth can't meet the needs of data communication, poor communication quality, low speed and so on becomes crucial [9].

To sum up, the fine integral method of fuzzy fractional-order ordinary differential equations is widely used in the fields of dynamic response of structures, optimal mechanical control and structural optimization design, etc., but there are few relevant researches in the field of overall information system. Based on this, this research optimizes the matrix exponential calculation by using the fine integral method of fuzzy fractional order ordinary differential equations, and establishes the optimal relationship between the weighted parameter N and the expanded term number q under the specified precision. Combined with the software-defined network framework, an information application bandwidth optimization system is proposed, which greatly improves users' communication service experience and provides a new perspective for bandwidth optimization under the background of mass data.

#### 2. Methodology

2.1. Fine integral method and parameter optimization for fuzzy fractional order ordinary differential equations

Firstly, the precise integration method of fractional order fuzzy differential equation based on Pade series approximation is constructed and deduced. Then, combined with the error analysis theory of matrix function approximation, the

optimization formulas of adaptive selection of weighted parameter N and expanded term q are derived.

Firstly, the fine integral method of fuzzy fractional order ordinary differential equations based on Pade series approximation is deduced. The matrix exponential e<sup>A</sup> (p, q) order [10] Pade approximation of the fuzzy fractional order ordinary differential equation is defined as:

$$R_{pq}(A) = \left[ D_{pq}(A) \right]^{-1} N_{pq}(A) \tag{1}$$

Among them,

$$Npq(A) = \sum_{j=0}^{p} \frac{(p+q-j)!p!}{(p+q)!j!(p-j)!} A^{j}$$
 (2)

$$Dpq(A) = \sum_{i=0}^{q} \frac{(p+q-j)!q!}{(p+q)!j!(q-j)!} (-A^{j})$$
(3)

Using the property of matrix exponential, the weighted matrix  $A'/2^N$  [11] exponential is approximated by Pade series of order (p, g). The approximate value of matrix exponential  $e^A$  is obtained, denoted as  $F_{pq}(A)$ . Then the equation is obtained:

$$Fpq(\mathbf{A}) = \left\{ R_{pq} \left\{ \frac{\mathbf{A}}{2^N} \right\} \right\} 2^N = \left\{ Rpq(\mathbf{A}) \right\} \tag{4}$$

In Eq. (4),  $A' = A/2^N$ . N is called the weighting coefficient of matrix A; (p, q) is the number of expansion terms approximated by Pade series. In the actual operation, supposing p = q, the diagonal Pade is approached.

Similarly, for the weighted matrix A', after fine division, Pade series is used to approximate the calculation of  $N_{qq}(A')$ . In the process of  $D_{qq}(A')$ , the rounding error will increase and the value will not be stable. Therefore, incremental storage technology is applied here to separate the main part of the q-order diagonal Pade series approximation of A' unit from the increment, which is expressed as:

$$Rqq(\mathbf{A}') = I + R_a \tag{5}$$

Correspondingly, it can be denoted as:

$$N_{a} = \sum_{k=1}^{q} \frac{(2q-k)!q!}{(2q)!k!(q-k)!} A^{k}$$
 (6)

$$D_{a} = \sum_{k=1}^{q} \frac{(2q-k)!q!}{(2q)!k!(q-k)!} (-A')^{k}$$
 (7)

Substituting Eqs. (5) and (6) into Eq. (1), it can be obtained after relatively simple calculation:

$$R_a = (I + D_a) - 1(N_a - D_a)$$
 (8)

It can be concluded that the calculation of the initial increment of the above equation requires inverting the matrix  $(I + D_a)$ . Considering that  $D_a$  of the subdivision of the post-weighted matrix A' is a very small quantity, the matrix  $(I + D_a)$  is obviously dominant diagonally and has a very good numerical behavior, which will not cause numerical stability problems.

After the above steps, the addition theorem and incremental storage can be used for multiple calculations. Then the matrix exponential based on Pade series approximation is obtained:

$$F_{aa}(A) = I + R_a \tag{9}$$

This is the fine integral formula of fuzzy fractional ordinary differential equations based on Pade series approximation. It can be concluded from the equation that the matrix exponential fine integral method of the fuzzy fractional order ordinary differential equation is calculated by approximately O[(N + q + 1/3)  $n^3$ ]. The accuracy of the exponential of the equation matrix is mainly affected by the approximate accuracy of the initial increment, so the incremental accuracy of the initial Pade series approximation can be improved by increasing N and q, but the disadvantage is that the calculation amount will increase. At present, most studies analyze and select the weighted parameter N under the condition of specifying expansion term q. However, the combination of (N, q) obtained by experience alone can't guarantee the minimum computation of the algorithm. Therefore, the weighted parameter N and the expanded term q are considered together. Under the specified precision EPS, the relation equation between N and q can be defined as:

$$Err(N,q) = \varepsilon(N,q) \| A \|_{\infty} < EPS$$
 (10)

The required relative error accuracy is denoted as EPS, and Err  $(N,\,q)$  is the actual calculated relative error. The previous analysis has pointed out that the total computation of the matrix exponential fine integral method for the fuzzy fractional order ordinary differential equation is  $O[(N+q+1/3)~n^3]$ . Therefore, it is necessary to minimize the value of N+q under the relation of Eq. (10). The effect of the change of q can be analyzed first or the effect of N on the error accuracy of the actual operation when the weighted parameter N or the expanded term q is the stable value. First of all, g is kept as a constant, and N increases, then:

$$\frac{Err(N+1,q)}{Err(N,q)} = \frac{\epsilon(N+1,q)}{\epsilon(N,q)} = \frac{1}{2^{2q}} = \rho_N(N,q) \tag{11} \label{eq:11}$$

N is kept as a constant, and q increases, then the equation is obtained:

$$\begin{split} \frac{\operatorname{Err}(\mathbf{N} \ + \ 1, \mathbf{q})}{\operatorname{Err}(\mathbf{N}, \mathbf{q})} &= \frac{\varepsilon(\mathbf{N} \ + \ 1, \mathbf{q})}{\varepsilon(\mathbf{N}, \mathbf{q})} \\ &= \left\{ \frac{\parallel \mathbf{A} \parallel \infty}{2^N} \right\}^2 \times \frac{1}{4(2\mathbf{q} + 1)(2\mathbf{q} + 3)} = \rho_{\mathbf{q}}(\mathbf{N}, \mathbf{q}) \end{split} \tag{12}$$

Eqs. (11) and (12) represent the effect of changes in N and q on the relative error accuracy of the actual operation. It is directly concluded that  $\rho_N(N,\,q)<1,\,\rho_q(N,\,q)<1.$  In other words, any change in N or q will increase the accuracy of the relative error, so what's talked about here is which of the two will increase the accuracy of the relative error more, and that's the basis for increasing N or q.

 $\rho_N\left(N,\,q\right)$  is a function of improving the accuracy of relative error with increasing N as an exponential, while  $\rho_q(N,\,q)$  is a function of improving the accuracy of relative error with q as a power of q. Therefore, if initial value q=1, there must be  $\rho_N\left(N,\,1\right)<\rho_q(N,\,1).$  And as q goes up, the exponential function  $\rho_N$  goes down faster. Therefore, there must be  $q^*\left(N\right)>1,$  which results in  $q>q^*(N),$  and  $\rho_N\left(N,\,q^*\right)<\rho_q\left(N,\,q^*\right).$ 

Therefore, in theory, there's q \* N:

$$\rho_{\alpha}(\mathbf{N}, \mathbf{q}*) = \rho_{N}(\mathbf{N}, \mathbf{q}*) \tag{13}$$

The basis for increasing N or q can be obtained: when q is less than or equal to  $q^*(N)$ , the expansion item number q needs to be extended; when  $g > q^*(N)$ , the weighted parameter N needs to be increased until the specified precision is reached. The recalculation of  $\rho_q$  and  $\rho_N$  is too tedious, so the adaptive selection equations of them are directly derived from Eqs. (11) and (12) here:

$$\rho_{N}(N, q + 1) = \frac{1}{4}\rho_{N}(N, q), \rho_{q}(N, q + 1)$$

$$= \frac{2q + 1}{2q + 5}\rho_{q}(N, q)$$
(14)

$$\rho_{N}(N + 1, q) = \rho_{N}(N, q), \, \rho_{q}(N, q + 1) = \frac{1}{4}\rho_{q}(N, q)$$
 (15)

When q = q0 = 1 and N = N0, is the initial value, which conforms to the requirement of relative error accuracy, then the equation below is obtained:

$$N_0 = \max(0, \log_2^{\|A\|_{\infty}} + 1) \tag{16}$$

Eqs. (11)–(15) give the recursive equation that the error precision changes with the increase of the weighted parameter N or the expansion term q. The flow chart of adaptive selection of N and q at fixed EPS can be obtained from the previous derivation steps. As shown in Fig. 1 below, the calculation amount of the whole parameter selection is mainly the calculation before the fixed EPS. Considering that recursive Eqs. (11)–(15) are used in the whole process, it can be concluded that the total calculation amount of the adaptive selection is O (N + q-1/n<sup>0</sup>). Compared with the calculated amount O [(N + q-1/3) n<sup>3</sup>] based on Pade series approximation, it is completely negligible.

### 2.2. Software defines network framework technology

SDN is a very powerful network control framework that can highly flexibly manage the operation of the whole network and improve the efficiency of big data computing. Its operation mode is mainly divided into the management function and forwarding function of the network. The control module and the data module are operated separately, which greatly improves the overall operation efficiency of the system to the network. SDN provides a very flexible programming interface API for the upper applications, which greatly improves the automatic control function of the network and makes new network applications easier to deploy. The overall framework is shown in Fig. 2 below.

Open Flow is a key technology for software to define the network. It is the first interface for the connection between control module and data module in the whole framework. The body of Open Flow consists of the control center and the Open Flow switch, as shown in Fig. 3. The control center is the core control of the whole process running in the software-defined network, while the Open Flow switch is mainly responsible for the forwarding of traffic data. In Open Flow, a general Flow table is used to exchange node information between each module. And remote access and monitoring are carried out through the Open Flow protocol interface. At the same time, by opening the north and south API interface,

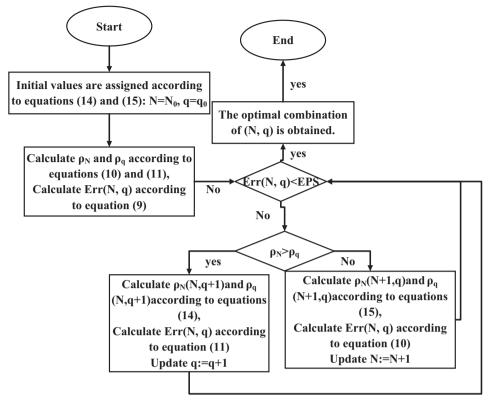


Fig. 1 Flow chart for adaptive selection of parameters (N, q).

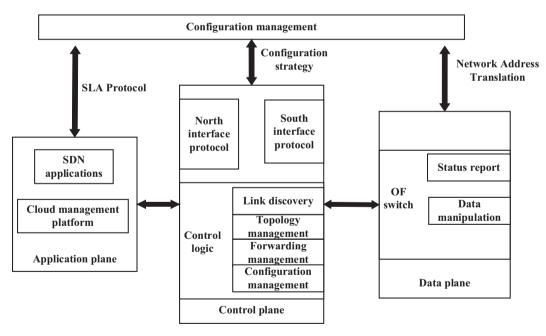


Fig. 2 SDN overall structure plan.

the network can be seamlessly connected to various business needs, such as latency, bandwidth, billing, and so on, so that the whole network has programmable capability. Users can use these programmable interfaces to call resources freely to develop network business, which shortens the online cycle of new business and promotes the better development of network.

#### 2.3. Information application system design

Information application system is built on the basis of software definition network. The overall process is shown in Fig. 4 below, which is logically divided into control module and data transmission module. The client submits the task to

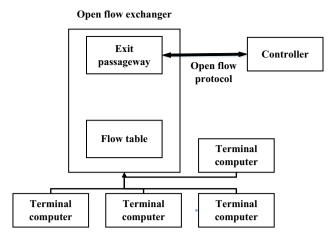


Fig. 3 Open Flow network structure diagram.

be processed and its data with the help of a Work user. The Work task in the node uses the task manager to schedule the task. At the same time, the result of adjustment is sent to the controller module, and then the controller calculates the result and sends it to the behavior processing module through the flow table item. Open Vswitch forwards the data through the flow table item issued by the controller, and the Work task on the node performs the specific task. The important steps can be divided into: submitting tasks, scheduling and distributing tasks, Open Flow system processing data, executing and completing tasks, and other parts.

The control module is the brain of the whole system, which is responsible for task regulation in the whole framework and can make full use of network data resources. It is also responsible for making the Open Flow network flexibly adjust its operation mode to meet the above needs, so that the cooperation between the two parties can achieve a more harmonious working state. As shown in Fig. 5 below, the whole process of task operation scheduling consists of the Run tracker node and the controller in Open Flow. The whole task scheduling adopts the mechanism of pulling tasks from computing resources to the scheduler. Considering the fault-tolerant mechanism of Run Tracker, if a task handed over by one node is faced with running failure, it will be transferred to the other nodes to run again. The Open Flow controller is kept connected to the Run Tracker node at the same frequency. The controller periodically sends query information to the Run Tracker node to ask if there is a new task scheduling assignment. When free resources appear, new tasks can be pulled. While the Run Tracker node recovers the task, the IP of the node and the task ID executing the task on the node are sent to the Open Flow controller.

The data transfer module of information application system plays the role of task executor in the whole frame system. As shown in Fig. 6 below, after receiving the Flow data set at the system port, the Open Flow switch adds data to the cache location. The packets in the cache are then processed. If the data can't match the appropriate flow table item, the packet can be determined to be unknown data. And then it is transmitted to the controller. Then, the controller formulates the

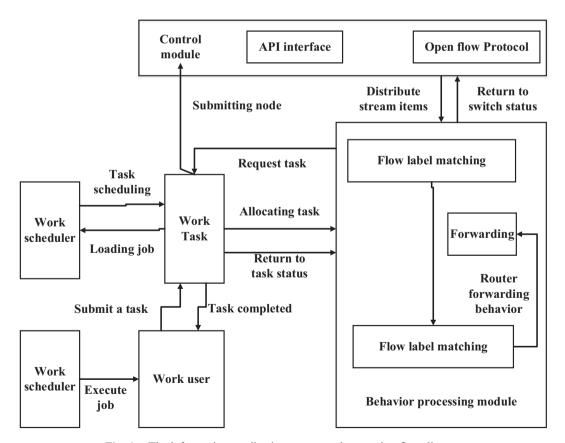


Fig. 4 The information application system task execution flow diagram.

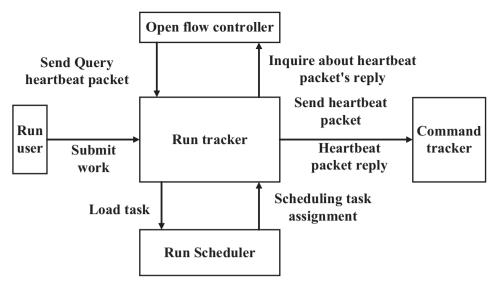


Fig. 5 The control module task scheduling diagram.

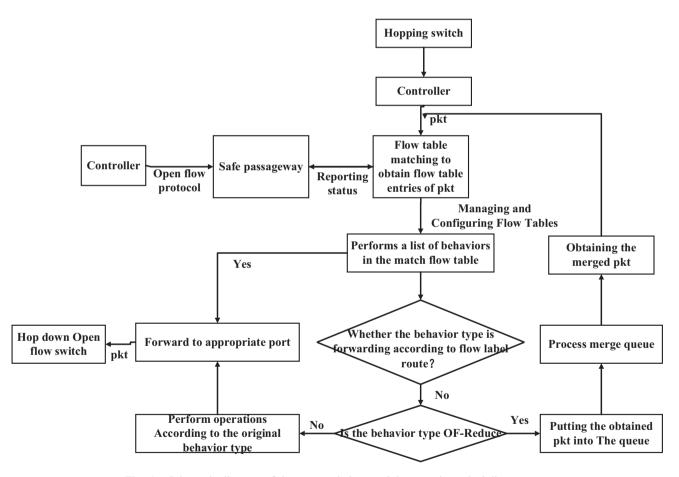


Fig. 6 Schematic diagram of data transmission module operation scheduling process.

corresponding routing operation strategy for the unknown data to convey how the Open Flow switch handles the unknown data. If the appropriate flow table entry is matched, then the task in the flow table entry is run. The data task is run directly or the behavior is added to the behavior set of the packet based on the type of task.

# 3. Results and discussion

# 3.1. Example simulation analysis of fine integration optimization

Two examples are given to verify the correctness and effectiveness of the adaptive selection method of weighted parameter N

and expanded item number q, which is calculated on the MATLAB platform.

Example 1: the matrix is constructed:  $A = \begin{bmatrix} n & m \\ -(n-1) & -(m-1) \end{bmatrix}$ . Among them, m > n, then  $\exp(A)$  is calculated. The eigenvalue of the matrix is:  $\operatorname{eig}(A) = [1, -(m-n)]$ . The analytical solution is obtained by characteristic analysis method:

$$\exp(A) = \frac{1}{n - m - 1} \begin{bmatrix} (n - 1)e^{n - m} - me & me^{n - m} - me \\ -(n - 1)e^{n - m} + (n - 1)e & -me^{n - m} + (n - 1)e \end{bmatrix}$$
(17)

From the eigenvalues of the matrix, it can be concluded that with m > n, the rigidity of the matrix increases continuously, which can be used to test the performance of the algorithm: if n = 2, then  $m = 2^0, 2^1, 2^2, 2^3, \dots 2^5$  and it increases gradually. Supposing the fixed accuracy EPS =  $10^{-12}$ , the adaptive selection calculation of parameters (N, q) is carried out by the algorithm before and after the improvement. And it is compared with the analytic solution respectively. As shown in Fig. 7 below, it can be concluded that under different matrix rigidity, the numerical precision of the improved algorithm and the error of the analytical solution basically remain unchanged, while the error value of the improved algorithm and the analytical solution increases with the increase of matrix rigidity. Moreover, the error between the numerical precision and the analytical solution of the improved algorithm is always smaller than that of the algorithm before the improvement, and the difference is statistically significant (P > 0.05).

Example 2: under the specified precision EPS =  $10^{-14}$ , the random matrix with different dimensions is the dimension of the matrix: B = rand (n, n), n = 60:60:350. The computing time of the algorithm before and after optimization in MATLAB platform is compared.

As shown in Fig. 8, the computing time required by the improved algorithm is not significantly different from that of the improved algorithm when the dimensions are 60, 120 and 180 (P > 0.05), while the computing time required by the improved algorithm is significantly lower when the dimensions are 240, 300 and 360 (P < 0.05).

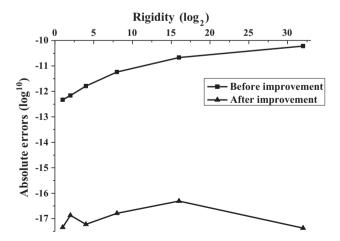
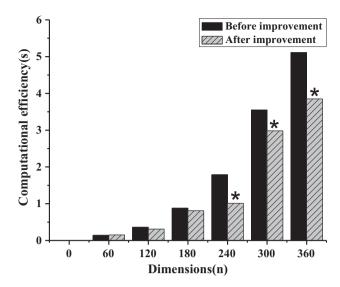


Fig. 7 The error comparison between numerical precision and analytical solution before and after the algorithm improvement under different rigidity.



**Fig. 8** Comparison of computing efficiency of different matrix dimensions before and after algorithm improvement.

It can be concluded from example 1 and example 2 that the improved fine integral algorithm is far more efficient than the improved algorithm in both numerical accuracy and higher dimensions. It is sufficient to prove that the improvement of the fine integral method to complete the fractional order fuzzy differential equation by optimizing the weighted parameter N and the expanded term number q through adaptive selection is real and feasible.

# 3.2. Information application system bandwidth optimization verification

This test is run on a real data center platform, the operating system is Ubuntu, the SDN network bandwidth parameter value is set as 200bps, and the data block backup number is 5. The Open Vswitch is run on the Open Flow switch machine, and the Open Flow controller runs Open Daylight. In order to verify the bandwidth optimization performance of the system, two sets of experiments are conducted to detect the system port traffic size and data merge rate of different input traffic data, and the time required to complete the system tasks under different bandwidth.

This research uses the task module of the system to test. The load of the task in turn is increase to test the port data flow size and data merge rate of the original information application system, and the improved information application system respectively when the input flow is 800, 1600, 6000, 14,000 and 2000 m. The data flow size and data merge rate of the system port represent the performance of the system in traffic transmission processing. The smaller the port data volume is, the higher the merge rate is, and the better the system performance is.

Fig. 9 shows that the port data flow of both the improved system and the original system increases with the increase of the input flow data, and the port flow of the improved system is gradually smaller than that of the original system. And the gap is widening. In terms of data merge rate, the improved system has been significantly higher than the original system.

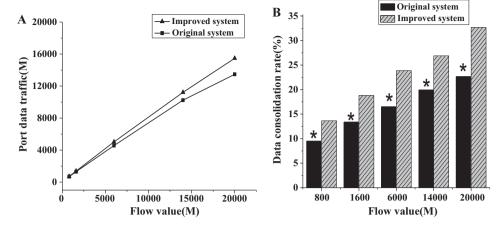
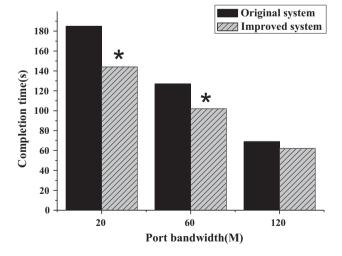


Fig. 9 The port data size and merge rate of the improved system comparison with the original system. Note: A refers to port data with different flow values between the improved system and the original system; B is the merging rate of port data between the improved system and the original system. \* indicates that there is a statistically significant difference between the statements and the improved system (P < 0.05).

In the second experiment, the task module of the system is also taken as the test sample. Under the condition that the input traffic data is fixed at 15000 M, the port bandwidth of switch Open Vswitch is changed from small to large to 20 M, 60 M and 120 M, and the time required to complete the tasks of the two information application systems under different bandwidth conditions is detected and recorded. The time required to complete the task represents the operating efficiency of the system. The less time requires, the higher the operating efficiency of the system.

As shown in Fig. 10 below, when the improved system is running at 20 M and 60 M bandwidth, the time required to complete the task is far less than the time spent by the original system. While at 120 M bandwidth, the task completion time is also lower than the original system, but the difference is not large.



**Fig. 10** The task completion time under different bandwidth comparison with the original system.

# 4. Discussion

Among the computing methods of fractional order fuzzy differential equations, there are many researches on fine integral calculation based on matrix exponential. Compared with the simplest Tabor series approximation fine integral algorithm, the Pade series approximation algorithm can save more computation and improve computational efficiency under the same precision. Therefore, this research constructs and deduces the fine integral method based on Pade series approximation [12]. However, many studies show that the parameter selection method in the fine integral method of fuzzy fractional order ordinary differential equations can not only improve the calculation efficiency but also make the fine integral method independent of matrix behavior and independent of step size. The weighted matrix system N and series expansion terms q can directly affect the numerical accuracy and calculation efficiency of the equation [13]. Therefore, combined with the error analysis theory of matrix function approximation of differential equations, the optimization formulas of adaptive selection of weighted parameter N and expanded term q are derived. The numerical accuracy of the improved algorithm and the computational efficiency under different dimensions are tested by simulation analysis of numerical examples. The results show that the improved fine integral algorithm is far more efficient than the improved algorithm in both numerical accuracy and high dimension. It is sufficient to prove that the improvement of the fine integral method to complete the fractional order fuzzy differential equation by optimizing the weighted parameter N and the expanded term number q through adaptive selection is real and feasible [14].

With the development of computer network data, the insufficient network bandwidth of information application system has become the key problem to limit the processing of large-scale data. As the research core of the new network system that many scholars have paid great attention to in recent years, SDN network framework has become an important benchmark for the development of the new network structure in

the future [15]. The Open Flow controller in the SND framework can provide a very powerful capability for the management of the entire network topology, while the Open Flow switch can handle the docking network traffic with a high sensitivity. Therefore, this research proposes a new information application system using the improved fuzzy fractional order ordinary differential equation fine integration method combined with the SDN network framework. The task board of the system is taken as the test sample, and the simulation platform is tested from the two perspectives of system port data merging rate of different input flows and task completion efficiency of different bandwidths. The results show that the improved system is far better than the original system in terms of processing efficiency of network data flow and task completion efficiency under limited bandwidth, which indicates that the fine integration method of fuzzy fractional order ordinary differential equations and the SDN network framework set can well solve the problems of low communication transmission efficiency and slow data processing caused by limited network bandwidth under the background of mass data [16].

#### 5. Conclusion

In this study, a bandwidth-optimized information application system is proposed by optimizing the fine integration method of fuzzy fractional ordinary differential equations and combining with the SDN network framework to solve the problem that the channel bandwidth of the traditional system communication network can't meet the needs of data communication and the network data flow processing efficiency is low. However, this study still has some shortcomings. Considering that the research on SDN is still within the scope of theoretical research in academia and has not been popularized in practical application, this research is an attempt to combine the fine integration method with the SDN network framework. In the future, it should be considered to explore the problem that the task completion efficiency of this system is not ideal under the condition of sufficient bandwidth. In a word, the information application system proposed in this study performs well in the efficiency of processing and merging network data flow and in the efficiency of task completion under the circumstance of limited bandwidth, which can well solve the problems of poor communication quality and low speed caused by limited network bandwidth under the background of mass data, which affect the communication service experience of users. The research provides a new way and perspective for big data bandwidth optimization.

# **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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